General Discussion on $g - 2$

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2nd Workshop on flavour changing and conserving processes (FCCP2017) – Anacapri, September 7 - 9, 2017

What is new?

What is new?

What needs to be done?

What is new since FCCP2015?

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What needs to be done for FCCP2019?

General context: rather intriguing...

Direct searches at colliders (the energy frontier) have so far not provided evidence of physics beyond the SM

Indirect indications exist, either from the cosmic frontier (dark matter,...) or from the precision frontier

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Among them, LHCb anomalies seen in several observables related to flavour-changing neutral-current $b \to s\ell^+\ell^-$ transitions have recently attracted the attention $\longrightarrow$ cf. Marina Calvi's talk

Deviations occur in the decay channels with $\mu^+\mu^-$, the corresponding channels with e^+e^- looking SM-like
$\longrightarrow$ violation of LFU? $\longrightarrow$ cf. Ferruccio Feruglio's talk

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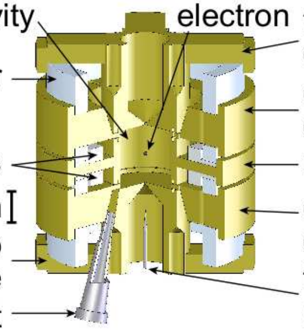
A coherent picture when combined with another discrepancy, at the level of $\sim 3.5\sigma$, between the measured value of the anomalous magnetic moment of the muon and its predicted value in the SM, whereas $a_e^{\rm exp} - a_e^{\rm SM} = -0.9(8) \cdot 10^{-12}$

a_μ and a_e are measured to very high precisions



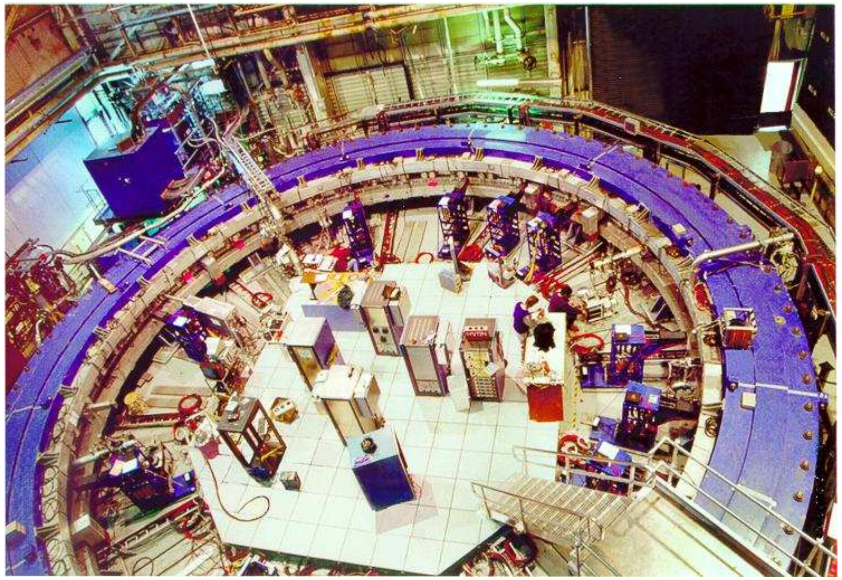
$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13} \text{ [0.24ppb]}$$

D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)

- $\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}$ s

$$\gamma \sim 29.3,\ p \sim 3.094 \text{ GeV/c}$$



$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \text{ [0.54ppm]}$$

G. W. Bennett et al, Phys Rev D 73, 072003 (2006)

- $\tau_\tau = (290.6 \pm 1.1) \times 10^{-15}$ s

$-0.007 < a_\tau^{\text{exp}} < +0.005$

G. A. Gonzalez-Springberg, A. Santamaria, J. Vidal, Nucl. Phys. B 582, 3 (2000)

theory: $a_\tau = 117721(5) \cdot 10^{-8}$

S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)

S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)

