Muon-electron scattering at NLO in QED

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Muon-electron scattering:
Theory kickoff workshop

Padova, September 4-5, 2017

with M. Alacevich, M. Chiesa, G. Montagna, O. Nicrosini, F. Piccinini

Outline

- Introduction
- Calculation details
- Numerical results
- Conclusions and outlook

Relevant literature:

- G. Abbiendi *et al.*,
  Measuring the leading hadronic contribution to the muon g-2 via μe scattering
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ex]
- C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

$\mapsto$ G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini,
Matching perturbative and parton shower corrections to Bhabha process at flavour factories
Nucl. Phys. B **758** (2006) 227 - hep-ph/0607181

Alternative approach to $a_\mu^{\rm HLO}$: space-like evaluation

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx(x-1)\bar{\Pi}_{\rm had}(t(x)) = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\rm had}(t(x))$$

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_{-\infty}^0 \frac{dt}{\beta t}\left(\frac{1-\beta}{1+\beta}\right)^2 \bar{\Pi}_{\rm had}(t) = -\frac{\alpha}{\pi}\int_{-\infty}^0 \frac{dt}{\beta t}\left(\frac{1-\beta}{1+\beta}\right)^2 \Delta\alpha_{\rm had}(t)$$

where

$$t(x) = \frac{x^2 m_\mu^2}{x-1} \quad \beta(t) = \sqrt{1-\frac{4m_\mu^2}{t}} \quad x(t) = \frac{t\,(1-\beta(t))}{2m_\mu^2} \quad t = \begin{cases} 0^- & \text{for } x \to 0^+ \\ -\infty & \text{for } x \to 1^- \end{cases}$$

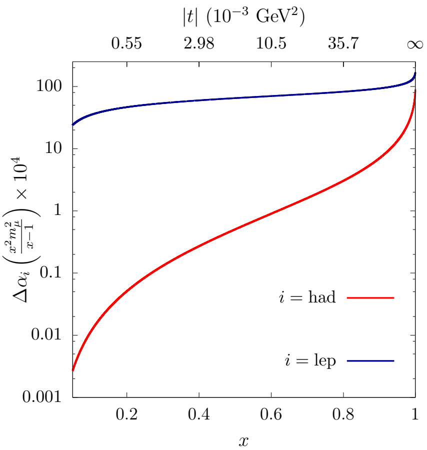
$\Delta\alpha_{\rm had}(t)$ is the hadronic contribution to the running of $\alpha_{\rm QED}(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)}$

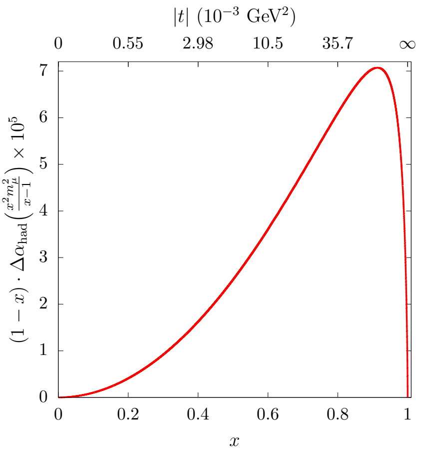
- ⋆ $\Delta\alpha_{\rm had}(t)$ in the integrand is evaluated in the space-like region (negative transfer momenta) where it is a smooth function
- ⋆ we propose to *measure* the running of $\alpha_{\rm QED}(t<0)$ to evalutate $a_\mu^{\rm HLO}$

see also G.V. Fedotovich, CMD-2 Collaboration, Nucl. Phys. Proc. Suppl. 181-182 (2008) 146

similar approach also used for lattice calculations of $a_\mu^{\rm HLO}$

General considerations





- $\Delta\alpha_{\text{had}}(t(x))$ (red) as a function of x
- $\Delta\alpha_{\text{lep}}(t(x))$ (blue) as a function of x

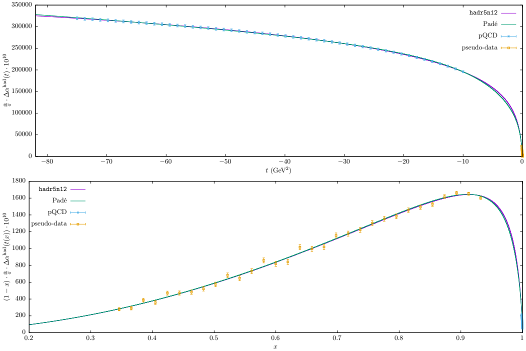
- integrand function $(1-x)\Delta\alpha_{\text{had}}(t(x))$

$$x_{\text{peak}} \simeq 0.914$$
$$t_{\text{peak}} \simeq -0.108 \text{ GeV}^2$$

General considerations

- To get $\Delta\alpha_{\rm had}(t)$, the goal is to measure the (absolute) running of $\alpha_{\rm QED}(t)$ from the distribution of the t variable in
  - $\rightarrow$ Bhabha events at e^+e^- (low-energy) colliders **original proposal**
  - $\rightarrow$ or μe scattering events in a fixed target experiment **new proposal**

$$\alpha(t) = \frac{\alpha}{1 - \Delta\alpha_{\rm other}(t) - \Delta\alpha_{\rm had}(t)} \qquad \Delta\alpha_{\rm had}(t) = 1 - \Delta\alpha_{\rm other}(t) - \frac{\alpha}{\alpha(t)}$$



