γ-RAY TESTS OF LORENTZ INVARIANCE AND QUANTUM GRAVITY MODELS

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Lorentz invariance violation (LIV)?

Suggestions for Lorentz invariance violation come from:

- need to cut off UV divergences of QFT & BH entropy
- tentative calculations in various QG scenarios, e.g.
    - semiclassical spin-network calculations in Loop QG
    - string theory tensor VEVs
    - non-commutative geometry
    - some brane-world backgrounds

Theoretical Frameworks for Lorentz Invariance Violation

- Effective Field Theory (EFT)
- “Deformed Relativity” (DSR)
- Stochastic spacetime “foam”
- Loop Quantum Gravity (LQG)
- String Inspired Models (D-branes)

Why Use High Energy Astrophysical Observations?

- Lorentz invariance implies scale-free spacetime.
- The group of Lorentz transformations is unbounded.
- Very large boosts probe physics at ultra-short distances, λ.
- To probe physics at these distances we need to go to ultrahigh energies $E = 1/\lambda$.
- Cosmic γ-rays and cosmic rays provide the highest observable energies in the universe.
- Planck scale physics such as quantum gravity may lead to the breaking or deformation of Lorentz invariance.

THE "ALMOST STANDARD" RENORMALIZABLE MODEL

WITH LORENTZ INVARIANCE VIOLATION

General Free Particle Lagrangian with dimension four operators:

$$\mathcal{L} \;=\; \partial_\mu \Psi^* \mathbf{Z} \partial^\mu \Psi \;-\; \Psi^* \mathbf{M^2} \Psi$$

Add very small Lorentz invariance violating term:

$$\mathcal{L} \;\Rightarrow\; \mathcal{L} \;+\; \partial_i \Psi^* \epsilon \partial^i \Psi$$

where ϵ is dimensionless and $[\epsilon, \mathbf{M}] = 0$.

This gives a new propagator

$$-iD^{-1} \;=\; (p_{(4)}^2 \;-\; m^2) \;+\; \epsilon p^2.$$

so that

$$p_{(4)}^2 \;=\; E^2 \;-\; p^2 \;\Rightarrow\; m^2 \;+\; \epsilon p^2.$$

which can be rewritten in the "conventional" form:

$$E^2 \;=\; p^2 \;+\; m^2$$

where

$$m \;\Rightarrow\; \frac{m}{(1+\epsilon)^2} \;\simeq\; m(1-2\epsilon),$$

and

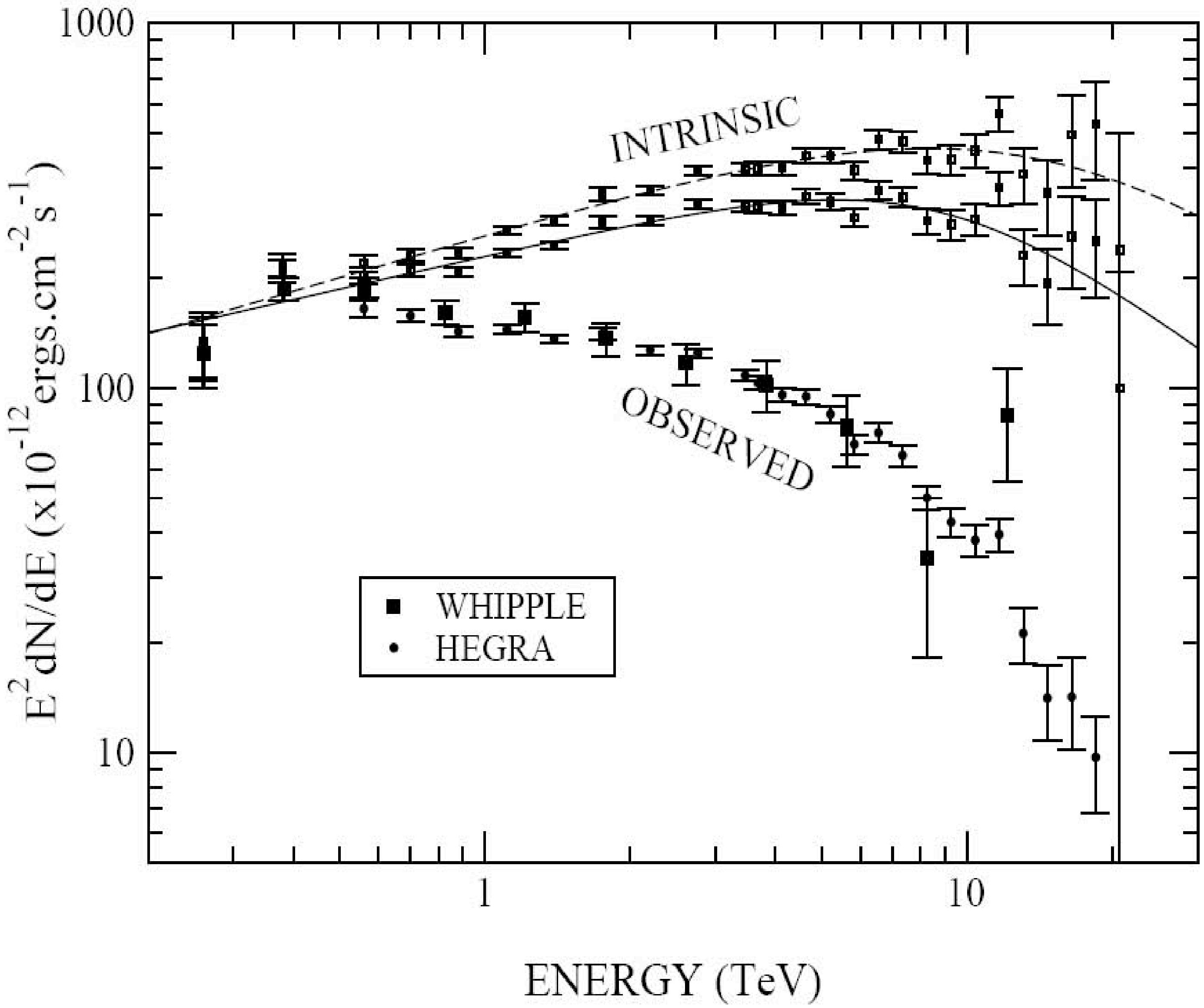
$$c^2 \;\Rightarrow\; 1+\epsilon$$

Thus, the maximum photon velocity ($c = 1$) has changed by $\epsilon/2$.

