Effect of the atomic wave function in μ scattering off bound e^-

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Present status of my ongoing discussion with myself about

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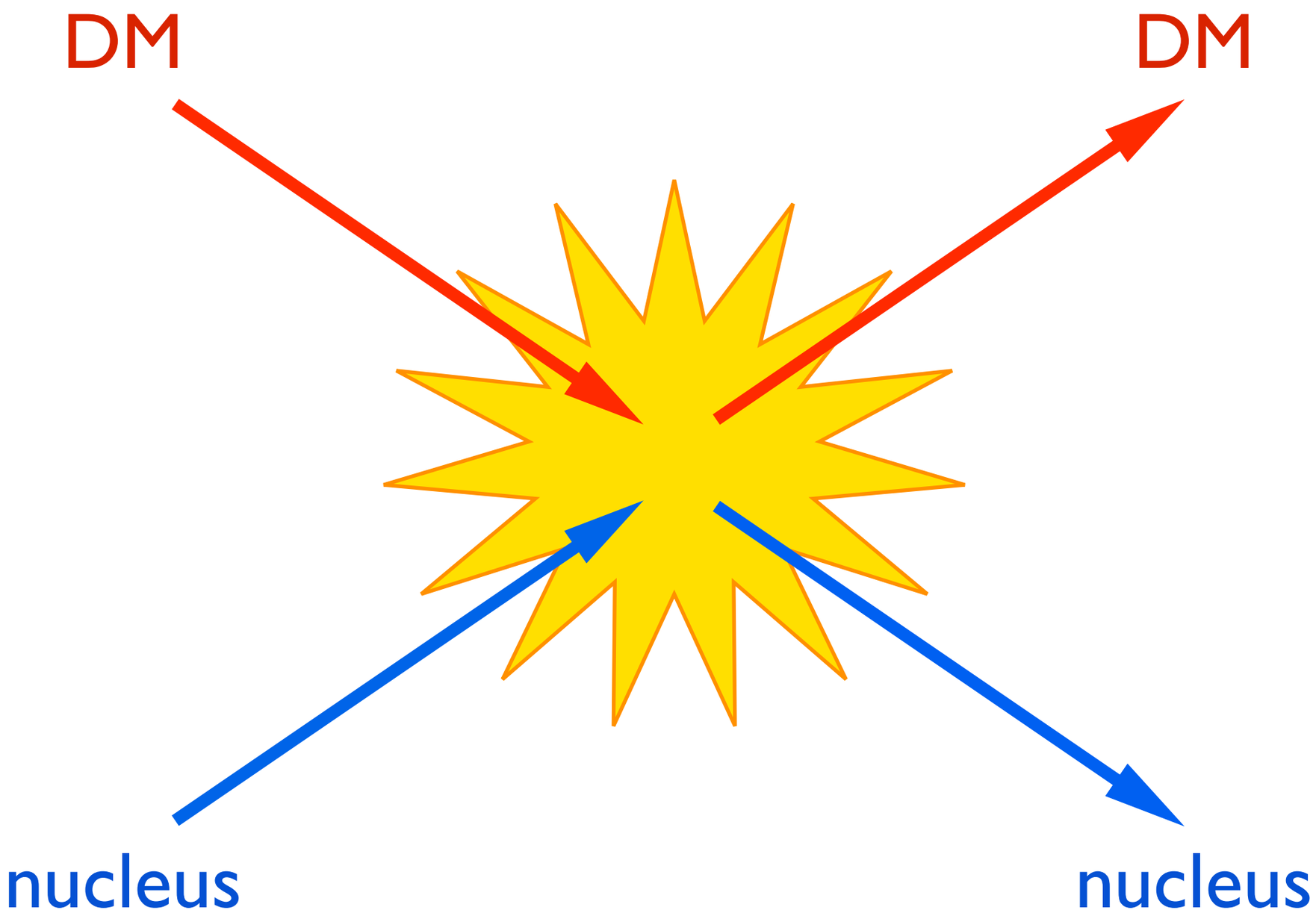
From Dark Matter to muons



