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Relation between experimental and theoretical distributions in $\mu^- e^-$ scattering

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disclaimer: *as a beginner on this topic, I just did a trivial, naive, hopefully not wrong exercise, counting higher-order contributions to this scattering process*

Accessing α(t) from the muon dσ/dt distribution: LO in QED and SM

- At LO there is only α(0) in the theoretical prediction → inadequate to study α(t)

- The Dyson resummation of the photon propagator yields a subset of O(α) corrections iterated to all orders, which are expressed with an effective running coupling α(t)

$$\frac{d\sigma^{exp}}{dt} = \frac{d\sigma_{LO}}{dt}\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

- Is it possible to preserve the factored dependence on α(t), also upon inclusion of higher-order radiative corrections? What is the residual dependence on *t* and possibly on α(t)?

  f_1 is wrong, f_2 considers only a residual kinematical dep. on t
  f_3 considers the presence of a subleading dep. on α(t)

$$\frac{d\sigma^{exp}}{dt} = f_1(s,m_\mu,m_e)\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

$$\frac{d\sigma^{exp}}{dt} = f_2(s,t,m_\mu,m_e)\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

$$\frac{d\sigma^{exp}}{dt} = f_3(s,t,\alpha(t),m_\mu,m_e)\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

- The LO amplitude includes, in the full SM, also the exchange of a Z boson in the *t* channel
- At $q^2 \ll M_Z^2$ we expect a strong suppression of the squared diagrams with Z exchange
- The γ-Z interference term at small |*t*| receives a milder suppression
- The rescaling factor (α(t)/α(0)) appears now with different powers in the cross section

$$\frac{d\sigma^{exp}}{dt} = \int d\Phi_2 \left\{ |\mathcal{M}_\gamma|^2 \left(\frac{\alpha(t)}{\alpha(0)}\right)^2 + 2\Re e(\mathcal{M}_\gamma \mathcal{M}_Z^\dagger)\left(\frac{\alpha(t)}{\alpha(0)}\right) + |\mathcal{M}_Z|^2 \right\}$$

Accessing $\alpha(t)$ from the muon $d\sigma/dt$ distribution: fixed NLO-QED (1)

- At fixed NLO-QED order we consider the two processes
  $\mu^- e^- \to \mu^- e^-$ including Born and $O(\alpha)$ virtual corrections
  $\mu^- e^- \to \mu^- e^- \gamma$ at tree level

  the pure fixed-order calculation has to be done with $\alpha(0)$ everywhere
  in order to exactly fulfil the IR cancellations between real and virtual corrections

- one can try to promote the LO couplings to running couplings
  rescaling by ($\alpha(t)/\alpha(0)$)^2 (factorizable reducible higher-order terms)
  removing the double counting from the virtual part where the first self-energy is also present

$$\frac{d\sigma^{exp}}{dt} = \left[\frac{d\sigma_{LO}}{dt} + \frac{d\sigma_{\alpha,virt}}{dt} - \frac{d\sigma_{\alpha,ct}^{doublecount}}{dt} + \frac{d\sigma_{\alpha,real}}{dt}\right]\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

- rescaling introduces higher-order real-virtual effects

- subtracting the double counted terms removes the dependence on the light quark contribution

- do Feynman diagrams yield this structure?

Accessing $\alpha(t)$ from the muon $d\sigma/dt$ distribution: fixed NLO-QED (2)

- 1-loop boxes M(box) do not contain “tree-level” photon propagators
  if we consider 2-loop and higher-order diagrams with self-energy insertions along one photon line
  we achieve the Dyson resummation which yields an additional factor $\alpha(k^2)$ in the integrand, with k the loop momentum

  the factored Ansatz $\alpha(t)/\alpha(0)$ M(box) differs (???) w.r.t. integrating the loop with the additional factor $\alpha(k^2)$ in the integrand

  if proven, such terms break the simple factored cross section structure
  → one may try to compute the difference between exact and factored and rearrange the xsec

Accessing $\alpha(t)$ from the muon $d\sigma/dt$ distribution: fixed NNLO-QED

- At fixed NNLO-QED order we consider the three processes

  $\mu^- e^- \to \mu^- e^-$ including Born, $O(\alpha)$ and $O(\alpha^2)$ virtual corrections
  $\mu^- e^- \to \mu^- e^- \gamma$ including Born and $O(\alpha)$ virtual corrections
  $\mu^- e^- \to \mu^- e^- \gamma\gamma$ at tree level

  all computed with $\alpha(0)$ for the coupling

- Rescaling by $(\alpha(t)/\alpha(0))^2$ is possible, provided that again double counting is avoided at 1- and 2-loop level

- Rescaling with $\alpha(q^2)$ evolving at 2-loop level should reabsorb in the overall factor (to all orders) the contribution of the 2-loop irreducible self-energy insertions

- The criticism about the contributions due to box diagrams should apply also here

Accessing α(t) from the muon dσ/dt distribution: NLO-QED + QED-PS

- At NLO-QED + QED-PS the matching *à la* BhabhaYaga, Horace, … yields factorizable h.o. terms by iteration of the QED-PS with the virtual part of the NLO calculation

$$d\sigma^{\infty}_{matched} =$$
$$\Pi_S(Q^2) F_{SV} \sum_{n=0}^{\infty} d\hat{\sigma}_0 \frac{1}{n!} \prod_{i=0}^{n} \left(\frac{\alpha}{2\pi} P(x_i)\, I(k_i)\, dx_i\, d\cos\theta_i\, F_{H,i} \right)$$

- The sum runs over all final state real photon multiplicities
- In the Sudakov form factor $\Pi_S(Q^2)$ we include real and virtual unresolved photons (IR limit)
- In the F_{SV} we include IR-finite virtual corrections, also those responsible for the redefinition of the LO couplings
- A rescaling by $(\alpha(t)/\alpha(0))^2$ in F_{SV} is possible, same discussion as at fixed order NLO-QED higher-order real-virtual effects, to all orders, are introduced by the factored structure

Which is the “true” value of *t*, in presence of real radiation?

- In a 2→2 scattering kinematics is clearly specified

- If there were only FSR photons, they could be recombined with final state leptons yielding eventually an effective 2→2 kinematics
- If there were only ISR photons, they would modify the $\mu^- e^-$ invariant mass and transverse momentum Would in this case the Collins-Soper frame be a solution to identify the “true” *t* value?
- At NLO (and beyond) the assignment of a photon to initial or final state is not possible, or only with approximations

→ a unique unambiguous definition of *t* is not possible

Conclusions

- The attempt to extract a common factor ($\alpha(t)/\alpha(0)$)^2 in front of the best available cross section is very appealing, because it would simplify the data-theory comparison
  but

- The intrinsic impossibility to define a value of the variable *t* beyond 2→2 scattering is a very basic QM problem
  → a factored formulation loses its appeal, because it becomes "ambiguous"

- The best available improvements in the existing codes
  +
  a template fit procedure (cfr. C. Carloni talk and comments)
  are a better defined way to acquire sensitivity to $\Delta\alpha_{had}$