# Muon pair production with hadronic vacuum polarization

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## Outline

- Introduction
- Theory of $e^+e^- \to \mu^+\mu^-$ for KLOE (2004)
- Results
- Running QED coupling at energy of $\rho$ (KLOE 2016)
- Data and extraction of $\Pi_h$

# Introduction

based on : "Hadronic vacuum polarization in $e^+e^- \to \mu^+\mu^-$ process bellow 3 GeV, V.S. arXiV 1704.01887, 1708.03616"

Test of SM is based on comparison between theory and experiment.

$a_\mu$, $\alpha_{M_Z}$,....

$\Pi_h$ plays crucial role

$\Pi^{\mu\nu}(x) = \Sigma_q Tr < j_q^\mu(x) j_q^\nu(0) >$

$\Pi^{\mu\nu}(q) = P_T^{\mu\nu}\Pi_h(q^2) = Tr \int_k \Gamma^\mu(k,p) S_q(k) \gamma^\mu S_q(p)$

# Theory of $e^+e^- \to \mu^+\mu^-$ for KLOE

Interference effects between hadronic and leptonic vacuum polarization are observed in close vicinity of narrow resonances: $J/\Psi$,$\Psi$, $\Upsilon$ as well as $\phi$ and $\omega$.

They have unique interference pattern and appears in all predominantly QED process in the timelike momentum channel. 1963 estimate : In vicinity of resonances the charges changes like :

$$e^2 \to e^2 \left( 1 - \frac{3 m_V \Gamma_{ll} \alpha^{-1}}{m_V^2 - s - i m \Gamma} \right) , \tag{1}$$

Recently $\phi$ meson sector measured by KLOE in 2004 and SND 2001.

Experiment. data: F. A. Ambrosino, et. al. Phys. Lett. B 608 (2005). see also M.N. Achasov NPA 675, 2000, PRL 86 (2001)

$\phi$-peak feedback is visibale almost everywhere today , $e^+e^- \to 4\pi$ by CMD-3 arXiv:1612.04483

Using $+\gamma$ process the running coupling has been measured by KLOE 2015.

To see the effect, the high precision measurement is required

$\sigma_{stat}(m_\phi) = 0.1nb, \delta_{sys} \simeq 1\%$ with KLOE experimental setup, $\sigma(m_\phi) \equiv 40nb$.

Using SM, the cross section reads (in next to (next to) leading order)

$$\sigma(s) = \frac{4\pi C_t}{|1-\Pi(s)|^2}\left[\sigma_A(s)\left(2-\beta_\mu^2(1-\frac{C_t^2}{3})\right)+\sigma_B(s)\right], \tag{2}$$

where $C_t = \cos(\theta_{min})$ with $\theta_{min} = 50^0$ ($\theta_{max} = 140^o$), which is KLOE experimental cut on polar scattering angle between $\mu^-$ and $e^-$ particles and $\beta_\mu = \sqrt{1-4m_\mu^2/s}$.

$$\sigma_B(s) = -\frac{\alpha^3}{4\pi s}(1-\beta_\mu^2)\ln\frac{1+\beta_\mu}{1-\beta_\mu}. \tag{3}$$

$\sigma_A(s)$- the dominant term, listed completely in Arbuzov1997 (9702262), collects all leading logs of Dirac and Pauli form factors and the known soft photon contributions for which we take $ln\frac{\Delta\epsilon}{\epsilon} = 0.05$ (15 MeV cut on c.m.s. soft photon energy at $\phi$ peak).

# Theory of $e^+e^- \to \mu^+\mu^-$ for KLOE

Nontrivial information is in $\Pi(s) = \Pi_l(s) + \Pi_h(s)$

which completes $\alpha(s) = \frac{\alpha}{1-\Pi(s)}$

$\alpha = \alpha(0) = 1/137.0359991390$

$\Pi_l(s)$- includes complete one loop + leading log in $(\alpha)^2$

hadronic vacuum polarization $\Pi_h$-driven by strong coupling QCD

$$\Pi_h(s) = \frac{s}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} d\omega \frac{\sigma_h(\omega)\left[\frac{\alpha}{\alpha(\omega)}\right]^2}{\omega - s + i\epsilon}\,. \tag{4}$$

$\sigma_h = \sigma(e^+e^- \to hadrons)$

# Theory of $e^+e^- \to \mu^+\mu^-$ for KLOE

For $\sigma_h$ dominant bellow 3GeV we use

this millennium published data for exclusive channels:

$e^+e^-$–> $\pi\pi$, $K^+K^-$, $K_LK_S$, $\pi\pi\pi$, $\pi$ and $\eta\gamma$

from **SND, CMD, CMD2, KLOE, BABAR , BESSIII** collab. selection rule introduced , excluding meassuremnts with large errors or large incompatibilty

For each channel we find fit valid for all $s$ and generate quasidata, from which we calculate $\Pi_h$.

