A space-like determination of a_μ^{HLO} via μ-e scattering data

G. Venanzoni

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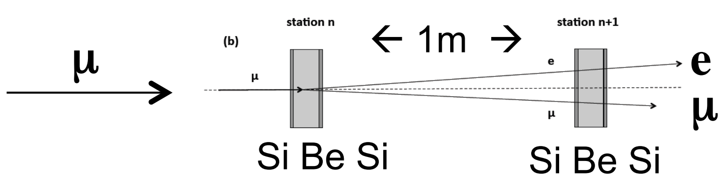
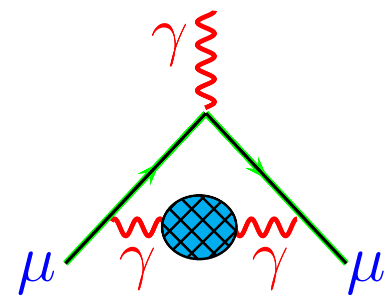
In collaboration with: G. Abbiendi, C.M.Carloni Calame, F.V. Ignatov, M. Incagli, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, C. Patrignani, F. Piccinini, F. Pisani, R. Tenchini, L. Trentadue





Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

Padova

Europe/Rome timezone

Muon g-2: summary of the present status

- E821 experiment at BNL has generated enormous interest:

$$a_\mu^{E821} = 11659208.9(6.3)\times 10^{-10} \quad \text{(0.54 ppm)}$$

- Tantalizing ~3σ deviation with SM (persistent since >10 years):

$$a_\mu^{SM} = 11659180.2(4.9)\times 10^{-10}\,(DHMZ)$$

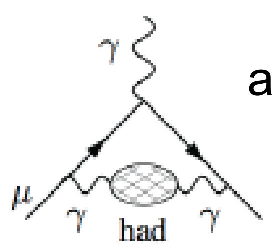
M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C71 (2011)

$$\boxed{a_\mu^{E821} - a_\mu^{SM} \sim (28 \pm 8)\times 10^{-10}}$$

- Current discrepancy limited by:
  - Experimental uncertainty→ New experiments at FNAL and J-PARC **x4** accuracy
  - Theoretical uncertanty→ limited by hadronic effects

$$a_\mu^{SM} = a_\mu^{QED} + \boxed{a_\mu^{HAD}} + a_\mu^{Weak}$$

Hadronic Vacuum polarization (HLO)



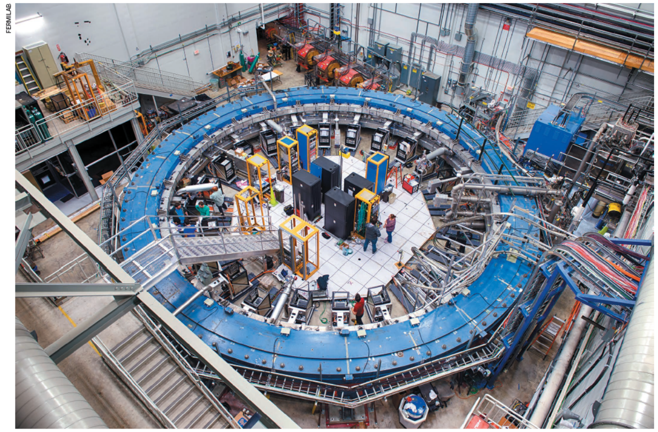
$a_\mu^{HLO} = (692.3\pm4.2)10^{-10}$

$\delta a_\mu^{HLO}/a_\mu^{HLO}$~0.6%

$(g\text{-}2)_\mu$: a new experiment at FNAL (E989)

- New experiment at FNAL (E989) at magic momentum, consolidated method. **20 x stat.** w.r.t. E821. First result with BNL accuracy (0.54 ppm) expected in 2018-19.

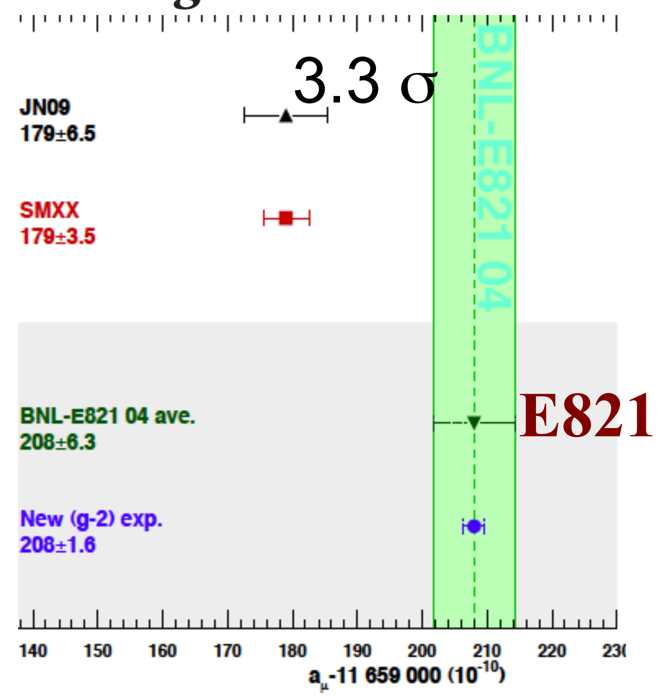
$\rightarrow$ Ultimate precision: δa_μx4 improvement (0.14ppm)



The Muon g−2 experiment will look for deviations from the standard model by measuring how muons wobble in a magnetic field.

PARTICLE PHYSICS

Muons' big moment



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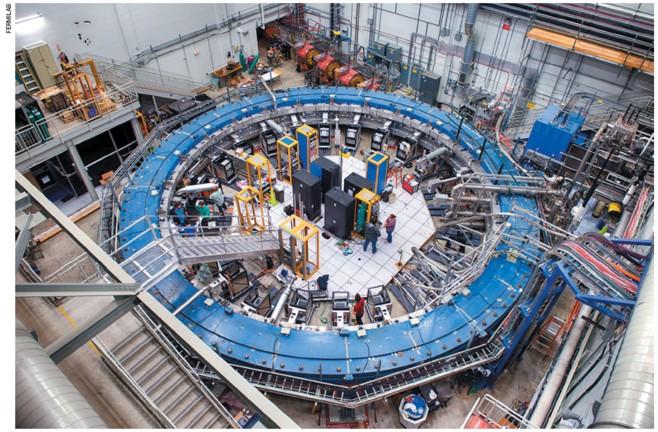
$\rightarrow$ Ultimate precision: δa_μx4 improvement (0.14ppm)

If the central value remains the same $\Rightarrow$ 5-8σ from SM* (enough to claim discovery of New Physics!)