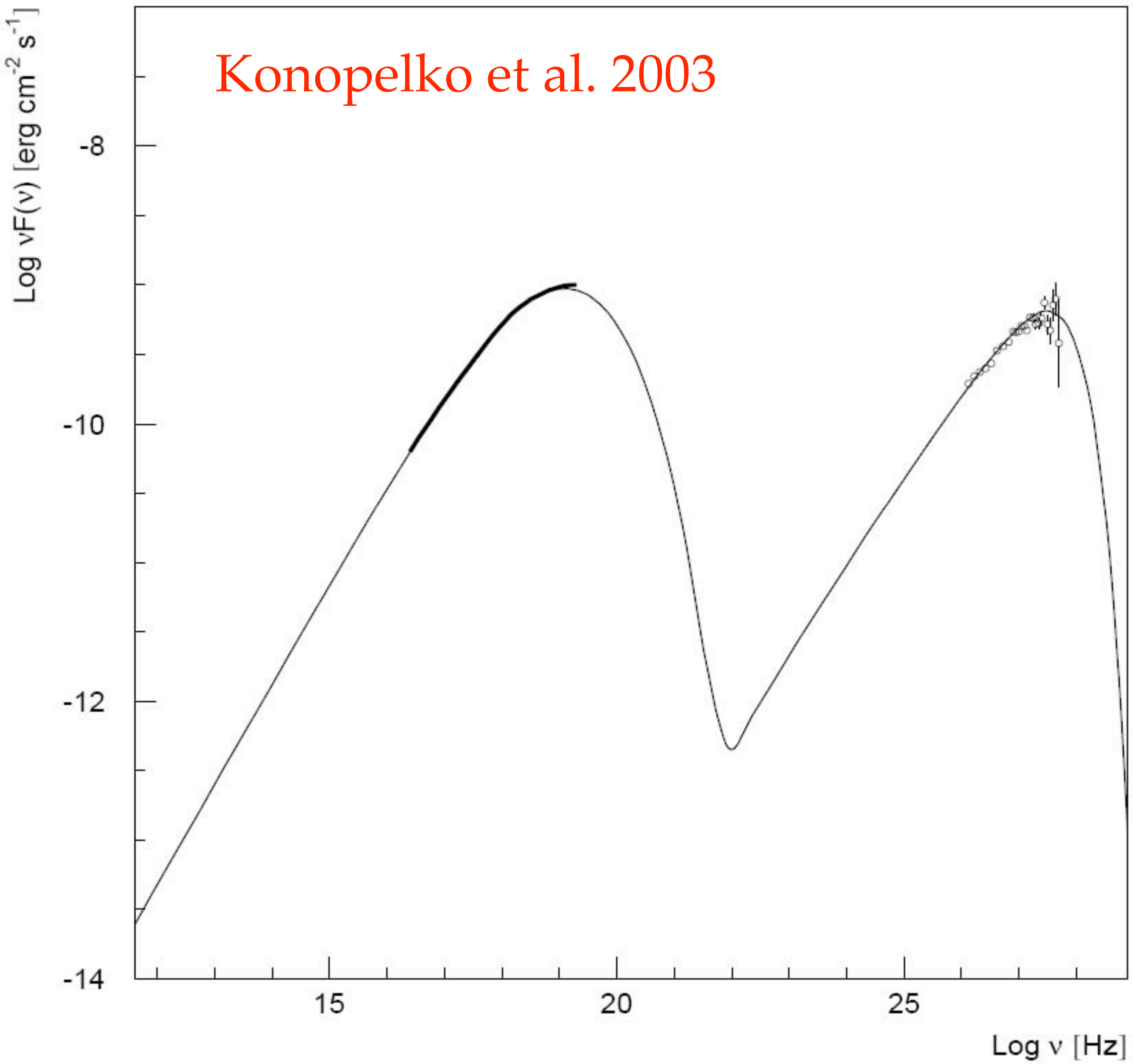
Other effects become important when $p^2 \;\simeq\; m^2/\epsilon$.

Implied Constraints on Lorentz Invariance Violation (LIV) from Very High Energy γ-ray Observations of Nearby Blazars

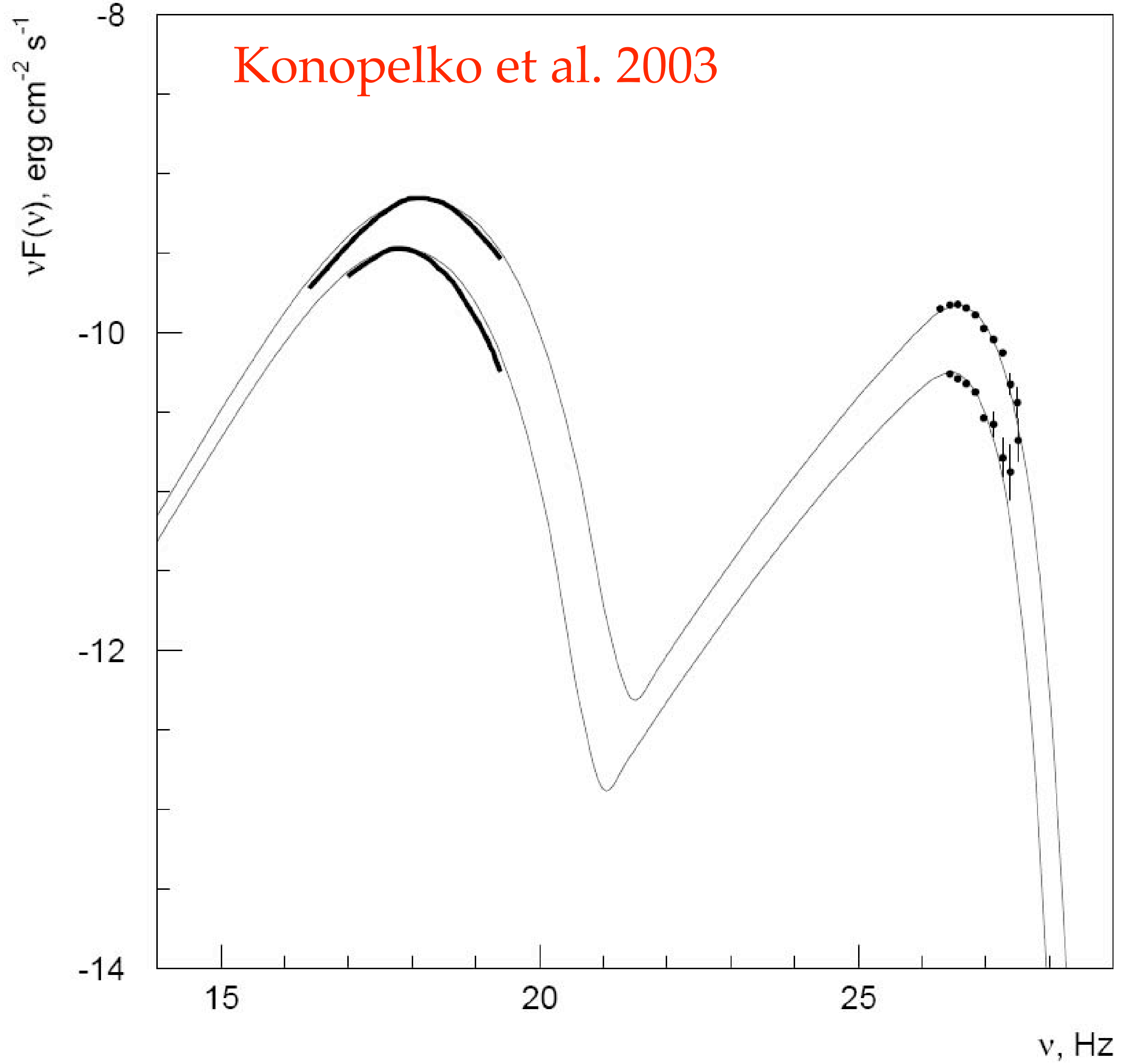
Flaring Spectrum of Mrk 501



Deabsorbed Mrk 501 Spectrum and Theoretical SSC Model Predictions



Deabsorbed Mrk 421 Spectra and Theoretical SSC Model Predictions



γ-Ray Astrophysics Limit on LIV from Blazar Absorption Features

Let us characterize Lorentz invariance violation by the parameter $\delta = \varepsilon/2$ such that

$$c_e \equiv c_\gamma (1 + \delta)$$

(Coleman & Glashow 1999). If $\delta > 0$, the γ-ray photon propagator in the case of pair production

$$\gamma + \gamma \rightarrow e^+ + e^-$$

is changed by the quantity $\varepsilon p_\gamma^2 = -2E_\gamma^2 \delta$

And the threshold energy condition is given by

$$2\varepsilon E_\gamma^2 (1 - \cos\theta) > 4m_e^2 + 2E_\gamma^2 \delta.$$

γ-Ray Limit on LIV from Blazar Absorption from Coleman-Glashow Modified Threshold

The pair production threshold is raised significantly if

$$\delta > \frac{2m_e^2}{E_\gamma^2}.$$

The existence of electron-positron pair production for γ-ray energies up to ~20 TeV in the spectrum of Mkn 501 therefore gives an upper limit on δ at this energy scale of

$$\delta < 1.3 \times 10^{-15}$$

(Stecker & Glashow 2001).

