

Intro
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SMEFT
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From EW to EM
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Background
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Conclusion
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CLFV: precise background calculation and EFT interpretation

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Standard Model and open issues

The SM does not take into account the following observations:

- neutrino oscillations;
 - dark matter observation;
 - baryogenesis;
 - gravity.

It does not provide a convincing explanation for:

- hierarchy problem;
 - flavour puzzle;
 - QCD theta term;
 - gauge couplings unification.

Lepton Flavour Violation: a conceptual challenge

The Dim-4 SM provides an accidental flavour symmetry:

- it holds in QCD and EM interactions;
 - in the quark sector, it's broken by EW interactions.

The lepton sector strictly conserves flavour and CP.

At the same time, we have remarkable phenomenological evidences of FV in the neutrino sector, but...

... No evidence of the following phenomenological realisations:

- $l_h^\pm \rightarrow \gamma + l_i^\pm$ where $h, i = e, \mu, \tau,$
 - $l_h^\pm \rightarrow l_i^\pm l_j^\pm l_k^\mp$ where $h, i, j, k = e, \mu, \tau,$
 - $Z \rightarrow l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau,$
 - $H \rightarrow l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau.$

Lepton flavour and CP violation are new physics

Leptons come in three generations and mix: CPV is expected.

Neutral sector: neutrino mass generation mechanism

ν oscillation is a BSM signal, but what is the underlying picture?

Charged sector: lepton flavour and CP puzzle

cLFV & CPV are severely constrained, why BSM is so elusive?

The handheld: leptonic electric dipole moment

[“]The KM phase in the quark sector can induce a lepton EDM via a diagram with a closed quark loop, but a non-vanishing result appears first at the four-loop level and therefore is even more suppressed, below the level of

$$d_e^{\text{CKM}} \leq 10^{-38} e \text{ cm},$$

and so small that the EDMs of paramagnetic atoms and molecules would be induced more efficiently by e.g. Schiff moments and other CP-odd nuclear momenta. [...] The electron EDM is not the best way to probe CP violation in the lepton sector.

M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119

A selection of limits on leptonic observables

Lepton EDMs:

- $d_e < 0.87 \times 10^{-28} \text{ ecm}$ at the 90% C.L.
ACME Collaboration, Science **343** (2014) 269;
 - $d_\mu < (-0.1 \pm 0.9) \times 10^{-19} \text{ ecm}$ at the 90% C.L.
Muon $(g - 2)$ Collaboration, Phys. Rev. D **80** (2009) 052008;
 - $-0.22 \times 10^{-16} \text{ ecm} < d_\tau < 0.45 \times 10^{-16} \text{ ecm}$ at the 95% C.L.
Belle Collaboration, Phys. Lett. B **551** (2003) 16.

cLFV in the muon sector:

- $\text{BR}(\mu \rightarrow 3e) < 1.0 \times 10^{-12}$ at the 90% C.L.
SINDRUM collaboration, Nucl. Phys. B **299** (1988) 1;
 - $\sigma(\mu^- \rightarrow e^-)/\sigma(\text{capt.})|_{\text{Au}} < 7.0 \times 10^{-13}$ at the 90% C.L.
SINDRUM II collaboration, Eur. Phys. J. C **47** (2006) 337;
 - $\text{BR}(\mu \rightarrow \gamma + e) < 4.2 \times 10^{-13}$ at the 90% C.L.
MEG collaboration, Eur. Phys. J. C **76** (2016) 434;

Recent developments

One can contribute in two ways:

- 1 performing precise calculations for backgrounds;
- 2 interpreting properly the current absence of signals.

1) Typical low-energy cLFV background computations:

- **radiative decays**, $l_1 \rightarrow l_2 + \gamma + 2\nu$;
- **rare decays**, $l_1 \rightarrow 3l_2 + 2\nu$, $l_1 \rightarrow 2l_2 + l_3 + 2\nu$.

2) Typical interpretive approaches:

- **bottom-up**, effective field theoretical formulations;
- **top-down**, UV-complete extensions of the SM.

Extending the interactions of the SM

Assumptions: SM is merely an effective theory, valid up to some scale Λ . It can be extended to a field theory that satisfies the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
 - all the SM degrees of freedom must be incorporated;
 - at low energies (i.e. when $\Lambda \rightarrow \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

Dimension-five operator

Only one dimension 5 operator is allowed by gauge symmetry:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T Cl_r^n \equiv (\tilde{\varphi}^\dagger l_p)^T C(\tilde{\varphi}^\dagger l_r).$$

After the EW symmetry breaking, it can generate neutrino masses and mixing (no other operator can do the job).

Its contribution to LFV has been studied since the late 70s:

- in the context of higher dimensional effective realisations;
S. T. Petcov, Sov. J. Nucl. Phys. **25** (1977) 340 [Yad. Fiz. **25** (1977) 641]
 - in connection with the “see-saw” mechanism.
P. Minkowski, Phys. Lett. B **67**, 421 (1977)

[...] This effect is beyond the reach of presently planned experiments."

J. P. Archambault, A. Czarnecki and M. Pospelov, Phys. Rev. D **70** (2004) 073006

Dimension-six operators

2-leptons

$$Q_{eW} \quad = \quad (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu};$$

$$Q_{eB} = (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}.$$

$$Q_{\varphi l}^{(1)} = (\varphi^\dagger iD_\mu \varphi) (\bar{l}_p \gamma^\mu l_r)$$

$$Q_{\langle\varphi l\rangle}^{(3)} = (\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi) (\bar{l}_p \tau^I \gamma^\mu l_r)$$

$$Q_{\varphi e} = (\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi) (\bar{e}_p \gamma^\mu e_r)$$

$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$$

4-fermions

$$Q_{la}^{(1)} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$

$$Q_{la}^{(3)} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$

$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$

$$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$$

$$Q_{lu} = (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$$

$$Q_{ld} = (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$$

$$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$$

4-leptons

$$Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

$$Q_{ee} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$

$$Q_{le} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

$$Q_{ledq} = (\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$$

$$Q_i^{(1)} \equiv (\bar{l}_r^j e_r) \varepsilon_{ijk} (\bar{q}_s^k u_t)$$

$$Q^{(3)}_+ \equiv (\bar{l}_-^j \sigma_{\mu\nu} e_x) \varepsilon_{ijk} (\bar{q}_-^k \sigma^{\mu\nu} u_t)$$

Leptonic tensorial current at the tree level

One dimension-six operator can produce tensorial current:

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

Working in the physical basis, we consider:

$$C_{eB} \rightarrow C_{e\gamma} c_W - C_{eZ} s_W,$$

$$C_{eW} \rightarrow -C_{e\gamma}s_W - C_{eZ}c_W,$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle.

