Lepton Flavour Universality in B decays

[on the importance of EW corrections for B-anomalies]

Anacapri, September 9th 2017

Workshop on
``Flavour changing and conserving processes"
(FCCP2017)

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Based on:
- F., P. Paradisi and A. Pattori 1606.00524 and 1705.00929
- C. Cornella, F., P. Paradisi, in preparation

Plan

- Hints of LFU violation and New Physics above the ew scale
- Electroweak corrections from the NP scale Λ down to scales $\leq m_b$
- Impact on LFU-violating and LFV transitions
- Discussion

Main message
- relevance of EW corrections when addressing B-anomalies
- simultaneous explanation of both $R_{K(*)}$ and $R_{D(*)}$ anomalies through V-A interactions challenged by electroweak RGE effects

Hints of violation of LFU in semileptonic B decays

NC b -> s [1-loop in SM]

[see talk by Marta Calvi]

$$R_{K^*}^{\mu/e} = \frac{\mathcal{B}(B \to K^* \mu\bar{\mu})_{\rm exp}}{\mathcal{B}(B \to K^* e\bar{e})_{\rm exp}}\Bigg|_{q^2\in[1.1,6]{\rm GeV}} = 0.69\ {}^{+\ 0.11}_{-\ 0.07}\,({\rm stat}) \pm 0.05\,({\rm syst})$$

[LHCb, 1705.05802
SM at 2.4-2.5σ]

$$R_K^{\mu/e} = \frac{\mathcal{B}(B \to K \mu\bar{\mu})_{\rm exp}}{\mathcal{B}(B \to K e\bar{e})_{\rm exp}}\Bigg|_{q^2\in[1,6]{\rm GeV}} = 0.745^{+0.090}_{-0.074} \pm 0.036\ ,$$

[LHCb, 1406.6482
SM at 2.6σ]

- allowing NP, global fits to b -> s transitions is consistent.
- solutions have a pull ~4-5σ w.r.t. the SM and prefer NP in muon channel.

CC b -> c [tree-level in SM]

$$R_{D^*}^{\tau/\ell} = \frac{\mathcal{B}(B \to D^* \tau\bar{\nu})_{\rm exp}/\mathcal{B}(B \to D^* \tau\bar{\nu})_{\rm SM}}{\mathcal{B}(B \to D^* \ell\bar{\nu})_{\rm exp}/\mathcal{B}(B \to D^* \ell\bar{\nu})_{\rm SM}} = 1.23 \pm 0.07\ ,$$

$$R_D^{\tau/\ell} = \frac{\mathcal{B}(B \to D \tau\bar{\nu})_{\rm exp}/\mathcal{B}(B \to D \tau\bar{\nu})_{\rm SM}}{\mathcal{B}(B \to D \ell\bar{\nu})_{\rm exp}/\mathcal{B}(B \to D \ell\bar{\nu})_{\rm SM}} = 1.34 \pm 0.17\ ,$$

[HFAG averages
of Babar, Belle and
LHCb, 1612.07233
SM at 3.9σ]

- theoretical uncertainties largely drop in these ratios and R≈1 is expected

[Bordone, Isidori, Pattori, 1605.07633]

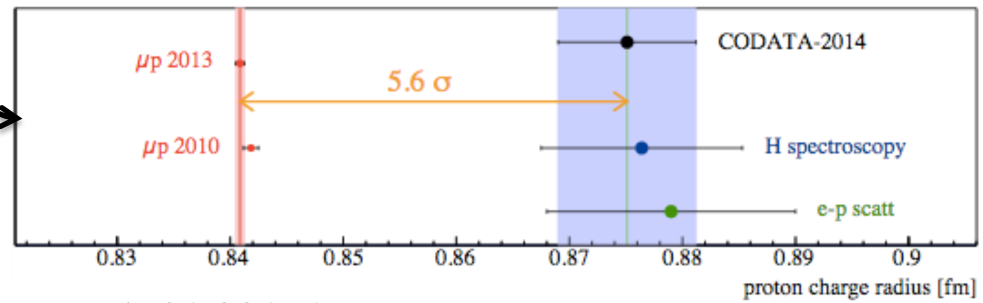
R ≠ 1 → violation of LFU and New Physics

- no conclusive NP signal from individual measurements
- significant discrepancy from the SM predictions comes from average and/or global fits

other hints of LFU violation

e-μ universality

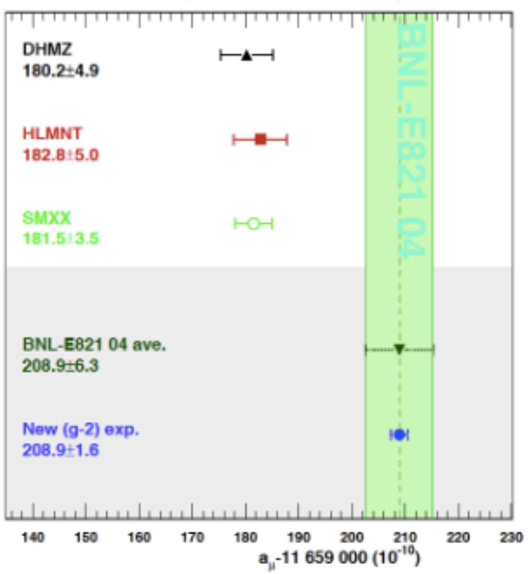
proton radius



[arXiv:1706.00696]

muon (g-2)

T. Blum et al. (arXiv:1311.2198)



τ-e and τ-μ universality

W leptonic decays

$$\frac{2\mathcal{B}(W^+ \to \tau^+\nu_\tau)}{\mathcal{B}(W^+ \to e^+\nu_e) + \mathcal{B}(W^+ \to \mu^+\nu_\mu)} = 1.067 \pm 0.029 \, .$$

Global Fit

- $B \to K^{(*)}\ell^+\ell^-$

$$\boxed{\begin{aligned}\mathcal{O}_9 &= \frac{\alpha}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma_\mu\ell)\\ \mathcal{O}_{10} &= \frac{\alpha}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5\ell)\end{aligned}} \qquad \begin{aligned}\mathcal{O}'_9 &= \frac{\alpha}{4\pi}(\bar{s}_R\gamma_\mu b_R)(\bar{\ell}\gamma_\mu\ell)\\ \mathcal{O}'_{10} &= \frac{\alpha}{4\pi}(\bar{s}_R\gamma_\mu b_R)(\bar{\ell}\gamma_\mu\gamma_5\ell)\end{aligned} \qquad \begin{aligned}\mathcal{O}_{7\gamma} &= \frac{e}{4\pi^2}m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu}\\ \mathcal{O}'_{7\gamma} &= \frac{e}{4\pi^2}m_b(\bar{s}_R\sigma^{\mu\nu}b_L)F_{\mu\nu}\end{aligned}$$

$$\Downarrow$$

- ▹ $C_9^{NP} \neq 0$
- ▹ $\boxed{C_9^{NP} = -C_{10}^{NP} \neq 0}$

} good fits of: ▹ R_K ▹ P'_5 (et al.)