Limit on the Quantum Gravity Scale

For pair production, $\gamma + \gamma \rightarrow e^{+} + e^{-}$ the electron (& positron) energy $E_e \sim E_\gamma / 2$. For a third order QG term in the dispersion relation, we find

$$\delta = \frac{E_\gamma}{2M_{QG}} - \frac{2m^2}{E_\gamma^2},$$

Threshold energy from Stecker and Glashow (2001)

$$\frac{E_\gamma^2 \delta}{2} \leq \frac{m^2}{E_\gamma}$$

reduces to

$$M_{QG} \geq \frac{E_\gamma^3}{8m^2}$$

Limit on the Quantum Gravity Scale

(Stecker 2003):

Since pair production occurs for energies of at least E_γ = 20 TeV, we then find the numerical constraint on the quantum gravity scale

$$M_{QG} > 0.3\ M_{Planck}$$

Arguing against some TeV scale quantum gravity models involving extra dimensions!

Fermi Launch: June 11, 2008

The *Fermi* Mission

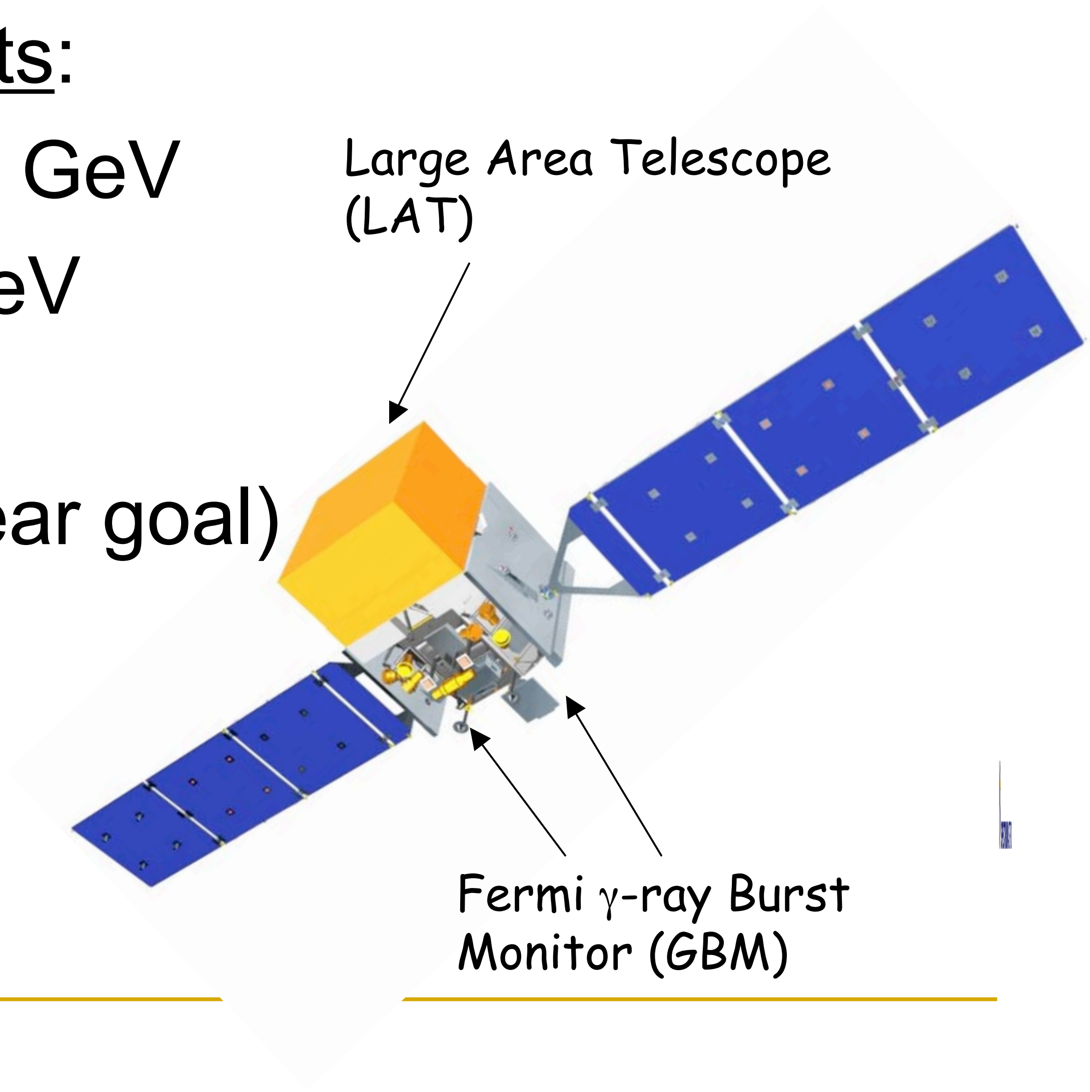
Two *Fermi* instruments:

LAT: 20 MeV – >300 GeV

GBM: 10 keV – 25 MeV

Launch: June, 2008

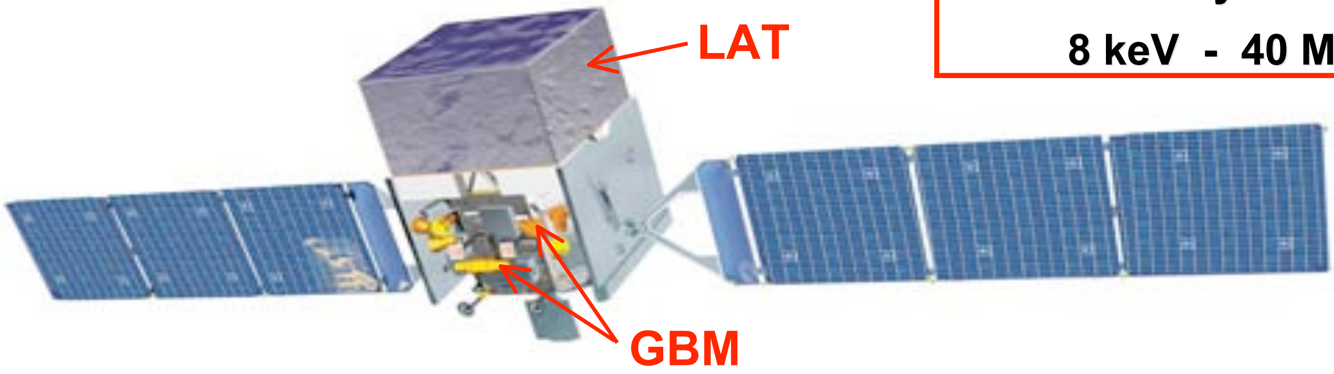
5-year mission (10-year goal)



Fermi LAT:

Two *Fermi* instruments:
• Large Area Telescope (LAT)
20 MeV - >300 GeV

• Gamma-ray Burst Monitor (GBM)
8 keV - 40 MeV



The Fermi-LAT consists of three subsystems:

•An anti coincidence detector consisting of segmented plastic scintillators for cosmic-ray background rejection.

•A tracker consisting of silicon strip detectors and tungsten foil converters for determining the identification and direction of γ-rays.

•An imaging calorimeter consisting of cesium iodide scintillators.

Testing Lorentz Invariance with *Fermi*

Some classes of quantum gravity models postulate or imply a photon velocity dispersion relation with a pertubative term which may be linear with energy (e.g. , Amelino-Camelia et al. 1998; Ellis et al. 2008).

$$v_\gamma = c\,[1 - \xi\,(E_\gamma/M_{QG})]$$

Using Fermi data for distant γ-ray bursts the difference in arrival times of γ-rays of different energies could be > 100 ms.

Time of flight constraint

arrival time delay

Constraints from blazar flares and GRB's.