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi F_{\mu\nu} + \text{h.c.}$$

Lepton dipole moments

Dimension-six operators contribute to the Wilson coefficients C_{TL} and C_{TR} of the dipole interaction:

$$V^\mu = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} (C_{TL}(p_\gamma^2) \omega_L + C_{TR}(p_\gamma^2) \omega_R) (p_\gamma)_\nu.$$

Anomalous magnetic and electric-dipole moments:

$$\begin{aligned} a_l &\propto \Re(C_{TR} + C_{TL})|_{p_\gamma^2 \rightarrow 0} & \text{CPC} \\ d_l &\propto \Im(C_{TR} - C_{TL})|_{p_\gamma^2 \rightarrow 0} & \text{CPV} \end{aligned}$$

If flavour is not diagonal, then the momenta are “transitional”.

In all generalities, UV-complete theories produce both CPV and FV effective dipole contributions.

Low-energy LFV observables

Neutrinoless radiative decay

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{\alpha_e m_\mu^5}{\Lambda^4 \Gamma_\mu} \left(|C_L^D|^2 + |C_R^D|^2 \right).$$

Neutrinoless three-body decay

$$\begin{aligned} \text{Br}(\mu \rightarrow 3e) &= \frac{\alpha_e^2 m_\mu^5}{12\pi\Lambda^4\Gamma_\mu} \left(|C_L^D|^2 + |C_R^D|^2 \right) \left(8 \log \left[\frac{m_\mu}{m_e} \right] - 11 \right) \\ &+ \frac{m_\mu^5}{3(16\pi)^3\Lambda^4\Gamma_\mu} \left(|C_{ee}^{S\,LL}|^2 + 16 |C_{ee}^{V\,LL}|^2 + 8 |C_{ee}^{V\,LR}|^2 \right. \\ &\quad \left. + |C_{ee}^{S\,RR}|^2 + 16 |C_{ee}^{V\,RR}|^2 + 8 |C_{ee}^{V\,RL}|^2 \right). \end{aligned}$$

Coherent conversion in nuclei

$$\Gamma_{\mu \rightarrow e}^N = \frac{m_\mu^5}{4\Lambda^4} \left| e C_L^D D_N + 4 \left(G_F m_\mu m_p \tilde{C}_{(p)}^{SL} S_N^{(p)} + \tilde{C}_{(p)}^{VR} V_N^{(p)} + p \rightarrow n \right) \right|^2 + L \leftrightarrow R.$$

High-energy LFV observables

Flavour-violating Z decays can be parametrised at the tree level by means of the following four operators:

$$\begin{aligned}\Gamma(Z \rightarrow l_1^\pm l_2^\mp) = & \frac{m_Z^3 v^2}{12\pi\Lambda^4} \left(|C_{eZ}^{12}|^2 + |C_{eZ}^{21}|^2 \right. \\ & \left. + |C_{\varphi e}^{12}|^2 + \left| C_{\varphi l(1)}^{12} \right|^2 + \left| C_{\varphi l(3)}^{12} \right|^2 \right),\end{aligned}$$

and all of their contributions occur at the same order. We have summed over the two possible final states, $l_1^+ l_2^-$ and $l_1^- l_2^+$.

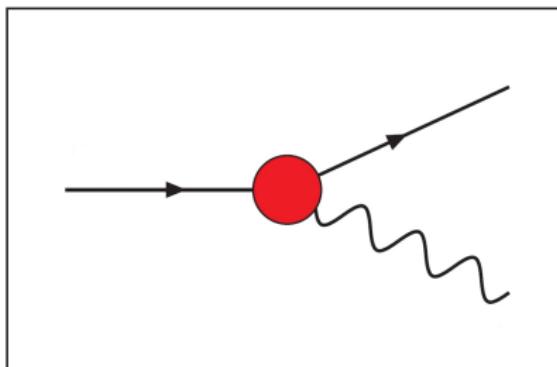
For the Higgs boson decay $H \rightarrow l_1^\pm l_2^\mp$, one has

$$\Gamma(H \rightarrow l_1^\pm l_2^\mp) = \frac{m_H v^4}{16\pi\Lambda^4} \left(|C_{e\varphi}^{12}|^2 + |C_{e\varphi}^{21}|^2 \right),$$

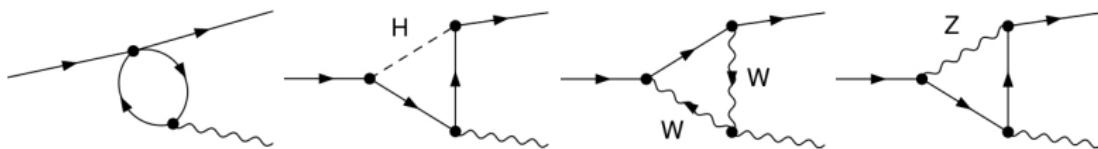
where only one operator contributes at tree level. Again, we have summed over the two possible decays $l_1^+ l_2^-$ and $l_1^- l_2^+$.

Dimension-six operators: lepton current at one loop

From a point-like interaction...



...to quantum fluctuations!



Effective coefficients and energy scale

The effective dipole coefficient can be written as

$$C_T^{(1)} = -\frac{v}{\sqrt{2}} \left(C_{e\gamma} \left(1 + e^2 c_{e\gamma}^{(1)} \right) + \sum_{i \neq e\gamma} e^2 c_i^{(1)} C_i \right).$$

In general, the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ contain UV singularities, i.e. a renormalisation of $C_{e\gamma}$ is required.

Such procedure makes the scale dependence explicit via the *anomalous dimensions* of the coefficient.

At the end of the day, the renormalised effective coefficients and the C_{TL} and C_{TR} are running quantities.

