Hadronic Leading Order Contribution to the Muon $g-2$

Daisuke Nomura (Nat'l Inst. Tech., Kagawa)

talk at FCCP 2017 @ Capri Island, Italy

September 7, 2017

Based on collaboration with
Alex Keshavarzi and Thomas Teubner **(KNT)**

http://www-conf.kek.jp/muonHVPws/index.html

Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

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About

The muon g-2 is arguably one of the most important observables in contemporary particle physics. The long-standing anomaly at the level of 3 standard deviations between the experimental value and the Standard Model (SM) prediction of the muon g-2 may indicate the existence of new physics beyond the SM, which has

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an activity of the $g-2$ Theory Initiative

Muon $g-2$: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma}\ \text{(spin)}, \qquad g = 2 + 2F_2(0)\right)$$

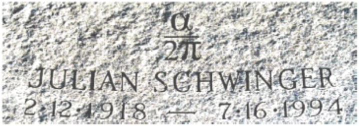
where

$$\overline{u}(p+q)\Gamma^\mu u(p) = \overline{u}(p+q)\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_\mu^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}]$$

(Bennett *et al*)

★ **Extremely useful** in **probing/constraining physics beyond the SM**

a_μ^{SM}: update HLMNT11 → KNT17 presented @ TGM2

	2011		2017	*to be discussed
QED	11658471.81 (0.02)	⟶	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	⟶	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	⟶	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*

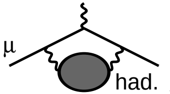
	HLMNT11		**KNT17**		Davier et al (2017)
LO HVP	694.91 (4.27)	⟶	692.23 **(2.54)**	this work*	693.1 (3.4)
NLO HVP	-9.84 (0.07)	⟶	-9.83 (0.04)	this work*	

NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *

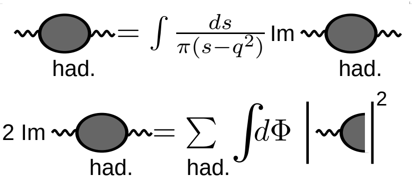
Theory total	11659182.80 (4.94)	⟶	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	⟶	28.1 (7.3)	this work
Δa_μ	3.3σ	⟶	3.9σ	this work

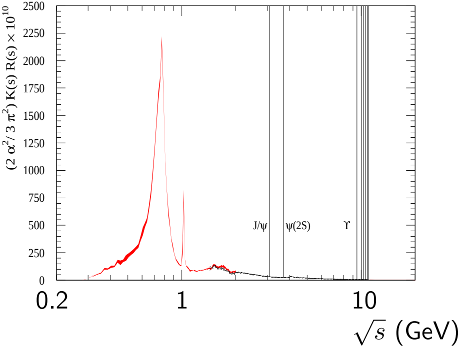
Slide by T. Teubner (Liverpool), at PhiPsi17, June 26-29, 2017

The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.





$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

Introduction

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To ensure reliable results with increasing levels of precision, what are *now* the main points of concern when correcting, combining and integrating data to evaluate $a_\mu^{\text{had, VP}}$?

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⇒ Radiative corrections of data and the corresponding error estimate

Slide by A. Keshavarzi (Liverpool), at '1st Workshop of Muon $g-2$ Theory Initiative' June 3-6, 2017

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→ ...how to best amalgamate large amounts of data from different experiments

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⇒ The reliability of the integral and error estimate

Slide by A. Keshavarzi (Liverpool), at '1st Workshop of Muon $g-2$ Theory Initiative' June 3-6, 2017

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⇒ The reliability of the integral and error estimate

⇒ The choices when estimating unmeasured hadronic final states

Slide by A. Keshavarzi (Liverpool), at '1st Workshop of Muon $g-2$ Theory Initiative' June 3-6, 2017

The previous analysis... [HLMNT(11), J. Phys. G**38** (2011), 085003]

⇒ **Back in 2011...**

→ Cross section measurements from radiative return

→ Correlated experimental uncertainties* !!