***Depending on the progress on Theory**



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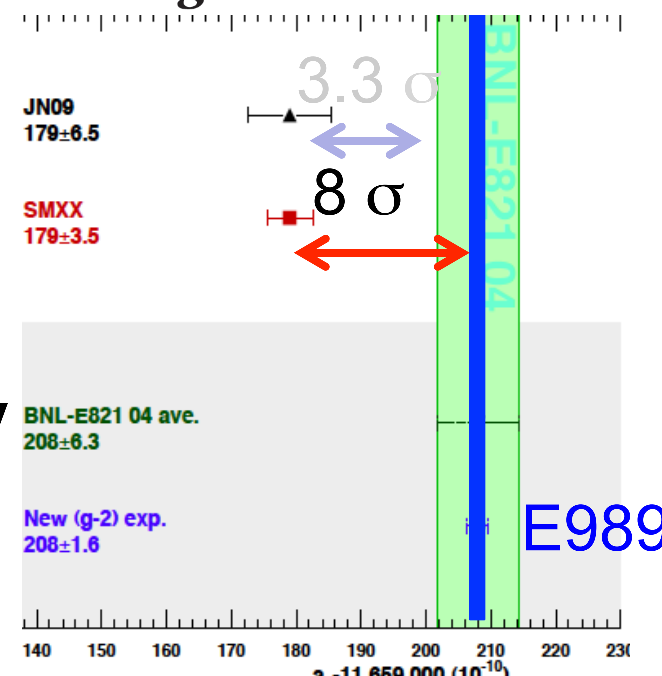
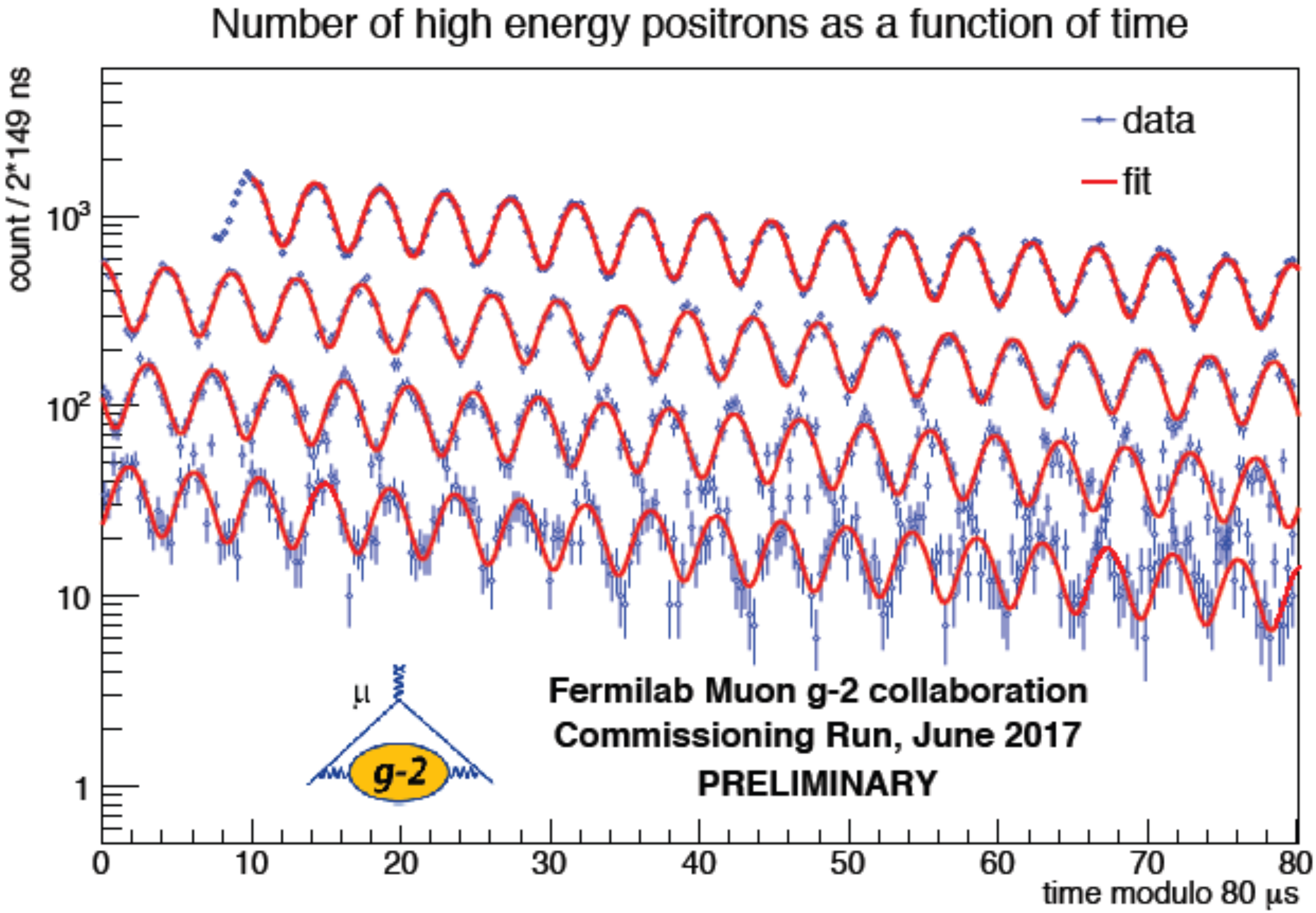


Thomas Blum; Achim Denig; Ivan Logashenko; Eduardo de Rafael; Lee
oberts, B.; Thomas Teubner; Graziano Venanzoni (2013). "The Muon (g-2)
heory Value: Present and Future". arXiv:1311.2198 [hep-ph].

Complementary proposal at J-PARC in progress

$(g\text{-}2)_\mu$: First wiggle plot at FNAL (E989)

- 

$\frac{\alpha}{2\pi}$

89

140 150 160 170 180 190 200 210 220 23(

a_μ-11 659 000 (10^{-10})

Complementary proposal at J-PARC in progress

Three Recent papers relevant for g-2!

20 years effort!

25 April 2017

High-precision calculation of the 4-loop contribution to the electron g-2 in QED

1100 digits!

Stefano Laporta*

Dipartimento di Fisica, Università di Bologna,
Istituto Nazionale Fisica Nucleare, Sezione di Bologna,
Via Irnerio 46, I-40126 Bologna, Italy

Abstract

I have evaluated up to 1100 digits of precision the contribution of the 891 4-loop Feynman diagrams contributing to the electron g-2 in QED. The total 4-loop contribution is

$$a_e = -1.912245764926445574152647167439830054060873390658725345\ldots\left(\frac{\alpha}{\pi}\right)^4 .$$

I have fit a semi-analytical expression to the numerical value. The expression contains harmonic polylogarithms of argument $e^{\frac{i\pi}{3}}$, $e^{\frac{2i\pi}{3}}$, $e^{\frac{i\pi}{2}}$, one-dimensional integrals of products of complete elliptic integrals and six finite parts of master integrals, evaluated up to 4800 digits.

Eur. Phys. J. C (2017) 77:139
DOI 10.1140/epjc/s10052-017-4633-z

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Experimental Physics

Measuring the leading hadronic contribution to the muon g-2 via μe scattering

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$\delta a_\mu^{HLO}/a_\mu^{HLO} \rightarrow 0.3\%_{stat}$

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The hadronic vacuum polarization contribution to the muon $g-2$ from lattice QCD

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Abstract

We present a calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment, $a_\mu^{\rm hvp}$, in lattice QCD employing dynamical up and down quarks. We focus on controlling the infrared regime of the vacuum polarization function. To this end we employ several complementary approaches, including Padé fits, time moments and the time-momentum representation. We correct our results for finite-volume effects by combining the Gounaris-Sakurai parameterization of the timelike pion form factor with the Lüscher formalism. On a subset of our ensembles we have derived an upper bound on the magnitude of quark-disconnected diagrams and found that they decrease the estimate for $a_\mu^{\rm hvp}$ by at most 2%. Our final result is $a_\mu^{\rm hvp} = (654 \pm 32\,^{+21}_{-23}) \cdot 10^{-10}$, where the first error is statistical, and the second denotes the combined systematic uncertainty. Based on our findings we discuss the prospects for determining $a_\mu^{\rm hvp}$ with sub-percent precision.

$\delta a_\mu^{HLO}/a_\mu^{HLO} \sim 6\%$

(see talks from U. Marconi, S. Laporta and M. Marinkovic)

a_μ^{HLO} calculation, traditional way: time-like data

[C. Bouchiat, L. Michel,'61; N. Cabibbo, R. Gatto 61; L. Durand '62-'63; M. Gourdin, E. De Rafael, '69; S. Eidelman F. Jegerlehner '95,. . . .]

- Optical theorem and analyticity:

$$\sigma(s)_{(e^+e^-\to had)} = \frac{4\pi}{s}\,\mathrm{Im}\,\Pi_{hadron}(s)$$

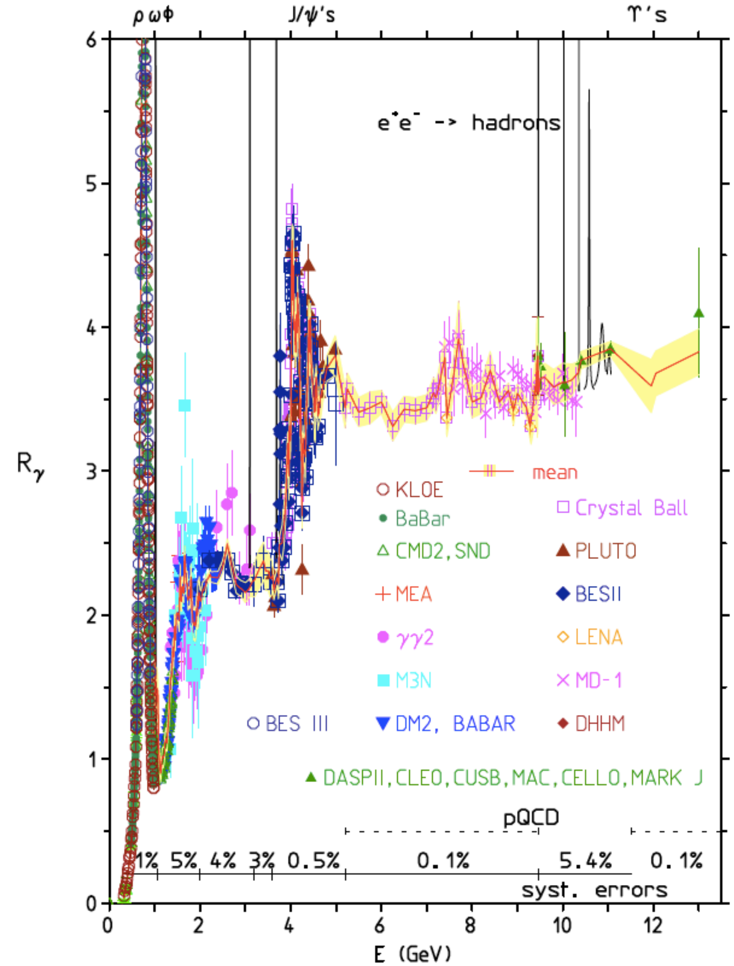
$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\,K(s)\cdot\sigma(s)_{(e^+e^-\to had)}$$

- The main contribution is in the highly fluctuating low energy region.