S. Descotes-Genon , L. Hofer, J. Matias, J. Virto (2015)

$$\Downarrow$$

$(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}_L\gamma_\mu\ell_L)$ $\Rightarrow$ left-handed current

Altmannshofer, Stangl and Straub, 1704.05435;
Celis, Fuentes Martin, Vicente and Virto, 1704.05672;
Capdevila, Crivellin, Descotes-Genon, Matias and Virto, 1704.05340;
D'Amico, Nardecchia, Panci, Sannino, Strumia, Torre and Urbano, 1704.05438;
Ciuchini, Coutinho, Fedele, Franco, Paul, Silvestrini and Valli 1704.05447;
G. Hiller and I. Nisandzic, 1704.05444 [hep-ph].

Are the NC and CC anomalies related?

- both NC and CC anomalies can be explained by NP occurring (above the EW scale) purely in V-A combinations

$$\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\mu}_L\gamma^\mu \mu_L\right)$$

$$\left(\bar{c}_L\gamma_\mu b_L\right)\left(\bar{\tau}_L\gamma^\mu \nu_{\tau L}\right)$$

not the only possibility:
- V lepton current (O_9 operator) by itself provides a good fit
- tensor operator vanish at LO when SU(2)xU(1) is enforced
- scalar operators are constrained by B leptonic decays
- right quark helicities disfavored after R_{K*} measurement

- the two operators are related by
  - $SU(2)_L$ gauge invariance
  - transformations in flavour space

This suggests to start from operators
- (V-A)
- SU(2)xU(1)-invariant
- involving only the 3rd generation

[U(2), U(1),...]

$$O_{ql}^{(1,3)} = \left(\bar{q}'_{3L}\gamma_\mu A\, q'_{3L}\right)\left(\bar{\ell}'_{3L}\gamma^\mu A\, \ell'_{3L}\right) \qquad A = \left(1, \sigma^a\right)$$

couplings to lighter generations $\Leftrightarrow$ misalignment between mass and interaction bases

- welcome since small mixing angles can suppress the contribution to $R_{K(*)}$ compared to $R_{D(*)}$, as in the SM

Starting point $L^0_{NP}(\Lambda) = \frac{C_1}{\Lambda^2} O^{(1)}_{ql} + \frac{C_3}{\Lambda^2} O^{(3)}_{ql} =$

$$\mathcal{L}^0_{NP}(\Lambda) = \frac{\lambda^e_{kl}}{\Lambda^2} [(C_1 + C_3)\ \lambda^u_{ij}\ \bar{u}_{Li}\gamma^\mu u_{Lj}\ \bar{\nu}_{Lk}\gamma_\mu \nu_{Ll} + (C_1 - C_3)\ \lambda^u_{ij}\ \bar{u}_{Li}\gamma^\mu u_{Lj}\ \bar{e}_{Lk}\gamma_\mu e_{Ll} + (C_1 - C_3)\ \lambda^d_{ij}\ \bar{d}_{Li}\gamma^\mu d_{Lj}\ \bar{\nu}_{Lk}\gamma_\mu \nu_{Ll} + (C_1 + C_3)\ \lambda^d_{ij}\ \bar{d}_{Li}\gamma^\mu d_{Lj}\ \bar{e}_{Lk}\gamma_\mu e_{Ll} + 2C_3\ (\lambda^{ud}_{ij}\ \bar{u}_{Li}\gamma^\mu d_{Lj}\ \bar{e}_{Lk}\gamma_\mu \nu_{Ll} + h.c.)\ ]$$

(limit of massless neutrinos)

mixing among generation encoded in matrices $\lambda^{e,d,u}$

$$\lambda^{d,e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & \vartheta^2_{d,e} & \vartheta_{d,e} \\ 0 & \vartheta_{d,e} & 1 \end{pmatrix}$$

$\lambda^u = V^+_{CKM} \lambda^d V_{CKM}$

4 parameters $\frac{C_1}{\Lambda^2}, \frac{C_3}{\Lambda^2}, \vartheta_d, \vartheta_e$

both $R_{K(*)}$ and $R_{D(*)}$ can be explained

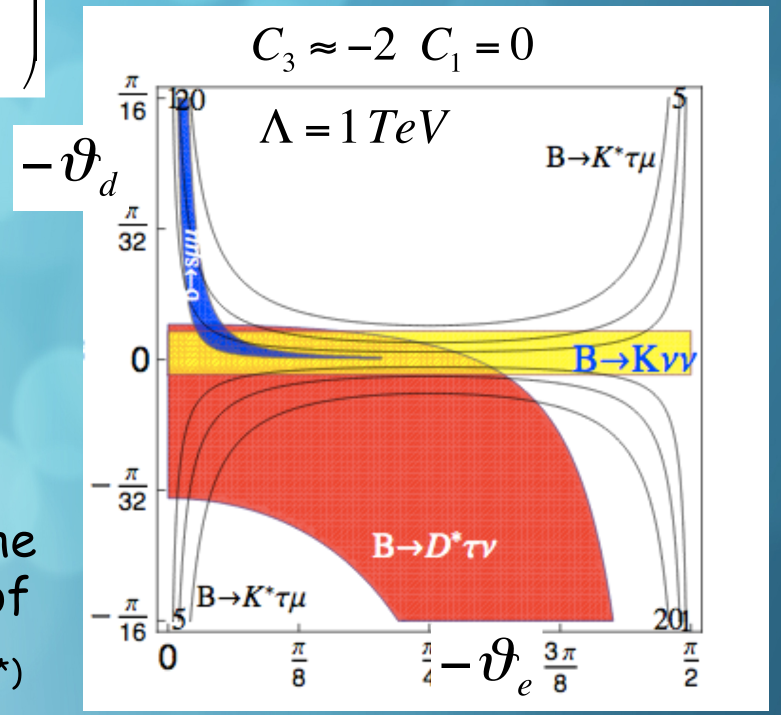
$\Lambda \approx 1\, TeV$ $C_3, C_1 = O(1)$

$\vartheta_d = O(0.01) \approx V_{cb}$

$\vartheta_e = O(0.3) \approx U^{PMNS}_{ij}$

(ϑ_d x ϑ_e^2) provides the needed suppression of $R_{K(*)}$ compared to $R_{D(*)}$

[Calibbi, Crivellin, Ota, 1506.02661]



Constraints (tree-level)