→ Large radiative correction uncertainties*

→ Constant cross section clusters*

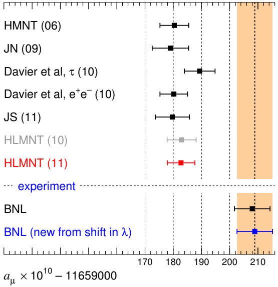
→ Non-linear χ^2 minimisation fitting nuisance parameters*

→ Trapezoidal rule integration

→ Reliance on isospin estimates* !!

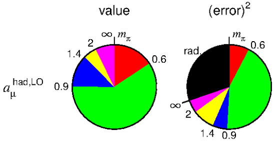
$$a_\mu^{\rm had,LOVP} = 694.9 \pm 3.7_{\rm exp} \pm 2.1_{\rm rad} = 694.9 \pm 4.3_{\rm tot}$$

$$a_\mu^{\rm had,NLOVP} = -9.8 \pm 0.1$$



*** Areas for improvement!!**

⇒ Changes in any of these areas can have drastic effect on mean value and error



!! e.g. - KNT 16/03/17 result - $693.9 \pm 2.6_{\rm tot}$!!

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Vacuum polarisation corrections (!!)

⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0]

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\rm had}(s) = \sigma_{\rm had}(s)|1-\Pi(s)|^2$

⇒ Applied to all dressed experimental data in all channels

→ Accurate to $\mathcal{O}(1‰)$ precision

⇒ If correcting data, apply corresponding radiative correction uncertainty

→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

⇒ Influence/need for VP corrections has changed over time

→ Less prominent in some dominant channels

⇒ Undressing of narrow resonances must be done *excluding* the contribution from the resonance

→ ...or would double count contribution

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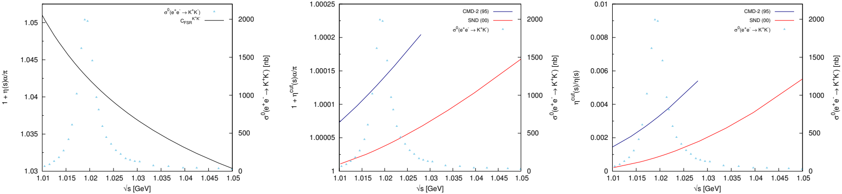
Final state radiation corrections

⇒ For $\pi^+\pi^-$, FSR more frequently included

→ If not, must include through sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For K^+K^-, is there available phase space for the creation of hard photons?



⇒ Choose to no longer apply FSR correction for K^+K^-

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

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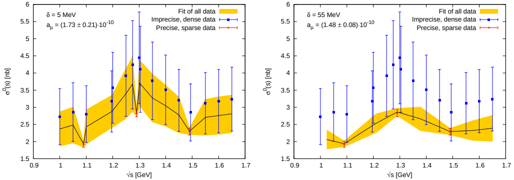
Clustering data

⇒ Re-bin data into *clusters*

Better representation of data combination through adaptive clustering algorithm



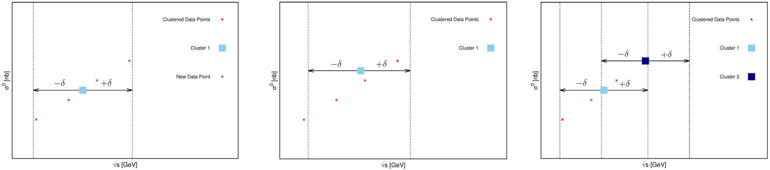
→ More and more data ⇒ risk of over clustering

⇒ loss of information on resonance

→ Scan cluster sizes for optimum solution (error, χ^2, check by sight...)

⇒ Scanning/sampling by varying bin widths

→ Clustering algorithm now adaptive to points at cluster boundaries



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Correlation and covariance matrices

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

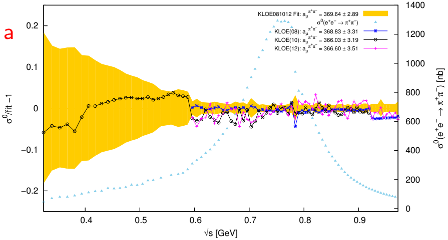
KNT17 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Matrices **must** satisfy properties of a covariance matrix

e.g. - KLOE $\pi^+\pi^-(\gamma)$ combination covariance matrices update

⟹ Originally, NOT a positive semi-definite matrix:

(This is not an example of bias)