$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)(s/m^2)} \sim \frac{1}{s}$$

The enhancement at low energy implies that the $\rho \to \pi^+\pi^-$ resonance is dominating the dispersion integral (~ 75 %). Current precision at 0.6%→ need to be reduced by a factor **~2**

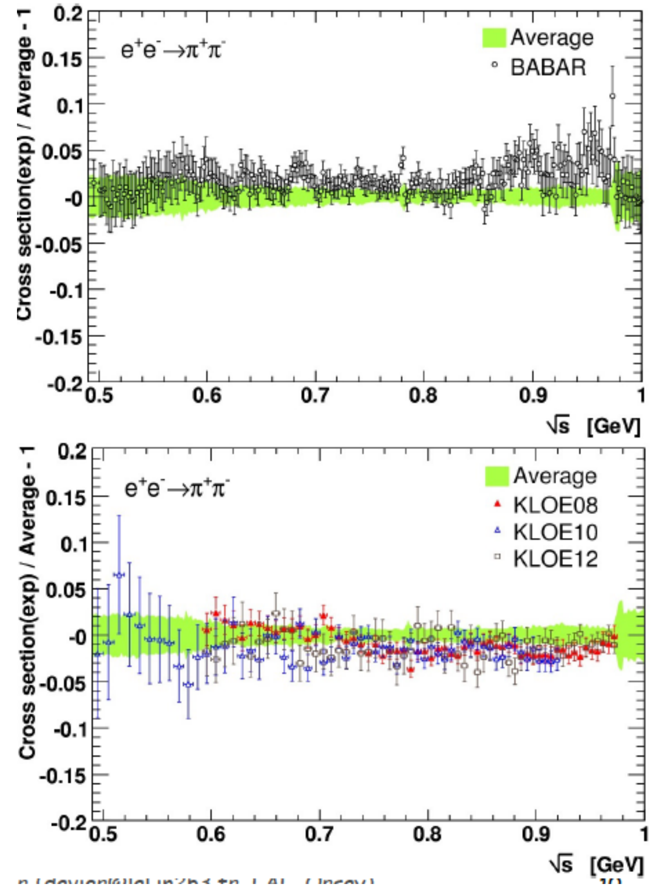
Collection of many experimental results



The high-energy tail of the integral is calculated using pQCD

Timelike data aiming at 0.2% on a_μ^{HLO}?

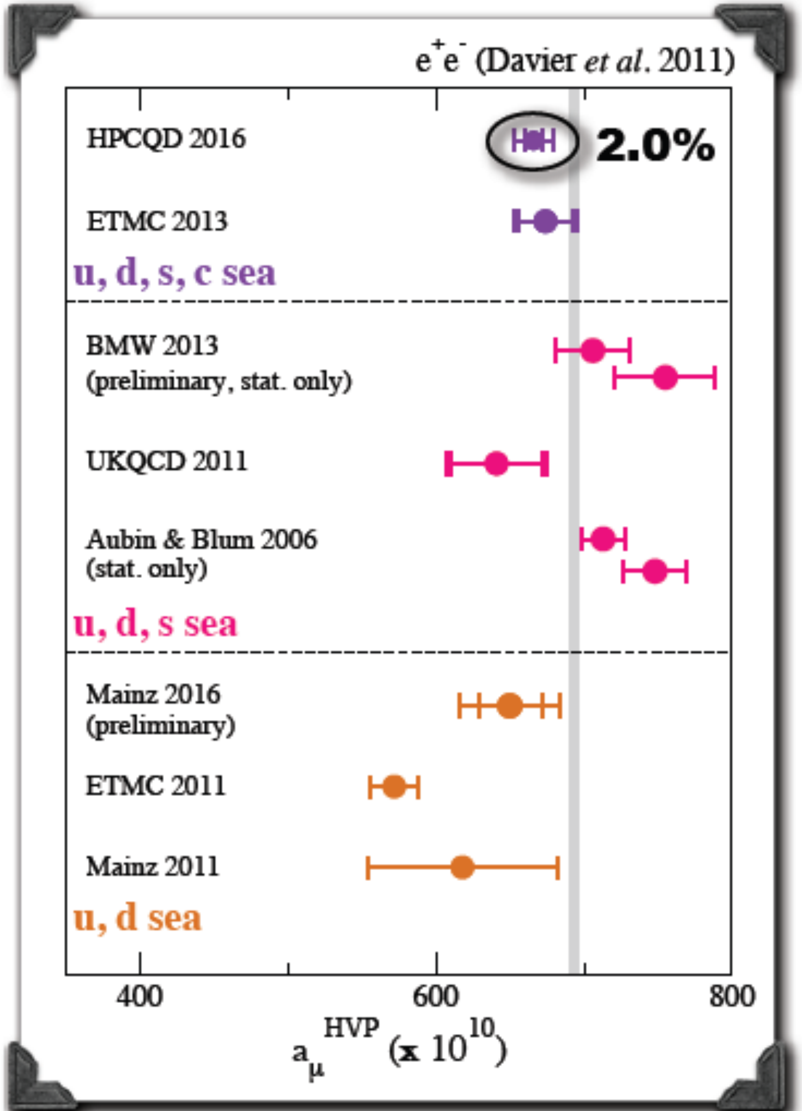
- Not an easy task!
  - >30 channels to keep under control (at (sub)percent level)
  - local discrepancies in main channels (2π (KLOE/Babar), K^+K^- CMD2/Babar)
  - Isospin corrections for not measured channels
  - Treatment of narrow resonances? (See F. Jegerlehner, ArXiv:1511.04473)



M. Davier, TAU16 WS

An independent/complementary approach is highly desirable!

Lattice-QCD progress on a_μ^{HVP}



- Can calculate nonperturbative vacuum polarization function $\Pi(Q^2)$ directly in lattice QCD from simple 2-point correlation function of EM quark current [Blum, PRL 91 (2003) 052001]
- Several ongoing lattice efforts yielding new results since ICHEP 2014 including:

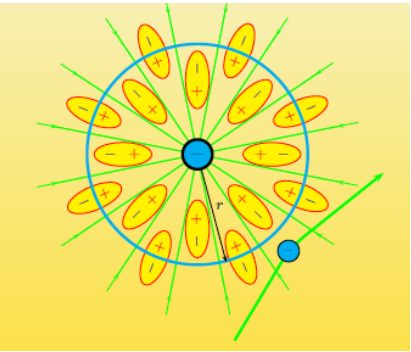
(1) First calculation of quark-disconnected contribution [RBC/UKQCD, PRL116, 232002 (2016)]

(2) Second complete calculation of leading-order a_μ^{HVP} [HPQCD, arXiv:1601.03071]

- First to reach precision needed to observe significant deviation from experiment

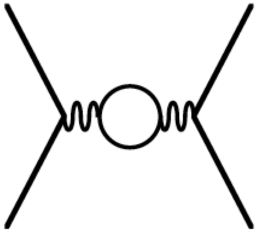
- ~1% total uncertainty by 2018 possible
- Sub-percent precision will require inclusion of isospin breaking & QED, and hence take longer

α_{em} running and the Vacuum Polarization



- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.