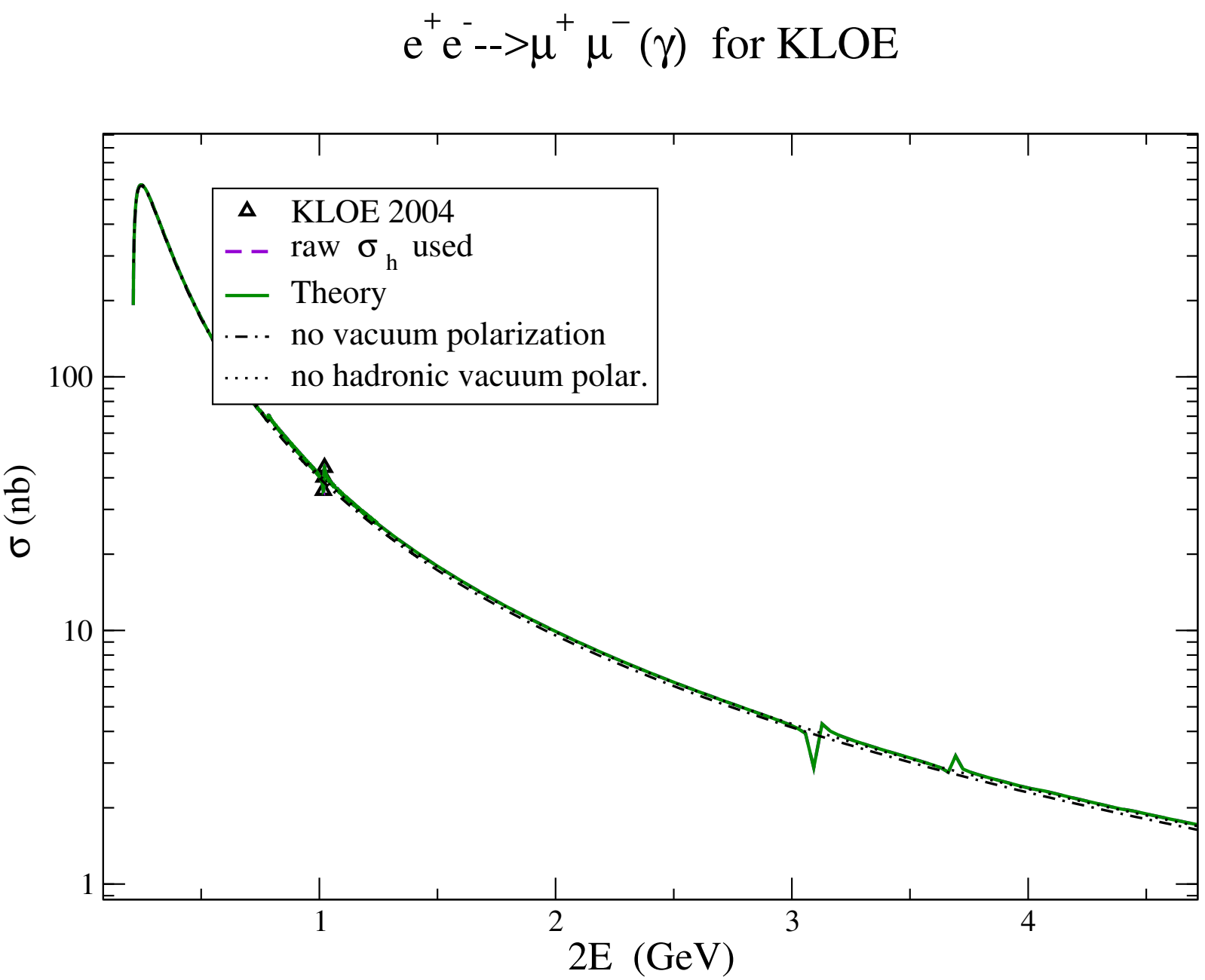
Existing fits for $\sigma_{hex}$ and $\epsilon_{hex}$ are explored and refitted.