Renormalisation Group Equations

$$16\pi^2 \frac{\partial C_{e\gamma}^{ij}}{\partial \log \lambda} \simeq \left(\frac{47e^2}{3} + \frac{e^2}{4c_W^2} - \frac{9e^2}{4s_W^2} + 3Y_t^2 \right) C_{e\gamma}^{ij} + 6e^2 \left(\frac{c_W}{s_W} - \frac{s_W}{c_W} \right) C_{eZ}^{ij}$$

$$+ 16eY_t C_{ijtt}^{(3)},$$

$$16\pi^2 \frac{\partial C_{eZ}^{ij}}{\partial \log \lambda} \simeq -\frac{2e^2}{3} \left(\frac{2c_W}{s_W} + \frac{31s_W}{c_W} \right) C_{e\gamma}^{ij} + 2e \left(\frac{3c_W}{s_W} - \frac{5s_W}{c_W} \right) Y_t C_{ijtt}^{(3)}$$

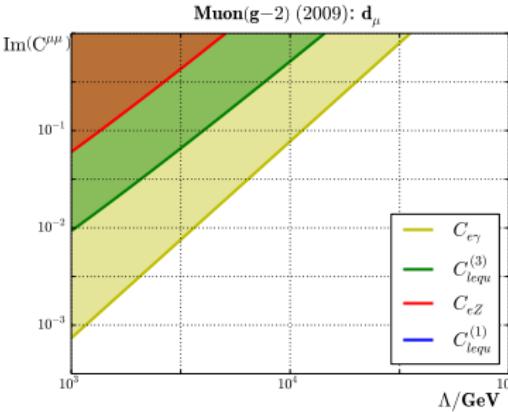
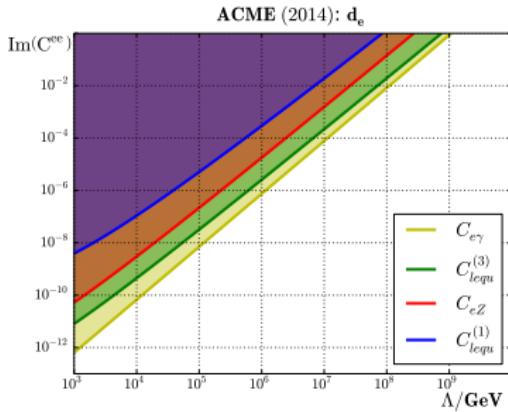
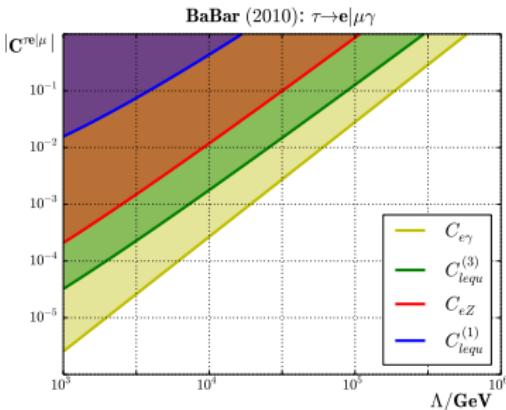
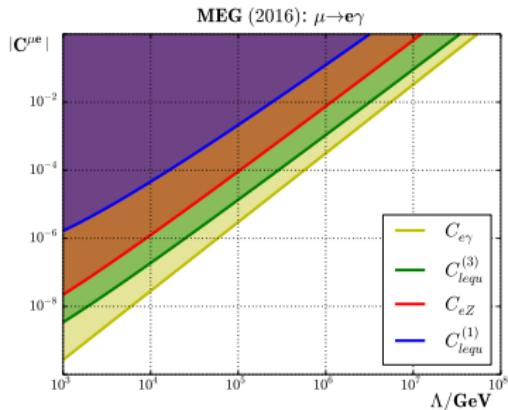
$$+ \left(-\frac{47e^2}{3} + \frac{151e^2}{12c_W^2} - \frac{11e^2}{12s_W^2} + 3Y_t^2 \right) C_{eZ}^{ij},$$

$$16\pi^2 \frac{\partial C_{ijtt}^{(3)}}{\partial \log \lambda} \simeq \frac{7eY_t}{3} C_{e\gamma}^{ij} + \frac{eY_t}{2} \left(\frac{3c_W}{s_W} - \frac{5s_W}{3c_W} \right) C_{eZ}^{ij} +$$

$$+ \left(\frac{2e^2}{9c_W^2} - \frac{3e^2}{s_W^2} + \frac{3Y_t^2}{2} + \frac{8g_S^2}{3} \right) C_{ijtt}^{(3)} + \frac{e^2}{8} \left(\frac{5}{c_W^2} + \frac{3}{s_W^2} \right) C_{ijtt}^{(1)},$$

$$16\pi^2 \frac{\partial C_{ijtt}^{(1)}}{\partial \log \lambda} \simeq \left(\frac{30e^2}{c_W^2} + \frac{18e^2}{s_W^2} \right) C_{ijtt}^{(3)} + \left(-\frac{11e^2}{3c_W^2} + \frac{15Y_t^2}{2} - 8g_S^2 \right) C_{ijtt}^{(1)}.$$