Strategy:

- measure $\Delta\alpha_{\rm had}(t)$ within the experimental kinematical range
- get large $|t|$ values from elsewhere
- fit $\Delta\alpha_{\rm had}(t)$
- get the integrand function and the value of $a_\mu^{\rm HLO}$

Roughly, to be competitive with the current evaluations, $\Delta\alpha_{\rm had}(t)$ needs to be know at some $\%$ level

New perspective: $\mu e \to \mu e$ scattering in fixed target experiment

$\mapsto$ A 150 GeV high-intensity ($\sim$1.3$\times 10^7$ μ's/s) muon beam is available at CERN NA

$\mapsto$ Muon scattering on a low-Z target ($\mu e \to \mu e$) looks an ideal process

- ⋆ it is a pure t–channel process $\to$

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha} \right|^2$$

- ⋆ Assuming a 150 GeV incident μ beam we have

$s \simeq 0.164 \text{ GeV}^2$ $\quad -0.143 \lesssim t < 0 \text{ GeV}^2$ $\quad 0 < x \lesssim 0.93$ **it spans the peak!**

Benefits:

- ⋆ the highly boosted kinematics allows to access a wide angular range in the CM
- ⋆ the same detector can be exploited for signal and normalization
- ⋆ the same process is used for signal and normalization:
  the region $x \lesssim 0.3$, where $\Delta\alpha_{\rm had}(t) < 10^{-5}$, can be used for normalization

μe scattering kinematics for leading order ($2 \to 2$, elastic process)

p_1, p_2 initial state μ and e $\quad$ p_3, p_4 final state μ and e

In the lab

$$\begin{aligned} p_1 &= (E_\mu^{beam}, 0, 0, p) \\ p_2 &= (m_e, 0, 0, 0) \\ p_3 &= p_1 + p_2 - p_4 \\ p_4 &= (E_e, p_e \sin\theta_e, 0, p_e \cos\theta_e) \end{aligned}$$

In the center of mass

$$\begin{aligned} p_1 &= (E^\mu_{CM}, 0, 0, p_{CM}) \\ p_2 &= (E^e_{CM}, 0, 0, -p_{CM}) \\ p_3 &= (E^\mu_{CM}, p_{CM} \sin\theta, 0, p_{CM} \cos\theta) \\ p_4 &= (E^e_{CM}, -p_{CM} \sin\theta, 0, -p_{CM} \cos\theta) \end{aligned}$$

Invariants:

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\ &= m_e^2 + m_\mu^2 + 2E^\mu_{CM} E^e_{CM} + 2p^2_{CM} \\ &= m_e^2 + m_\mu^2 + 2E_\mu^{beam} m_e \end{aligned}$$

$$\begin{aligned} t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\ &= -2p^2_{CM}(1 - \cos\theta) \\ &= 2m_e^2 - 2E_e m_e \end{aligned}$$

$$p_{CM} = \frac{1}{2}\sqrt{\frac{\lambda(s, m_\mu^2, m_e^2)}{s}}$$

$$t = m_\mu^2 \frac{x^2}{x-1} \propto E_e$$

$$E_e = m_e \frac{1 + r^2 \cos^2\theta_e}{1 - r^2 \cos^2\theta_e}$$

$$r \equiv \frac{\sqrt{\left(E_\mu^{beam}\right)^2 - m_\mu^2}}{E_\mu^{beam} + m_e}$$

Next-to-leading order calculation

- The μe cross section and distributions must be known as precisely as possible
  $\mapsto$ radiative corrections (RCs) are mandatory

⋆ First step are QED $\mathcal{O}(\alpha)$ (i.e. QED NLO, next-to-leading order) RCs

The NLO cross section is split into two contributions,

$$\sigma_{NLO} = \sigma_{2\to2} + \sigma_{2\to3} = \sigma_{\mu e\to\mu e} + \sigma_{\mu e\to\mu e\gamma}$$

$\mapsto$ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$

$\mapsto$ $[2\to2]/[2\to3]$ phase space splitting at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$

- $\mu e \to \mu e$

$$\sigma_{2\to2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2(|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^* \times \mathcal{A}_{NLO}^{virtual}(\boldsymbol{\lambda})])$$