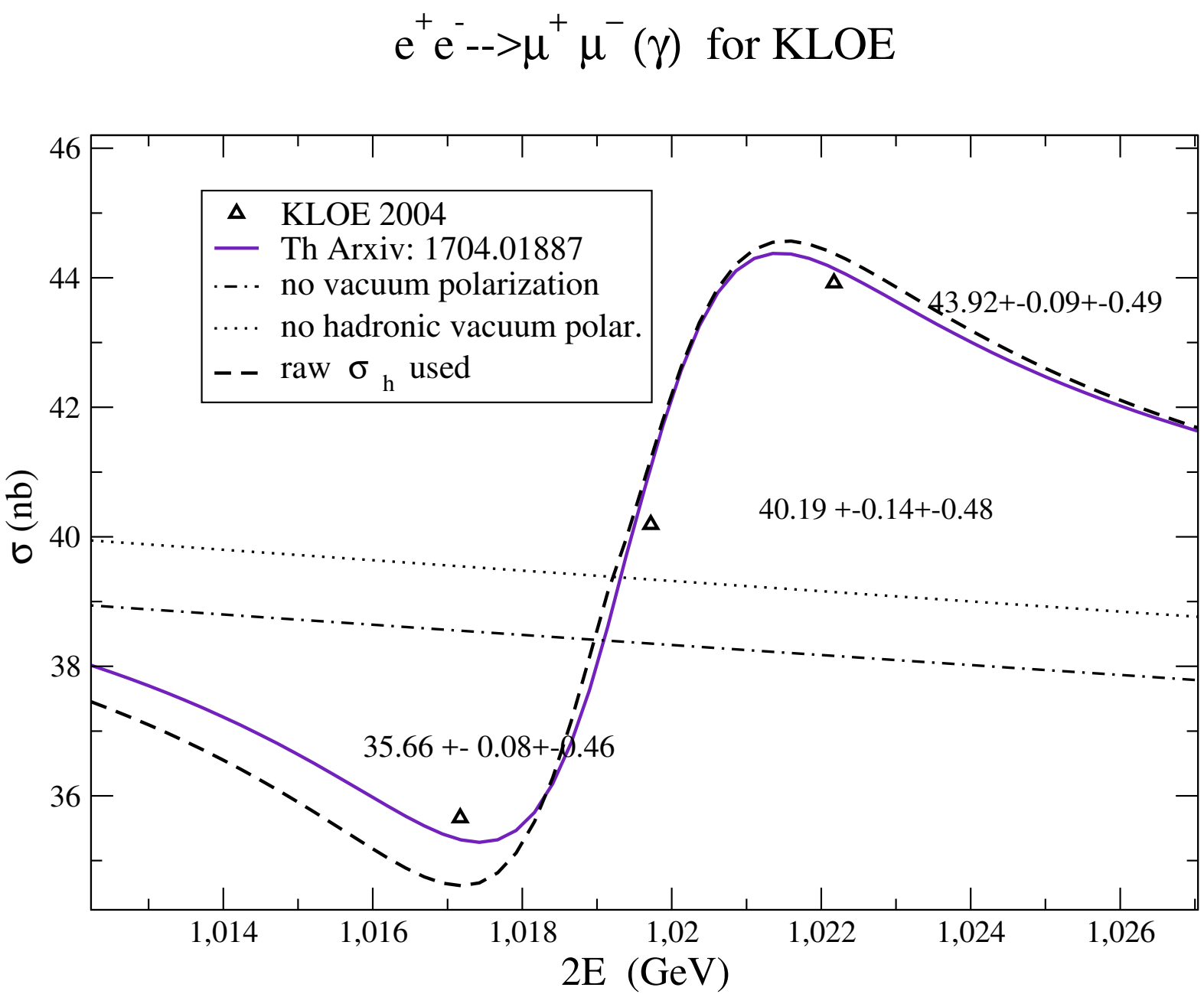
$\sigma_h^0 = \sigma_h(s)(|1 - \Pi(s)|)^2$ calculated

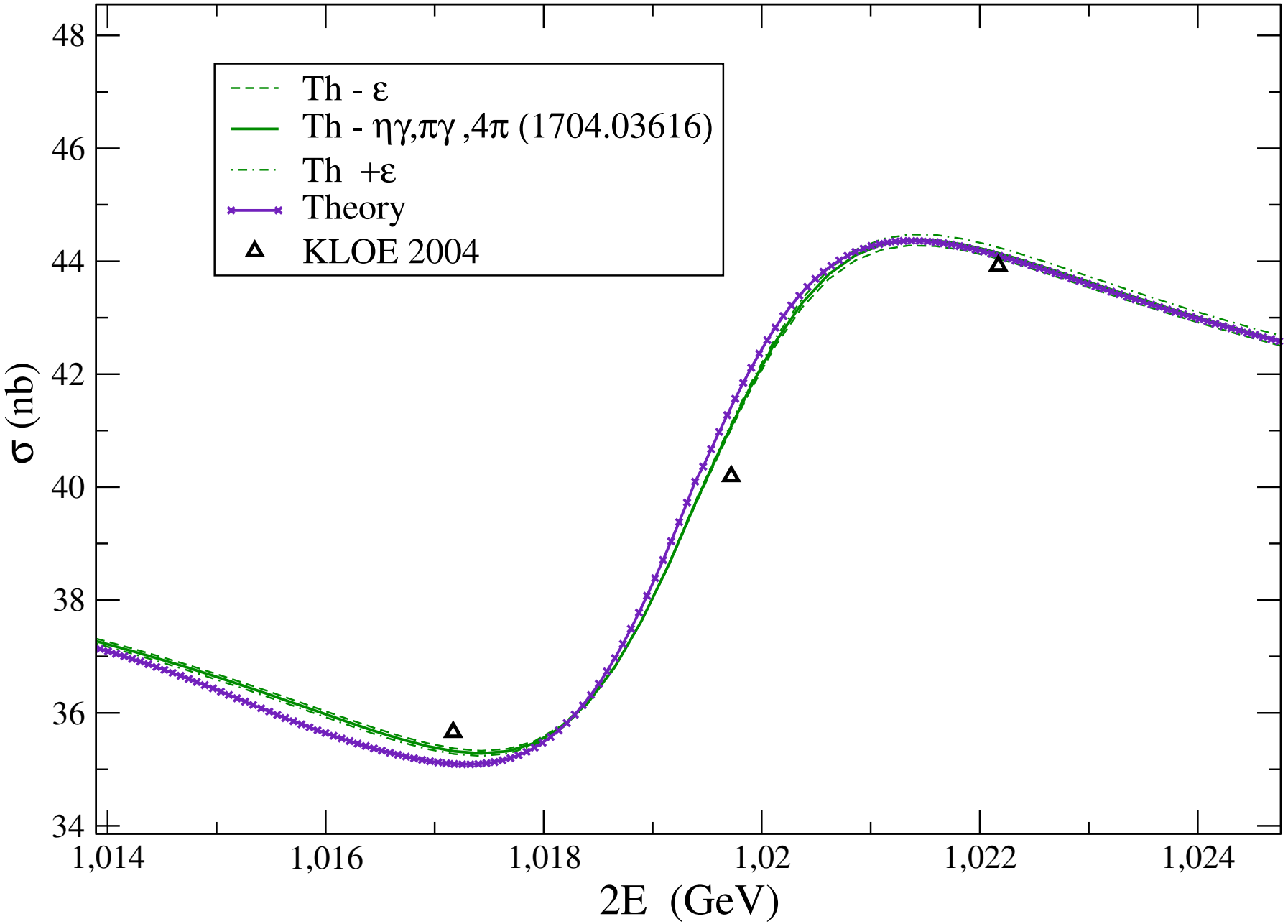
published alternatives: *[1] K. Hagiwara, J.Phys. G38 (2011) 085003 [2] F. Jegerlehner, J. Phys. G 29 (2003) 101. the updated code is available at http://www-com.physik.hu-berlin.de/ fjeger [3] The vacuum polarisation corrections*