Experimental limits “reinterpreted” at the EW scale



cLFV effective contributions to C_{TL} and C_{TR}

Operator	C_{TL} or $C_{TR}(l_2 \longleftrightarrow l_1)$	
$Q_{e\gamma}$	$-C_{e\gamma} \frac{\sqrt{2}m_W s_W}{e}$	
Q_{eZ}	$-C_{eZ} \frac{em_Z}{16\sqrt{2}\pi^2} \left(3 - 6c_W^2 + 4c_W^2 \log \left[\frac{m_W^2}{m_Z^2} \right] + (12c_W^2 - 6) \log \left[\frac{m_Z^2}{\lambda^2} \right] \right)$	
$Q_{\varphi l}^{(1)}$	$-C_{\varphi l}^{(1)} \frac{em_1 (1 + s_W^2)}{24\pi^2}$	
$Q_{\varphi l}^{(3)}$	$C_{\varphi l}^{(3)} \frac{em_1 (3 - 2s_W^2)}{48\pi^2}$	
$Q_{\varphi e}$	$C_{\varphi e} \frac{em_2 (3 - 2s_W^2)}{48\pi^2}$	
$Q_{e\varphi}$	$C_{e\varphi} \frac{m_W s_W}{48\sqrt{2}m_H^2 \pi^2} \left(4m_1^2 + 4m_2^2 + 3m_1^2 \log \left[\frac{m_1^2}{m_H^2} \right] + 3m_2^2 \log \left[\frac{m_2^2}{m_H^2} \right] \right)$	
$Q_{lequ}^{(3)}$	$-\frac{e}{2\pi^2} \sum_u m_u \left(C_{lequ}^{(3)} \right)^{21uu} \log \left[\frac{m_u^2}{\lambda^2} \right]$	
Operator	C_{TL}	C_{TR}
Q_{le}	$\frac{e}{16\pi^2} \left(m_e C_{le}^{2ee1} + m_\mu C_{le}^{2\mu\mu1} + m_\tau C_{le}^{2\tau\tau1} \right)$	$\frac{e}{16\pi^2} (m_e C_{le}^{1ee2} + m_\mu C_{le}^{1\mu\mu2} + m_\tau C_{le}^{1\tau\tau2})$

No correlation: limits from muonic cLFV

GMP and A. Signer
JHEP **1410** (2014) 014

F. Feruglio,
arXiv:1509.08428

GMP and A. Signer
EPJWC 118 (2016) 01031

Coefficient	MEG ($\mu \rightarrow e\gamma$) $BR \leq 5.7 \cdot 10^{-13}$	ATLAS ($Z \rightarrow e\mu$) $BR \leq 7.5 \cdot 10^{-7}$	SINDRUM ($\mu \rightarrow 3e$) $BR \leq 1.0 \cdot 10^{-12}$
$C_{eZ}^{\mu e}(m_Z)$	$1.4 \cdot 10^{-13} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.8 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi e}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.6 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{e\varphi}^{\mu e}$	$2.7 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$6.1 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{eeee\mu}$	$4.2 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$2.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{e\mu\mu\mu}$	$2.0 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{le}^{e\tau\tau\mu}$	$1.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{ee}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{ll}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$

No correlation: limits from tauonic cLFV

Coefficient	BaBar ($\tau \rightarrow \mu\gamma$) $BR \leq 4.4 \cdot 10^{-8}$	LEP ($Z \rightarrow \tau\mu$) $BR \leq 1.2 \cdot 10^{-5}$	BELL ($\tau \rightarrow 3\mu$) $BR \leq 2.1 \cdot 10^{-8}$	ATLAS&CMS ($H \rightarrow \tau\mu$) $BR \leq 1.85 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}(m_Z)$	$1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$6.1 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{\varphi l}^{(1)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{\varphi l}^{(3)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{\varphi e}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$9.5 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{e\varphi}^{\tau\mu}$	$1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$		$1.1 \cdot 10^{-5} \frac{\Lambda^2}{[\text{GeV}]^2}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{\mu ee\tau}$	$4.7 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$			
$C_{le}^{\mu\mu\mu\tau}$	$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$		$8.0 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{le}^{\mu\tau\tau\tau}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$			
$C_{ee}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	
$C_{ll}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	

No correlation: limits from EDM

Separate **hard** and **soft!** Regularisation scheme independent?

Coefficient	Contribution to d_l	Limits from d_e	Limits from d_μ
$C_{e\gamma}^{ii}$	$\frac{2\sqrt{2}m_W s_W}{e} \Im C_{e\gamma}^{ii}$	$5.8 \times 10^{-19} \frac{\Lambda^2}{[\text{GeV}]^2}$	$6.7 \times 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$
C_{le}^{1iii}	$\frac{e m_e}{8\pi^2} \Im C_{le}^{1iii}$	N/A	N/A
C_{le}^{2ii2}	$\frac{e m_\mu}{8\pi^2} \Im C_{le}^{2ii2}$	$5.5 \times 10^{-13} \frac{\Lambda^2}{[\text{GeV}]^2}$	N/A
C_{le}^{3iii3}	$\frac{e m_\tau}{8\pi^2} \Im C_{le}^{3iii3}$	$3.2 \times 10^{-14} \frac{\Lambda^2}{[\text{GeV}]^2}$	$3.7 \times 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{e\varphi}^{ii}$	$\frac{m_i^2 m_W s_W}{8\sqrt{2}m_H^2 \pi^2} \left(-3 + 2 \log \left[\frac{m_i^2}{m_H^2} \right] \right) \Im C_{e\varphi}^{ii}$	$3.8 \times 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$	N/A
C_{eZ}^{ii}	$\frac{\alpha}{8\pi c_W s_W} (-3c_W^2 + 3s_W^2) \Im C_{eZ}^{ij}$	$5.5 \times 10^{-16} \frac{\Lambda^2}{[\text{GeV}]^2}$	$6.3 \times 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$

See also: A. Crivellin *et al.*, JHEP **1404** (2014) 167
 Improving on d_τ : M. Fael *et al.*, JHEP **1603** (2016) 140

The good old k plot

Due to the **extremely-low accessible branching ratios**, CLFV muon channels can strongly **constrain** new physics models and scales.

Model independent Lagrangian:

$$\frac{m_\mu}{(\kappa + 1)\Lambda^2} \times \text{Diagram A} + \frac{\kappa}{(\kappa + 1)\Lambda^2} \times \text{Diagram B}$$

Dipole term
(e.g. SUSY)

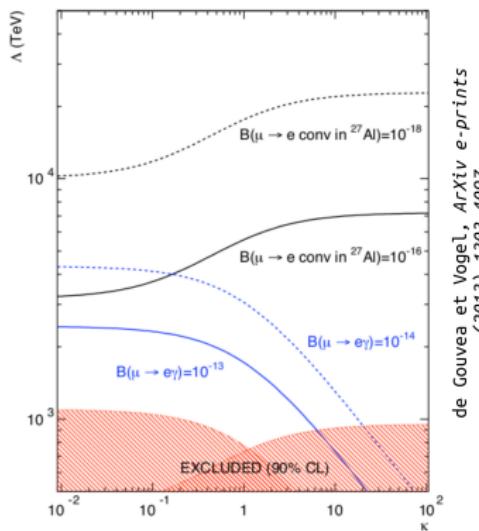
Contact term
(e.g. Z', LQ)

$\mu \rightarrow e\gamma$

$\mu \rightarrow eee$

$\mu - e$ conversion

Sensitive to high-mass New Physics!