$\Delta t = 20 \text{ ms } (M_{Pl}/M_{QG})\, d_{Gpc}\, \Delta E_{GeV}$

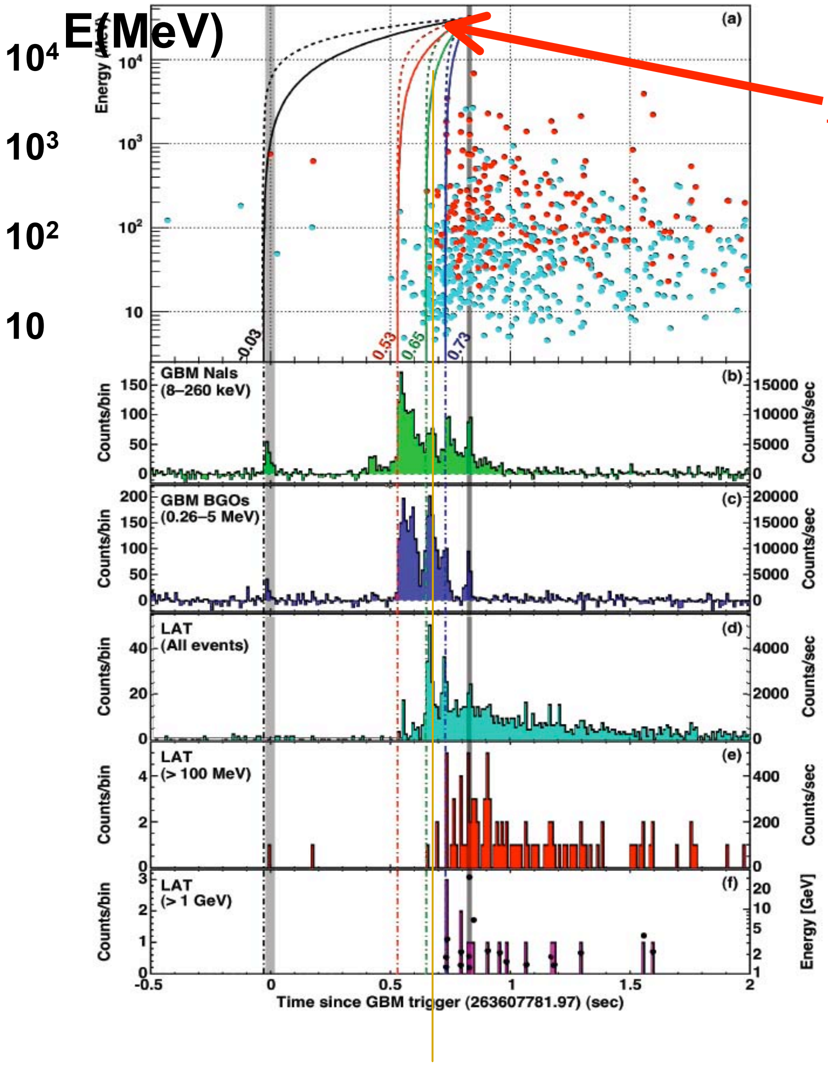
Longer distance and higher energy help, also intrinsically short bursts.

Energy Dependent Time Delay for ΛCDM Cosmology

$$\Delta t = \frac{\Delta E}{M_{QG} H_0} \int_0^z \frac{dz(1+z)}{\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}}.$$

require $M_{QG} \leq M_{Pl}$.

31 GeV photon from GRB 090510



31 GeV photon : 0.83 s after the trigger

This is the highest energy observed from short GRB

Thus, this photon can be used to constrain both the bulk Lorentz factor of the relativistic jet and Lorentz Invariance Violation (LIV)

Fermi Limits on LIV from GRB090510

- Uses 1 σ lower limit on 31 GeV γ of 28.0 GeV
- Uses 1 σ lower limit on redshift of 0.900

	(ms)	(ms)	or limit on Δt or $\lvert\Delta t/\Delta E\rvert$	(MeV)	for s_n*	confidence*	$M_{QG,1}/M_{Planck}$
(a)□	– 30	< 859	start of any < 1 MeV emission	0.1	1	very high	> 1.19
(b)□	530	< 299	start of main < 1 MeV emission	0.1	1	high	> 3.42
(c)□	630	< 199	start of main > 0.1 GeV emission	100	1	high	> 5.12
(d)□	730	< 99	start of > 1 GeV emission	1000	1	medium	> 10.0
(e)*	—	< 10	association with < 1 MeV spike	0.1	± 1	low	> 102
(f)*	—	< 19	If 0.75 GeV‡ γ-ray from 1st spike	0.1	– 1	low	> 1.33
(g)♠	$\lvert\Delta t/\Delta E\rvert$ < 30 ms/GeV		lag analysis of all LAT photons	—	± 1	very high	> 1.22

New Results from *Fermi* Observations of GRBs (*Fermi* Collaboration 2009)

- GRB 080916C:

$M_{QG} > 0.1\ M_{Planck}$ (where $M_{Planck} \sim 10^{19}$ GeV)

- GRB 090510 (short-hard burst):

$M_{QG} > 1.2\ M_{Planck}$

(probably larger, depending on the assumed start of the burst)

(Fermi collaboration, arXiv:0908.1832)

EFT of LV suppressed by E/M_P

(Myers & Pospelov, 2003)

Lorentz-violating dimension 5 terms that are gauge & rotation invariant, not reducible to lower order terms, and contribute to dispersion at order E/M_P:

$$\frac{\xi}{M} u^m F_{ma} (u \cdot \partial)(u_n \tilde{F}^{na}) \Longrightarrow \omega^2_{R,L} = k^2 \pm \frac{2\xi}{M} k^3$$

photon helicities

opposite

$$\frac{1}{M} u^m \bar{\psi} \gamma_m (\eta_1 + \eta_2 \gamma_5)(u \cdot D)^2 \psi \Longrightarrow E^2_{\pm} = p^2 + m^2 + \frac{2(\eta_1 \pm \eta_2)}{M} p^3$$

$(E \gg m)$

Electron helicities independent

All violate CPT

Vacuum birefringence constraint

$$\Delta\theta = \xi \, (E_2^2 - E_1^2) \, d/2M_P$$

relative polarization rotation

Polarized UV light from a galaxy at 300 Mpc yields

$$|\, \xi \,| \leq 2 \times 10^{-4}$$

Gleiser & Kozameh (2001)

Polarized soft γ-ray emission from the region of the Crab Nebula pulsar yields

$$|\, \xi \,| \; < \; 10^{-9}$$

Maccione et al. (2008)

Photomeson Production by Ultrahigh Energy Cosmic Rays off the Cosmic Microwave Background Radiation

$$\gamma_{CMB} + p \rightarrow \Delta \rightarrow N + \pi$$

Produces "GZK Cutoff" Effect

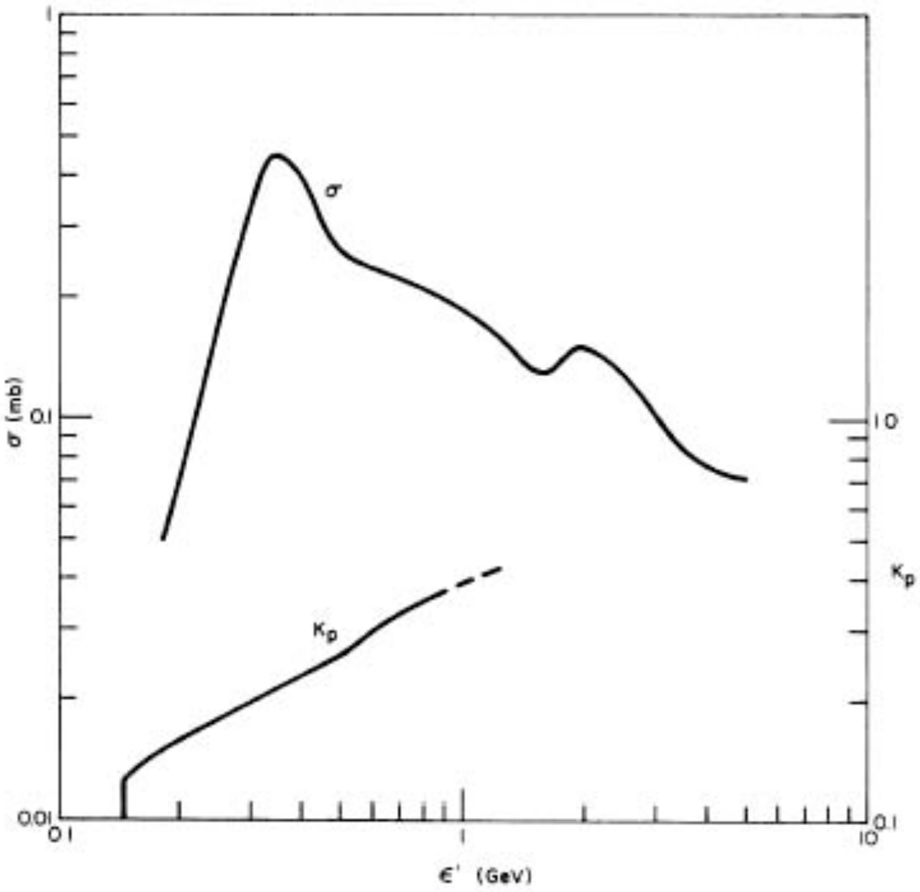


FIG. 1. Total photomeson production cross section and inelasticity as a function of gamma-ray energy in the proton rest system.