$R_K^{\mu/e}$ $R_{K^*}^{\mu/e}$	$(C_1+C_3)\vartheta_d\vartheta_e^2$		
$R_D^{\tau/\ell}$ $R_{D^*}^{\tau/\ell}$	C_3		
process	parameters	size	exp. bound
$R_{B_s\mu\mu} = \dfrac{\mathcal{B}(B_s\to\mu\bar{\mu})_{\rm exp}}{\mathcal{B}(B_s\to\mu\bar{\mu})_{\rm SM}}$	$(C_1+C_3)\vartheta_d\vartheta_e^2$	$O(0.1)$	$\mathcal{B}(B_s\to\mu\bar{\mu})_{\rm exp} = 2.8^{+0.7}_{-0.6}\times 10^{-9}$ $\mathcal{B}(B_s\to\mu\bar{\mu})_{\rm SM} = 3.65(23)\times 10^{-9}$
$R^{\tau/\mu}_{B\tau\nu} = \dfrac{\mathcal{B}(B\to\tau\nu)_{\rm exp}/\mathcal{B}(B\to\tau\nu)_{\rm SM}}{\mathcal{B}(B\to\mu\nu)_{\rm exp}/\mathcal{B}(B\to\mu\nu)_{\rm SM}}$	C_3	$O(0.1)$	Belle II ?
$R^{\nu\nu}_{K^{(*)}} = \dfrac{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})_{\rm SM}}$	$(C_1-C_3)\vartheta_d$	$O(1)$	$R^{\nu\nu}_{K^*} < 4.4$ $R^{\nu\nu}_{K} < 4.3$
$\mathcal{B}(B\to K\tau\mu)$ $\mathcal{B}(B\to\tau^\pm\mu^\mp)\approx\mathcal{B}(B\to K\tau^\pm\mu^\mp)$, $\mathcal{B}(B\to K^*\tau^\pm\mu^\mp)\approx 2\times\mathcal{B}(B\to K\tau^\pm\mu^\mp)$	$\left\|(C_1+C_3)\vartheta_d\vartheta_e\right\|^2$	$O(10^{-6\div 7})$	$\mathcal{B}(B\to K\tau\mu)\le 4.8\times 10^{-5}$
$\mu^+\mu^-$ and $\tau^+\tau^-$ Production at LHC	(C_1+C_3)		[Greljo, Marzocca 1704.09015]

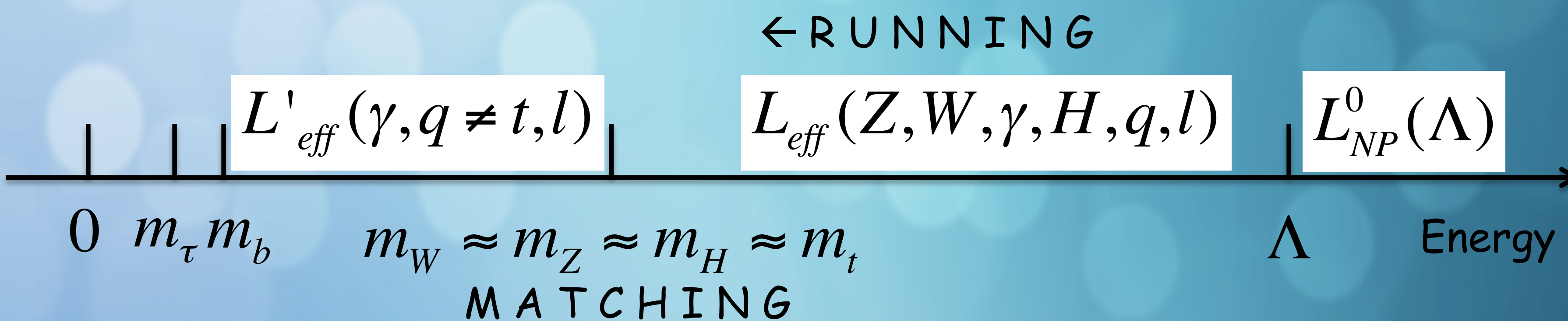
Constraints from quantum effects

$L_{NP}(m_b) = L_{NP}^0(\Lambda)$ + quantum corrections

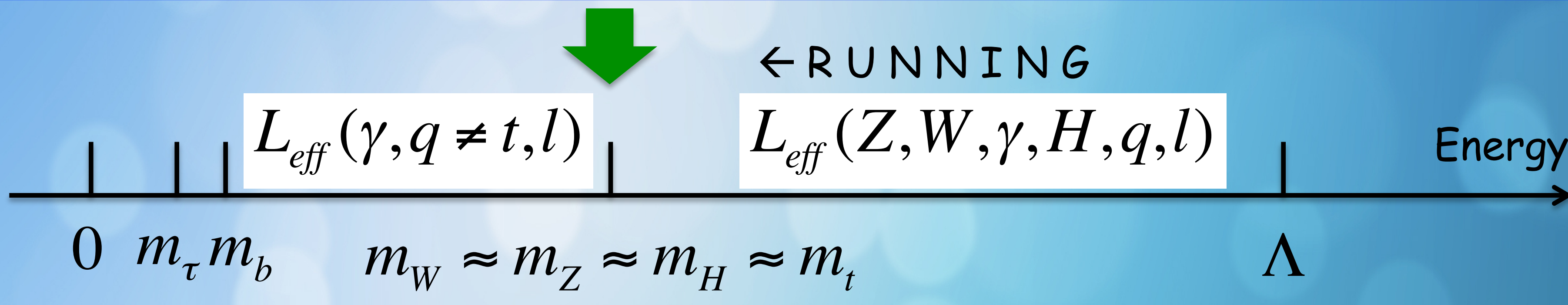
- How can quantum corrections ~ $\alpha/4\pi$ ~ 10^{-3} be relevant?