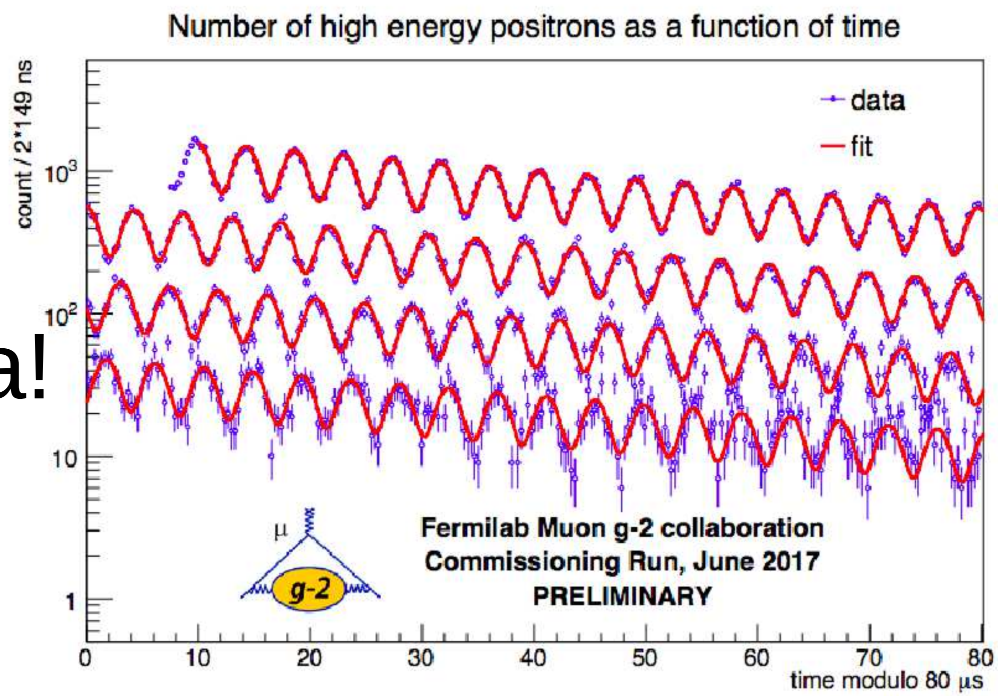
Differences (in m_ℓ, τ_ℓ, and hence in $\Delta a_\ell^{\text{exp}}$) come uniquely from the Yukawa couplings (the only source of LFU violation in the SM!)

Improve experimental precision on a_μ by a factor of four

FNAL E989:
There are new data!

⟶ cf. Tim Gorringe's talk

June 2017 ~ 700,000 positrons (~2 weeks)



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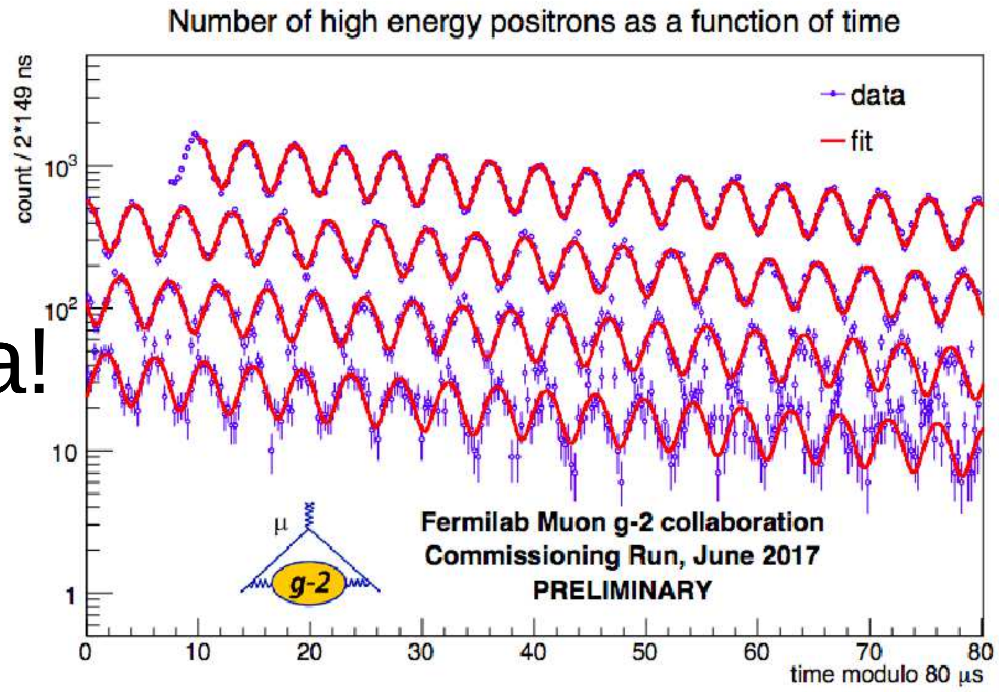