Below the EWSB scale (1)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} + \frac{1}{\Lambda^2} \sum_i C_i Q_i,$$

and the explicit structure of the operators is given by

Dipole	
$Q_{e\gamma}$	$em_r(\bar{l}_p \sigma^{\mu\nu} P_L l_r) F_{\mu\nu} + \text{H.c.}$
Scalar/Tensorial	
Q_S	$(\bar{l}_p P_L l_r)(\bar{l}_s P_L l_t) + \text{H.c.}$
	Q_{VLL} $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_L l_t)$
	Q_{VLR} $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
	Q_{VRR} $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{l}_s \gamma_\mu P_R l_t)$
$Q_{Slq(1)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_L q_t) + \text{H.c.}$
$Q_{Slq(2)}$	$(\bar{l}_p P_L l_r)(\bar{q}_s P_R q_t) + \text{H.c.}$
Q_{Tlq}	$(\bar{l}_p \sigma^{\mu\nu} P_L l_r)(\bar{q}_s \sigma_{\mu\nu} P_L q_t) + \text{H.c.}$
	Q_{VlqLL} $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
	Q_{VlqLR} $(\bar{l}_p \gamma^\mu P_L l_r)(\bar{q}_s \gamma_\mu P_R q_t)$
	Q_{VlqRL} $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_L q_t)$
	Q_{VlqRR} $(\bar{l}_p \gamma^\mu P_R l_r)(\bar{q}_s \gamma_\mu P_R q_t)$

Below the EWSB scale (2)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}}$$

$$\begin{aligned} &+ \frac{1}{\Lambda^2} \left\{ C_L^D O_L^D + \sum_{f=q,\ell} \left(C_{ff}^{V\,LL} O_{ff}^{V\,LL} + C_{ff}^{V\,LR} O_{ff}^{V\,LR} + C_{ff}^{S\,LL} O_{ff}^{S\,LL} \right) \right. \\ &\quad \left. + \sum_{h=q,\tau} \left(C_{hh}^{T\,LL} O_{hh}^{T\,LL} + C_{hh}^{S\,LR} O_{hh}^{S\,LR} \right) + C_{gg}^L O_{gg}^L + L \leftrightarrow R \right\} + \text{h.c.}, \end{aligned}$$

and the explicit structure of the operators is given by

$$O_L^D = e m_\mu (\bar{e} \sigma^{\mu\nu} P_L \mu) F_{\mu\nu},$$

$$O_{ff}^{V\,LL} = (\bar{e} \gamma^\mu P_L \mu) (\bar{f} \gamma_\mu P_L f),$$

$$O_{ff}^{V\,LR} = (\bar{e} \gamma^\mu P_L \mu) (\bar{f} \gamma_\mu P_R f),$$

$$O_{ff}^{S\,LL} = (\bar{e} P_L \mu) (\bar{f} P_L f),$$

$$O_{hh}^{S\,LR} = (\bar{e} P_L \mu) (\bar{h} P_R h),$$

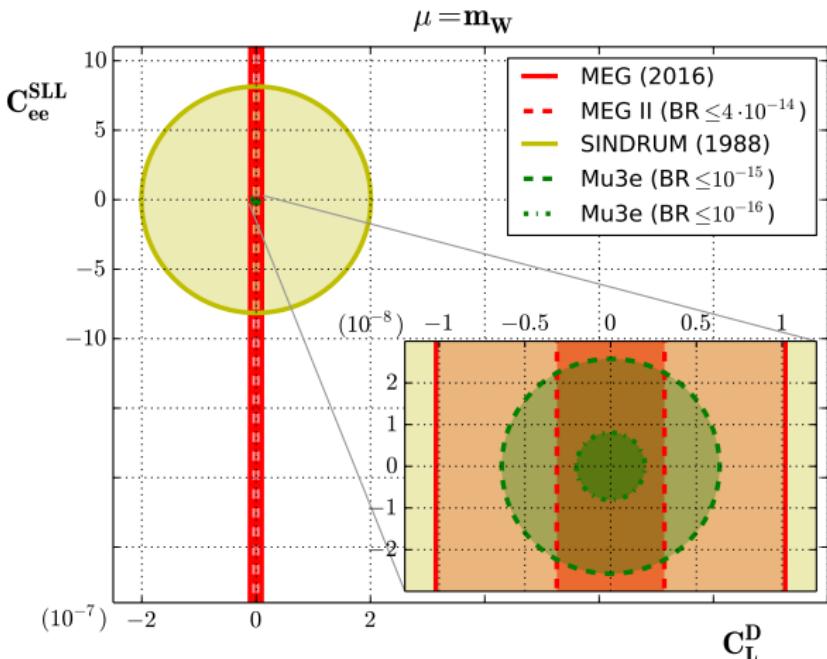
$$O_{hh}^{T\,LL} = (\bar{e} \sigma_{\mu\nu} P_L \mu) (\bar{h} \sigma^{\mu\nu} P_L h),$$

$$O_{gg}^L = \alpha_s m_\mu G_F (\bar{e} P_L \mu) G_{\mu\nu}^a G_a^{\mu\nu}.$$

Interplay between $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$

A. Crivellin, S. Davidson, GMP and A. Signer, arXiv:1611.03409 [hep-ph].

Below the EW scale, four-fermion vs dipole:



Dipole evolution below the EWSB scale

At the two-loop level, in the tHV scheme:

$$\begin{aligned}\dot{C}_L^D &= 16 \alpha_e Q_l^2 C_L^D - \frac{Q_l}{(4\pi)} \frac{m_e}{m_\mu} C_{ee}^{S\ LL} - \frac{Q_l}{(4\pi)} C_{\mu\mu}^{S\ LL} \\ &+ \sum_h \frac{8Q_h}{(4\pi)} \frac{m_h}{m_\mu} N_{c,h} C_{hh}^{T\ LL} \Theta(\mu - m_h) \\ &- \frac{\alpha_e Q_l^3}{(4\pi)^2} \left(\frac{116}{9} C_{ee}^{V\ RR} + \frac{116}{9} C_{\mu\mu}^{V\ RR} - \frac{122}{9} C_{\mu\mu}^{V\ RL} - \left(\frac{50}{9} + 8 \frac{m_e}{m_\mu} \right) C_{ee}^{V\ RL} \right) \\ &- \sum_h \frac{\alpha_e}{(4\pi)^2} \left(6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9} \right) N_{c,h} C_{hh}^{V\ RR} \Theta(\mu - m_h) \\ &- \sum_h \frac{\alpha_e}{(4\pi)^2} \left(-6Q_h^2 Q_l + \frac{4Q_h Q_l^2}{9} \right) N_{c,h} C_{hh}^{V\ RL} \Theta(\mu - m_h) \\ &- \sum_h \frac{\alpha_e}{(4\pi)^2} 4Q_h^2 Q_l N_{c,h} \frac{m_h}{m_\mu} C_{hh}^{S\ LR} \Theta(\mu - m_h) + [\dots].\end{aligned}$$

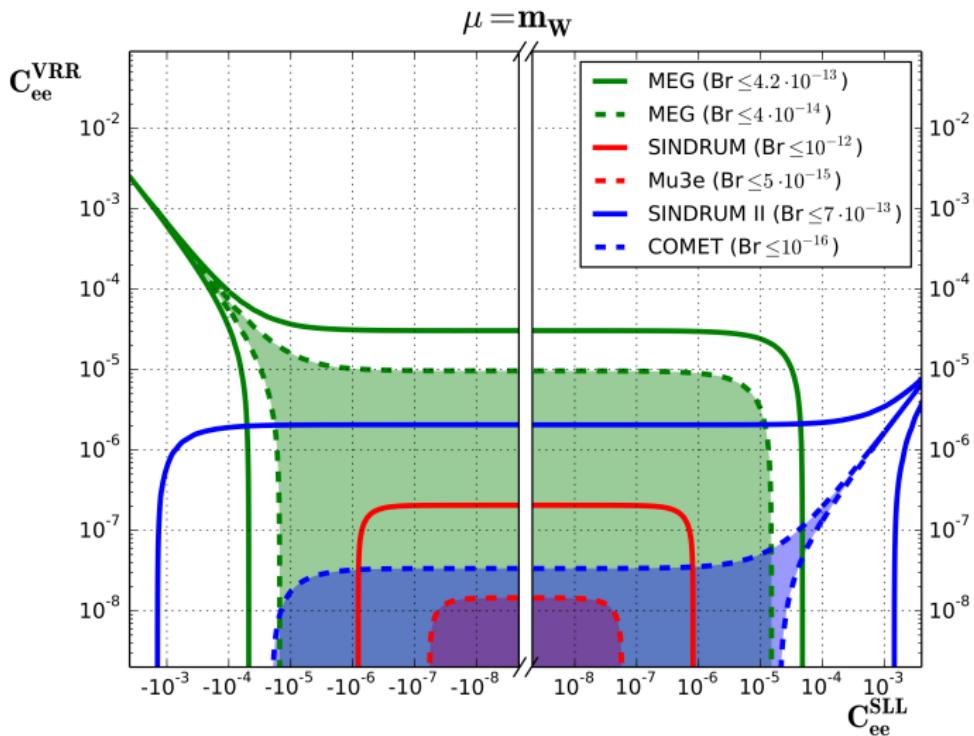
In absence of interplay at the EWSB scale

	$\text{Br}(\mu^+ \rightarrow e^+ \gamma)$ $4.2 \cdot 10^{-13}$ $4.0 \cdot 10^{-14}$		$\text{Br}(\mu^+ \rightarrow e^+ e^- e^+)$ $1.0 \cdot 10^{-12}$ $5.0 \cdot 10^{-15}$		$\text{Br}_{\mu \rightarrow e}^{\text{Au/Al}}$ $7.0 \cdot 10^{-13}$ $1.0 \cdot 10^{-16}$	
C_L^D	$1.0 \cdot 10^{-8}$	$3.1 \cdot 10^{-9}$	$2.0 \cdot 10^{-7}$	$1.4 \cdot 10^{-8}$	$2.0 \cdot 10^{-7}$	$2.9 \cdot 10^{-9}$
$C_{ee}^{S\,LL}$	$4.8 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$	$8.1 \cdot 10^{-7}$	$5.8 \cdot 10^{-8}$	$1.4 \cdot 10^{-3}$	$2.1 \cdot 10^{-5}$
$C_{\mu\mu}^{S\,LL}$	$2.3 \cdot 10^{-7}$	$7.2 \cdot 10^{-8}$	$4.6 \cdot 10^{-6}$	$3.3 \cdot 10^{-7}$	$7.1 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$
$C_{\tau\tau}^{S\,LL}$	$1.2 \cdot 10^{-6}$	$3.7 \cdot 10^{-7}$	$2.4 \cdot 10^{-5}$	$1.7 \cdot 10^{-6}$	$2.4 \cdot 10^{-5}$	$3.5 \cdot 10^{-7}$
$C_{\tau\tau}^{T\,LL}$	$2.9 \cdot 10^{-9}$	$9.0 \cdot 10^{-10}$	$5.7 \cdot 10^{-8}$	$4.1 \cdot 10^{-9}$	$5.9 \cdot 10^{-8}$	$8.5 \cdot 10^{-10}$
$C_{bb}^{S\,LL}$	$2.8 \cdot 10^{-6}$	$8.6 \cdot 10^{-7}$	$5.4 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$9.0 \cdot 10^{-7}$	$1.2 \cdot 10^{-8}$
$C_{bb}^{T\,LL}$	$2.1 \cdot 10^{-9}$	$6.4 \cdot 10^{-10}$	$4.1 \cdot 10^{-8}$	$2.9 \cdot 10^{-9}$	$4.2 \cdot 10^{-8}$	$6.0 \cdot 10^{-10}$
$C_{ee}^{V\,RR}$	$3.0 \cdot 10^{-5}$	$9.4 \cdot 10^{-6}$	$2.1 \cdot 10^{-7}$	$1.5 \cdot 10^{-8}$	$2.1 \cdot 10^{-6}$	$3.5 \cdot 10^{-8}$
$C_{\mu\mu}^{V\,RR}$	$3.0 \cdot 10^{-5}$	$9.4 \cdot 10^{-6}$	$1.6 \cdot 10^{-5}$	$1.1 \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	$3.5 \cdot 10^{-8}$
$C_{\tau\tau}^{V\,RR}$	$1.0 \cdot 10^{-4}$	$3.2 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$4.8 \cdot 10^{-6}$	$7.9 \cdot 10^{-8}$
$C_{bb}^{V\,RR}$	$3.5 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$	$6.7 \cdot 10^{-5}$	$4.8 \cdot 10^{-6}$	$6.0 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$
C_{bb}^{RA}	$4.2 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$6.5 \cdot 10^{-3}$	$4.6 \cdot 10^{-4}$	$1.3 \cdot 10^{-3}$	$2.2 \cdot 10^{-5}$
C_{bb}^{RV}	$2.1 \cdot 10^{-3}$	$6.4 \cdot 10^{-4}$	$6.7 \cdot 10^{-5}$	$4.7 \cdot 10^{-6}$	$6.0 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$