  - they generate terms that are absent in $L_{NP}^0(\Lambda)$ and new processes are affected
  - their order of magnitude is similar to accuracy in EWPT and in other tests of LFU
  - they are enhanced by logs: $\log(\Lambda^2/m_W^2)$ ~ 5-7

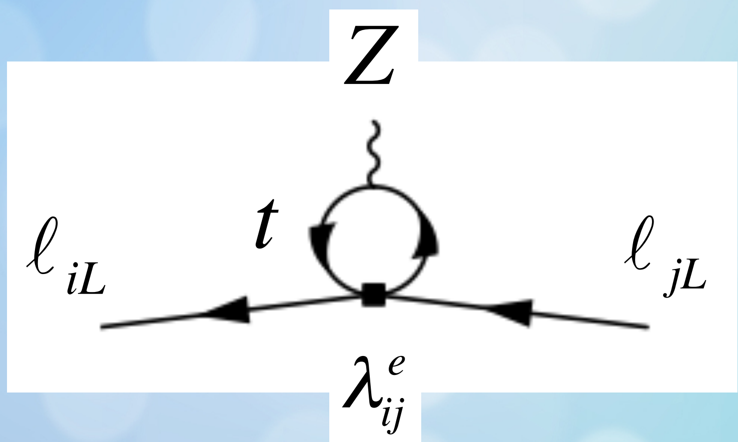
in the present framework - (V-A) semileptonic operators - corrections are dominated by electroweak interactions. They can be estimated by a well-known running and matching procedure. Here, Leading Log effects only



1st: the electroweak scale



1. modifications of the W,Z couplings to fermions by non-universal terms



$$\frac{a_\tau}{a_e} \approx 1 - 0.004 \, \frac{(C_1 - 0.8\, C_3)}{\Lambda^2(\mathrm{TeV}^2)}$$

$$\frac{v_\tau}{v_e} \approx 1 - 0.05 \, \frac{(C_1 - 0.8\, C_3)}{\Lambda^2(\mathrm{TeV}^2)}$$

$$a_\tau / a_e = 1.0019\ (15)$$

$$v_\tau / v_e = 0.959\ (29)$$

$$N_\nu \approx 3 + 0.008 \, \frac{(C_1 + 0.8\, C_3)}{\Lambda^2(\mathrm{TeV}^2)}$$

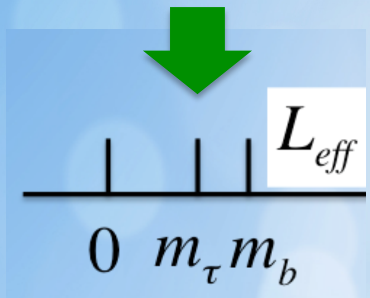
$$N_\nu = 2.9840 \pm 0.0082$$

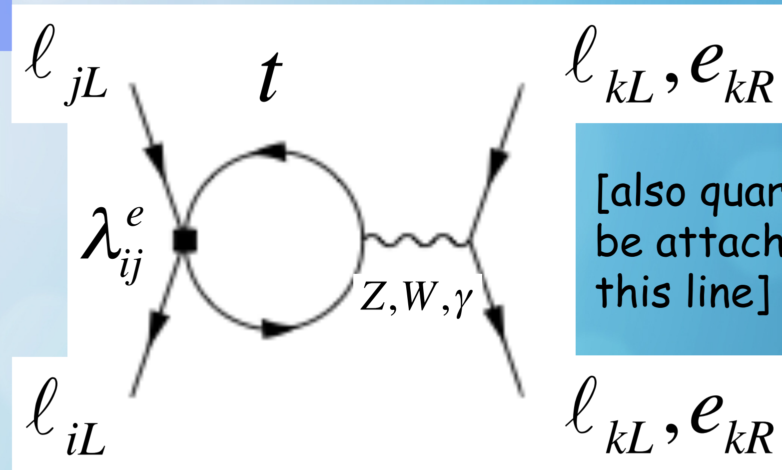
$$\mathcal{B}(Z \to \mu^\pm \tau^\mp) \approx 10^{-7}$$

$$\mathcal{B}(Z \to \mu^\pm \tau^\mp)_{\mathrm{exp}} \leq 1.2 \times 10^{-5}$$

2. generation of a purely leptonic effective Lagrangian at the scale ≤ m_b

2nd: m_τ

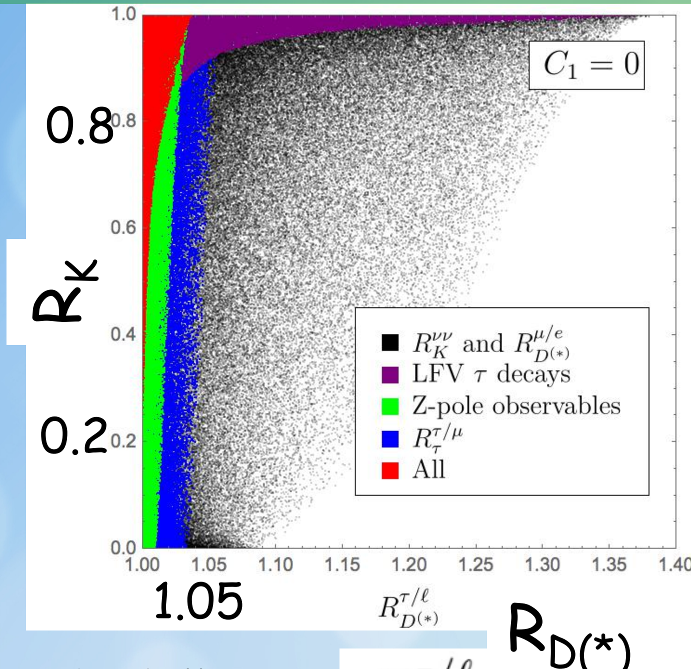


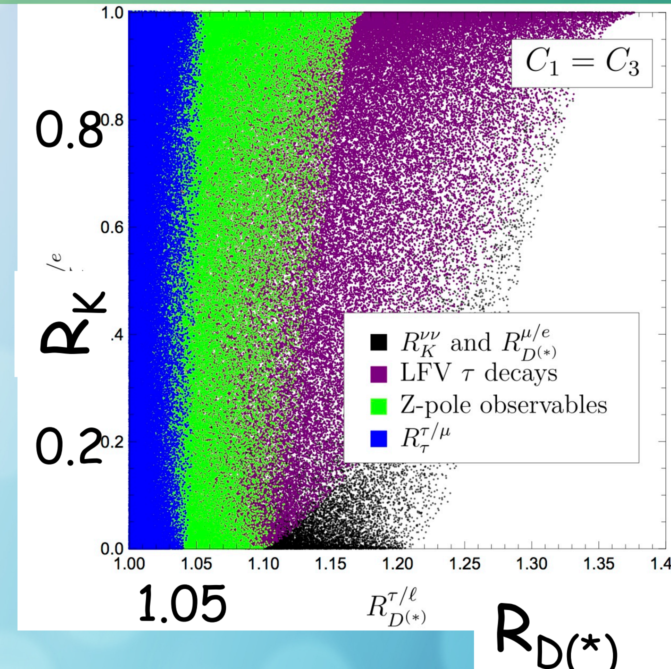


[also quarks can be attached to this line]