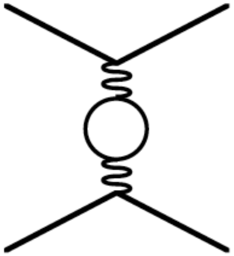
- The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:



$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

- Δa takes a contribution by non perturbative hadronic effects ($\Delta a^{(5)}_{had}$) which exibits a different behaviour in time-like and space-like region



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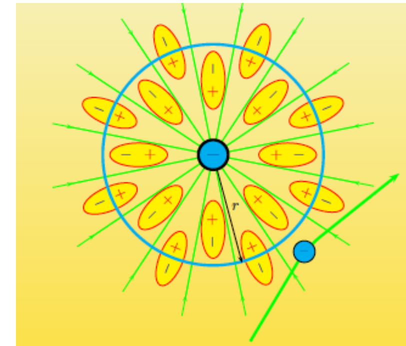

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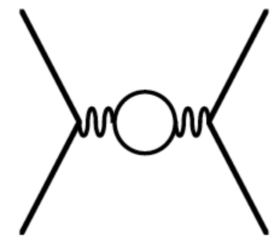

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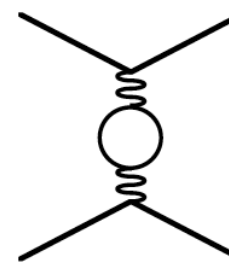
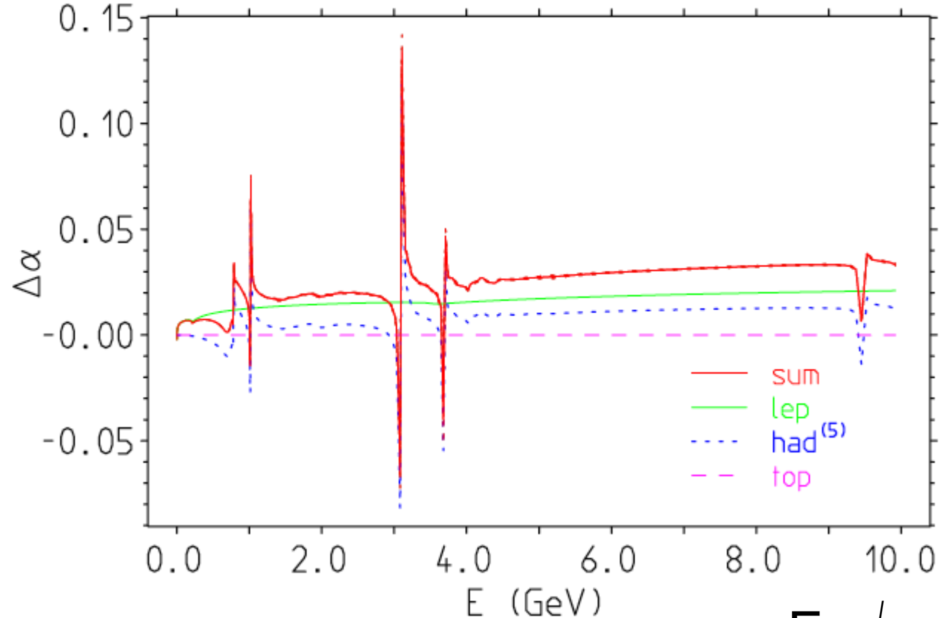
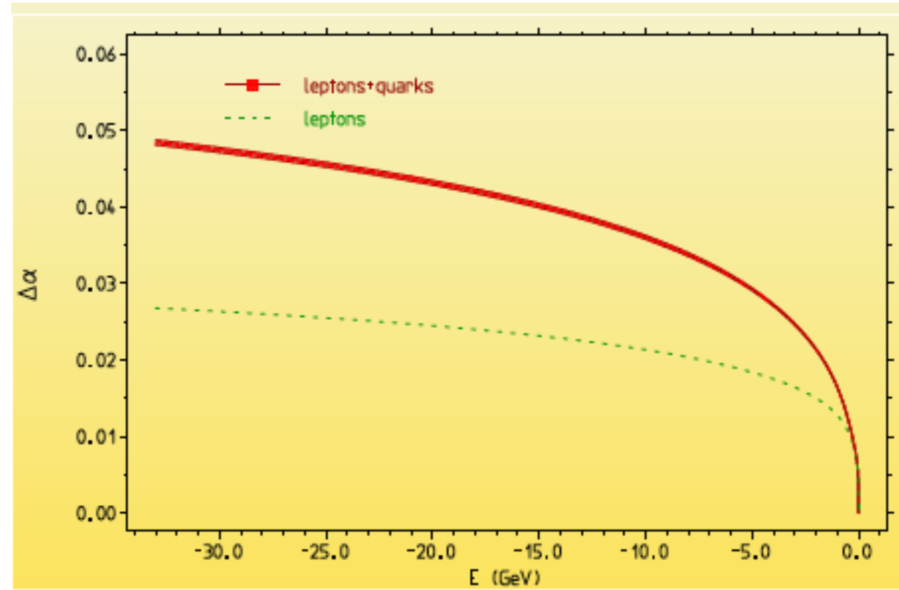


Running of α_{em}

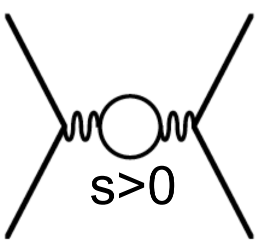




$E=\sqrt{s}$

Time-like

Behaviour characterized by the opening of resonances

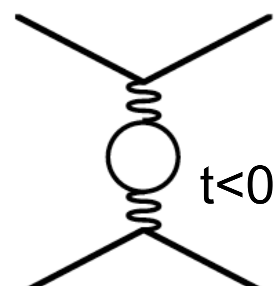


$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

$E=-\sqrt{-t}$

Space-like

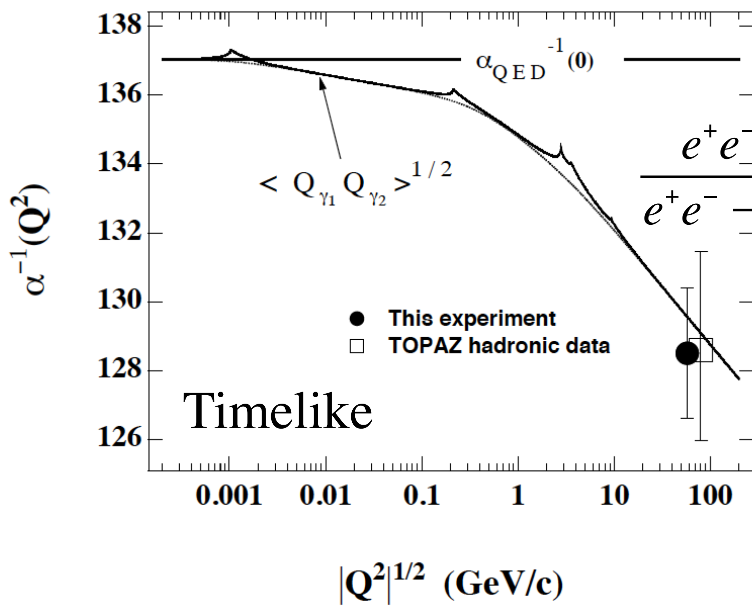
Very smooth behaviour



$$\Delta\alpha_{had}^{(5)}(-q_0^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s+q_0^2)}$$

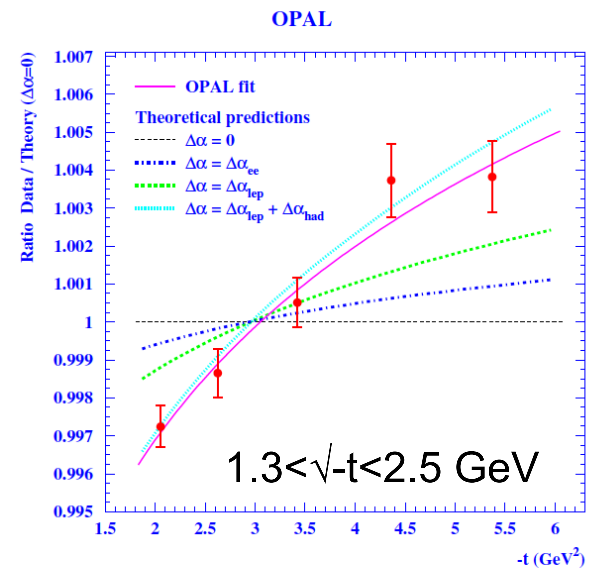
Direct measurement of α running

e+e- collider TRISTAN at √s=57.8 GeV,



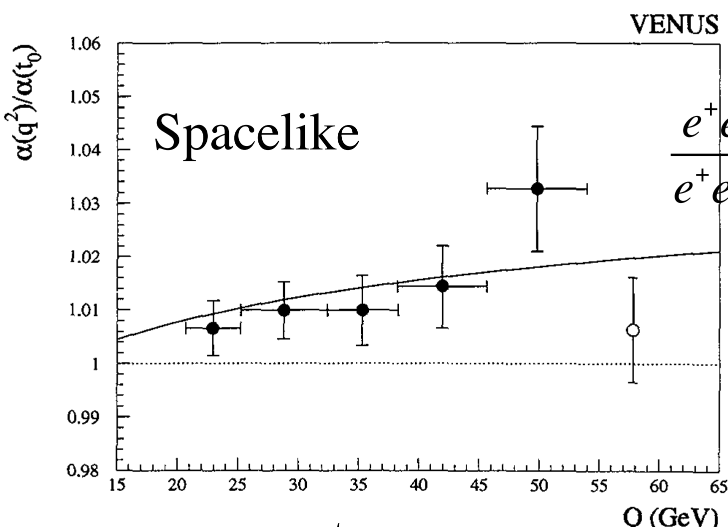
$$\frac{e^+e^- \to \mu^+\mu^-}{e^+e^- \to e^+e^-\mu^+\mu^-}$$