UHECR Attenuation (Stecker 1968)

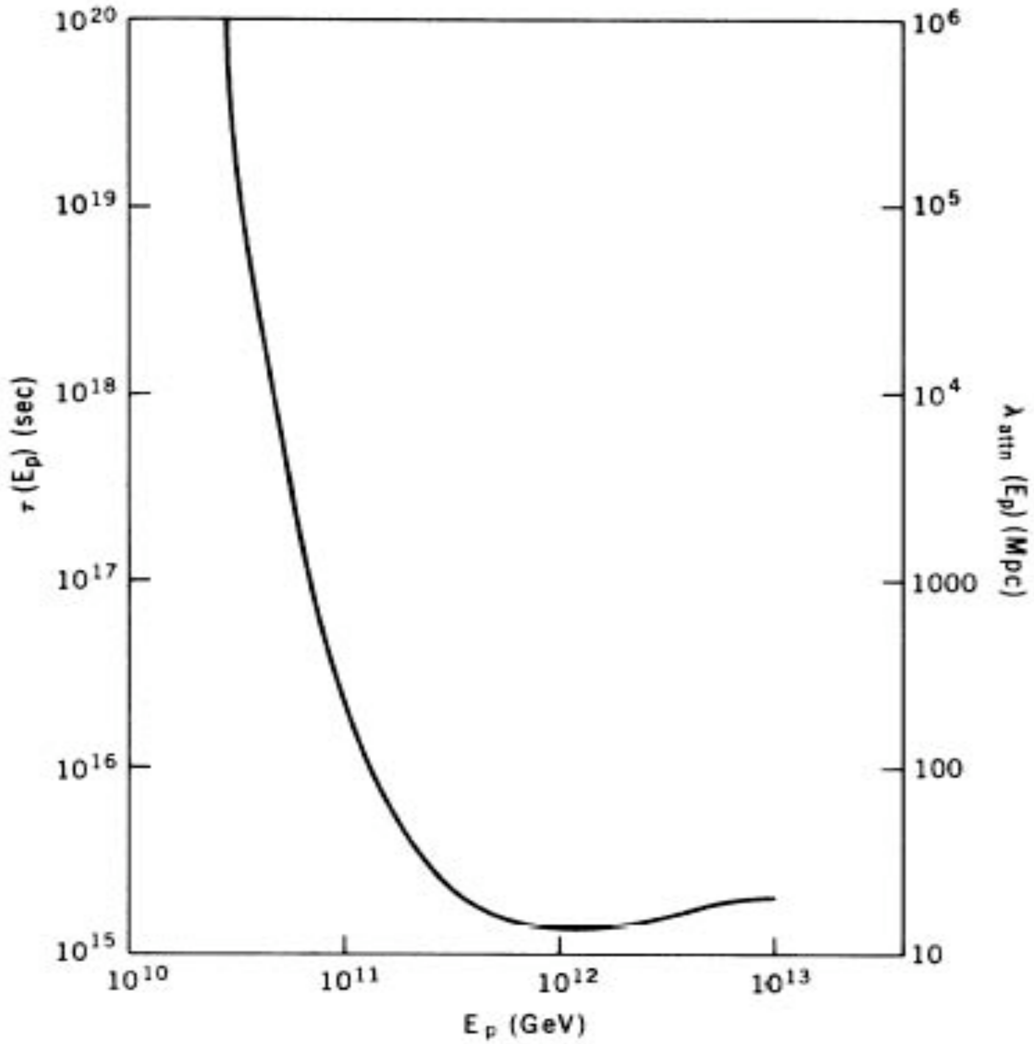


FIG. 2. Characteristic lifetime and attenuation mean free path for high-energy protons as a function of energy.

But, Cosmic Photomeson Interactions can be Modified by the Effects of LIV

Coleman-Glashow Formalism Revisited:

$$\mathcal{L} \to \mathcal{L} + \partial_i \Psi \epsilon \partial^i \Psi$$

The Lorentz violating perturbative term shifts the poles of the propagators, resulting in free particle dispersion relations of the form

$$E^2 = \vec{p}^{\,2} + m^2 + \epsilon \vec{p}^{\,2}. \quad (3)$$

These can be put in the standard form for the dispersion relations

$$E^2 = \vec{p}^{\,2} c_{MAV}^2 + m^2 c_{MAV}^4, \quad (4)$$

by shifting the renormalized mass by the small amount $m \to m/(1+\epsilon)$ and shifting the velocity from c (=1) by the amount $c_{MAV} = \sqrt{(1+\epsilon)} \simeq 1 + \epsilon/2$.

Since the group velocity is given by

$$\frac{\partial E}{\partial |\vec{p}|} = \frac{|\vec{p}|}{\sqrt{|\vec{p}|^2 + m^2 c_{MAV}^2}} c_{MAV} \quad \to \quad c_{MAV} \quad \text{as} \quad |\vec{p}| \quad \to \quad \infty, \quad (5)$$

Coleman and Glashow thus identify c_{MAV} as the maximum attainable velocity of the free particle.

Using this formalism, it becomes apparent that, in principle, different particles can have different maximum attainable velocities (MAVs) resulting from the individually distinguishable eigenstates of the ϵ matrix. These various MAVs can all be different from c as well as different from each other. Hereafter, we denote the MAV of a particle of type i by c_i and the difference

$$c_i - c_j = \frac{\epsilon_i - \epsilon_j}{2} \equiv \delta_{ij} \quad (6)$$

LIV Modified Interaction Threshold

Using the normal Lorentz invariant kinematics, the energy threshold for photomeson interactions of UHECR protons of initial laboratory energy E with low energy photons of the CBR with laboratory energy ω is determined by the relativistic invariance of the square of the total four-momentum of the proton-photon system. This relation, together with the threshold inelasticity relation $E_\pi = [m/(M+m)]E$ for single pion production, yields the threshold conditions for head on collisions in the laboratory frame. In terms of the pion energy for single pion production at threshold

$$4\omega E_\pi = \frac{m^2(2M+m)}{M+m}, \tag{8}$$

where M is the rest mass of the proton and m is the rest mass of the pion [4].

If LI is broken so that $c_\pi > c_p$, it follows from equation (3) that the threshold energy for photomeson production is altered because the square of the four-momentum is shifted from its LI form so that the threshold condition becomes

$$4\omega E_\pi = \frac{m^2(2M+m)}{M+m} + 2\delta_{\pi p}E_\pi^2 \tag{9}$$

Equation (9) is a quadratic equation with real roots only under the condition

$$\delta_{\pi p} \le \frac{2\omega^2(M+m)}{m^2(2M+m)} \simeq \omega^2/m^2. \tag{10}$$

Defining $\omega_0 \equiv kT = 2.35 \times 10^{-4}$ eV, equation (10) can be rewritten

$$\delta_{\pi p} \le 3.23 \times 10^{-24}(\omega/\omega_0)^2. \tag{11}$$

Modifying Photomeson Interactions with LIV

- With LIV, different particles, *i*, can have different maximum attainable velocities c_i. *(Coleman and Glashow 1999)*