J-PARC E34:
Positive recommendations from TDR Review Committee
Construction phase is starting

⟶ cf. Glen Marshall's talk

News from the theory front line (I): QED

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Issue: computing high perturbative orders

$\longrightarrow$ cf. Stefano Laporta's talk

Situation at FCCP2015

- Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)
C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)
A. Petermann, Helv. Phys. Acta 30, 407 (1957)
H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)
S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)
S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$ (not relevant at present precisions)

- Values of $C_\ell^{(8)}$ and $C_\ell^{(10)}$ computed numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); D 85, 093013 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012); Phys. Rev. D 91, 033006 (2015)

order $(\alpha/\pi)^4$: 891 diagrams

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373) \cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118) \cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28) \cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz et al., Phys. Rev. D 92, 073019 (2015); arXiv:1602.02785 [hep-ph]

Agreement at the level of accuracy required by present (and future) experiments for a_μ

Important cross-check, since $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim a_\mu^{\text{QED}}(\alpha^4)$

Cross-check for $C_\mu^{(10)}$? $C_\mu^{(10)}(\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$ $\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \longrightarrow \sim 1.6 \cdot 10^{-10}$

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A. Kurz et al., Phys. Rev. D 92, 073019 (2015); arXiv:1602.02785 [hep-ph]

$A_1^{(8)}$ has also been computed analytically! $A_1^{(8)} = -1.912\,245\,764\,926\,4\ldots$

S. Laporta, Phys. Lett. B 772, 232 (2017) [arXiv:1704.06996 [hep-ph]]

Main order α^4 contribution to a_e!

Cross-check for $C_e^{(10)}$? $C_e^{(10)}(\alpha/\pi)^5 \sim 5\cdot 10^{-13}$ $\Delta a_e^{\rm exp} = 2.8\cdot 10^{-13} \longrightarrow$?

News from the theory front line (II): HVP

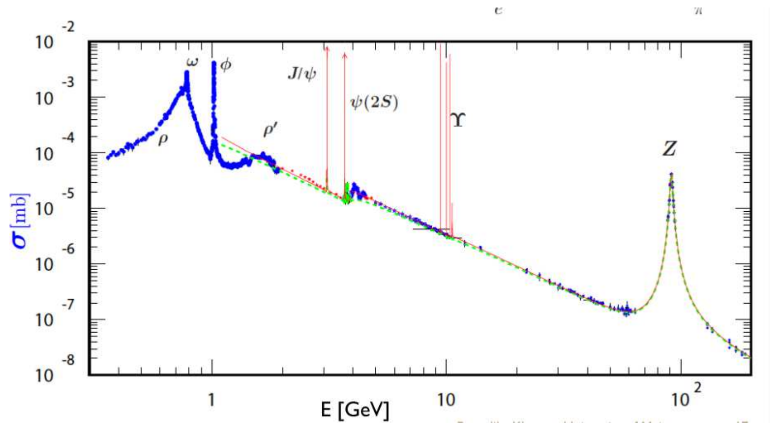
Issue: taming non-perturbative aspects of QCD

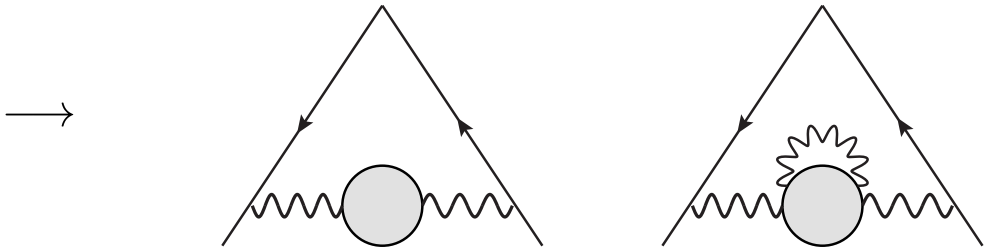
HVP from $e^+e^- \to \mathrm{hadrons}$

Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \qquad K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)