e+e- collider LEP at √s=189 GeV, using **Bhabha** events



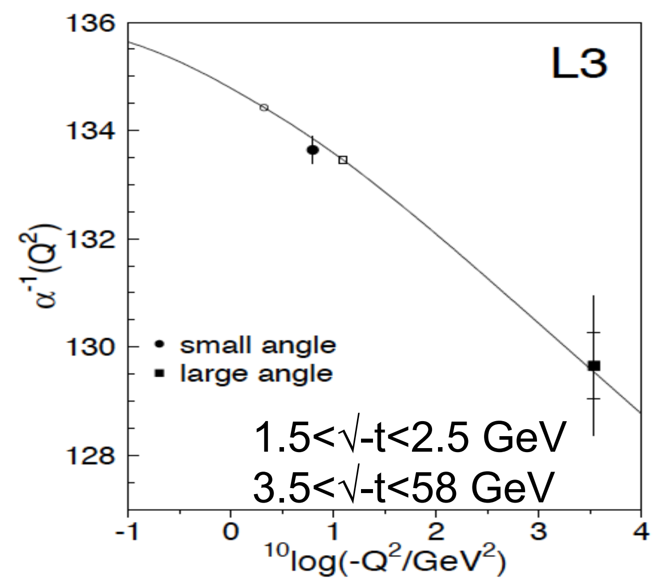
1.3<√-t<2.5 GeV

Spacelike



$$\frac{e^+e^- \to e^+e^-}{e^+e^- \to \mu^+\mu^-}$$

10<√-t<54 GeV



1.5<√-t<2.5 GeV
3.5<√-t<58 GeV

Measurement of the running of $\alpha(s)$ time-like at KLOE ($e^+e^- \rightarrow \mu^+\mu^-\gamma$)

Phys. Lett. B 767 (2017) 485-492

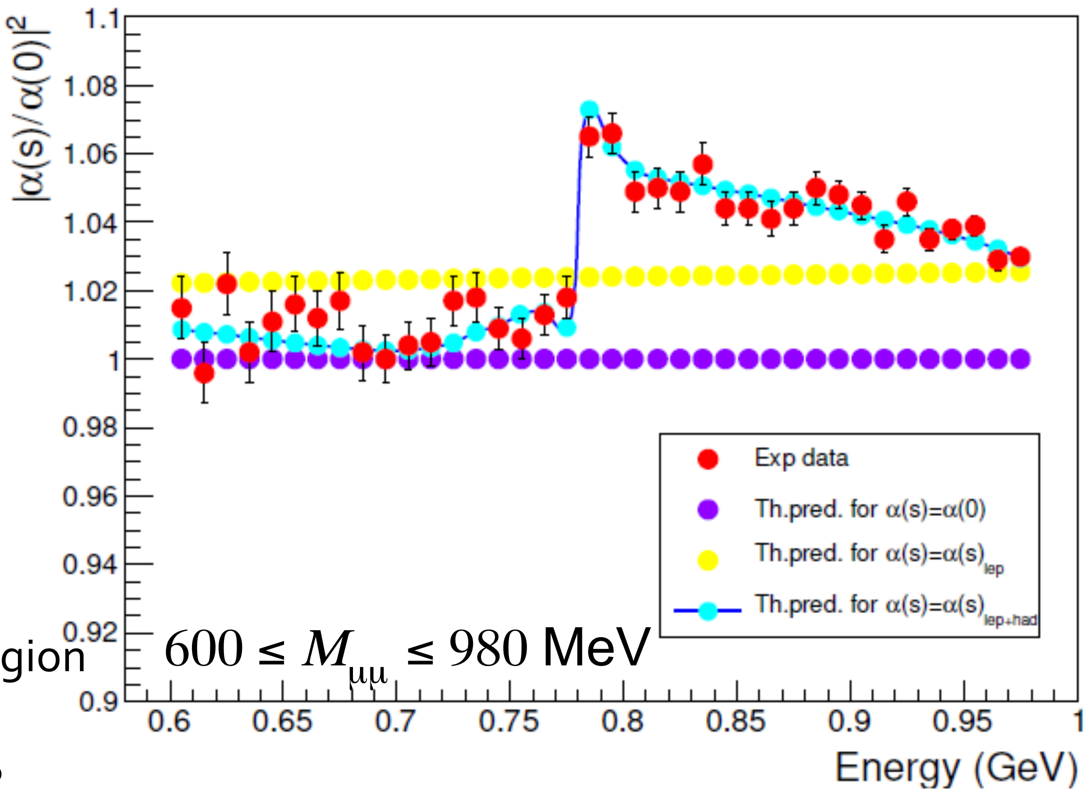
$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{\frac{d\sigma^{ISR}}{dM_{\mu\mu}}}{\frac{d\sigma^{MC}}{dM_{\mu\mu}}}$$

MC with $\alpha=\alpha(0)$

$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{1}{|1-\Delta\alpha(s)|^2}$$

$$\Delta\alpha(s) = \Delta\alpha_{lep} + \Delta\alpha_{had}$$

(we neglect the top contribution)



– Δa_{had} obtained by dispersive approach using data in time-like region provided by F. Jegerlehner

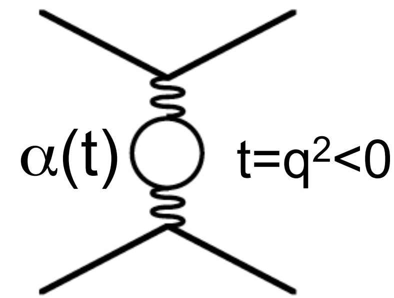
- Excellent agreement with other R compilation (Teubner / Ignatov)

Systematic error at ~1% level

$$\Delta\alpha_{had}(s) = -\left(\frac{\alpha(0)s}{3\pi}\right) Re \int_{m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\epsilon)} \qquad R(s) = \frac{\sigma_{tot}(e^+e^-\rightarrow\gamma*\rightarrow hadrons)}{\sigma_{tot}(e^+e^-\rightarrow\gamma*\rightarrow\mu^+\mu^-)}$$