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KLOE as an example: Constructing the KLOE $\pi^+\pi^-(\gamma)$ combination covariance matrices (!!) [preliminary]

⇒ Three measurements of $\sigma^0_{\pi\pi(\gamma)}$ by KLOE

→ KLOE08, KLOE10 and KLOE12

⇒ They are, in part, highly correlated → **must** be incorporated

→ e.g. - KLOE08 and KLOE12 share the same $\pi\pi(\gamma)$ data, with KLOE12 normalised by the measured $\mu\mu(\gamma)$ cross section

⇒ Must ensure construction satisfies required properties of covariance matrices

e.g. - KLOE0810

→ Correlated statistic and systematics

→ Correlations must cover entire data range

→ KLOE08 is more precise than KLOE10

⇒ **Expected influence on non-overlapping data region**

KLOE08 60×60	KLOE0810 60×75	KLOE0812 60×60
KLOE1008 75×60	KLOE10 75×75	KLOE1012 75×60
KLOE1208 60×60	KLOE1210 60×75	KLOE12 60×60

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Linear χ^2 minimisation

⇒ Redefine clusters to have linear cross section

→ Consistency with trapezoidal rule integration

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

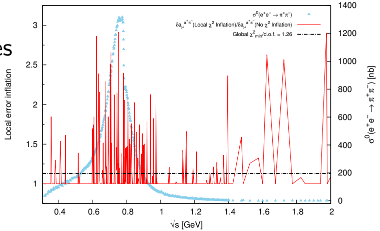
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right)\left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

⇒ Through correlations and linearisation, result is the minimised solution of all neighbouring clusters

→ ...and solution is the product of the influence of all correlated uncertainties

⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

→ ...and any data tensions are reflected in a local χ^2 error inflation



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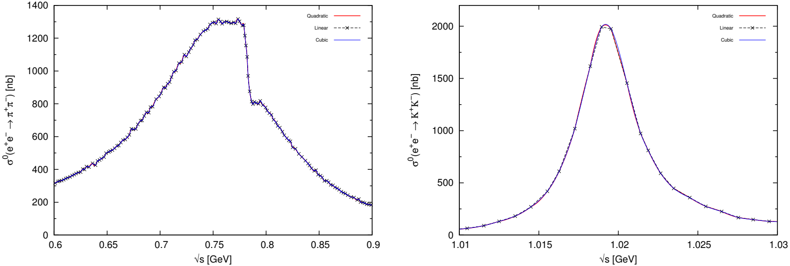
Integration

⇒ Trapezoidal rule integral

→ Consistency with linear cluster definition

→ High data population ∴ Accurate estimate from linear integral



→ Higher order polynomial integrals give (at maximum) differences of $\sim 10\%$ of error

⇒ Estimates of error non-trivial at integral borders

⟶ Extrapolate/interpolate covariance matrices

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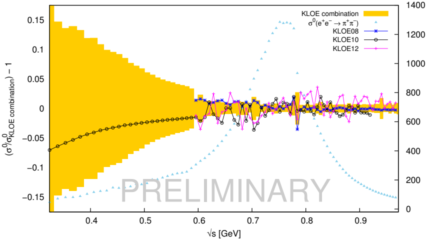
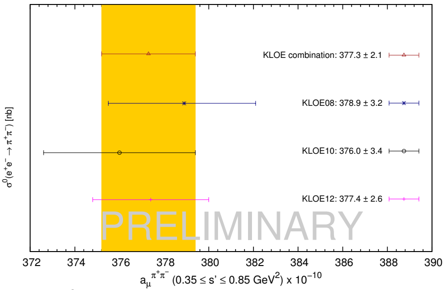
KLOE as an example: the resulting KLOE $\pi^+\pi^-(\gamma)$ combination (!!) [preliminary]

$\Rightarrow$ Combination of KLOE08, KLOE10 and KLOE12 gives 85 distinct bins between $0.1 \leq s \leq 0.95$ GeV^2





$\rightarrow$ Covariance matrix now correctly constructed

$\Rightarrow$ a positive semi-definite matrix

$\rightarrow$ Non-trivial influence of correlated uncertainties on resulting mean value

$$a_\mu^{\pi^+\pi^-}(0.1 \leq s' \leq 0.95 \text{ GeV}^2) = (489.9 \pm 2.0_{\text{stat}} \pm 4.3_{\text{sys}}) \times 10^{-10}$$