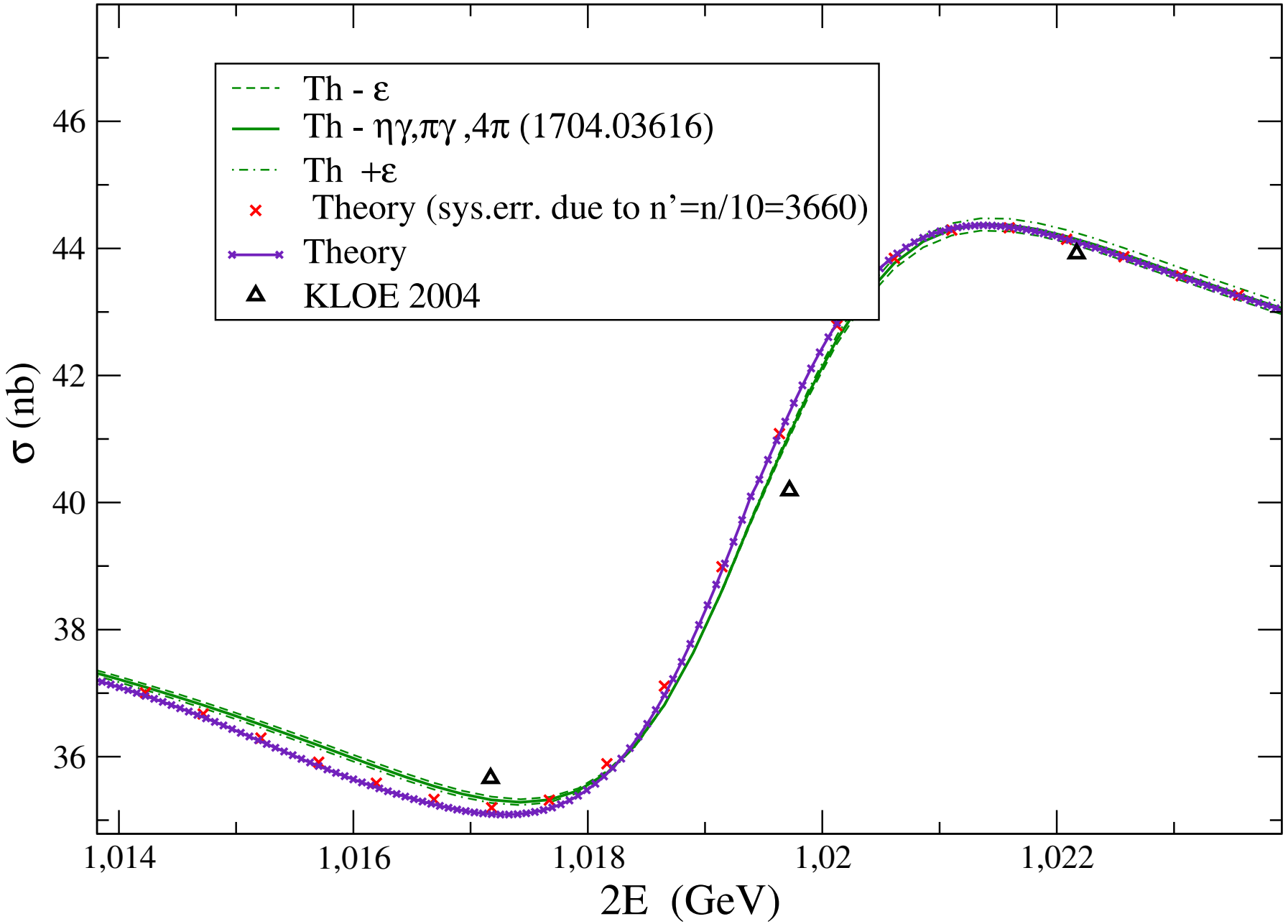
*used by Novosibirsk experiments (SND, CMD2, CMD3) http://cmd.inp.nsk.su/ ignatov/vpl [4] A. Anastasi et al. Phys.Lett. B767 (2017) 485.*

Above 3 GeV all important narrow quarkonia are included also 1 loops PT QCD for b,c are (uncorrectly) added for all s (almost no effect)


$e^+e^-\to\mu^+\mu^-(\gamma)$ for KLOE
KLOE 2004
raw $\sigma_h$ used
Theory
no vacuum polarization
no hadronic vacuum polar.
σ (nb)
100
10
1
1
2
3
4
2E (GeV)