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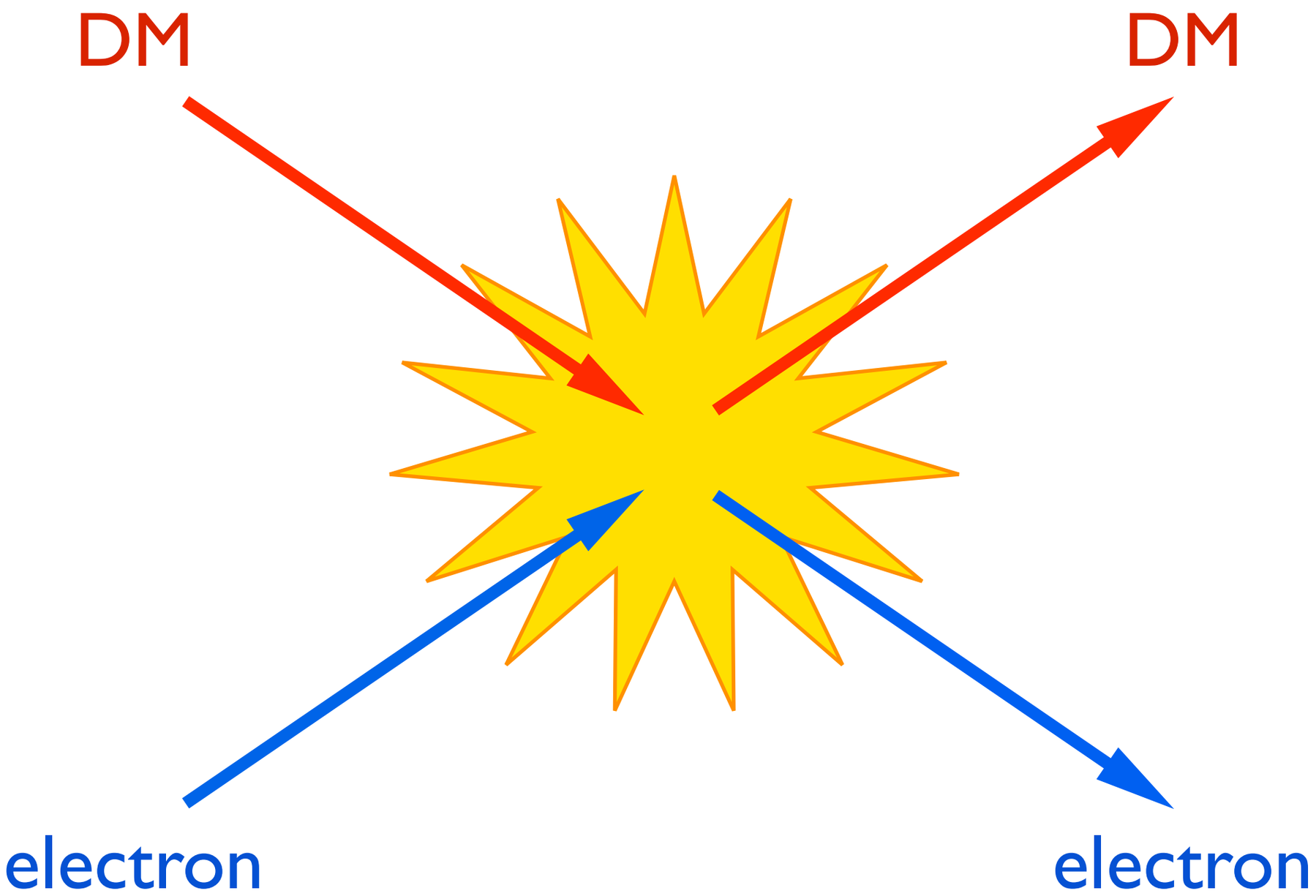