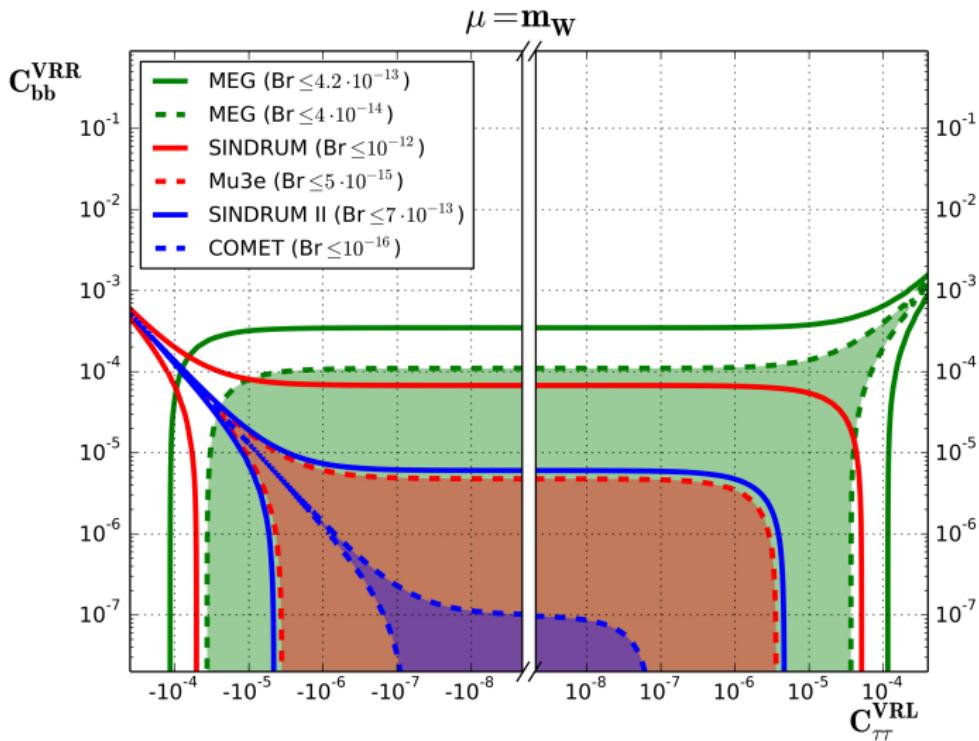
Limits on the various coefficients $C_i(m_W)$ from current and future experimental constraints, assuming that (at the high scale m_W) only one coefficient at a time is non-vanishing and not including operator-dependent efficiency corrections.