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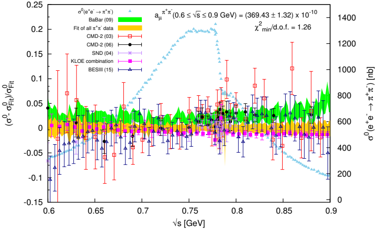
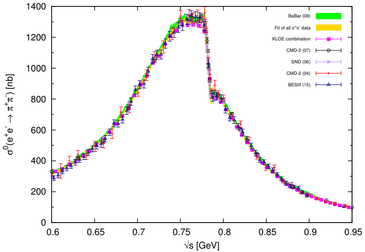
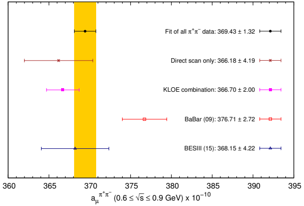
$\pi^+\pi^-$ channel (!!)

⇒ Large improvement for 2π estimate

→ BESIII [Phys.Lett. B753 (2016) 629-638] and KLOE combination provide downward influence to mean value







⇒ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}(0.305 \le \sqrt{s} \le 2.00\ \text{GeV}):$$

HLMNT11: 505.77 ± 3.09

KNT17: 502.85 ± 1.93 (!!)

(no radiative correction uncertainties)

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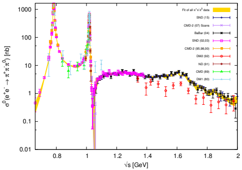
Other notable exclusive channels

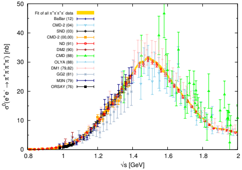
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT17: 47.68 ± 0.70

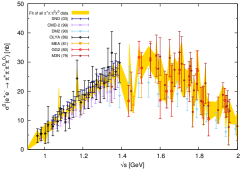
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT17: 15.18 ± 0.14

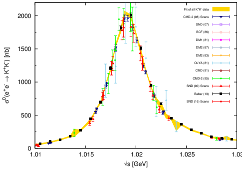
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT17: 20.07 ± 1.19

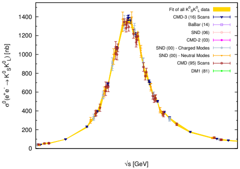
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT17: 22.76 ± 0.22

$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT17: 13.09 ± 0.12

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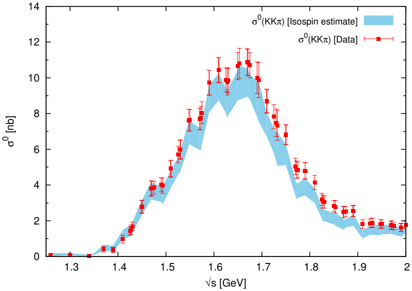
$KK\pi$, $KK\pi\pi$ and isospin (!!)

⇒ New BaBar data for $KK\pi$ and $KK\pi\pi$ removes reliance on isospin (only $K^0_S = K^0_L$)

$\underline{KK\pi}$

$K^0_S K^0_L \pi^0$ [Phys.Rev. D95 (2017), 052001]



HLMNT11: 2.77 ± 0.15

KNT17: 2.83 ± 0.14

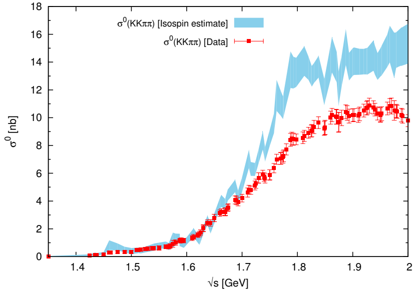
$\underline{KK\pi\pi}$

$K^0_S K^0_L \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002]

$K^0_S K^0_S \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002],

$K^0_S K^0_L \pi^0\pi^0$ [Phys.Rev. D95 (2017), 052001]

$K^0_S K^\pm \pi^\mp \pi^0$ [arXiv:1704.05009]



HLMNT11: 3.31 ± 0.58

KNT17: 2.42 ± 0.09

⇒ **But**, still reliant on isospin estimates for $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $KK3\pi$...