Note that some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons
- some radiative exclusive modes, e.g. $\pi^0\gamma$, $\eta\gamma$

$$a_\mu^{\pi^0\gamma}(600\ \mathrm{MeV} - 1030\ \mathrm{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

New determinations from $e^+e^- \to \text{hadrons}$

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)	
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)	$\sim 0.6\%$
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)	
$693.1(3.4)$	M. Davier et al., arXiv:1706.09436 [hep-ph]	
$692.23(2.54)$	A. Keshavarzi et al., talk at Fermilab g-2 Workshop, 2017	$\sim 0.4\%$
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.002633 [hep-ph]	

$\longrightarrow$ talks by Daisuke Nomura, Fedor Ignatov, Achim Denig

Some tensions remain

Experiment	$a_\mu^{\text{HVP}-\text{LO}\,2\pi}(600 - 900\ \text{MeV})$
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)
KLOE comb.	366.7(2.2) preliminary! S.E. Müller, Fermilab Worshop, June 2017
BaBar 09	376.7(2.0)(1.9)
BESIII 16	368.2(2.5)(3.3)
SND 04	371.7(5.1)
CMD-2 03,06	372.4 (3.5)

HVP from the space-like region (from Bhabha or μe scattering)?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\rm HVP} - \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\rm had}(-\frac{x^2}{1-x}m_\mu^2)$

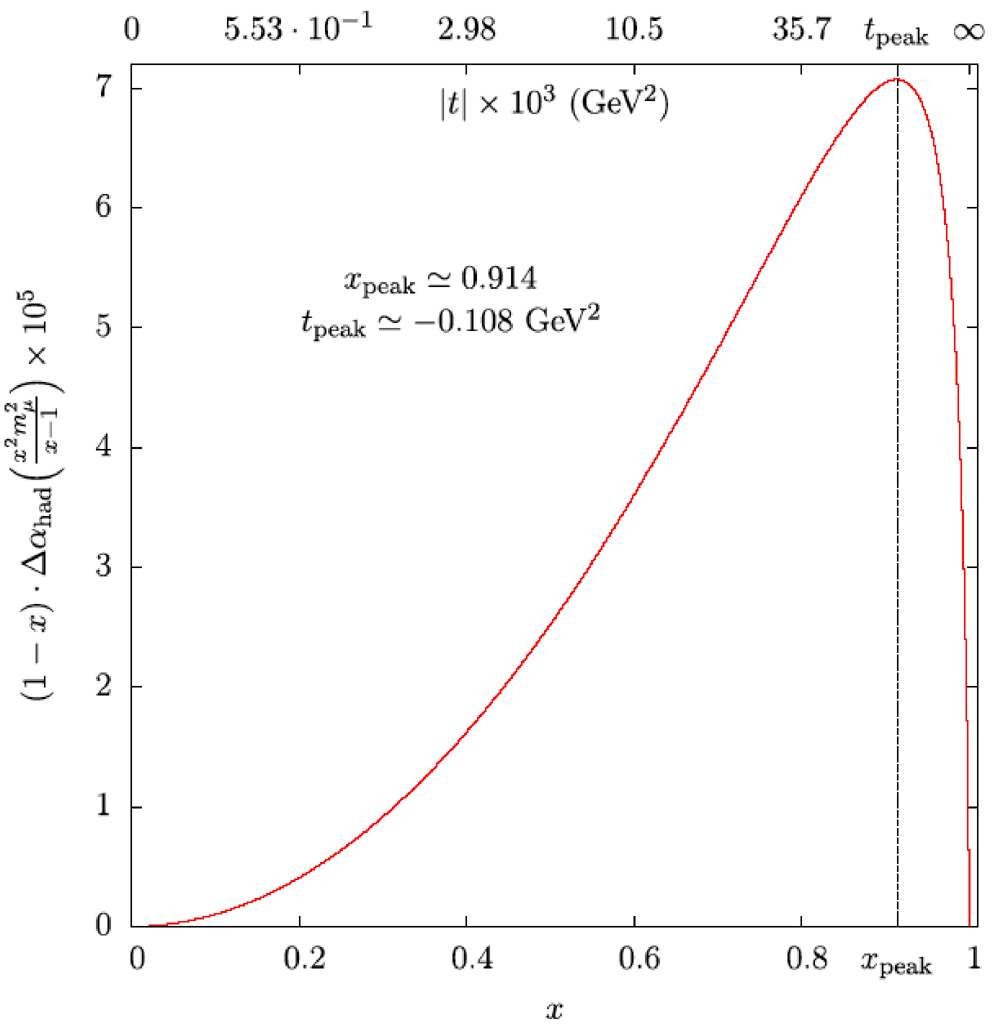
$t = \frac{x^2 m_\mu^2}{x-1}$ $0 \le -t < +\infty$ $0 \le x < 1$

