Paul Scherrer Institut PSI | Universität Zürich

Muon-electron scattering: Theory kickoff workshop

# Muon decay at NNLO
## plans and interim results

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# disclaimer

- this is not meant to be a polished talk with impressive results
- some preliminary thoughts about a project we (the PSI gang) have started that is:
    - simpler than $\mu\, e$ scattering
    - but has overlap
    - might be useful to learn things for $\mu\, e$ scattering
    - (and might be useful to learn other things)

Calculate muon (and maybe top) decay fully differential at NNLO with:

$m = 0$ ($\sim$ already done)

- scheme dependence of $\mathrm{d}\Gamma_{\mathrm{VV}}$ (known), $\mathrm{d}\Gamma_{\mathrm{RV}}$ and $\mathrm{d}\Gamma_{\mathrm{RR}}$
- play with $\gamma_5$

and $m > 0$ (but $m \ll M$)

- mirrors physical situation & cuts better
- three scales ($M$, $m$ and $s$) $\Rightarrow$ integrals not all known.
- no collinear divergence $\Rightarrow$ real corrections simple (modify FKS)
    - double soft limit factorises $\mathcal{S}_{56}\,\mathrm{d}\Gamma_{\mathrm{RR}} = \big(\mathcal{E}_5\,\mathcal{E}_6\big)\,\mathrm{d}\Gamma_{\mathrm{RR}}$
    - virtual-real soft limit is 'simple' $\mathcal{S}\,\mathrm{d}\Gamma_{\mathrm{RV}} = \mathcal{E}_{\text{Born}} \cdot \mathrm{d}\Gamma_{\mathrm{V}}$

focus on $m > 0$ but $m \ll M \sim s$

- main problem: $\mathrm{d}\Gamma_{\mathrm{VV}}$, i.e. master integrals (form factor with two different masses)
- strategy
  - expand $\mathrm{d}\Gamma_{\mathrm{VV}}$ ($\mathrm{d}\Gamma_{\mathrm{RV}}$, $\mathrm{d}\Gamma_{\mathrm{V}}$) in $z = m/M$ and $m^2/s$
  - drop terms $\mathcal{O}(z^n)$ (practically $n = 2$, but can go as high as 'we want') but keep phase space exact
  - check poles exact in $z$
  - compare with fragmentation function approach

Use SCET inspired way to relate $m = 0 \;\to\; m \neq 0$ [Becher, Melnikov 07]

Form factor: (only one external mass $m \ll Q^2 = s$)

$$F(s, m, \{m_i^2\}) = Z_J(m^2, \{m_i^2\})\, S(s, \{m_i^2\})\, \tilde{F}(s) + \mathcal{O}(m^2/s)$$

- $S(s, \{m_i^2\})$: soft function, only contributions from vacuum polarization diagrams with massive fermions, $\supset \ln(m^2/s)$
- $Z_J(m^2)$: jet fct., independent of hard scale $s$, $\supset \ln(m^2/m_i^2)$
- $\tilde{F}(s)$: massless form factor
- factorisation $\leftrightarrow$ resummation via RG equations

Bhabha scattering:

$$M(\{p_i\}, \{m_i^2\}) = Z_J^2(m^2)\, S(s, t, u, \{m_i^2\})\, \tilde{M}(\{p_i\}) + \mathcal{O}(m^2/\{s, t, u\})$$

## strategy

- try to adapt/generalize to case with large external mass, i.e. $m^2 \ll M^2 \sim s$
- start with 'form factor' i.e. $\sim$ muon decay
- later maybe generalize to $\mu\ e$ scattering
- start brute force, use method of regions (MoR) to compute form factor $F(s, m, M, \{m_i^2\})$
- find all relevant regions, final result: hard, collinear, soft, others? (case above has additional contributions in separate diagrams, but not in final result)

$\mu(p) \to e(q) + \nu\bar{\nu}$

- Qgraf $\to$ FORM $\to$ Reduze $\to$ FORM
- reduction introduces $1/\epsilon^2$ and $1/m^2$ poles, i.e. need (some) integrals up to $\mathcal{O}(z^{n+2}, \epsilon^2)$
- master integrals through method of regions
  - $q = (q_+, q_-, q_\perp) \sim (0, 1, \lambda^2)$ and $p \sim (1, 1, 0)$
  - simple examples:

$$\mathcal{I}^{(1)}_{\alpha\beta\gamma} = \int_k (k^2)^{-\alpha}(k^2 - 2k\cdot q)^{-\beta}(k^2 - 2k\cdot p)^{-\gamma}$$

$$\mathcal{I}^{(2)}_{\alpha\beta\gamma} = \int_{k_1\,k_2} ((k_1 - k_2)^2)^{-\alpha}(k_2^2 - 2k_2\cdot q)^{-\beta}(k_1^2 - 2k_1\cdot p)^{-\gamma}$$