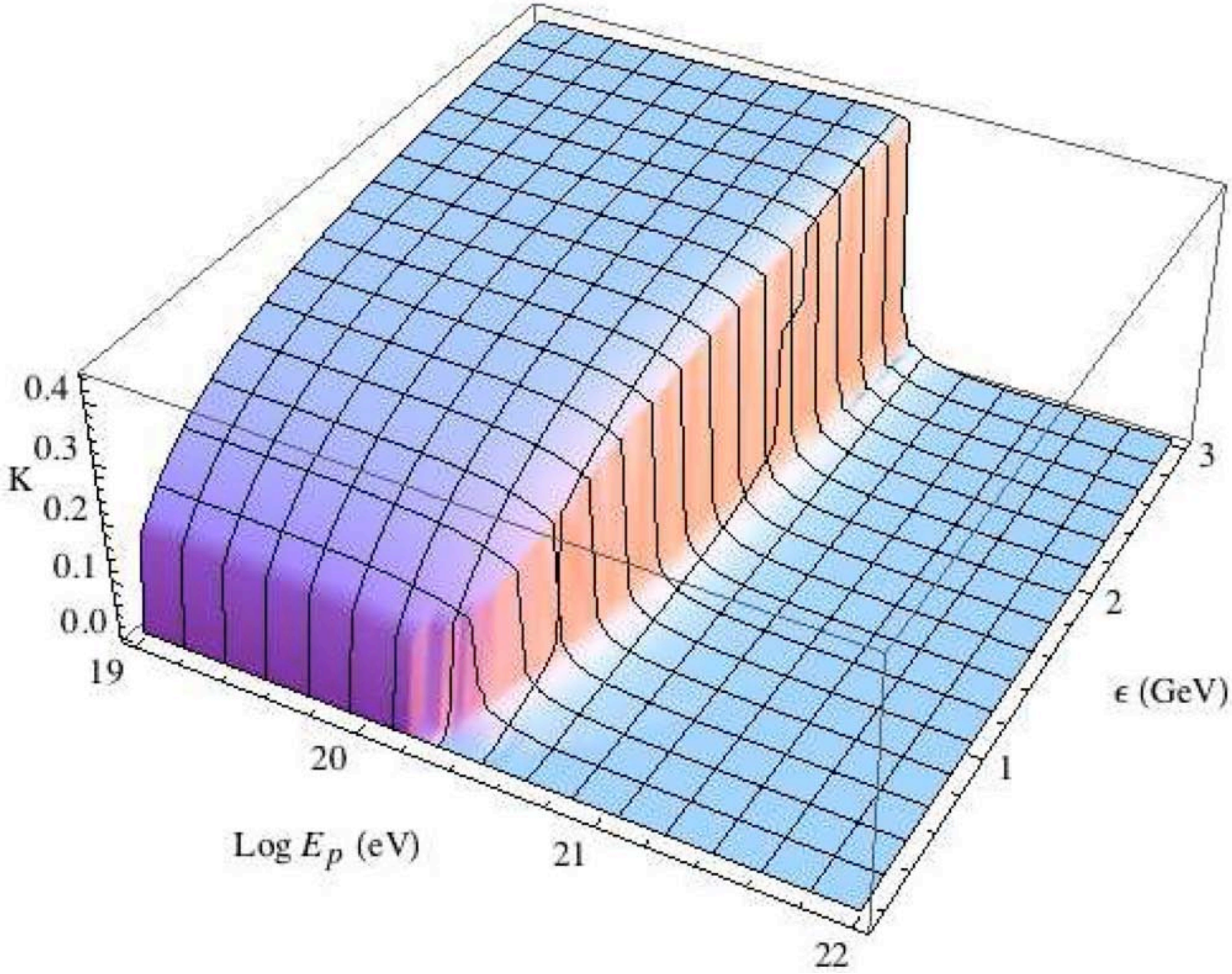
- Photomeson production interactions of ultrahigh energy cosmic rays are disallowed if

$$\delta_{\pi p} = c_\pi - c_p > 3.23 \times 10^{-24} (\omega / T_{CBR})^2$$

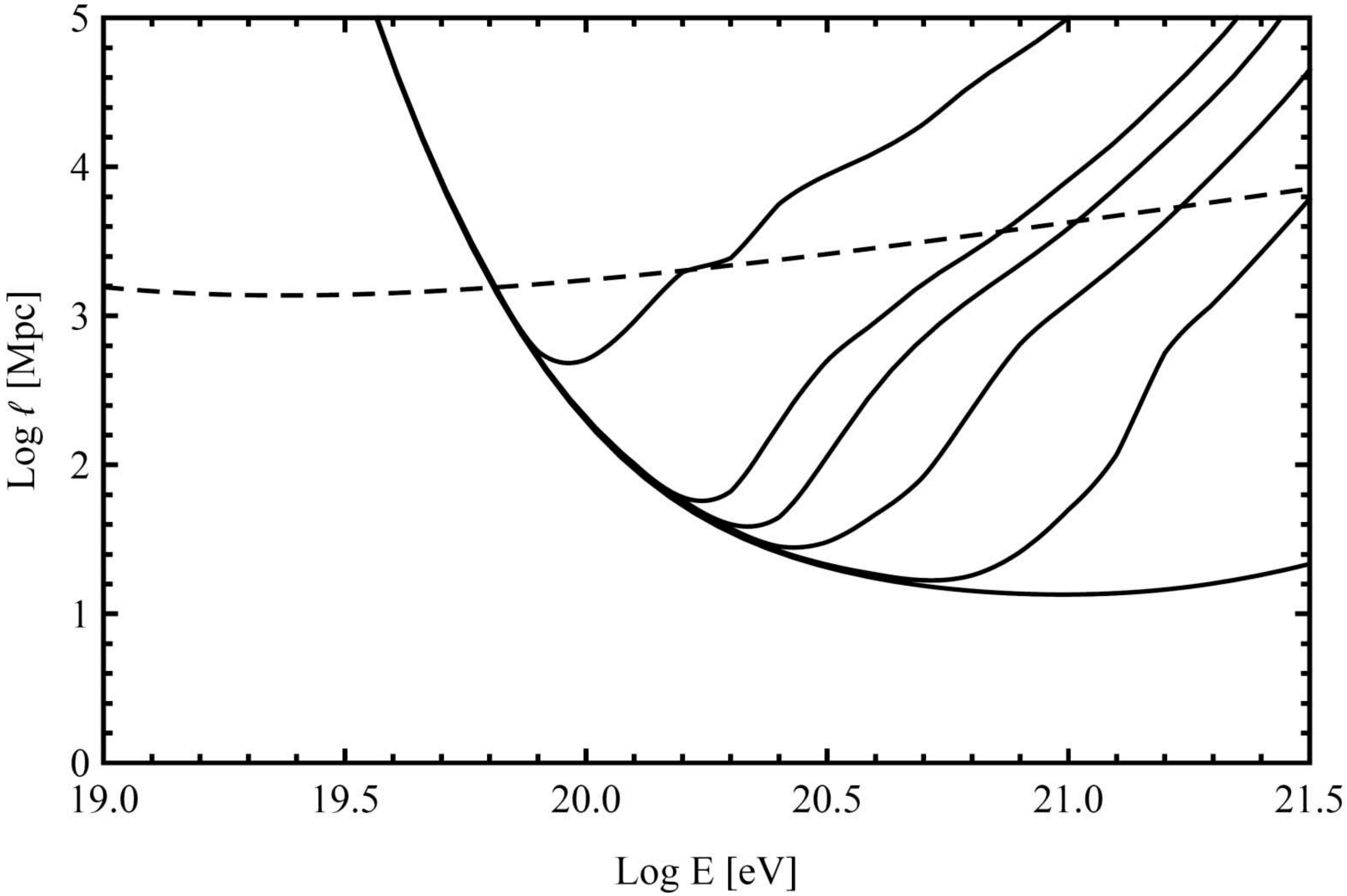
- The higher the value of δ, the higher the photon energy ω required for the interactions to occur.