$R^{\tau/e}_\tau = \dfrac{\mathcal{B}(\tau\to\mu\nu\bar\nu)_{\text{exp}}/\mathcal{B}(\tau\to\mu\nu\bar\nu)_{\text{SM}}}{\mathcal{B}(\mu\to e\nu\bar\nu)_{\text{exp}}/\mathcal{B}(\mu\to e\nu\bar\nu)_{\text{SM}}}$ $R^{\tau/\mu}_\tau = \dfrac{\mathcal{B}(\tau\to e\nu\bar\nu)_{\text{exp}}/\mathcal{B}(\tau\to e\nu\bar\nu)_{\text{SM}}}{\mathcal{B}(\mu\to e\nu\bar\nu)_{\text{exp}}/\mathcal{B}(\mu\to e\nu\bar\nu)_{\text{SM}}}$	$\approx 1 + \dfrac{0.008\, C_3}{\Lambda^2(\text{TeV}^2)}$	$R^{\tau/e}_\tau = 1.0060 \pm 0.0030$ $R^{\tau/\mu}_\tau = 1.0022 \pm 0.0030$ [A.Pich, 1310.7922]
$\mathcal{B}(\tau\to 3\mu)$	$\approx 5\times 10^{-8}\dfrac{(C_1 - C_3)^2}{\Lambda^4(\text{TeV}^4)}\left(\dfrac{\vartheta_e}{0.3}\right)^2$	$\mathcal{B}(\tau\to 3\mu) \le 1.2\times 10^{-8}$ [HFAG, 1412.7515]
$\mathcal{B}(\tau\to \mu\rho)$	$\approx 5\times 10^{-8}\dfrac{(C_1 - 1.3\, C_3)^2}{\Lambda^4(\text{TeV}^4)}\left(\dfrac{\vartheta_e}{0.3}\right)^2$	$\mathcal{B}(\tau\to \mu\rho) \le 1.5\times 10^{-8}$
$\mathcal{B}(\tau\to \mu\pi)$	$\approx 8\times 10^{-8}\dfrac{(C_1 - C_3)^2}{\Lambda^4(\text{TeV}^4)}\left(\dfrac{\vartheta_e}{0.3}\right)^2$	$\mathcal{B}(\tau\to \mu\pi) \le 2.7\times 10^{-8}$ [HFAG, 1412.7515]

Putting everything together

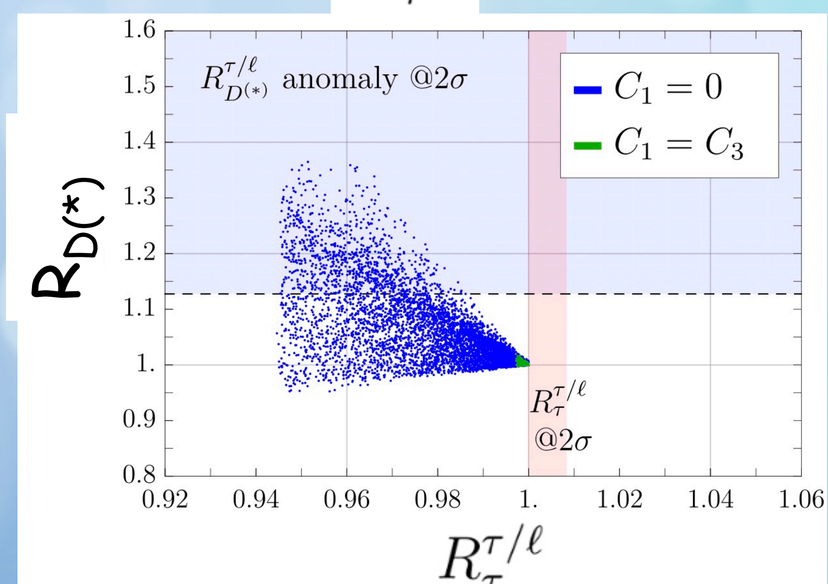




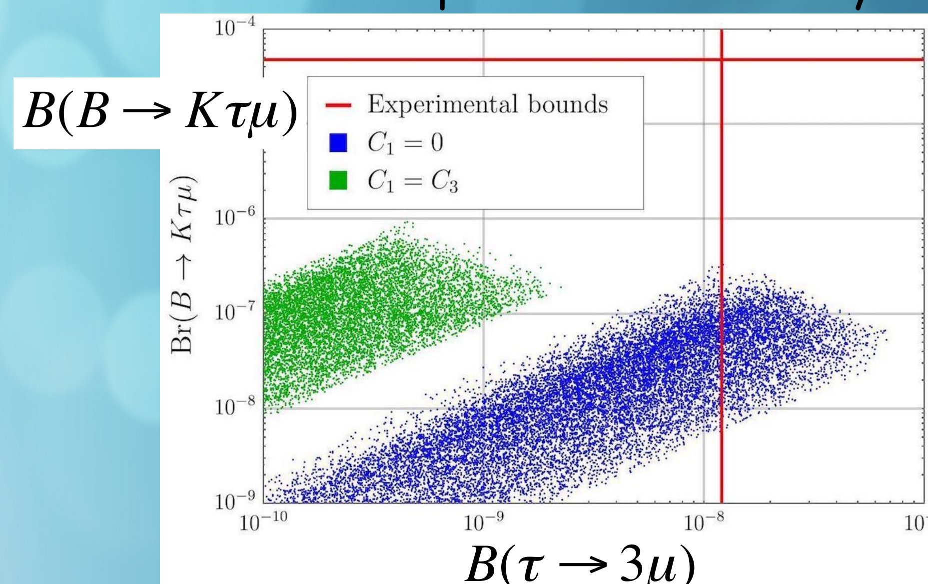
$$\left|\frac{C_{1,3}}{\Lambda^2}\right| \le 4\,TeV^{-2}$$

$$\left|\vartheta_{d,e}\right| \le 0.5$$

the killer is $R_\tau^{\tau/\ell}$!



LFV better probed in tau decays



A more general setup

[Cornella, F. Paradisi in preparation]

$$\mathcal{L}_{\rm NP}^{0} = \frac{1}{\Lambda^2}(C_1[Q_{lq}^{(1)}]_{3333} + C_3[Q_{lq}^{(3)}]_{3333} + C_4[Q_{ld}]_{3333} + C_5[Q_{ed}]_{3333} + C_6[Q_{qe}]_{3333})$$

$$[Q_{lq}^{(1)}]_{3333} = (\bar{\ell}'_{3L}\gamma^\mu \ell'_{3L})(\bar{q}'_{3L}\gamma^\mu q'_{3L})$$
$$[Q_{lq}^{(3)}]_{3333} = (\bar{\ell}'_{3L}\gamma^\mu \tau^a \ell'_{3L})(\bar{q}'_{3L}\gamma^\mu \tau^a q'_{3L})$$
$$[Q_{ld}]_{3333} = (\bar{\ell}'_{3L}\gamma^\mu \ell'_{3L})(\bar{d}'_{3R}\gamma d'_{3R})$$
$$[Q_{ed}]_{3333} = (\bar{e}'_{3R}\gamma^\mu e'_{3R})(\bar{d}'_{3R}\gamma d'_{3R})$$
$$[Q_{qe}]_{3333} = (\bar{q}'_{3L}\gamma q'_{3L})(\bar{e}'_{3R}\gamma^\mu e'_{3R})$$