- $a_\mu^{\rm HVP}$ given by the integral

- measurement of $\Delta\alpha_{\rm had}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 $\mu/{\rm s}$ (CERN)



$\longrightarrow$ talks by Luca Trentadue, Umberto Marconi, Pierpaolo Mastrolia, Fulvio Piccinini

If feasible, interesting cross-check of time-like determinations of HVP

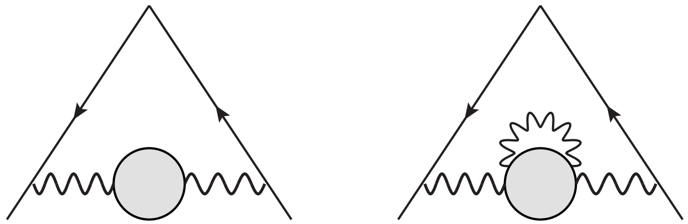
HVP from lattice QCD determinations?

Several groups are producing results, e.g. $\longrightarrow$ more in Marina Marinkovic's talk

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$	
$667(6)(12)$	HPQCD, C. Davies, Fermilab Workshop, June 2017
$654(32)^{+21}_{-23}$	H. Wittig, Fermilab Workshop, June 2017, arXiv:1705.01775 [hep-lat]
$704(9)(7)(13)(6)$	BMWc, L. Lellouch, Fermilab Workshop, June 2017, arXiv: 1612.02364 [hep-lat]

Still quite large uncertainties, but steady progress

Comparison with data at the sub-percent level requires to deal with isospin breaking effects (radiative corrections)



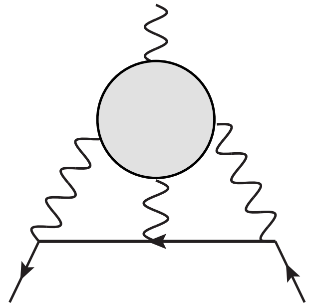
(Experimentalists don't live in the theoretician's paradise)

News from the theory front line (III): HLxL

Issue: taming non-perturbative aspects of QCD

Not related, as a whole, to an experimental observable...

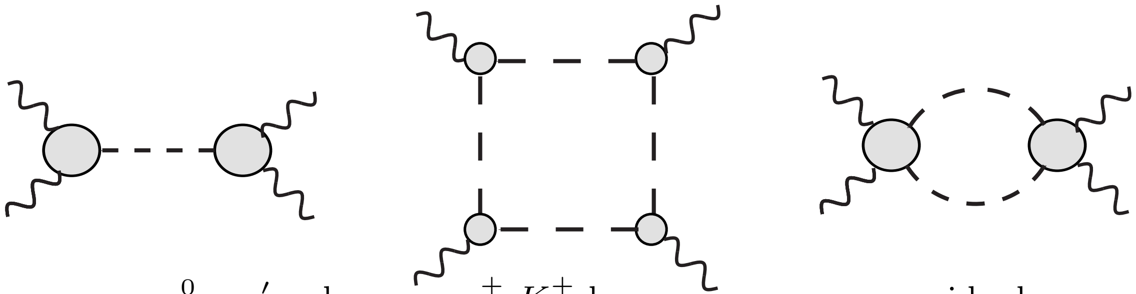
? $\longrightarrow$



Involves a much more complicated object, the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

Many individual contributions have been identified...