Since $s \sim \omega E_p$, and there is a peak in the photomeson cross section at a fixed value of s corresponding to the Δ-resonance energy, interactions occur for lower values of E_p near the GZK "cutoff" energy but are suppressed at higher values of E_p

LIV Modified Proton Inelasticity for $\delta = c_{\pi} - c_{p} = 3 \times 10^{-23}$



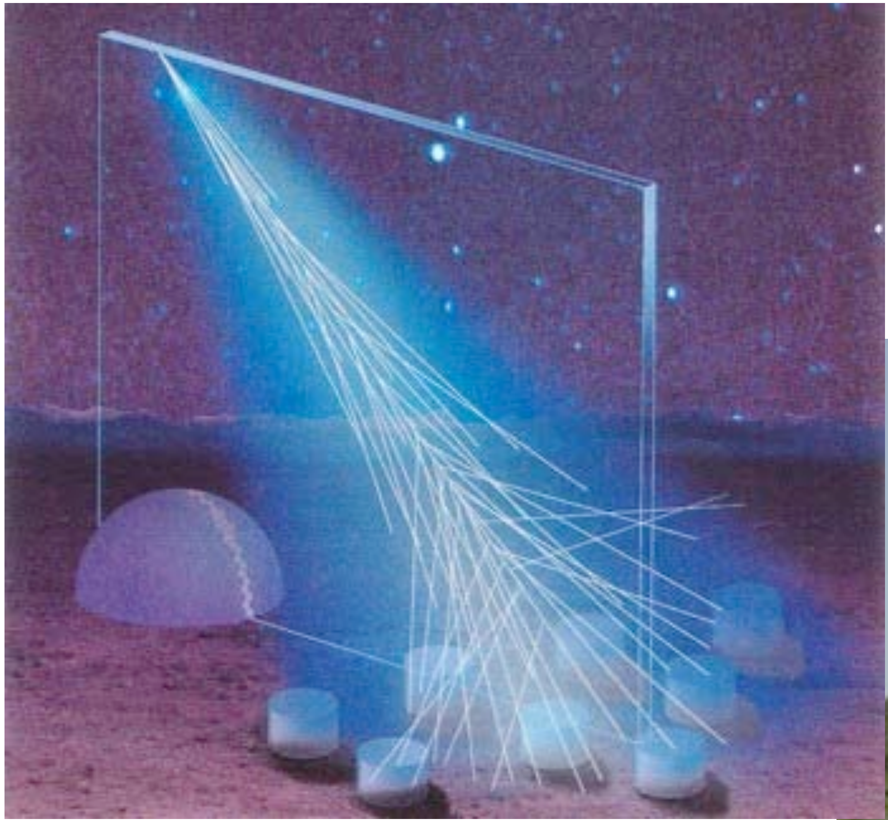
LIV Modified Proton Attenuation Lengths



IMPORTANT RESULT:

You can have (and indeed expect to have) *both* GZK suppression just above ~70 EeV *and* LIV that can be manifested as a "recovery" in the spectrum at higher energies caused by the suppression of the GZK (photomeson interaction) suppression!