Interplay at the EWSB scale

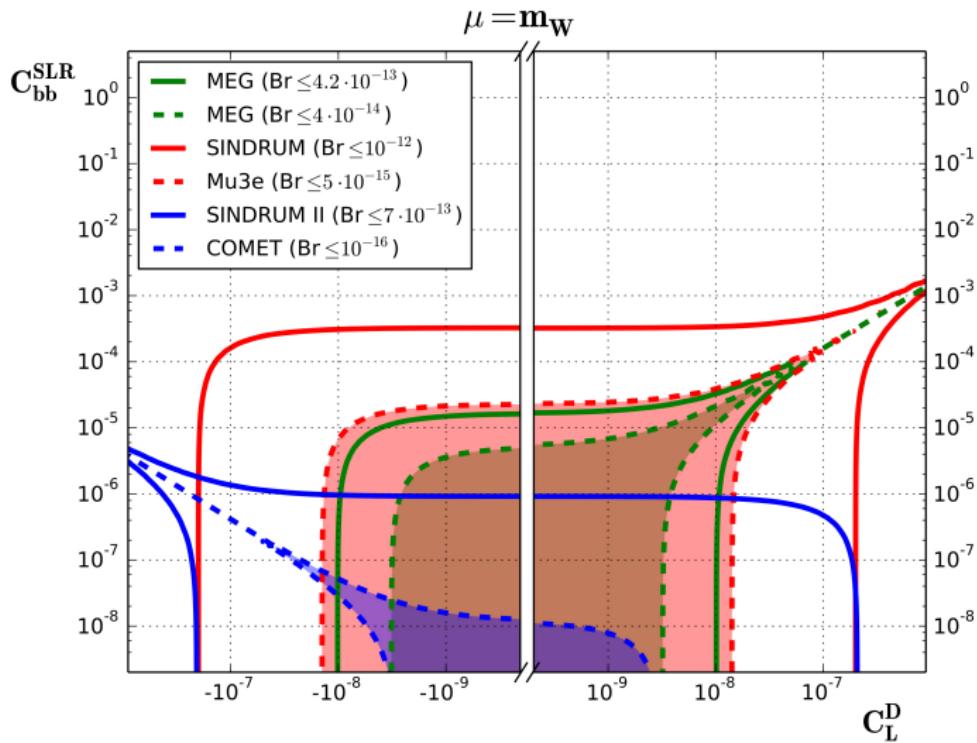
Mu3e money plot



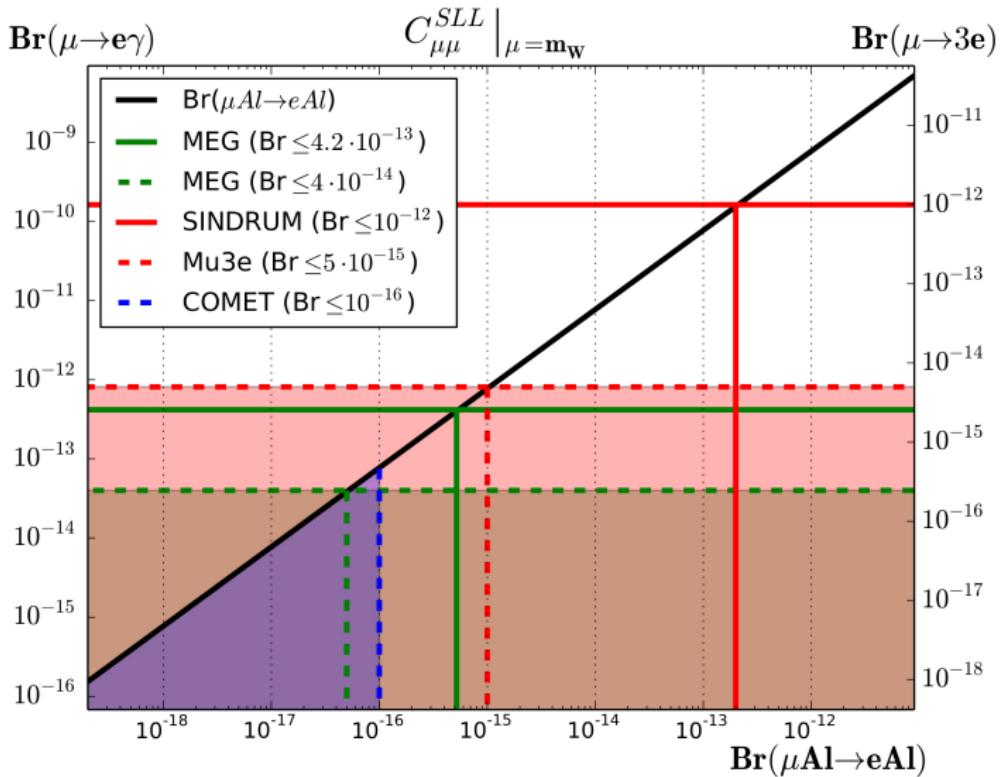
Interplay at the EWSB scale COMET/Mu2e money plot (1)



Interplay at the EWSB scale COMET/Mu2e money plot (2)



MEG/MEG-II money plot



Precise calculations for cLFV backgrounds

Leptonic radiative and rare decays are known at the Next-to-leading order in the Fermi Theory.

- $l_1 \rightarrow l_2 + \gamma + 2\nu$

M. Fael, L. Mercolli and M. Passera, JHEP **07** (2015) 153

GMP, A. Signer and Y. Ulrich, Phys. Lett. B **772** (2017) 452

- $l_1 \rightarrow 3l_2 + 2\nu$

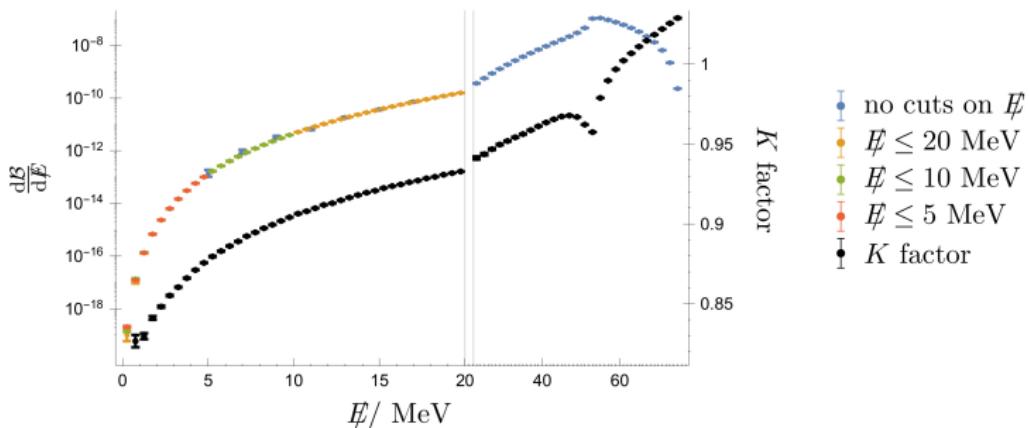
M. Fael, C. Greub, JHEP **1701**, 084 (2017)

GMP, A. Signer and Y. Ulrich, Phys. Lett. B **765** (2017) 280

Fully differential NLO Monte Carlo for the radiative/rare decays of a polarised lepton is now available.

Predictions can be tailored on future experiments, arbitrary kinematic cuts can be implemented!

Rare decay at the NLO: the outcome



The differential decay distribution w.r.t. the invisible \not{E} at NLO in blue, orange, green and red (see text) and the K factor NLO/LO in black. To emphasize the low energy tail, the scaling is broken at $\not{E} = 20$ MeV. The error bars indicate the numerical error of the Monte Carlo integration.

There are substantially fewer background events than expected from the naive tree-level simulations.

Conclusion

- ✓ CPV and LFV phenomena are forbidden in the minimal SM
 - Neutrino sector seems to ignore this fact, calling for something **more** than the minimal theoretical setup
 - Charged sector seems to take the job seriously
 - ✓ If NP lives at very high energy, then consistent EFT techniques can be adopted to extract information of new physics at high scale from low-energy observables
 - ✓ Precise background calculations are important to improve the experimental limits
 - ✓ From limits on leptonic FV and EDM one can gain information on the parameter space of possible UV-complete BSM theories
 - ✓ “MEG tests the dipole interaction, and Mu2e & Mu2e test the contact interaction” is wrong, false, misleading, *et cetera*.

Acknowledgements

“Lepton flavour violation in an effective-field-theory
approach with dimension 6 operators”

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