- $\mu e \to \mu e\gamma$

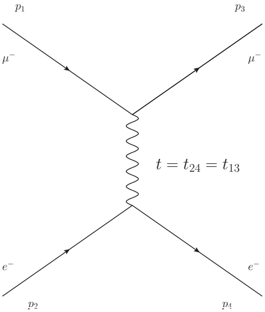
$$\sigma_{2\to3} = \frac{1}{F}\int_{\omega>\boldsymbol{\lambda}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2\right)$$

$$= \Delta_s(\boldsymbol{\lambda},\boldsymbol{\omega_s})\int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2$$

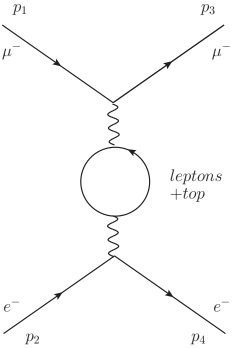
- the integration over the 2/3-particles phase space is done with MC techniques and fully-exclusive events are generated

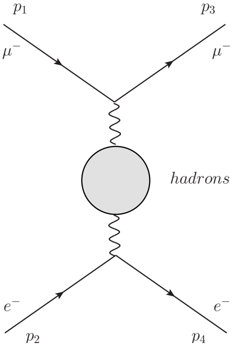
LO and NLO vacuum polarization diagrams

- $\mathcal{A}_{LO}$

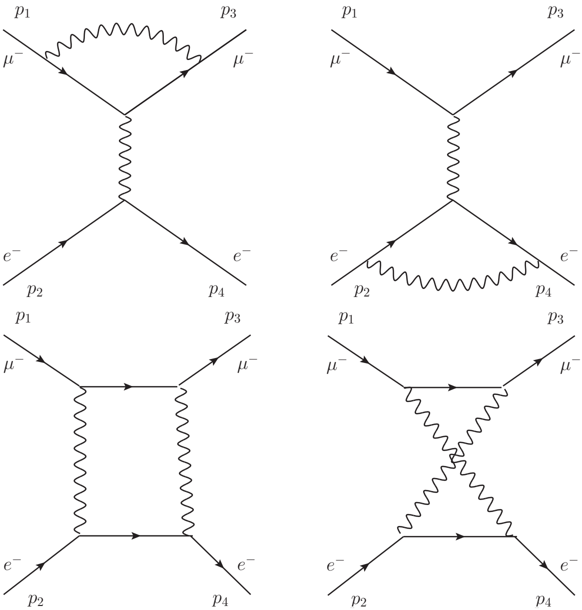


- $\mathcal{A}_{NLO}^{virtual}$



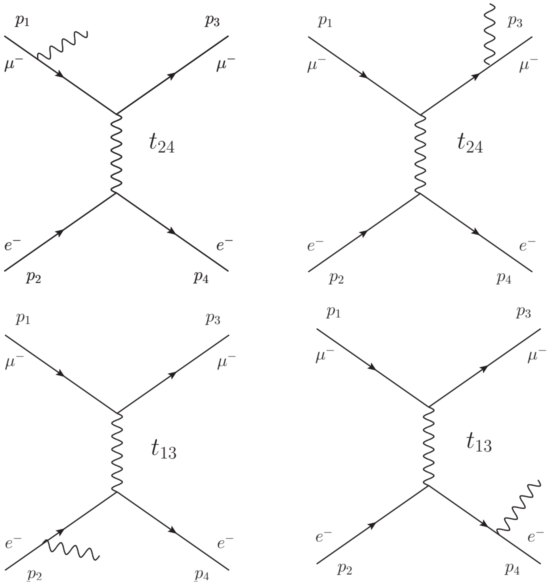


NLO virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)



+ counterterms

NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$



Method and cross-checks

- Calculation performed in the on-shell renormalization scheme
- Full mass dependency kept everywhere, fermions' helicities kept explicit
- Diagrams manipulated with the help of **FORM**, independently by at least two of us [perfect agreement]

  J. Vermaseren, **https://www.nikhef.nl/~form**

- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with **LoopTools** and **Collier** libraries [perfect agreement]

  T. Hahn, **http://www.feynarts.de/looptools**

  A. Denner, S. Dittmaier, L. Hofer, **https://collier.hepforge.org**

- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations [perfect agreement]
- Comparison with past (and present) independent results *[to be done]*

  P. Van Nieuwenhuizen, Nucl. Phys. B **28** (1971) 429

  T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725

  D. Y. Bardin and L. Kalinovskaya, DESY-97-230, **hep-ph/9712310**

  N. Kaiser, J. Phys. G **37** (2010) 115005

  Fael, Passera *et al.*

Further cross-checks and (simple) simulation setup

- Good agreement with NLO Bhabha (only t-channel diagrams) if $m_\mu \to m_e$ as implemented in **BabaYaga@NLO** (within known approximations)
- Everything calculated in the center-of-mass frame, and boosted in the lab if needed
- Implemented in a MC event generator (*temporary* extension of **BabaYaga@NLO**)
- ω_s independence

$$\sigma_{LO} = 245.039062\ \mu\text{b}$$

$$\sigma_{NLO}(\omega_s = 5\cdot 10^{-5} \times \sqrt{s} \equiv \omega_1) \quad = \quad 244.3957 \pm .0005\ \mu\text{b}$$