$e^+e^-\to\mu^+\mu^-(\gamma)$ for KLOE
KLOE 2004
Th Arxiv: 1704.01887
no vacuum polarization
no hadronic vacuum polar.
raw $\sigma_h$ used
43.92+-0.09+-0.49
40.19 +-0.14+-0.48
35.66 +- 0.08+-0.46
σ (nb)
2E (GeV)
46
44
42
40
38
36
1,014
1,016
1,018
1,02
1,022
1,024
1,026

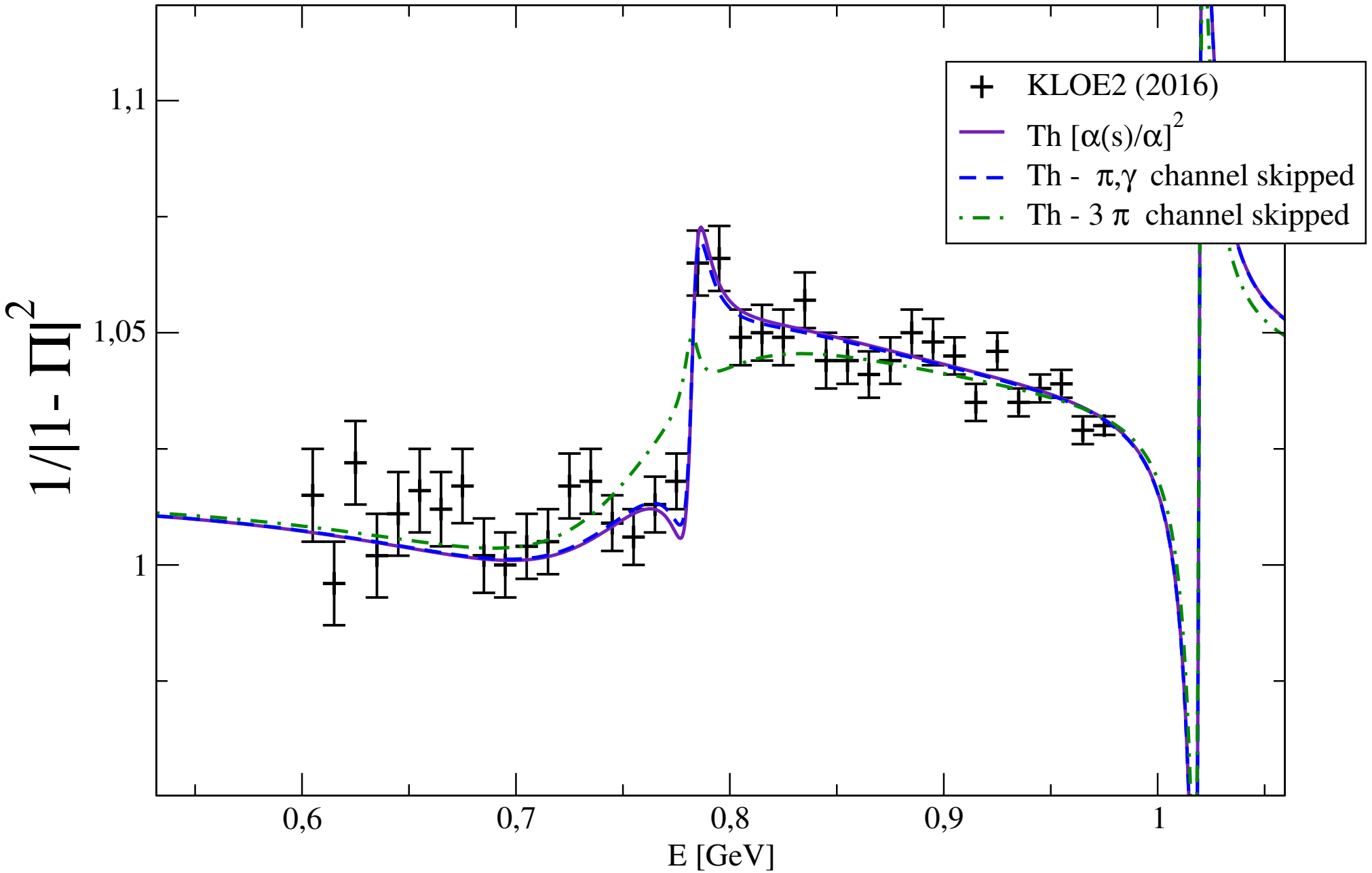

Th - ε
Th - ηγ,πγ ,4π (1704.03616)
Th +ε
Theory
KLOE 2004
σ (nb)
2E (GeV)
48
46
44
42
40
38
36
34
1,014
1,016
1,018
1,02
1,022
1,024


Th - ε
Th - ηγ,πγ ,4π (1704.03616)
Th +ε
Theory (sys.err. due to n'=n/10=3660)
Theory
KLOE 2004
σ (nb)
46
44
42
40
38
36
1,014
1,016
1,018
1,02
1,022
2E (GeV)

# Running $\alpha$ at $\omega$

Never measured precisely in muon pair production

2016 KLOE collaboration studied process $e^-e^+ \to \mu\mu\gamma(\gamma)$


KLOE2 (2016)
Th [α(s)/α]^2
Th - π,γ channel skipped
Th - 3 π channel skipped
1/|1 - Π|^2
1,1
1,05
1
0,6
0,7
0,8
0,9
1
E [GeV]