  - these only have hard $h_i : k_i \sim (1, 1, 1)$ and collinear $c_i$: $k_i \sim (\lambda^2, 1, \lambda)$ contributions

## explicit example $\mathcal{I}_{111}^{(2)}$

- Write $p = p_+ + p_-$, $q = q_- + q_\perp$

  $\Rightarrow M^2 = p^2 = 2p_+ \cdot p_-$, $m^2 = q^2 = q_\perp^2$, $s = 2p_+ \cdot q_-$

$$\mathcal{I}_{111}^{h_1\text{-}h_2} = \int_{k_1\, k_2} \frac{1}{(k_1 - k_2)^2} \frac{1}{k_1^2 - 2k_1 \cdot p} \times \left( \frac{1}{k_2^2 - 2k_2 \cdot q_-} + \lambda^2 \, \frac{4(k_2 \cdot q_\perp)^2}{(k_2^2 - 2k_2 \cdot q_-)^3} + \mathcal{O}(\lambda^4) \right)$$

$$\mathcal{I}_{111}^{h_1\text{-}c_2} = \int_{k_1\, k_2} \frac{1}{k_1^2 - 2k_1 \cdot p} \frac{1}{k_2^2 - 2k_2 \cdot q} \times \left( \frac{1}{k_1^2 - 2k_1 \cdot k_2^-} + \lambda^2 \left[ \frac{4(k_1 \cdot k_2^\perp)^2}{(k_1^2 - 2k_1 \cdot k_2^-)^3} + \frac{2k_1 \cdot k_2^+ - k_2^2}{(k_1^2 - 2k_1 \cdot k_2^-)^2} \right] \right)$$

- $\mathcal{I}_{111}^{h_1\text{-}h_2}$ naive polynomial expansion in $m^2$, $\mathcal{I}_{111}^{h_1\text{-}c_2}$ contains $\ln(z)$
- both parts very simple (but tedious), scales separated

Currently semi-automatic in Mathematica

- try all regions $k_2 \sim (\lambda^a, \lambda^b, \lambda^{(a+b)/2})$ with $0 \le a, b \le 4$
- `expanded = Series[expr/.{`
  `pl[k2]` $\to \lambda^a$ `pl[k2], mi[k2]` $\to \lambda^b$ `mi[k2],`
  `perp[k2]` $\to \lambda^{(a+b)/2}$ `perp[k2]}, {`$\lambda$`,0,0}]`
- ```
  reg = SubloopSchwinger[k2,
    SubloopSchwinger[k1, expanded, 2]/.{
      mi[k2]·r_Symbol → k2·pl[r]
    }, 2]
  ```
- so far: `Integrate[reg, {x,0,∞}, {y,0,∞}]`
- leading term of $h_1$-$h_2$ is usually $\mathcal{I}(m \to 0, q \to q_-)$

$\mathcal{I}^{(2)}_{111}$ agrees surprisingly well with SecDec ($z = 13/25$)

$$1 - \frac{\text{MoR}}{\text{SecDec}} = \frac{2{\cdot}10^{-4}}{\epsilon^2} + \frac{4{\cdot}10^{-4}}{\epsilon} + 7\cdot 10^{-4} + 2\cdot 10^{-3}\epsilon + 3\cdot 10^{-3}\epsilon^2 + \mathcal{O}(\epsilon^3, z^6)$$
  ```

Two-loop calculation

- cross check with FIESTA;
  sometimes we have trouble to deal with non-trivial masses
- cross check with SecDec + Normaliz + Dreadnaut;
  works fine as long as $z$ isn't to small.
- cross check (find all regions) in Feynman-parameter space:
  Smirnov's asy: tried for $\mathcal{I}_{111}^{(2)}$, had to send $z \equiv \lambda \to \lambda^2$
- better automation / implementation:
  $\Rightarrow$ Master student starting 25th of September

Real calculation

- scheme dependence of $\mathrm{d}\Gamma_{\mathrm{CT}}$ and $\int \mathrm{d}\Gamma_{\mathrm{CT}}$
- implement FKS properly