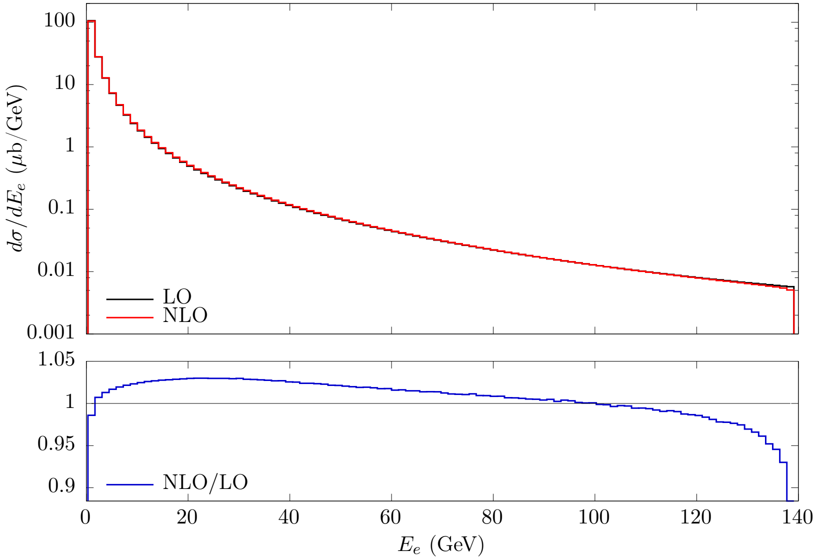
$$\sigma_{NLO}(\omega_s = 1\cdot 10^{-6} \times \sqrt{s} = \omega_1/50) \quad = \quad 244.3948 \pm .0007\ \mu\text{b}$$

⋆ Simulation setup for $\mu^- e^- \to \mu^- e^- \ (\gamma)$

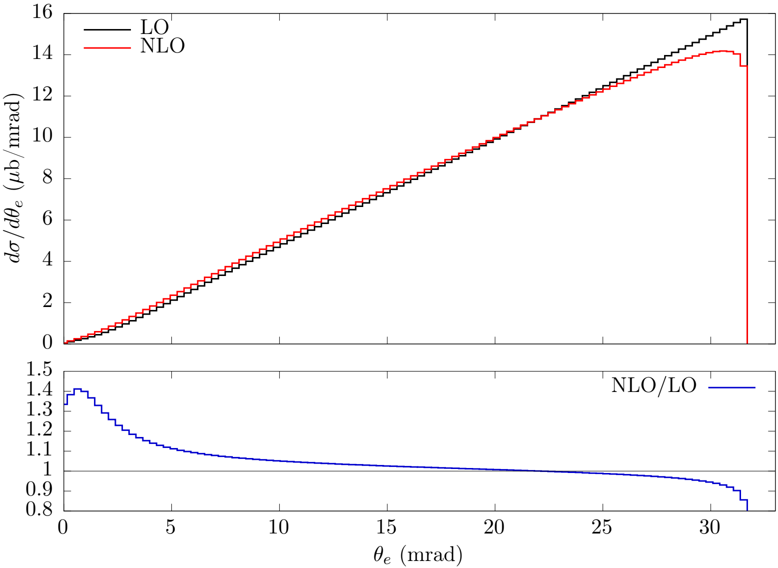
- $E_\mu^{beam} = 150$ GeV $\quad \to \quad \sqrt{s} \simeq 0.4055$ GeV
- $t_{min} = -\lambda(s, m_\mu^2, m_e^2)/s \simeq -0.1429\ \text{GeV}^2$
- $t_{max} \simeq -1.021\cdot 10^{-3}\ \text{GeV}^2$ (i.e. $E_e > 1$ GeV in the lab)

$\mapsto$ **Cuts:** $t_{min} \le t_{24} \le t_{max} \qquad t_{min} \le t_{13} \le t_{max}$