# Data and extraction of $\Pi_h$

Fits for $\sigma_h$ was made for each $\sigma_{hex}$

Sum of (un)dressed BW with complex phases

Gounaris-Sakurai VMD, Kuhn-Santamaria model

BABAR 2012 fit $\pi\pi$

modified BABAR fits 2013 for $K^+K^-$

modified SND fit for $K_LK_S$

own fits for $3\pi, 4\pi$ and $\eta\gamma, \pi\gamma$

Inflated Error $\epsilon(s)$ for each $\sigma_{hex}(s)$ (IE is fit of upper estimate of the error)

Selection Rule 1.

$$\epsilon[nb] = c\sqrt{\sigma_h[nb]}\ ; c = 0.8nb^{1/2} \tag{5}$$

such that $\epsilon^2 > \sigma^2_{TOT} = \sigma^2_{syst} + \sigma^2_{stat.}$

Selection Rule 2.

For combined data inflate further error function such that minimized $\chi^2 = 1$ for combined data with $\epsilon$ used in $\chi^2$.

Exclude data with a large incompatibility or inflate the error function

A: Exclusion of given data set is a choice based on statistic or decision

B: Inflation of error is a decision to ignore good data

due to the singular integration A,B is a must in order to gain the error of Pi

Hollinde trick is used to solve integral Eq. for $\Pi$

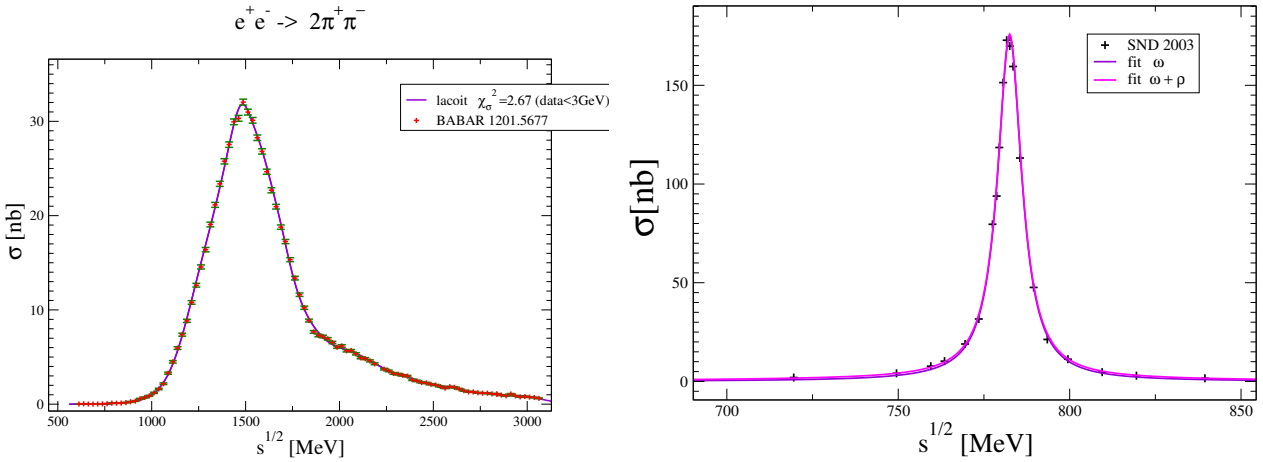



Figure 6: $4\pi$ and $\gamma\pi$ channels

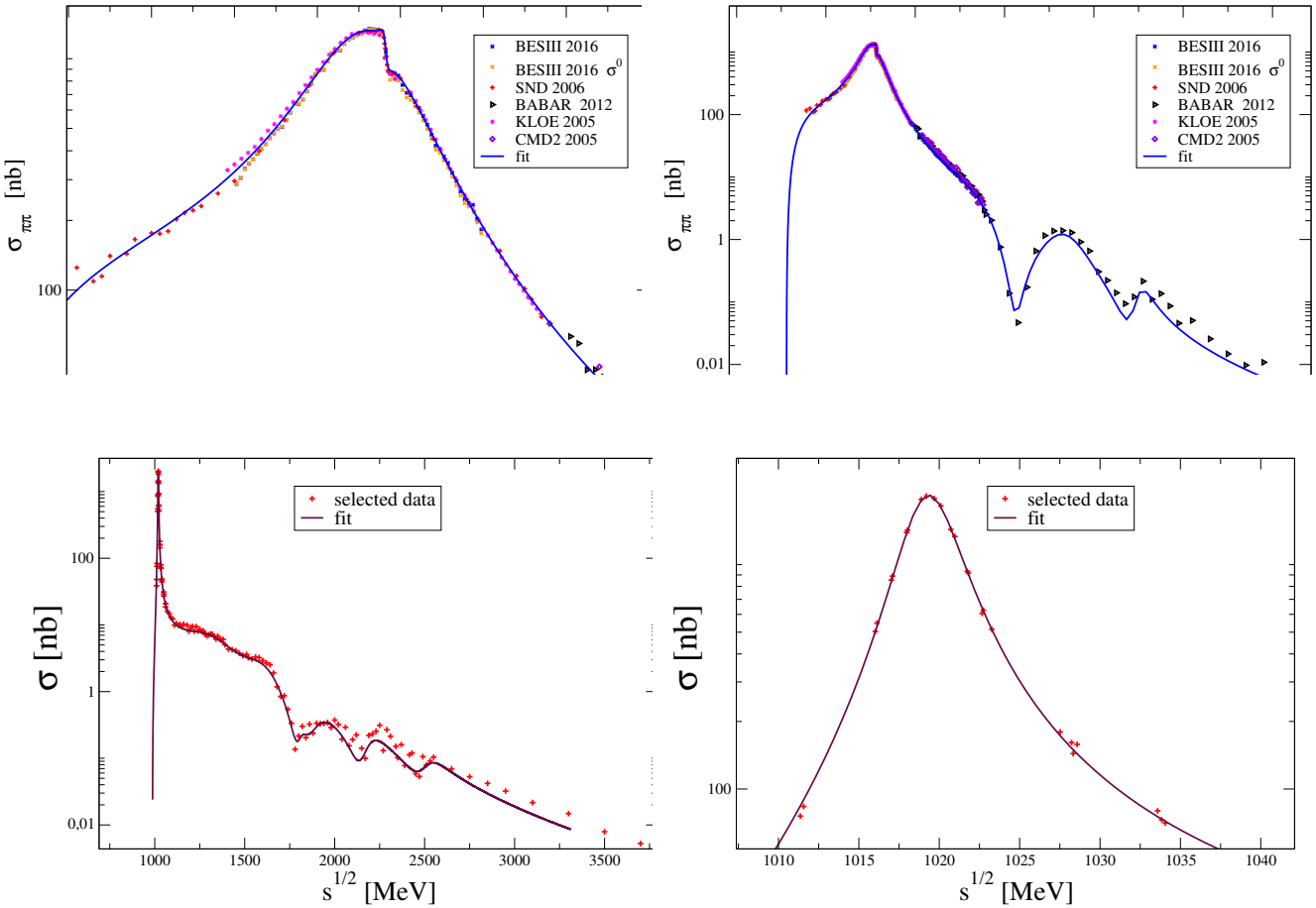



Figure 7: $\pi\pi$ and $K^+K^-$

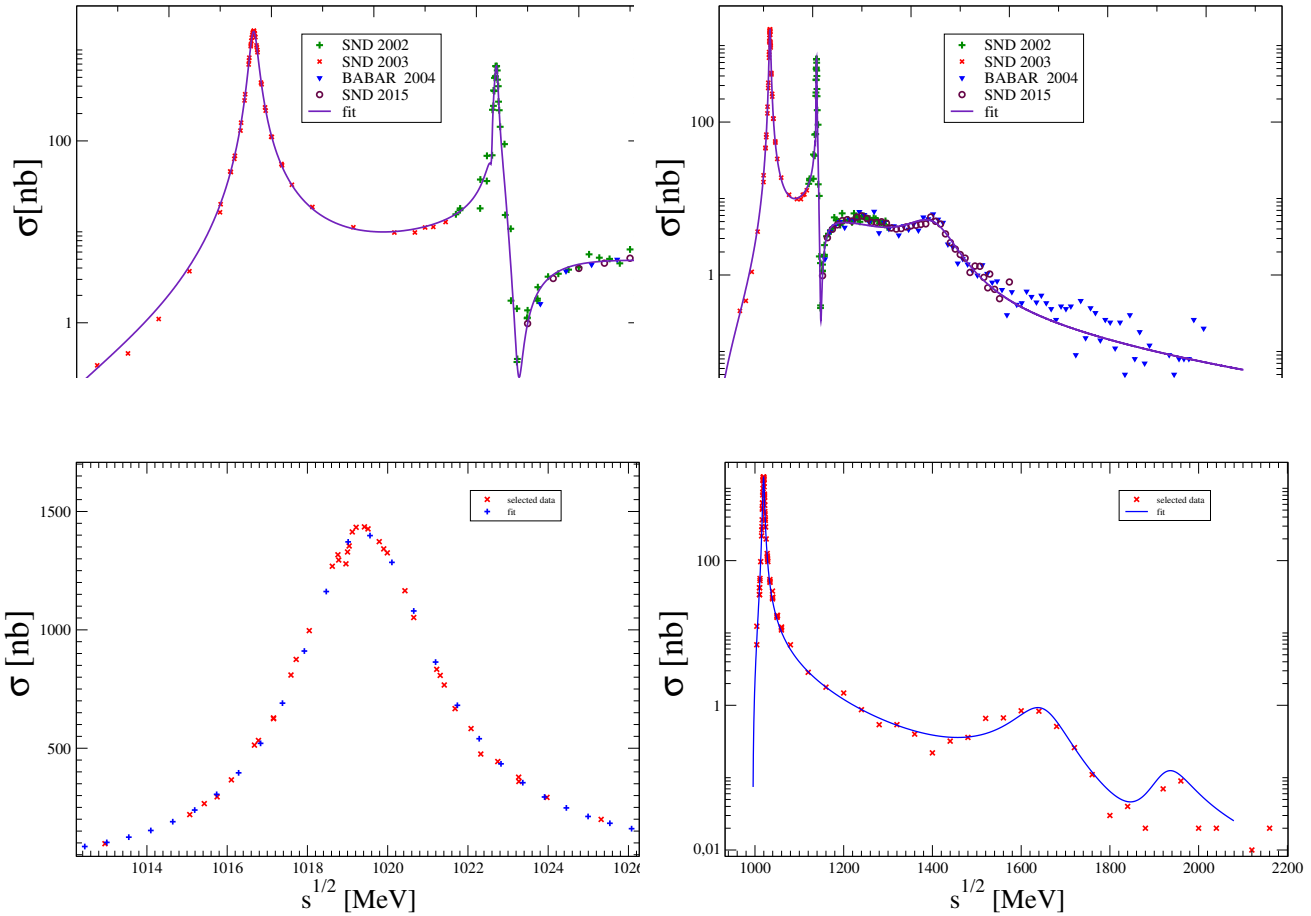



Figure 8: $\pi\pi$ and $K^+K^-$

# Conclusion

$\alpha_{QED}$ is complex at the timelike scale and runs in accordance with SM with