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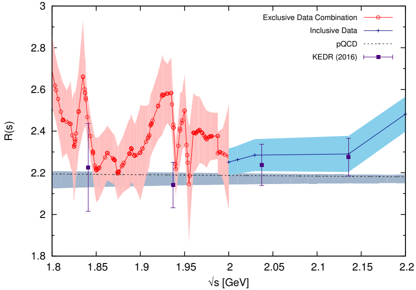
Inclusive

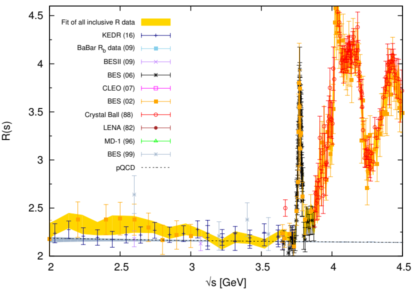
⇒ New KEDR inclusive R data ranging $1.84 \leq \sqrt{s} \leq 3.05$ GeV [Phys.Lett. B770 (2017) 174-181] and $3.12 \leq \sqrt{s} \leq 3.72$ GeV [Phys.Lett. B753 (2016) 533-541]



$a_\mu^{\text{had, LOVP}}(1.84 \leq \sqrt{s} \leq 2.00$ GeV):

pQCD : 6.42 ± 0.03

Data : 6.88 ± 0.25



$a_\mu^{\text{had, LOVP}}(2.60 \leq \sqrt{s} \leq 3.73$ GeV):

pQCD (inflated errors) : 10.82 ± 0.38

Data : 11.20 ± 0.14

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

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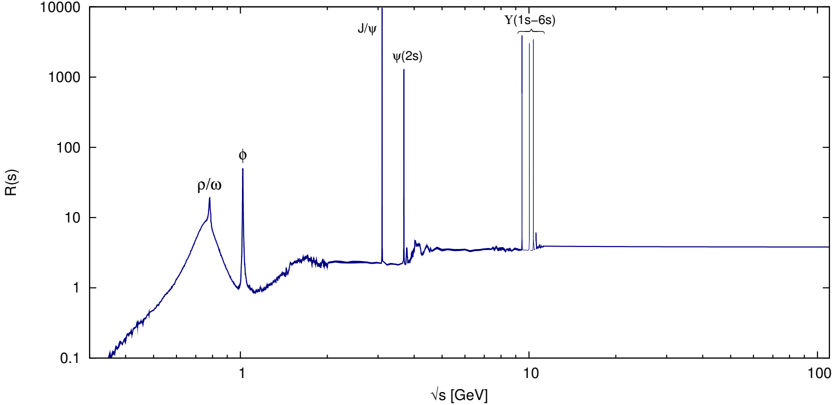
$R(s)$ for $m_\pi \leq \sqrt{s} < \infty$



⇒ Full compilation data set for hadronic R-ratio to be made available soon...

⟹ **...complete with full covariance matrix**

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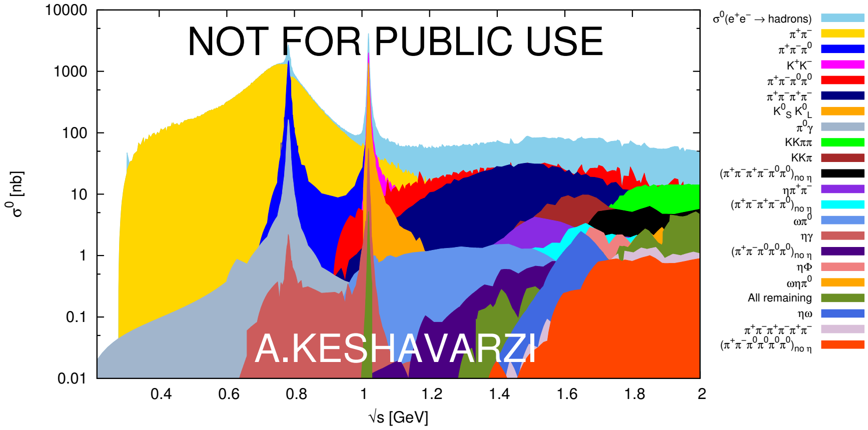
Contributions to mean value below 2GeV