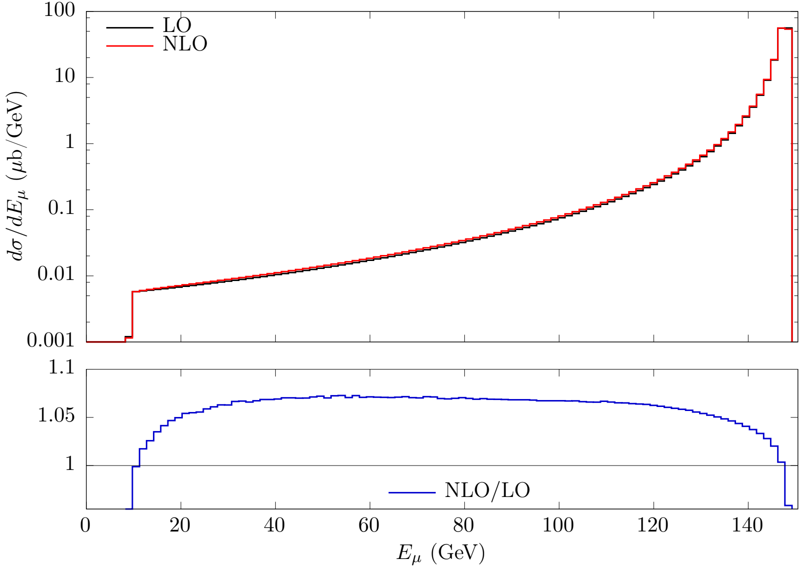
E_e distribution and corrections in the lab



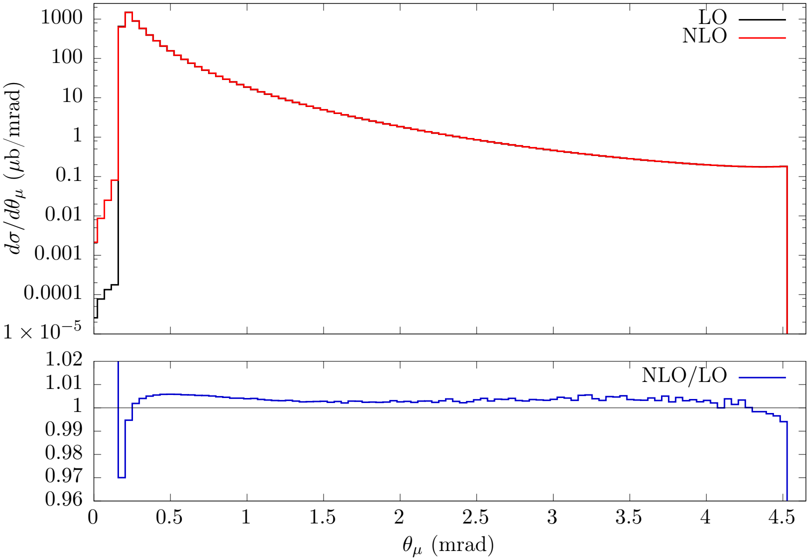
θ_e distribution and corrections in the lab



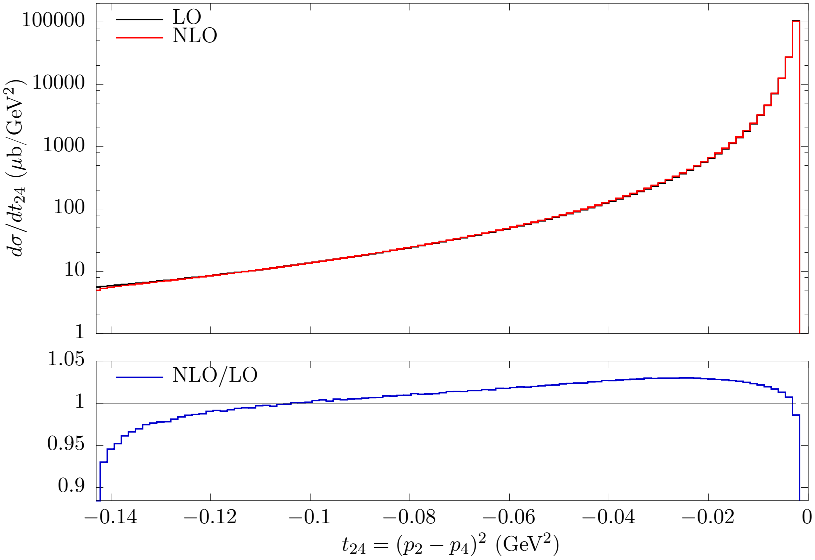
E_μ distribution and corrections in the lab