most general set of SU(2)xU(1) – invariant semileptonic operators involving the 3rd generation

the main effects are 1. and 2., as before

an example

$$C_1 + C_3 = C_6 \qquad C_4 = C_5 = 0\,.$$

$$\Rightarrow \quad O^9 = \frac{e^2}{16\pi^2}(\bar{s}_L\gamma_\mu b_L)(\bar{e}_i\gamma^\mu e_i)$$

we find

$$\frac{v_\tau}{v_e} = 1 - \frac{0.05\,\lambda_{33}^e}{\Lambda^2}(2\,C_1 + 0.2\,C_3 + 0.02\,(2\,C_1 + C_3))$$
$$\frac{a_\tau}{a_e} = 1 + 0.007\,\lambda_{33}^e\frac{C_3}{\Lambda^2}$$

directly correlated to $R_\tau^{\tau/\ell}$ and $R_{D(*)}$

forces $\delta R^{\tau/\ell}_{D^{(*)}}$ to be $\lesssim 0.02.$

Discussion

- log effects discussed here can be cancelled/suppressed by finite terms, not captured by this approach [require knowledge of the complete UV theory]

- the starting point adopted here can be generalized by allowing more SU(2)xU(1) invariant operators at the scale Λ, making it possible cancellation/suppression of log effects

- different generation pattern in $O_{lq}^{(1,3)}$ can help in evading the bounds most of flavour schemes adopted in model building - $U(1)_{FN}$, U(2), Partial Compositeness – prefer NP coupled mainly to third generation.

- $R_D^{\tau/\ell}$ $R_{D^*}^{\tau/\ell}$ alone can be explained in present framework

  e.g. $\vartheta_d \approx 1$, $\vartheta_e \ll \alpha_{em}$, $\Lambda \approx 5$ TeV → loop effects decouple as v^2/Λ^2

- $R_K^{\mu/e}$ $R_{K^*}^{\mu/e}$ alone can be explained in present framework

  e.g. $\vartheta_d \approx 1$, $\vartheta_e \approx 1$, $\Lambda \approx 30$ TeV → loop effects decouple as v^2/Λ^2

conclusion

- B anomalies extensively studied in literature
  simultaneous $R_{K(*)}$ and $R_{D(*)}$ explanation is appealing

- the estimate of quantum corrections is crucial to asses the viability of proposed solutions

- in the example discussed here (NP in 3rd generation V-A currents) purely leptonic LFUV/LFV transitions are generated and strong constraints arise

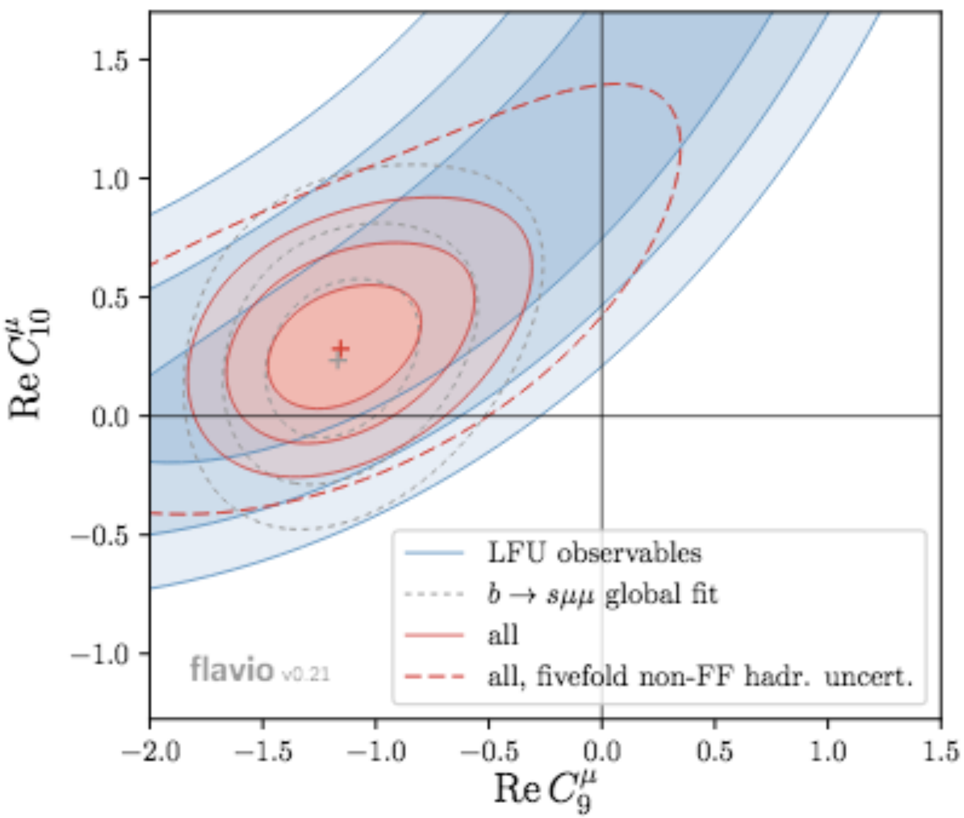
- this is not a no-go theorem:
  - ways out are possible but require some conspiracy by UV physics.

Back-up slides

Global Fit

Coeff.	best fit	1σ	2σ	pull
C_9^μ	-1.59	$[-2.15, -1.13]$	$[-2.90, -0.73]$	4.2σ
C_{10}^μ	$+1.23$	$[+0.90, +1.60]$	$[+0.60, +2.04]$	4.3σ
C_9^e	$+1.58$	$[+1.17, +2.03]$	$[+0.79, +2.53]$	4.4σ
C_{10}^e	-1.30	$[-1.68, -0.95]$	$[-2.12, -0.64]$	4.4σ
$C_9^\mu = -C_{10}^\mu$	-0.64	$[-0.81, -0.48]$	$[-1.00, -0.32]$	4.2σ
$C_9^e = -C_{10}^e$	$+0.78$	$[+0.56, +1.02]$	$[+0.37, +1.31]$	4.3σ
$C_9'^\mu$	-0.00	$[-0.26, +0.25]$	$[-0.52, +0.51]$	0.0σ
$C_{10}'^\mu$	$+0.02$	$[-0.22, +0.26]$	$[-0.45, +0.49]$	0.1σ
$C_9'^e$	$+0.01$	$[-0.27, +0.31]$	$[-0.55, +0.62]$	0.0σ
$C_{10}'^e$	-0.03	$[-0.28, +0.22]$	$[-0.55, +0.46]$	0.1σ

TABLE I. Best-fit values and pulls for scenarios with NP in one individual Wilson coefficient.