Alternative approach: a_μ^{HLO} from space-like region

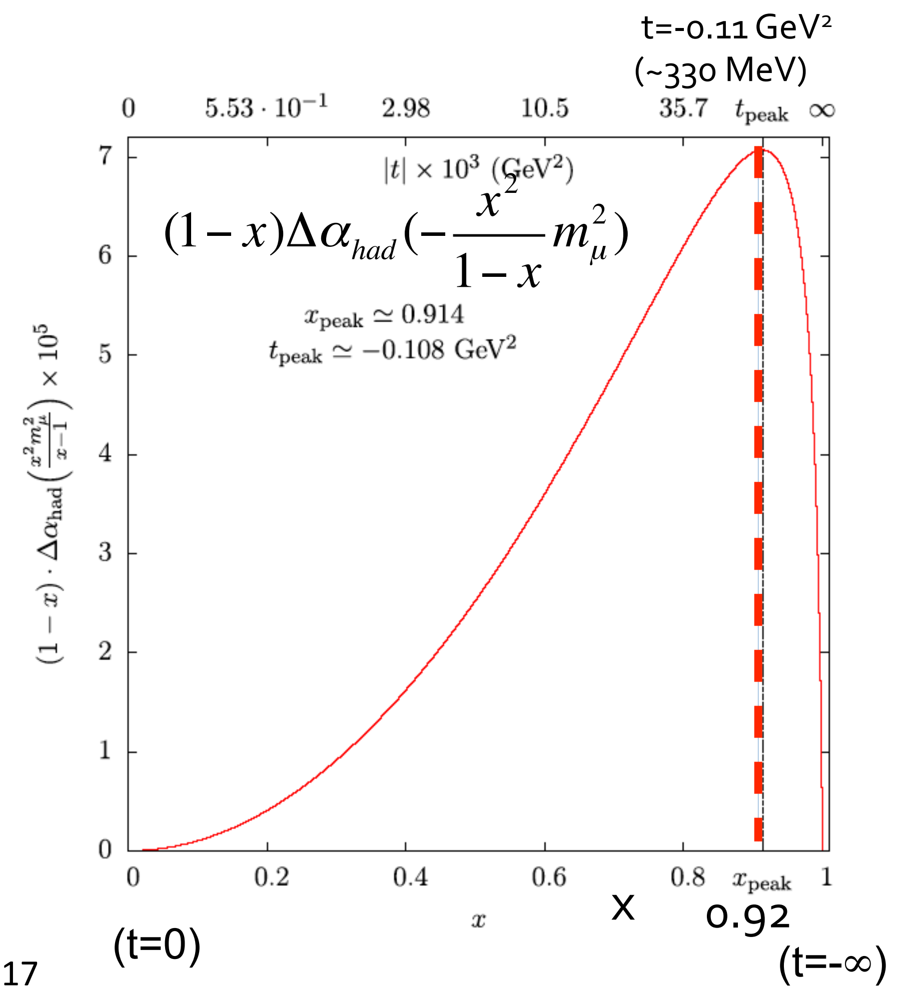
$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$



$$t = \frac{x^2 m_\mu^2}{x-1} \quad 0 \le -t < +\infty$$

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \quad 0 \le x < 1;$$

- a_μ^{HLO} **is given by the integral of the curve** (smooth behaviour)
- **It requires a measurement of** the hadronic contribution to the effective electromagnetic coupling in the space-like region **$\Delta\alpha_{had}$(t) (t=q²<0)**
- **It enhances** the contribution from **low** q² region (below 0.11 GeV²)
- **Its precision** is determined by the **uncertainty** on $\Delta\alpha_{had}$ (t) in this region

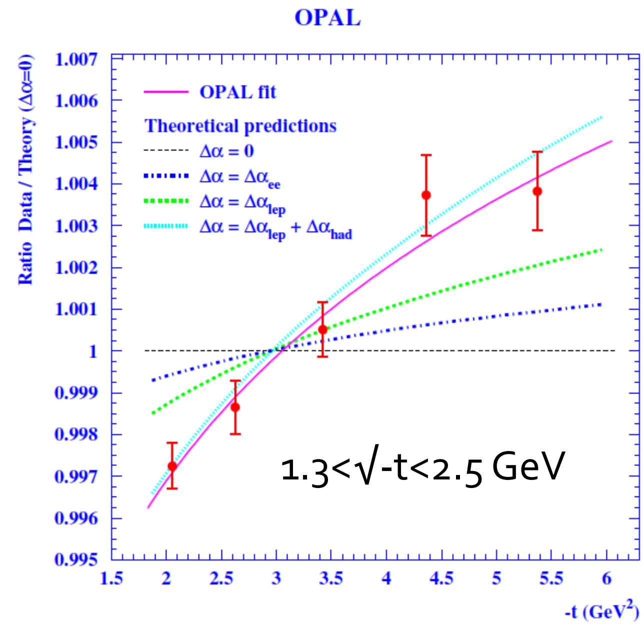


Measurement of $\Delta\alpha_{had}(t)$ spacelike at LEP

- $\Delta\alpha_{had}$ (t) (t<o) has been measured at LEP using small angle Bhabha scattering

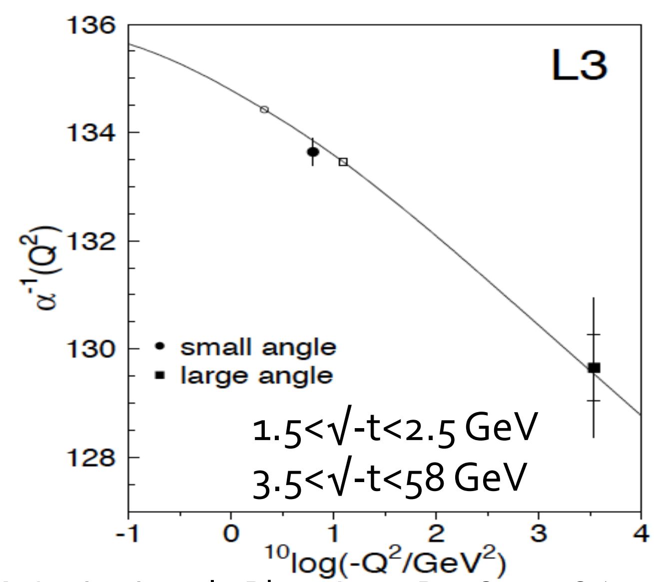
$$f(t) = \frac{N_{\rm data}(t)}{N^0_{\rm MC}(t)} \propto \left(\frac{1}{1-\Delta\alpha(t)}\right)^2 .$$

Accuracy at per mill level was achieved!



G. Abbiendi et al., Eur. Phys. J. C 45, 1–21 (2006)

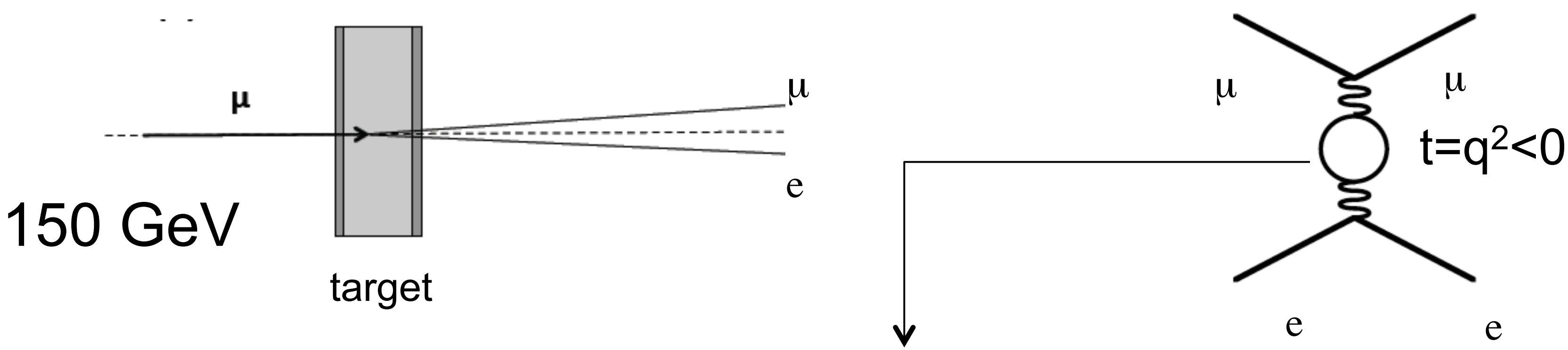
- For low t values (≤0.11 GeV²) and higher precision (~10⁻⁵) as in our case a different approach is needed!