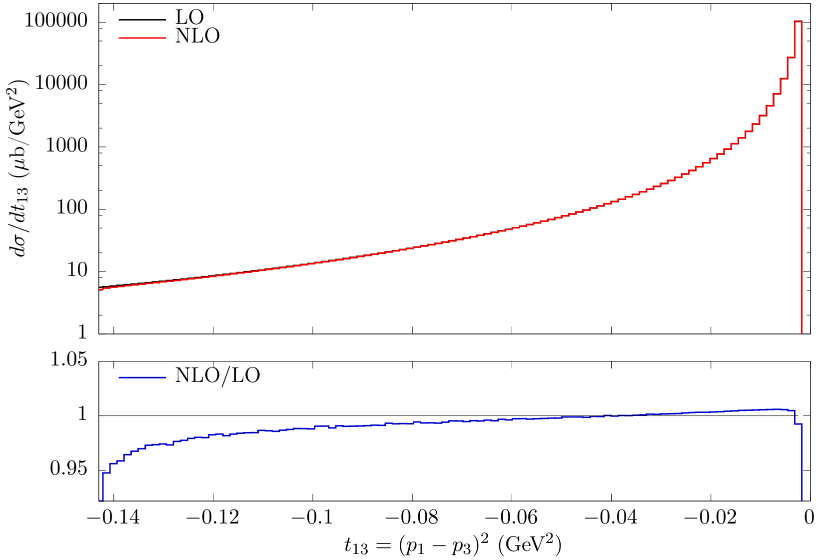
θ_μ distribution and corrections in the lab



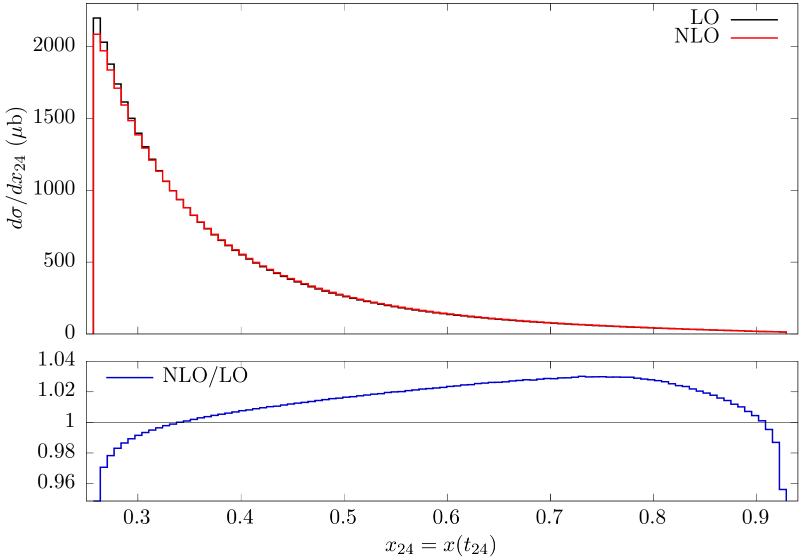
t_{24} (t on the “electron line”) distribution and corrections



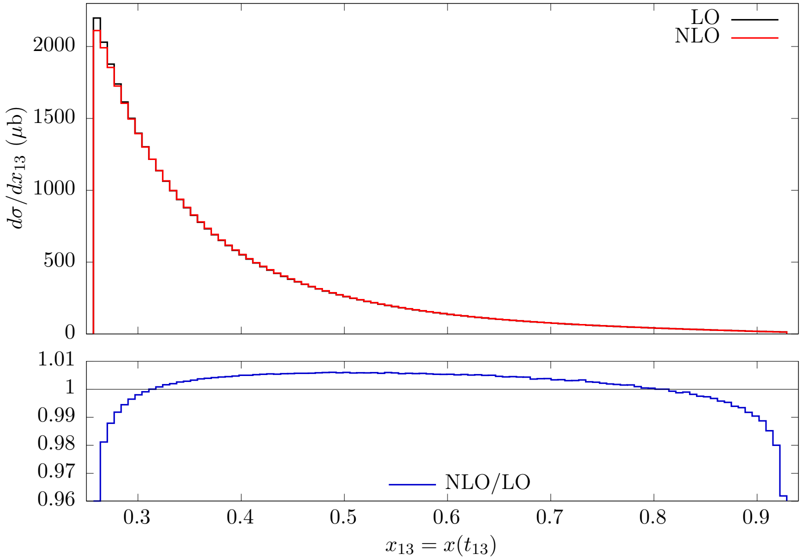
t_{13} (t on the “muon line”) distribution and corrections