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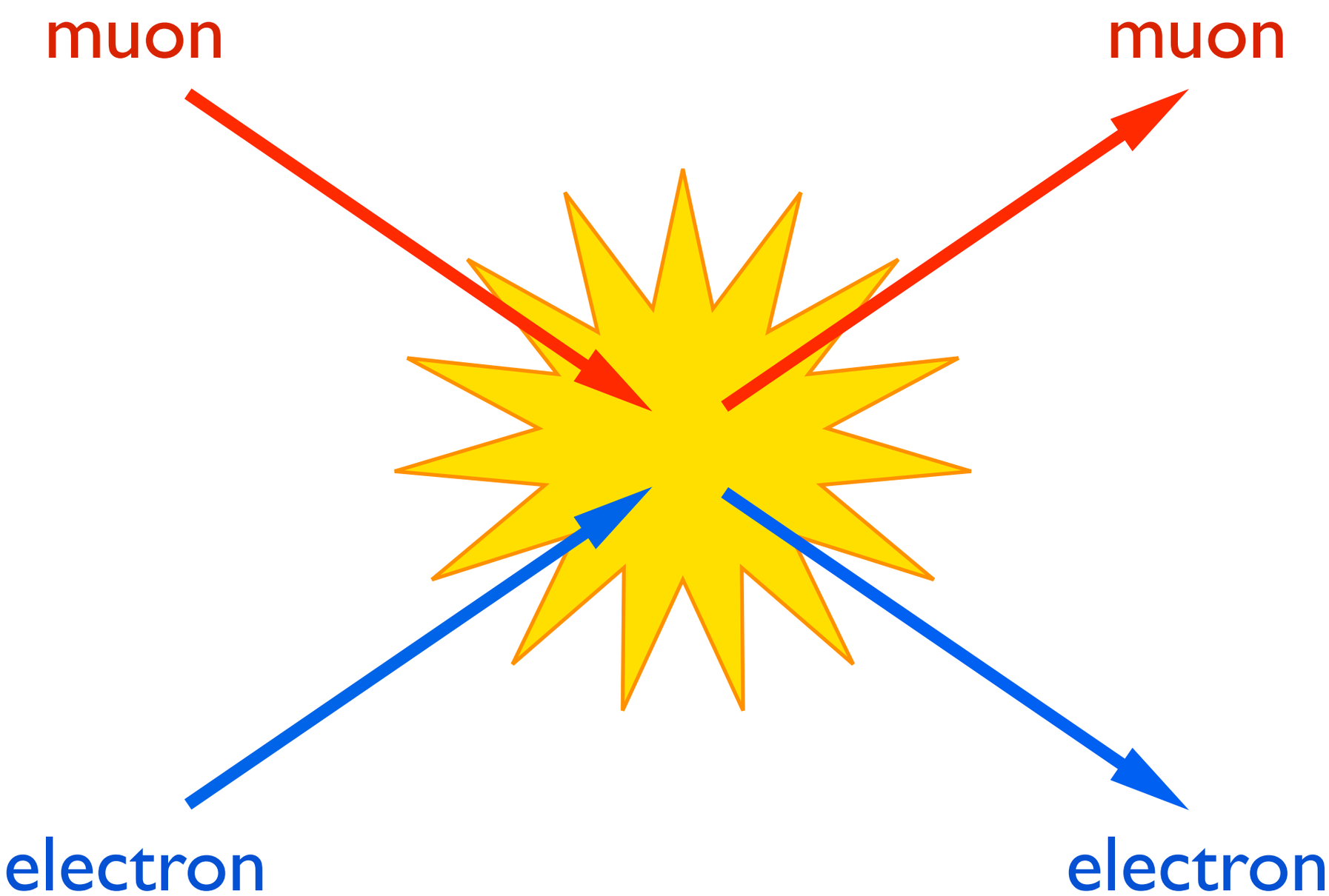
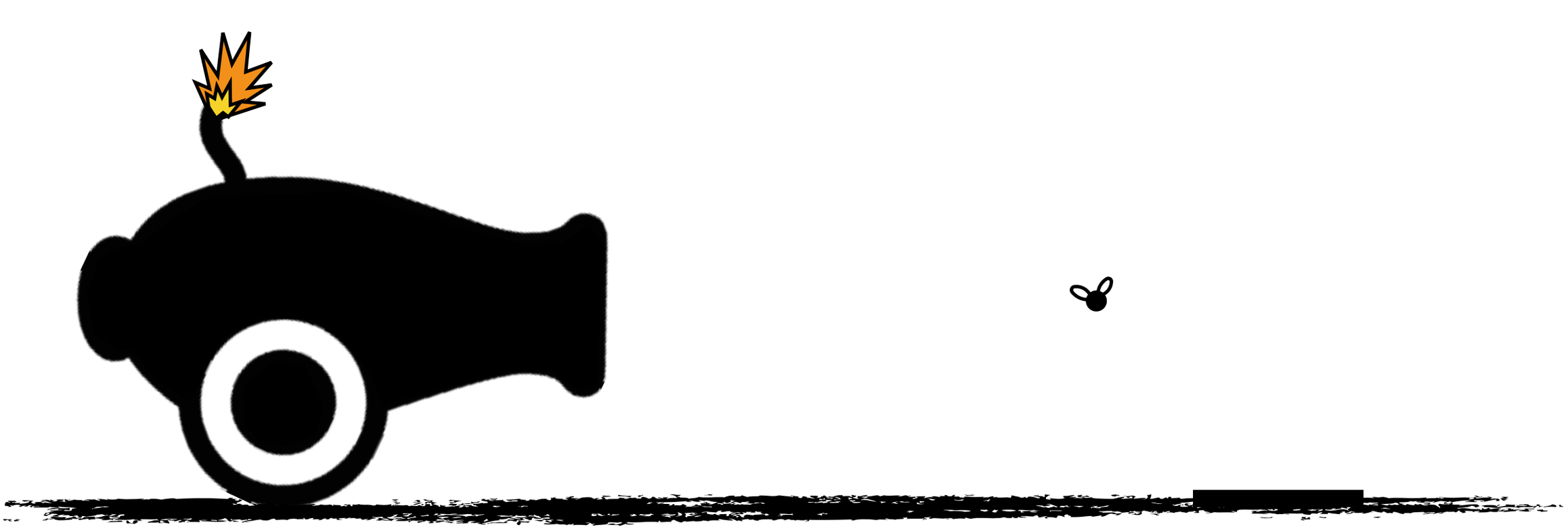