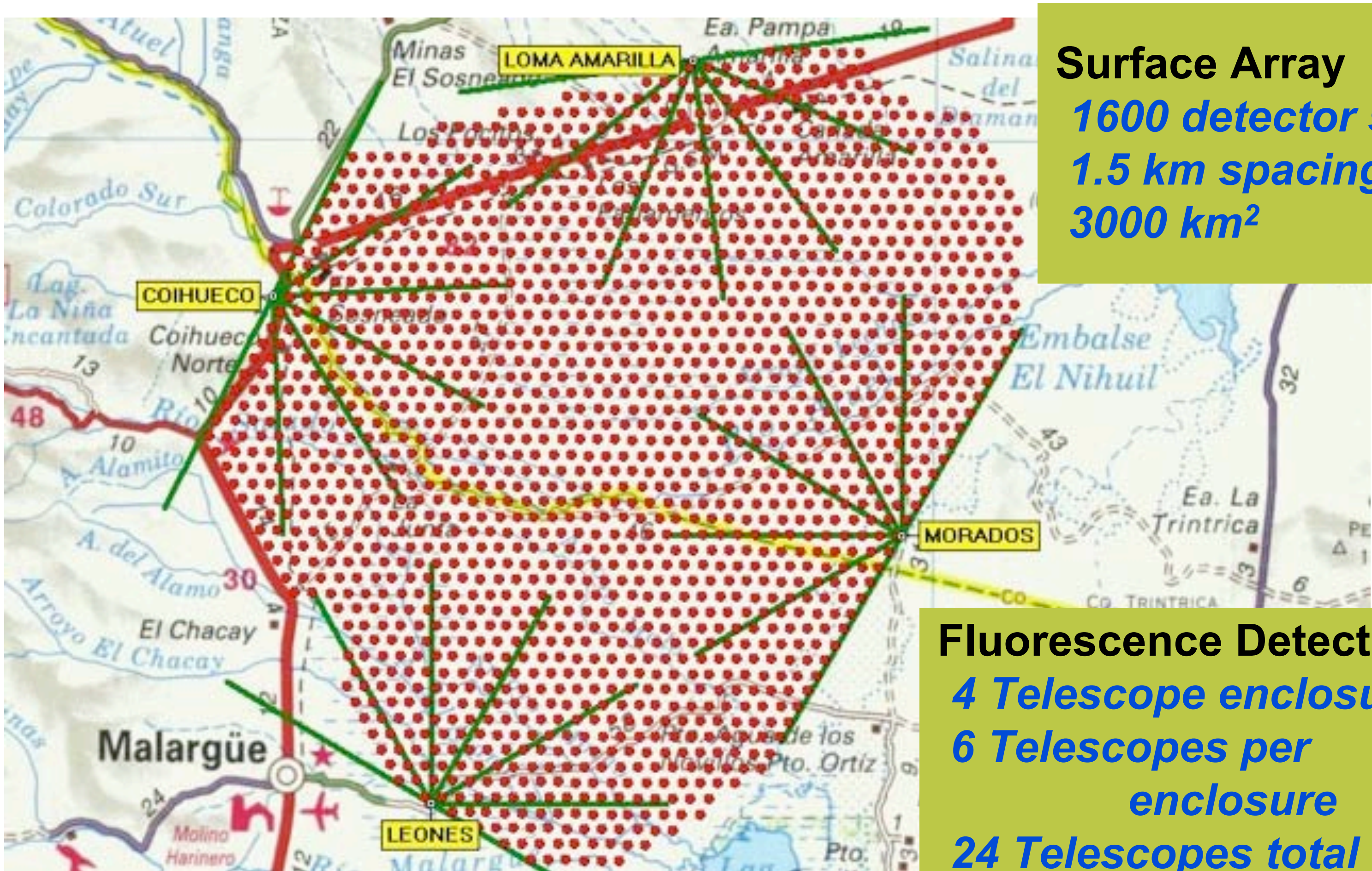
Pierre Auger Observatory



On the Pampa Amarilla in western Argentina

Slides from presentation of
Paul Mantsch, *Auger Project Manger*
www.auger.org

The Observatory Plan



Surface Array

1600 detector stations

1.5 km spacing

3000 km^2

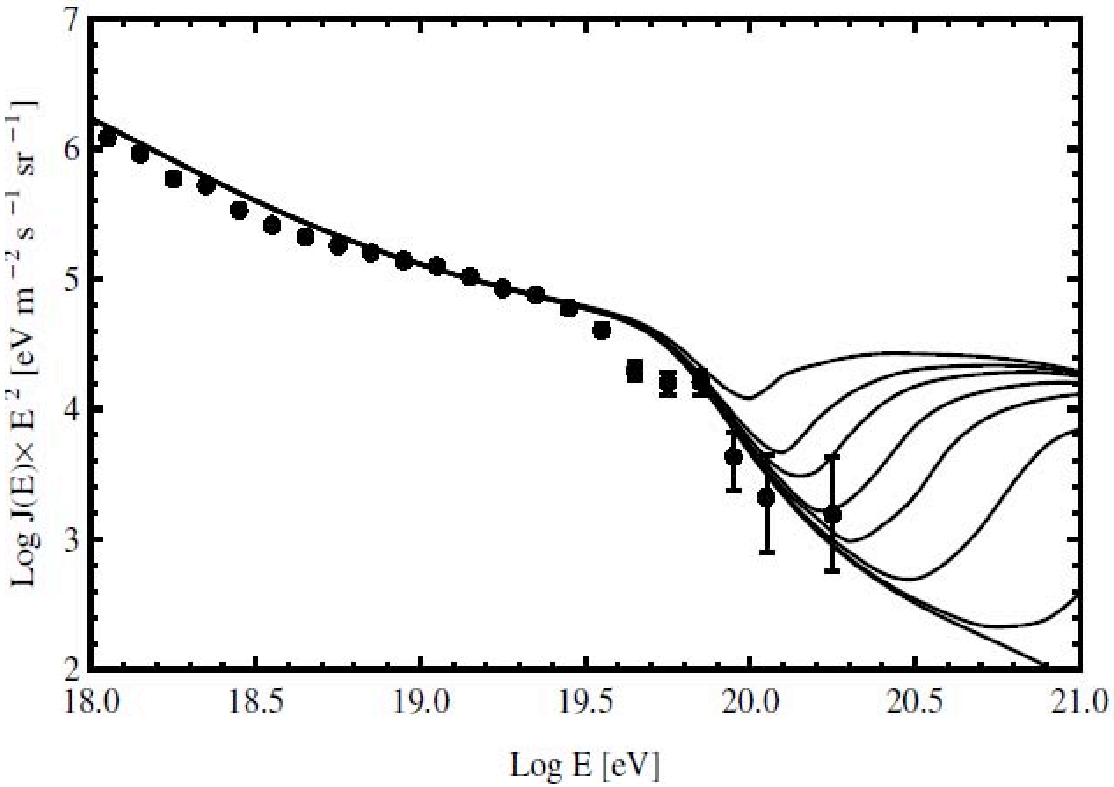
Fluorescence Detectors

4 Telescope enclosures

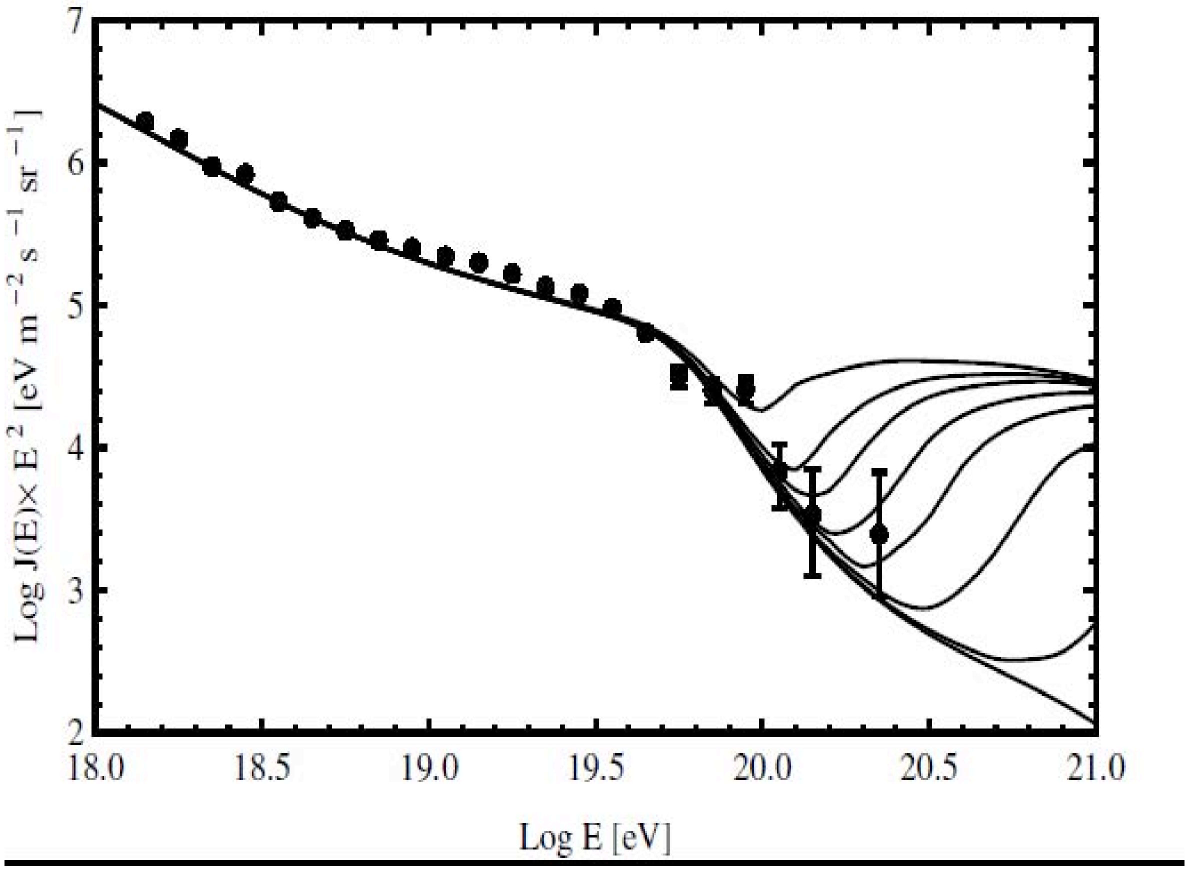
6 Telescopes per enclosure

24 Telescopes total

Auger spectrum with curves for various amounts of LIV (the lowest curve is for no LIV)



Same curves with Auger data shifted by 25%



UPPER LIMIT ON THE AMOUNT OF LORENTZ INVARIANCE VIOLATION FROM AUGER DATA

$d_{\pi p} < 4.5 \times 10^{-23}$ (Stecker & Scully 2009)

Implies LIV suppression by $\boldsymbol{O}(10^{-6})\ M_{Planck}^{-2}$
(Maccione et al. 2009; Stecker & Scully 2009)

High Energy Astrophysics Tests of Lorentz Invariance Violation (LIV)

- **Energy dependent time delay of γ-rays from GRB090510 (*Fermi* Collaboration 2009).**
- **Cosmic γ-ray decay constraints (Coleman & Glashow 1999, Stecker & Glashow 2001).**
- **Cosmic ray vacuum Cherenkov effect constraints (Coleman & Glashow 1999; Stecker & Glashow 2001).**
- **Shifted pair production threshold constraints from AGN γ-rays (Stecker & Glashow 2001).**
- **Long baseline vacuum birefringence constraints from GRBs (Jacobson, Liberati, Mattingly & Stecker 2004; Maccione et al. 2009).**
- **Electron velocity constraints from the Crab Nebula γ-ray spectrum (Jacobson, Liberati & Mattingly 2003).**
- **Ultrahigh energy cosmic ray spectrum GZK effect (Coleman & Glashow 1999; Stecker & Scully 2009).**

The Bottom Line!

- The Fermi timing observations of GRB090510 rule out simple QG and D-brane model predictions of a retardation of photon velocity proportional to E/M_{QG} because they would require $M_{QG} > M_{Planck}$
- More indirect results from γ-ray birefringence limits, the non-decay of 50 TeV γ–rays from the Crab Nebula, and the TeV spectra of nearby AGNs also place severe limits on violations of special relativity (LIV).
- Limits on Lorentz invariance violation from observations of ultrahigh energy cosmic-rays provide severe constraints for other quantum gravity models.
- ***Presently, we have no positive evidence for LIV!*** Theoretical models of Planck scale physics and quantum gravity need to meet all of the present observational constraints.

Supplemental Slides

Energy Dependent Time Delay for Λ CDM Cosmology

$$\Delta t = \frac{\Delta z}{H_0} = \frac{1+n}{2H_0}\left(\frac{E_0}{\xi E_{\rm pl}}\right)^n \int_0^z \frac{(1+z')^n\,{\rm d}z'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}.$$

where $v = c[1-\xi(E/E_{Pl})^n]$ and M_{QG} *is asssumed to be equal to* $E_{Pl} = M_{Pl}$ (Jacob & Piran 2008).

We consider here the important case $n=1$ and take $M_{QG} \lesssim M_{Pl}$.