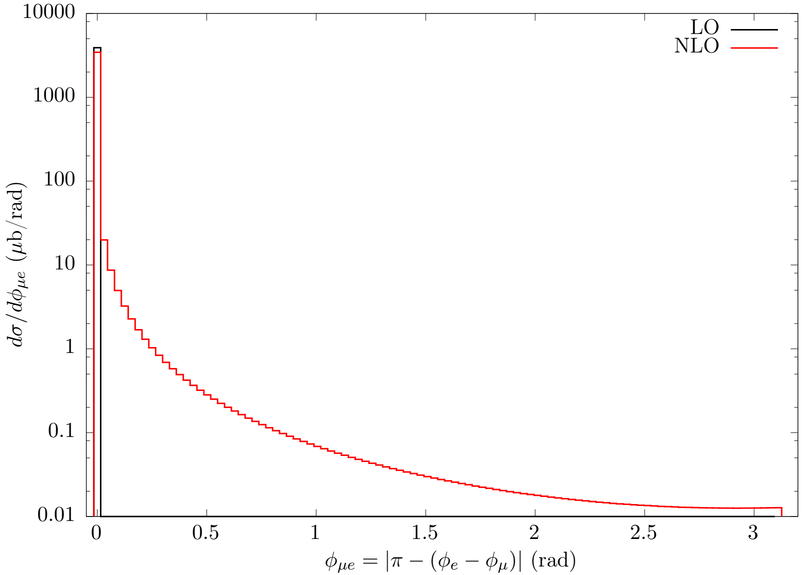


x_{24} distribution and corrections

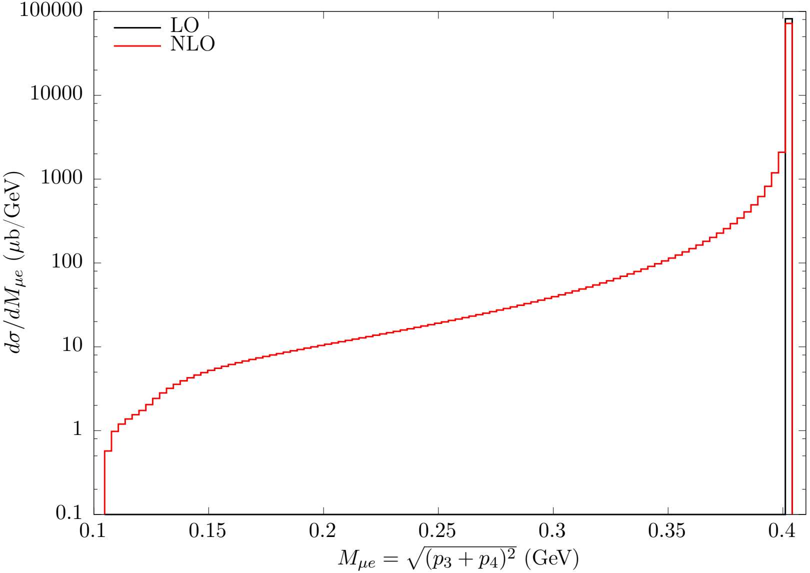


x_{13} distribution and corrections

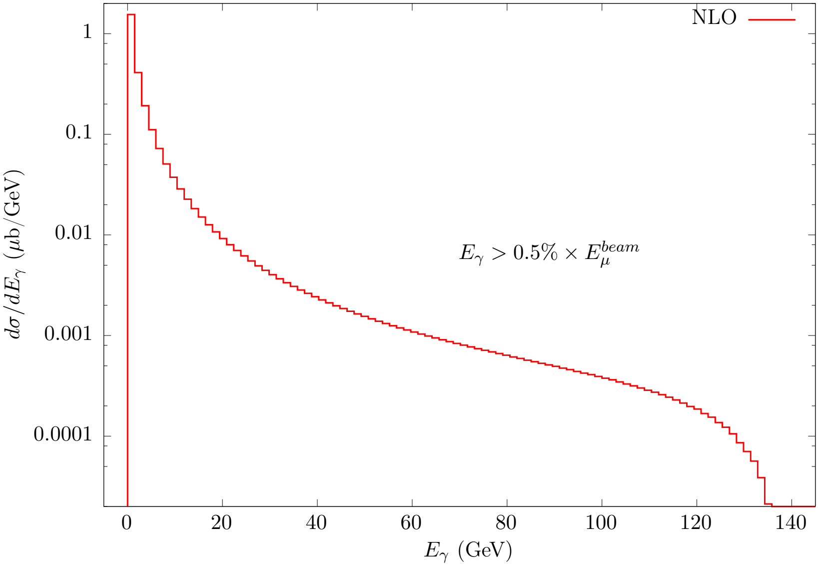



10000
1000
100
10
1
0.1
0.01
$d\sigma/d\phi_{\mu e}$ (μb/rad)
0
0.5
1
1.5
2
2.5
3
$\phi_{\mu e} = |\pi - (\phi_e - \phi_\mu)|$ (rad)
LO
NLO