M. Acciarri et al., Phys. Lett. B476 40-48 (2000)

Experimental approach:

Use of a 150 GeV μ beam on Be target at CERN (elastic scattering μe → μ e)

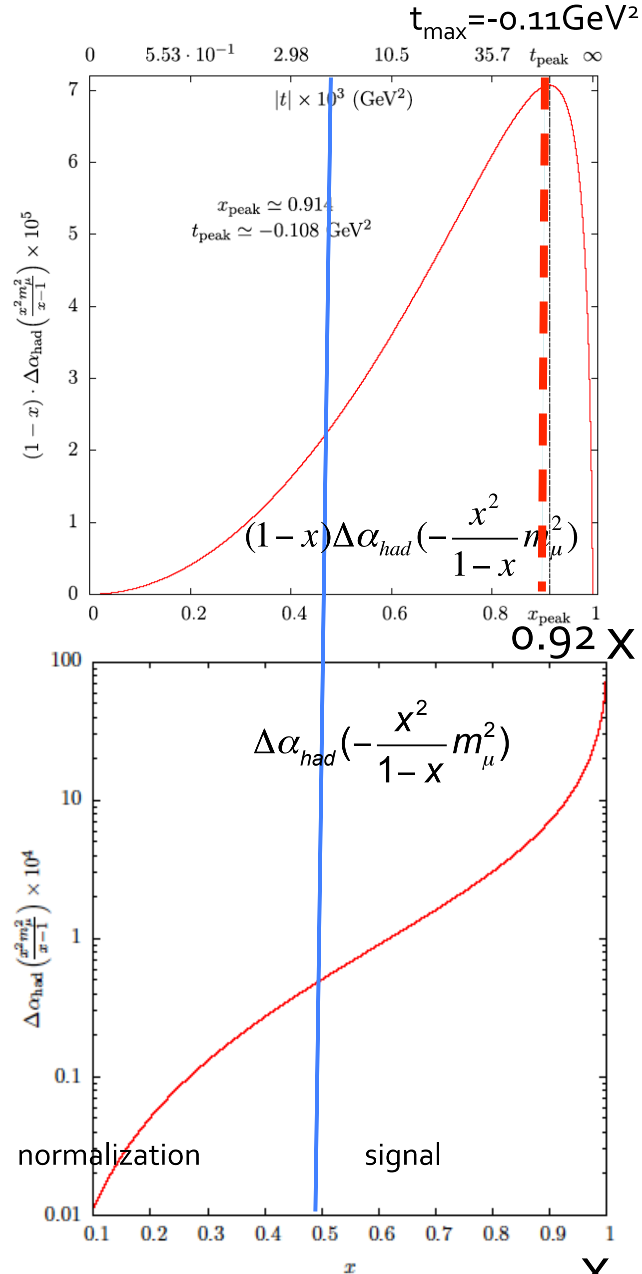


$$\left|\frac{\alpha(t)}{\alpha_0}\right|^2 = \left|\frac{1}{1-\Delta\alpha(t)}\right|^2$$

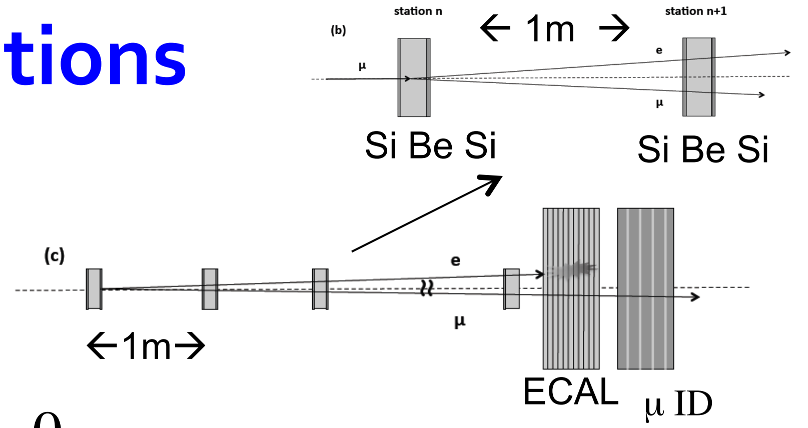
Why measuring $\Delta\alpha_{had}(t)$ with a 150 GeV μ beam on e^- target ?

It looks an ideal process!

- $\mu\, e \rightarrow \mu\, e$ is pure t-channel (at LO)
- Simple kinematics (2 body process, $t=-2m_eE_e<0$) allows to span the region $0<-t<0.143$ GeV² ($0<x<0.93$); 87% of total a_μ^{HLO} (the rest can be computed by pQCD/time-like data)
- Angular measurement: high boosted system gives access to all angles (t) in the cms region $\theta_e^{LAB}<32$ mrad ($E_e>1$ GeV) $\theta_\mu^{LAB}<5$ mrad
- It allows using the same detector for signal and normalization ($x<0.3$, $\Delta\alpha_{had}(t)<10^{-5}$) → cancellation of detector effects at first order



Detector considerations



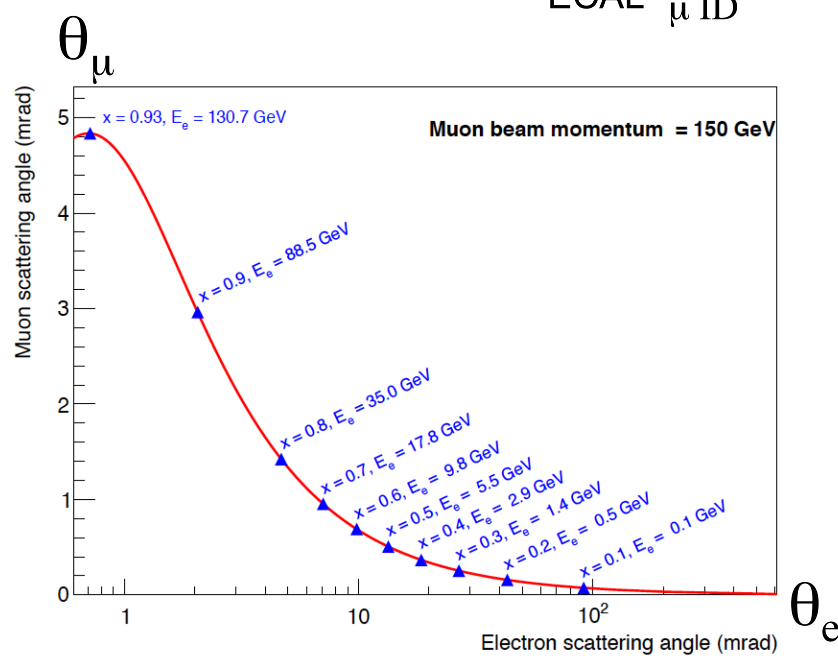
- Modular apparatus: 20 layers of 3 cm Be (target), each coupled to 1 m distant Si (0.3 mm) planes. It provides a 0.02 mrad resolution on the scattering angle

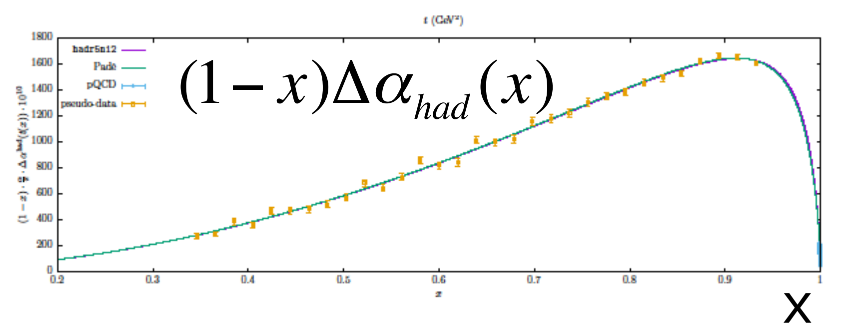
- The $t=q^2<0$ of the interaction is determined by the electron (or muon) scattering angle (a\` la NA7)

- ECAL and μ Detector located downstream to solve PID ambiguity below 5 mrad. Above that, angular measurement gives correct PID

- It provides uniform full acceptance, with the potential to keep the **systematic** errors at **10^{-5}**

- **Statistical** considerations show that a **0.3%** error can be achieved on a_μ^{HLO} in 2 years of data taking with 1.3×10^7 μ/s (available at CERN)

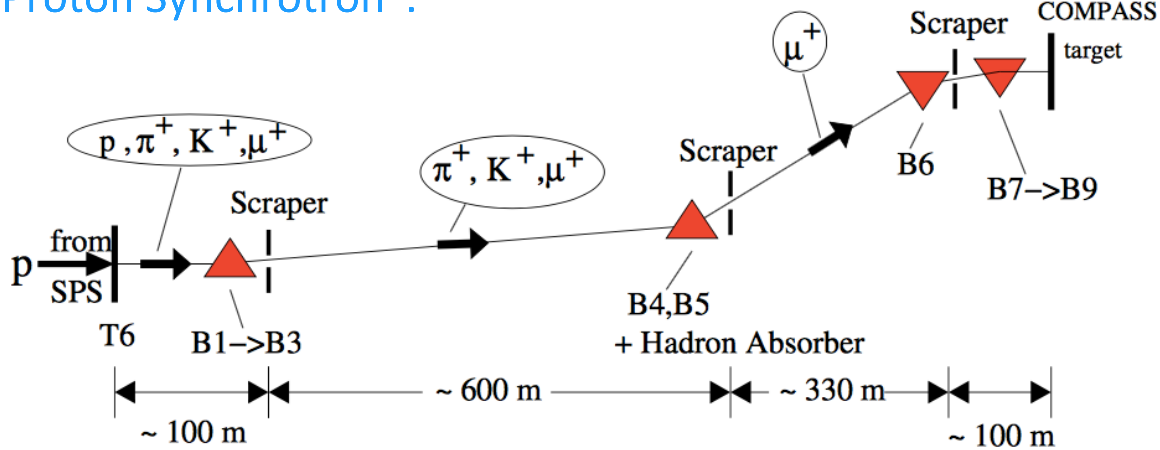




More details in Umberto's talk

Muon beam M2 at CERN

“Forty years ago, on 7 May 1977, CERN inaugurated the world’s largest accelerator at the time – the Super Proton Synchrotron”.



https://arxiv.org/pdf/hep-ex/0703049.pdf

Table 3
Parameters and performance of the 160 GeV/c muon beam.