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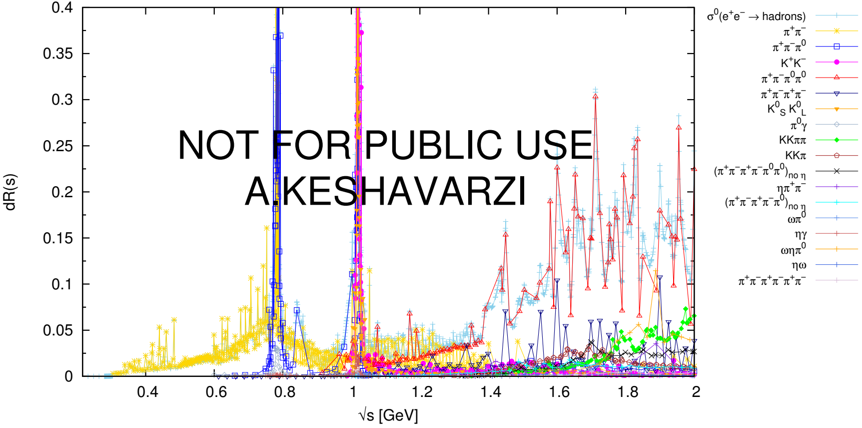
Contributions to uncertainty below 2GeV



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KNT17 $a_\mu^{\text{had, VP}}$ update (!!)

HLMNT(11): 694.91 ± 4.27

↓

!! KNT 16/03/17 result: $693.9 \pm 1.34_{\text{stat}} \pm 2.15_{\text{sys}} \pm 0.32_{\text{vp}} \pm 0.70_{\text{fsr}}$!!

↓

!! Updated KLOE combination covariance matrix construction !!

↓

!! $KK\pi\pi$ determination without isospin !!

↓

!! New VP iteration !!

↓

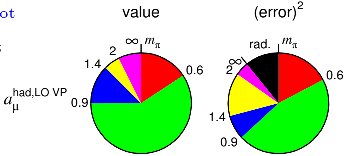
This work: $a_\mu^{\text{had, LOVP}} = 692.23 \pm 1.26_{\text{stat}} \pm 2.02_{\text{sys}} \pm 0.31_{\text{vp}} \pm 0.70_{\text{fsr}}$

$= 692.23 \pm 2.42_{\text{exp}} \pm 0.77_{\text{rad}}$

$= 692.23 \pm 2.54_{\text{tot}}$

$a_\mu^{\text{had, NLOVP}} = -9.83 \pm 0.04_{\text{tot}}$

⇒ Accuracy better then 0.4%
(uncertainties include all available correlations)



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KNT17 a_μ^{SM} update

	2011		2017	*to be discussed
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*
	HLMNT11		**KNT17**	
LO HVP	694.91 (4.27)	$\longrightarrow$	692.23 **(2.54)**	this work*
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.83 (0.04)	this work*
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	28.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.9σ	this work

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Conclusions

Question:
To ensure reliable results with increasing levels of precision, what is the KNT17 approach when correcting, combining and integrating data to evaluate $a_\mu^{\text{had, VP}}$?

✓ Necessary VP and FSR corrections carefully applied with conservative uncertainties

⇒ When combining data...

- ✓ ...adaptive clustering algorithm rebins data into appropriate clusters
- ✓ ...all covariance matrices are correctly constructed with a framework that can accommodate any available correlations
- ✓ ...employ a linear χ^2 minimisation that has been shown to be free from bias

✓ Reliable trapezoidal rule integral with mean value and error on solid ground

✓ Less reliance on isospin for estimated states with more measured final states

✓ Continuously adapt and improve...

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http://www-conf.kek.jp/muonHVPws/index.html

Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

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About

The muon g-2 is arguably one of the most important observables in contemporary particle physics. The long-standing anomaly at the level of 3 standard deviations between the experimental value and the Standard Model (SM) prediction of the muon g-2 may indicate the existence of new physics beyond the SM, which has

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an activity of the $g-2$ Theory Initiative