[Altmannshofer, Stangl and Straub, 1704.05435]

\`All' includes R_K, R_{K^*}, angular variables in $B \to K^* \mu^+ \mu^-$, differential BR in $B \to K^* \mu^+ \mu^-$, $B \to \varphi \mu^+ \mu^-$

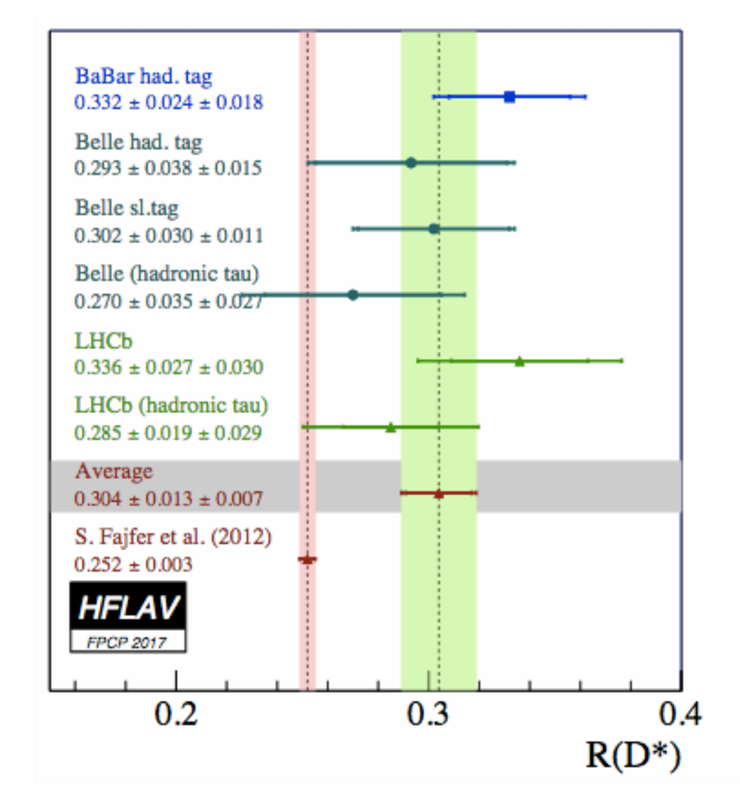
Global Fit

$$[R_K]_{[1,6]} \simeq 1.00(1) + 0.230(\mathcal{C}^{\rm NP}_{9\mu-e} + \mathcal{C}'_{9\mu-e}) - 0.233(2)(\mathcal{C}^{\rm NP}_{10\mu-e} + \mathcal{C}'_{10\mu-e})\,,$$

$$[R_{K^*}]_{[0.045,1.1]} \simeq 0.92(2) + 0.07(2)\mathcal{C}^{\rm NP}_{9\mu-e} - 0.10(2)\mathcal{C}'_{9\mu-e} - 0.11(2)\mathcal{C}^{\rm NP}_{10\mu-e} + 0.11(2)\mathcal{C}'_{10\mu-e} + 0.55(6)\mathcal{C}^{\rm NP}_{7}\,,$$

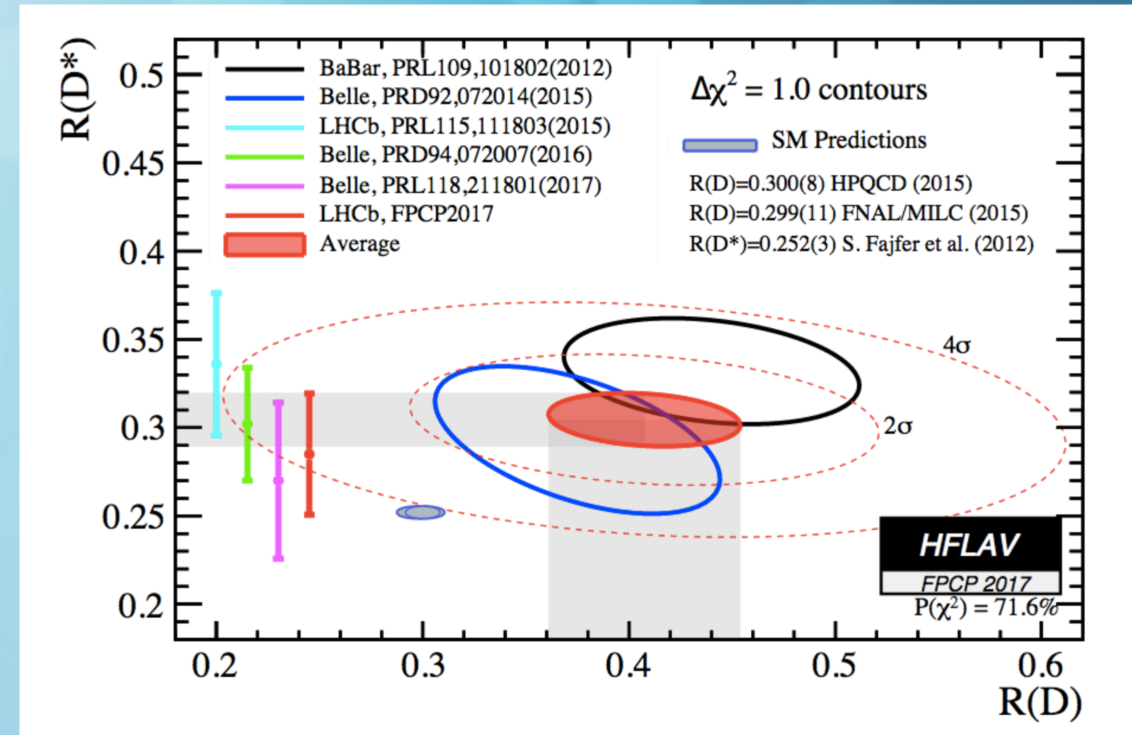
$$[R_{K^*}]_{[1.1,6]} \simeq 1.00(1) + 0.20(1)\mathcal{C}^{\rm NP}_{9\mu-e} - 0.19(1)\mathcal{C}'_{9\mu-e} - 0.27(1)\mathcal{C}^{\rm NP}_{10\mu-e} + 0.21(1)\mathcal{C}'_{10\mu-e}\,.$$