$+\cdots$

$$\Pi = \Pi^{\pi^0,\eta,\eta'\ \text{poles}} + \Pi^{\pi^\pm,K^\pm\ \text{loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

Π^{residual}: other intermediate states (3π)...

Present estimates still rely on two model-dependent calculation

$$a_\mu^{\text{HLxL}} = +(8.3 \pm 3.2) \cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\text{HLxL}} = +(89.6 \pm 15.4) \cdot 10^{-11}$$

M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)
M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

$\longrightarrow$ more in Johan Bijnens's talk

Provide useful hints on the expected sizes of various contributions

More recently other approaches have been developed:

- dispersive methods $\longrightarrow$ cf. Gilberto Colangelo's talk

  require input for form factors, either from experiment, or from lattice

  $\longrightarrow$ cf. Andreas Nyffeler's talk A. Gerardin, H. B. Meyer, A. Nyffeler, Phys. Rev. D 94 074507 (2016)

- lattice QCD $\longrightarrow$ cf. Christoph Lehner's talk

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)
N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

Recently:

- Reevaluation of single meson exchanges...

$$a_\mu^{\mathrm{HLxL}}(f_1, f_1') = 6.4(2.0) \cdot 10^{-11}; \; a_\mu^{\mathrm{HLxL}}(f_0, f_0', a_0) = (-1 \text{ to } -4) \cdot 10^{-11}; \; a_\mu^{\mathrm{HLxL}}(f_2, f_2', a_2, a_2') = 1.1(0.1) \cdot 10^{-11}$$

V. Pauk, M. Vanderhaeghen, Eur. Phys. J C 74, 3008 (2014)

$$a_\mu^{\mathrm{HLxL}}(a_1, f_1, f_1') = 7.51(2.71) \cdot 10^{-11}$$

F. Jegerlehner, EPJ Web Conf. 118 (2016)

- ...or of pion box and scalar-exchange (S-wave $\pi\pi$ rescattering) contributions (but only pion contribution to left-hand cut)

$$a_\mu^{\pi\ \mathrm{box}} \sim -24(1) \cdot 10^{-11}$$

G. Colangelo et al., JHEP 1704 (2017)

Updated estimate

$$a_\mu^{\mathrm{HLxL}} = +(10.3 \pm 2.9) \cdot 10^{-10}$$

F. Jegerlehner, arXiv:1705.002633 [hep-ph]

Conclusions and perspectives (for FCCP2019?)