small tension between Standard Theory and $e^+e^- \to \mu^+\mu^-$ 2004 KLOE experiment.

$$\Delta \simeq 1 - 2\sigma_{tot}^{KLOE} \simeq \sigma_{syst}^{KLOE}$$

In opposite to $a_\mu$ , the efect in $\sigma_{\mu\mu}$ seems to be overestimated

Another explanation and motivation: Dispersion relation validity? Boxes with $2\gamma$ are not treated and considered in $\sigma_h$, they are beyond single variable DR technique.

Confinement and its relation with analyticity!?? SDES?

$$\Pi^{\mu\nu}(q) = P_T^{\mu\nu}\Pi_h(q^2) = Tr \int_k \Gamma^\mu(k,p) S_q(k) \gamma^\mu S_q(p)$$

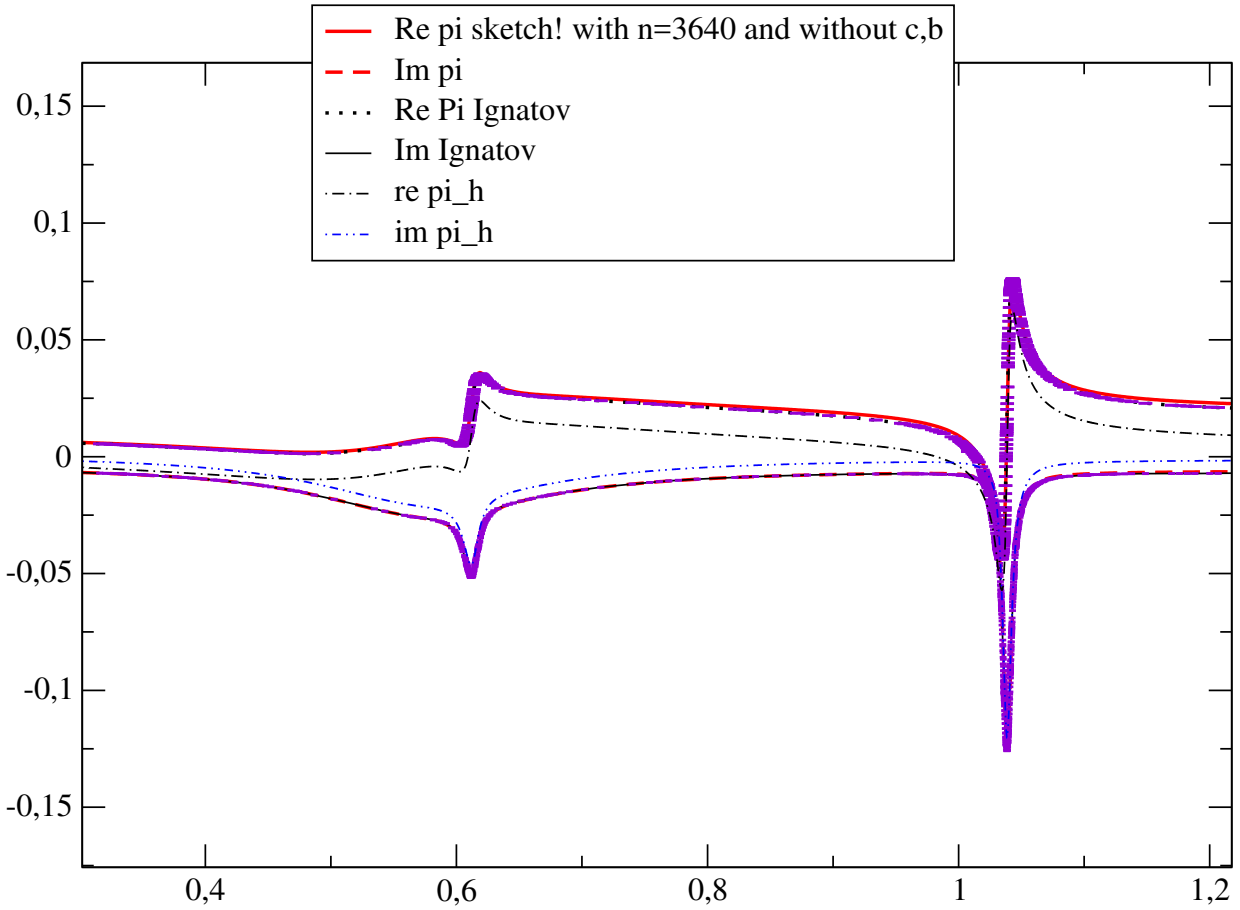



Figure 9: compare