μ off a free e^-

- Is a *free* e^- a good approximation to a *bound* e^-?
- What's the correction due to the e^- wave function?



μ off a bound e^-

- Is a *free* e^- a good approximation to a *bound* e^-?
- What's the correction due to the e^- wave function?

Effect of ψ: bound electron

- Not in a momentum eigenstate, not at rest
- Momenta distributed according to ψ
- Can have ~any momentum
- Not on the mass shell

Free electron at rest

1 dof

$$\theta_\mu$$

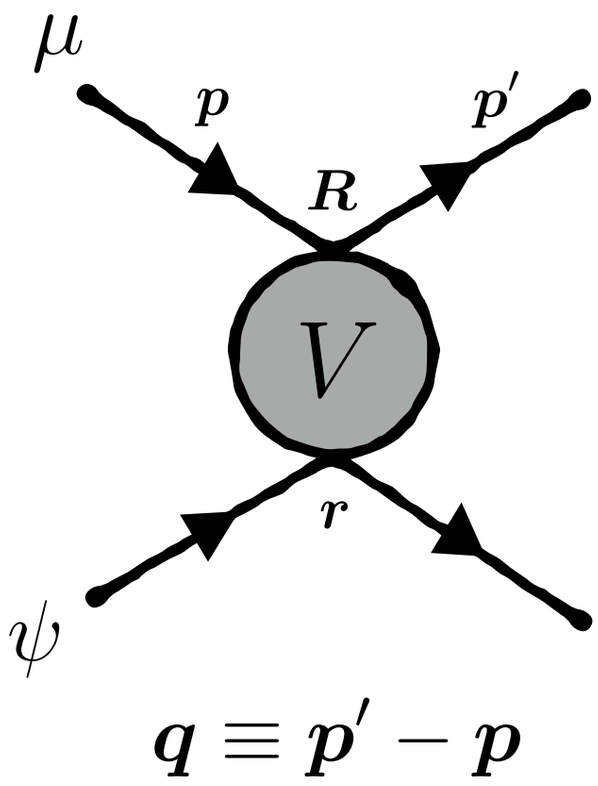
Bound electron

5 dof

$$E'_\mu, \theta_\mu, \phi_\mu, \theta_e, \phi_e$$

(possibly less, symmetries permitting)

Scattering cross section



In Born approximation

$$T_{\mathrm{fi}} = \int e^{-i\boldsymbol{p}'\cdot\boldsymbol{R}} \psi'^{*}(\boldsymbol{r})\, V(|\boldsymbol{r}-\boldsymbol{R}|)\, e^{i\boldsymbol{p}\cdot\boldsymbol{R}} \psi(\boldsymbol{r})\, \mathrm{d}^3R\, \mathrm{d}^3r$$

$$= \langle\psi'| \int V(|\boldsymbol{r}-\boldsymbol{R}|)\, e^{-i\boldsymbol{q}\cdot\boldsymbol{R}}\, \mathrm{d}^3R |\psi\rangle$$

$$= \tilde{V}(|\boldsymbol{q}|)\, \langle\psi'|e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}|\psi\rangle$$

$$\mathrm{d}\sigma = \frac{2\pi}{v_\mu} \left|\tilde{V}(|\boldsymbol{q}|)\, \langle\psi'|e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}|\psi\rangle\right|^2 \delta(\mathrm{Energy}) \frac{\mathrm{d}^3p'}{(2\pi)^3} \frac{\mathrm{d}^3\psi'}{(2\pi)^3}$$

Electron wave function

$$\psi(r) \sim e^{-m_e Z\alpha r}\,(2m_e Z\alpha r)^{\gamma-1}$$

non-relativistic ψ

relativistic correction

In *s* wave (zero orbital angular momentum)

$$\gamma = \sqrt{1-(Z\alpha)^2}$$

Non-relativistic expansion: $Z\alpha \sim 0$

Relativistic corrections become important when

$$2m_e Z\alpha r \leqslant e^{\mp 2/(Z\alpha)^2} \approx 10^{\mp 10^4/Z^2}$$

For $Z = 1$ $\quad r \leqslant 10^{\mp 10^4}$ fm

For $Z = 50$ $\quad r \leqslant 10^{\mp 3}$ fm

Shut up and integrate

Integrating the angular part of ψ

$$\int Y_\ell^m(\Omega)\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \mathrm{d}\Omega \sim \int P_\ell(\cos\theta)\, e^{-iqr\cos\theta}\, \mathrm{d}\cos\theta \sim j_\ell(qr)$$

Thus

$$\langle\psi'|e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}|\psi\rangle \sim \int R'^*(r)R(r)j_0(qr)\, r^2\, \mathrm{d}r \sim \int e^{-m_e Z\alpha r} r^{2\gamma} j_0(qr)\, \mathrm{d}r$$

= known ugly expression

Suppression at large momentum transfer:

$$\langle\psi'|e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}|\psi\rangle \sim \frac{1}{q^{3-(Z\alpha)^2}}$$

Open questions

- Does any of this actually matter?
- When is *free* a good approximation to *bound*?
- And how good an approximation is it?
  - Understand the kinematics: what q is most likely?
  - Effect of atomic recoil: does it matter?
  - O(10) estimated yet unknown issues