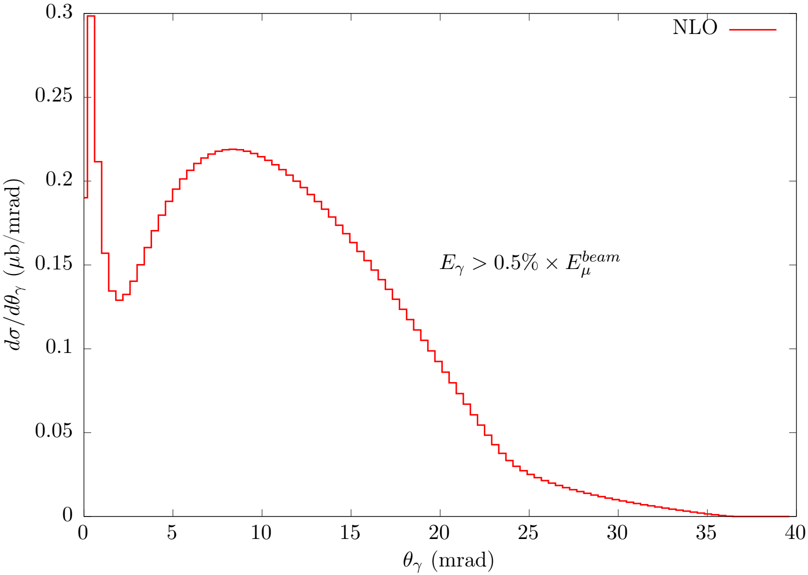
Final state μe invariant mass distribution



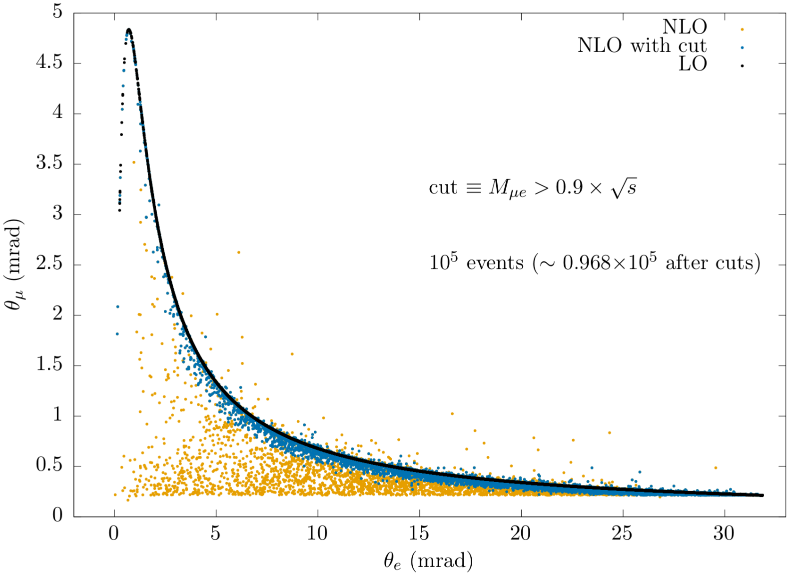
Photon energy distribution in the lab (radiative events)



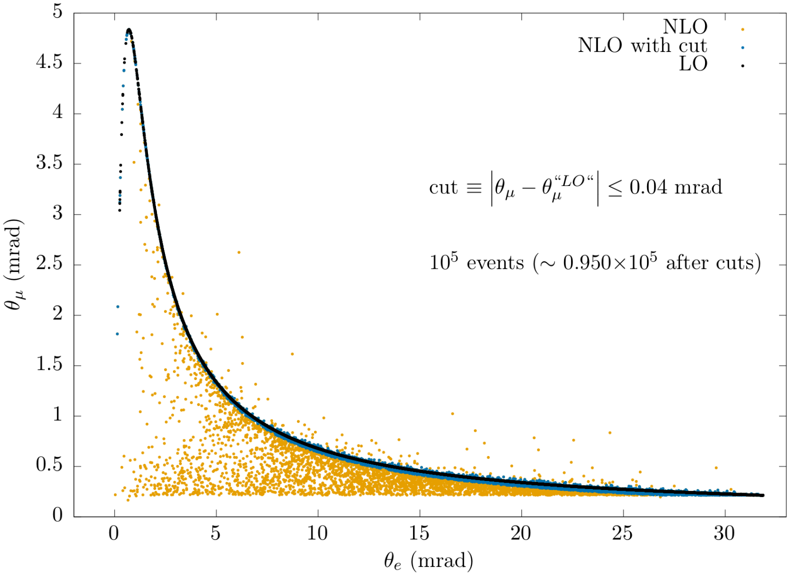
Photon angle distribution in the lab (radiative events)



μ-e angle correlation in the lab



μ-e angle correlation in the lab



Conclusions and Outlook

- (At least) one working QED NLO calculation is available in an event generator for our μe proposal
- The size of QED NLO RCs on the considered observables lies in the 1-5% range *(except in the low θ_e and θ_μ distributions)*, **within the loose cuts applied here**
- The “Bhabha experience” at LEP & flavour factories compels to include also higher-orders (h.o., beyond NLO) RCs to reach high theoretical accuracy
- Exact NNLO corrections are needed to reduce the theoretical uncertainty at the required level
- A QED Parton Shower approach (matched to NLO) could be used to resum h.o. *(multiple-photon emission effects)* preserving fully exclusive generation
  - $\mapsto$ needs to be re-thought for the inclusion of (muon) mass effects
  - $\mapsto$ needs to be extended to be matched to exact NNLO corrections