First data successfully taken by FNAL E989 this summer

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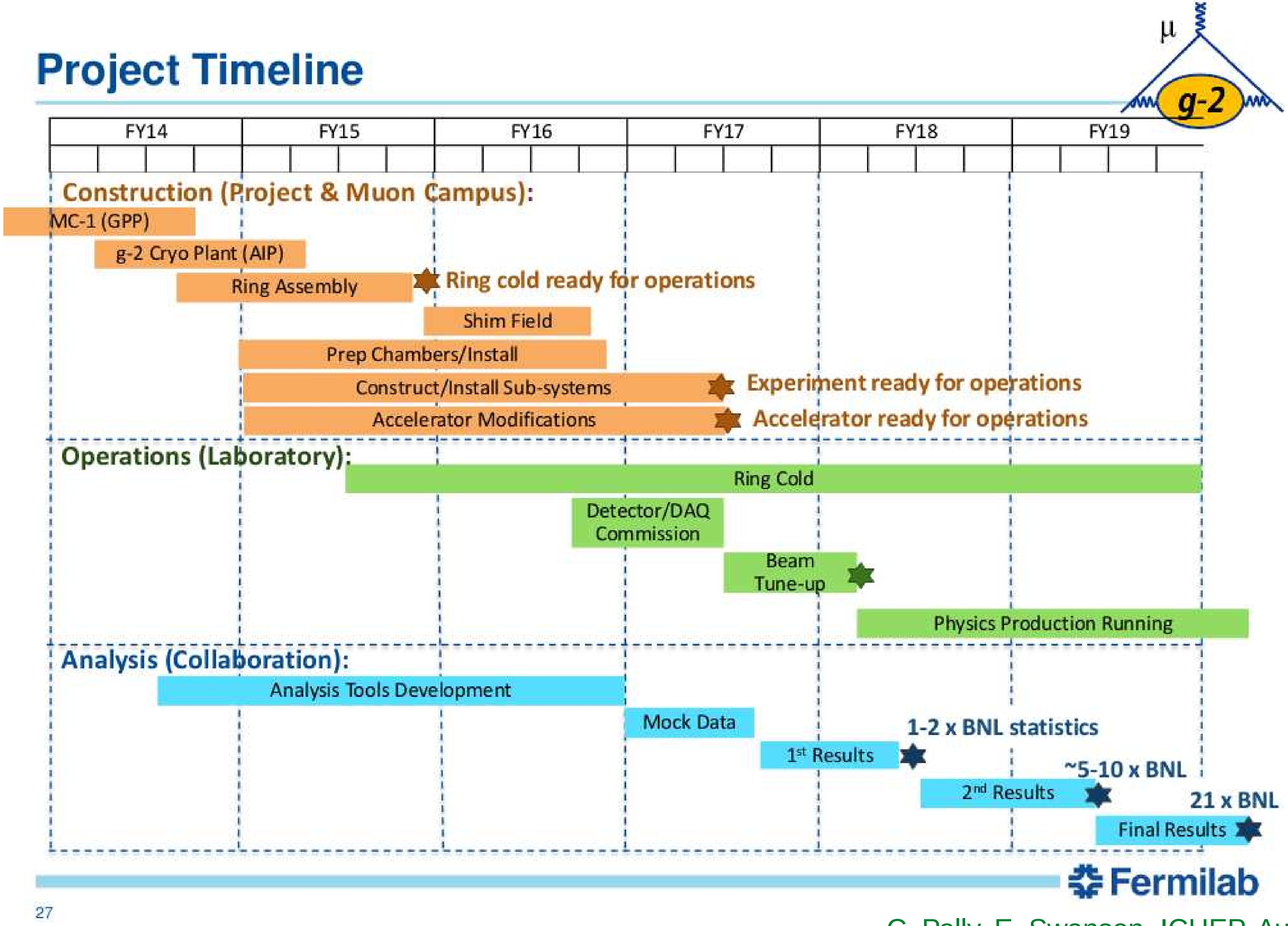


C. Polly, E. Swanson, ICHEP, Aug. 2016

Results with statistics comparable to BNL E821 available by the end of 2018?

Experiment E34 at J-PARC in progress (full approval and funding for FCCP2019?)

Operates under completely different experimental conditions

Important to provide independent cross-check of BNL E821 and FNAL E989 results

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

Next steps?

- $A_2^{(10)}(m_\mu/m_e)$ for a_μ
- $A_1^{(10)}$ for a_e

HVP

New estimates based on 39 measured exclusive channels

Precision below the 0.5% level in relative terms

Some tensions between data remain

$\longrightarrow$ would be interesting to see analysis for the $\pi^+\pi^-$ channel of the data collected at VEPP at FCCP2019 (or before...)

Interesting possibilities for cross-checks, either from

- Bhabha or $e\mu$ scattering

$\longrightarrow$ first data at FCCP2019?

- Lattice QCD

(several groups, different strategies to overcome challenging difficulties)

$\longrightarrow$ determination at 2% -3% with controlled systematics (incl. IB) at FCCP2019?