Beam parameters	Measured
Beam momentum $(p_\mu)/(p_\pi)$	$(160\,\mathrm{GeV}/c)/(172\,\mathrm{GeV}/c)$
Proton flux on T6 per SPS cycle	$1.2 \cdot 10^{13}$
Focussed muon flux per SPS cycle	$2 \cdot 10^{8}$
Beam polarisation	$(-80 \pm 4)\%$
Spot size at COMPASS target $(\sigma_x \times \sigma_y)$	$8 \times 8\,\mathrm{mm}^2$
Divergence at COMPASS target $(\sigma_x \times \sigma_y)$	$0.4 \times 0.8\,\mathrm{mrad}$
Muon halo within 15 cm from beam axis	16%
Halo in experiment $(3.2 \times 2.5\,\mathrm{m}^2)$ at $\lvert x, y \rvert > 15\,\mathrm{cm}$	7%

I_{beam}> 10^7 muon/s, E_μ = 150 GeV

Systematics

1. Acceptance
2. Tracking
3. Trigger
4. PID
5. Effects of E_e energy cut
6. Signal/Background:
   It requires a dedicated event generator.

Affordable by means of GEANT4 based simulations

7. Uncertainty in the location of interaction vertices: Segmented/active target to resolve the vertex position
8. Uncertainty in the muon beam momentum:
   Scattering kinematics to determine the beam momentum
9. Effects of Multiple Scattering (must be known at $\leq 1\%$):
   It requires dedicated work on simulation and measurements (test beam).
10. Theoretical uncertainty on the mu-e cross section (see later)

All the systematic effects must be known to ensure an errror on the cross section < 10ppm

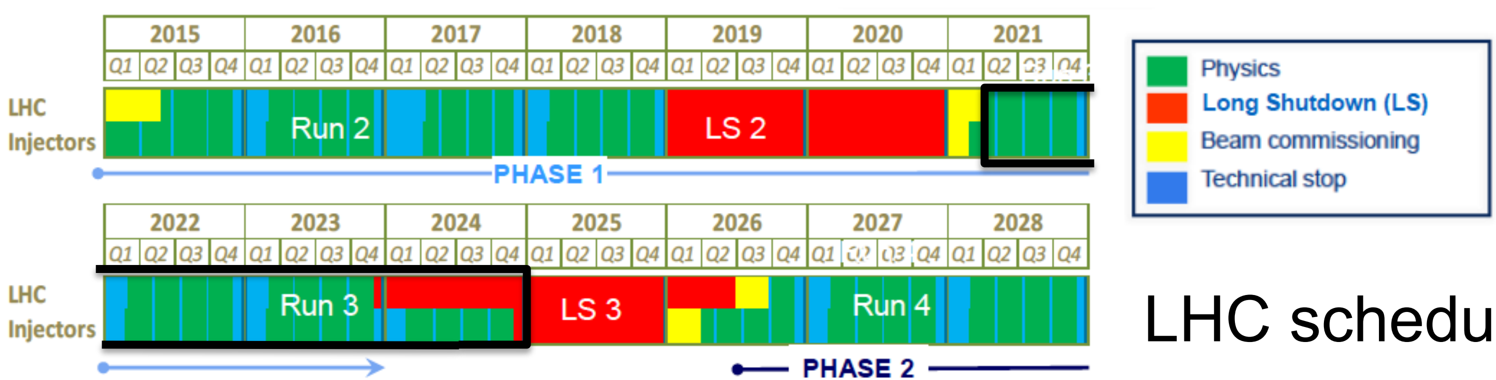
Activity on the theory side

1. Development of dedicated NNLO MC event generator for mu-e scattering. Ultimate precisione ≤10ppm. Many ingredients…see this workshop!!!

2. Theory workshop next year in Mainz (19-24 February 2018). You are all invited!

Plans

- **2017 - 2019**
  - Detector optimization studies
  - Test beams (first on 27 Sep-3 October 2017 at CERN)
  - Set up a collaboration
  - Theoretical studies
  - Letter of Intent to the SPSC
- **2020**
  - Detector construction and installation
    (a staged version of the detector may be)
- **2021 – 2024**
  - Start the data taking after LS2 to measure $\mathbf{a}_\mu^{\mathbf{HLO}}$
    (not necessarily the ultimate precision)



LHC schedule

Conclusion

- Proposal part of the CERN "Physics Beyond Collider Study Group" at CERN (http://pbc.web.cern.ch/)
- If approved (by CERN SPSC) first results in the same period of the g-2 measurements at Fermilab and J-Parc (first results around 2021-2024)
- Many experimental and theoretical challenges: very exciting!!
- We need your help!!

Thanks!

Reference papers

A new approach to evaluate the leading hadronic corrections to the muon g-2 ☆

C. M. Carloni Calame[a], M. Passera[b], L. Trentadue[c], G. Venanzoni[d]

[a] *Dipartimento di Fisica, Università di Pavia, Pavia, Italy*
[b] *INFN, Sezione di Padova, Padova, Italy*
[c] *Dipartimento di Fisica e Scienze della Terra "M. Melloni" Università di Parma, Parma, Italy and INFN, Sezione di Milano Bicocca, Milano, Italy*
[d] *INFN, Laboratori Nazionali di Frascati, Frascati, Italy*

Measuring the leading hadronic contribution to the muon g-2 via μe scattering

G. Abbiendi[1], C. M. Carloni Calame[2], U. Marconi[1], C. Matteuzzi[3], G. Montagna[4,2], O. Nicrosini[2], M. Passera[5], F. Piccinini[2], R. Tenchini[6], L. Trentadue[7,3], and G. Venanzoni[8]
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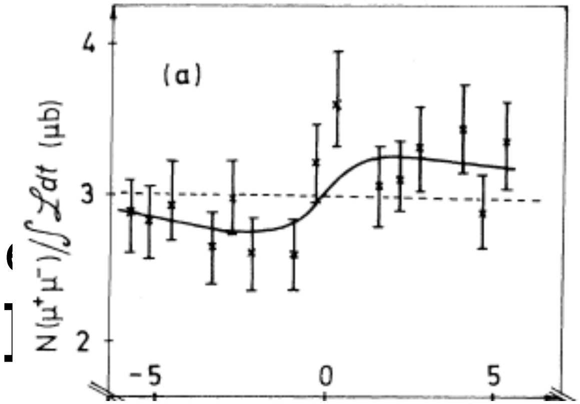
Phys. Lett. B 746 (2015) 325

Eur. Phys. J. C (2017) 77: 139.

Spare

Previous tests of the hadronic contribution to VP

1) '73: $\phi(1020)$ contribution to VP at ACO (Orsay e+e-) in the $e^+e^- \rightarrow \mu+\mu-$ process: evidence at 3σ in the region ±5 MeV around the ϕ peak [**Phys.Rev.Lett. 30 (1973) 462-464)**]



-5 MeV m_ϕ 5 MeV

2) 70's: g-2 experiment at CERN: evidence for hadronic contribution to g-2 at 6σ **[Phys. Lett. 67B (1977) 225; Phys. Lett. 68B (1977) 191]**

exp

$$a_\mu = 1\ 165\ 924\ (8.5) \times 10^{-9}\ (7\ \text{ppm}).$$

theo

QED terms	Muon	Numerical values ($\times 10^9$)	
2nd order : A	0.5	Total QED:	1 165 852 (1.9)
4th order : B	0.765 782 23	Strong interactions:	66.7 (8.1)
6th order : C	24.452 (26)	Weak interactions:	2.1 (0.2)
8th order : D	135 (63)	Total theory:	1 165 921 (8.3)
10th order : E	420 (30)		

3) 2006: OPAL at LEP: evidence for hadronic contributon Δa_{had} (t) (t<0) at 3σ in Bhabha scattering at small angle [**Eur.Phys.J. C45 (2006) 1-21]**