HLxL

Dispersive evaluation

- implementation of short-distance constraints
- estimate for Π^{residual}? Cf. axial vectors (leading in large-N_c) $\to 3\pi$ channel
- form factors to be provided from data and/or lattice QCD

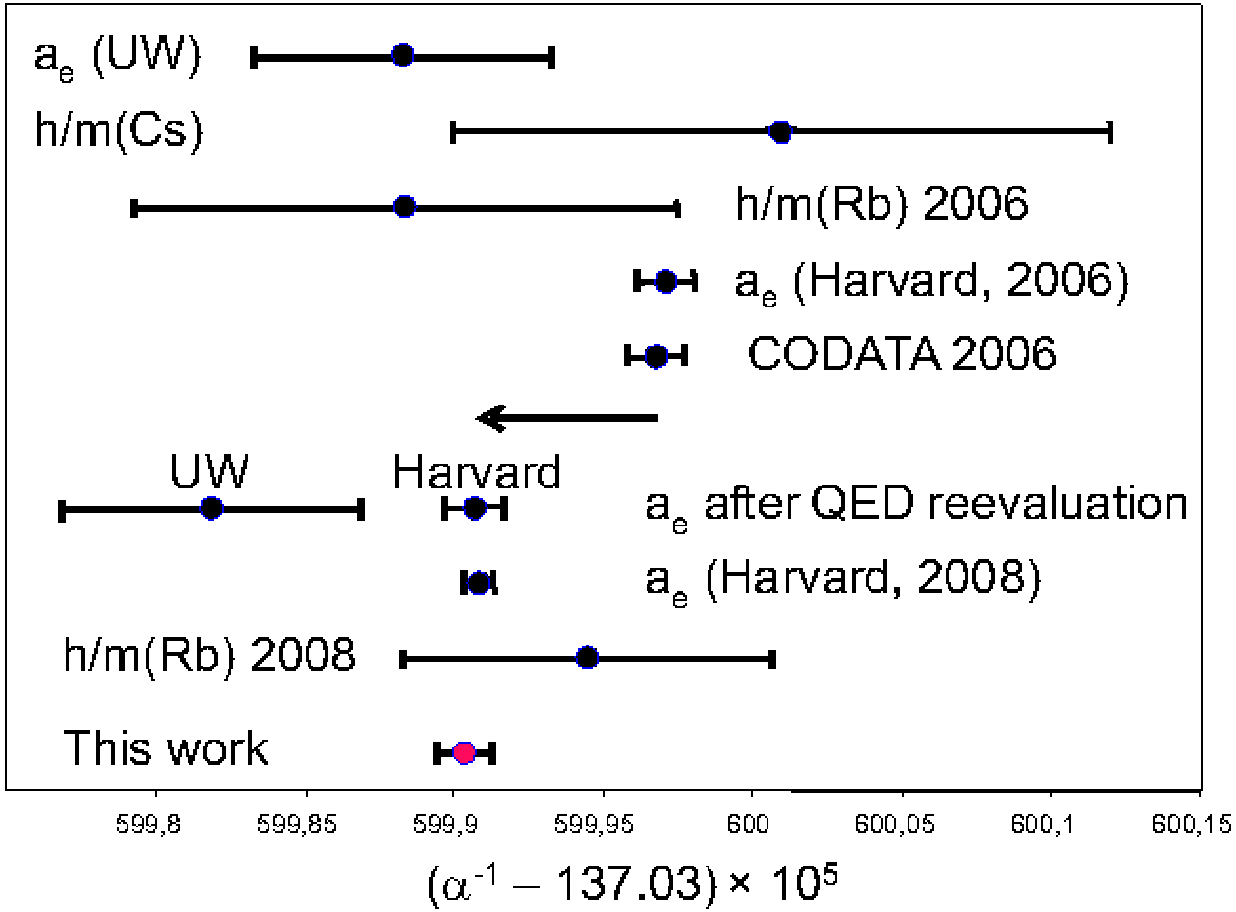
- Lattice QCD

(several groups, different strategies to overcome challenging difficulties)

$\longrightarrow$ determination of HLxL at ~ 10% with controlled systematics at FCCP2019?

Thanks for your attention!

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]} \\
&= -1.557(70) && \text{[Kinoshita (1995)]} \\
&= -1.4092(384) && \text{[Kinoshita (1997)]} \\
&= -1.5098(384) && \text{[Kinoshita (2001)]} \\
&= -1.7366(60) && \text{[Kinoshita (2005)]} \\
&= -1.7260(50) && \text{[Kinoshita (2005)]} \\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]} \\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \leftarrow \\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]} \\
&= -1.91298(84) && \text{[Aoyama et al., Phys. Rev. D 91, 033006 (2015)]}
\end{aligned}
$$



R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)