[Celis, Fuentes Martin, Vicente and Virto, 1704.05672]



individual measurements are compatible with the SM

when averaging, a 4σ discrepancy shows up



Dimension six operators

Semileptonic operators:	Leptonic operators:
$[O_{\ell q}^{(1)}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})\ (\bar{q}'_{sL}\gamma^\mu q'_{tL})$	$[O_{\ell\ell}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})\ (\bar{\ell}'_{sL}\gamma^\mu \ell'_{tL})$
$[O_{\ell q}^{(3)}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \tau^a \ell'_{rL})\ (\bar{q}'_{sL}\gamma^\mu \tau^a q'_{tL})$	$[O_{\ell e}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})\ (\bar{e}'_{sR}\gamma^\mu e'_{tR})$
$[O_{\ell u}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})\ (\bar{u}'_{sR}\gamma^\mu u'_{tR})$	
$[O_{\ell d}]_{prst} = (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})\ (\bar{d}'_{sR}\gamma^\mu d'_{tR})$	
$[O_{qe}]_{prst} = (\bar{q}'_{pL}\gamma_\mu q'_{rL})\ (\bar{e}'_{sR}\gamma^\mu e'_{tR})$	
Vector operators:	**Hadronic operators:**
$[O_{H\ell}^{(1)}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)\ (\bar{\ell}'_{pL}\gamma_\mu \ell'_{rL})$	$[O_{qq}^{(1)}]_{prst} = (\bar{q}'_{pL}\gamma_\mu q'_{rL})\ (\bar{q}'_{sL}\gamma^\mu q'_{tL})$
$[O_{H\ell}^{(3)}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}^a_\mu \varphi)\ (\bar{\ell}'_{pL}\gamma_\mu \tau^a \ell'_{rL})$	$[O_{qq}^{(3)}]_{prst} = (\bar{q}'_{pL}\gamma_\mu \tau^a q'_{rL})\ (\bar{q}'_{sL}\gamma^\mu \tau^a q'_{tL})$
$[O_{Hq}^{(1)}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)\ (\bar{q}'_{pL}\gamma_\mu q'_{rL})$	$[O_{qu}^{(1)}]_{prst} = (\bar{q}'_{pL}\gamma_\mu q'_{rL})\ (\bar{u}'_{sR}\gamma^\mu u'_{tR})$
$[O_{Hq}^{(3)}]_{pr} = (\varphi^\dagger i \overleftrightarrow{D}^a_\mu \varphi)\ (\bar{q}'_{pL}\gamma_\mu \tau^a q'_{rL})$	$[O_{qd}^{(1)}]_{prst} = (\bar{q}'_{pL}\gamma_\mu q'_{rL})\ (\bar{d}'_{sR}\gamma^\mu d'_{tR})$

Table 1: Minimal set of gauge-invariant operators involved in the RGE flow considered in this paper. Fields are in the interaction basis to maintain explicit $SU(2) \times U(1)$ gauge invariance. Our notation and conventions are as in [26].

Effective Lagrangian – ew scale

$$g_{fL,R} = g^{SM}_{fL,R} + \Delta g_{fL,R} \qquad g_{\ell,q} = g^{SM}_{\ell,q} + \Delta g_{\ell,q}$$

$$\Delta g^{ij}_{\nu L} = \frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}\left(\frac{1}{3}g_1^2 C_1 - g_2^2 C_3 + 3y_t^2\lambda^u_{33}(C_1 + C_3)\right)\lambda^e_{ij}$$

$$\Delta g^{ij}_{eL} = \frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}\left(\frac{1}{3}g_1^2 C_1 + g_2^2 C_3 + 3y_t^2\lambda^u_{33}(C_1 - C_3)\right)\lambda^e_{ij}$$

$$\Delta g^{ij}_{uL} = -\frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}\frac{1}{3}(g_1^2 C_1 + g_2^2 C_3)\lambda^u_{ij}$$

$$\Delta g^{ij}_{dL} = -\frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}\frac{1}{3}(g_1^2 C_1 - g_2^2 C_3)\lambda^d_{ij}$$

$$\Delta g^{ij}_{fR} = 0 \qquad (f = \nu, e, u, d)$$

$$\Delta g^{ij}_{\ell} = \frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}(-2g_2^2 C_3 + 6y_t^2\lambda^u_{33}C_3)\lambda^e_{ij}$$

$$\Delta g^{ij}_{q} = -\frac{v^2}{\Lambda^2}\frac{L}{16\pi^2}\frac{2}{3}g_2^2 C_3\lambda^{ud}_{ij} \quad . \qquad L = \log\frac{\Lambda}{\mu}$$

Effective Lagrangian - ew scale

$$\mathcal{L}^{EW}_{eff} = \mathcal{L}'_{SM} + \mathcal{L}^0_{NP} + \frac{1}{16\pi^2\Lambda^2}\log\frac{\Lambda}{m_{EW}}\sum_i \xi_i Q_i$$

Q_i	ξ_i
$(\bar\nu_{iL}\gamma_\mu\nu_{jL})\,(\bar\nu_{kL}\gamma^\mu\nu_{nL})$	$\lambda^e_{ij}\,\delta_{kn}\,[-6y_t^2\lambda^u_{33}(C_1+C_3)]$
$(\bar\nu_{iL}\gamma_\mu\nu_{jL})\,(\bar e_{kL}\gamma^\mu e_{nL})$	$\lambda^e_{ij}\,\delta_{kn}\,[\frac{4}{3}e^2(C_1+3C_3)-12\,(-\frac{1}{2}+s^2_\theta)\,y_t^2\lambda^u_{33}(C_1+C_3)]$ $+\delta_{ij}\,\lambda^e_{kn}\,[-6y_t^2\lambda^u_{33}(C_1-C_3)]$
$(\bar\nu_{iL}\gamma_\mu\nu_{jL})\,(\bar e_{kR}\gamma^\mu e_{nR})$	$\lambda^e_{ij}\,\delta_{kn}\,[\frac{4}{3}e^2(C_1+3C_3)-12\,s^2_\theta\,y_t^2\lambda^u_{33}(C_1+C_3)]$
$(\bar e_{iL}\gamma_\mu e_{jL})\,(\bar e_{kL}\gamma^\mu e_{nL})$	$\lambda^e_{ij}\,\delta_{kn}\,[\frac{4}{3}e^2(C_1-3C_3)-12(-\frac{1}{2}+s^2_\theta)\,y_t^2\lambda^u_{33}(C_1-C_3)]$
$(\bar e_{iL}\gamma_\mu e_{jL})\,(\bar e_{kR}\gamma^\mu e_{nR})$	$\lambda^e_{ij}\,\delta_{kn}\,[\frac{4}{3}e^2(C_1-3C_3)-12\,s^2_\theta\,y_t^2\lambda^u_{33}(C_1-C_3)]$
$(\bar\nu_{iL}\gamma_\mu e_{jL})\,(\bar e_{kL}\gamma^\mu\nu_{nL})$	$(\lambda^e_{ij}\,\delta_{kn}+\delta_{ij}\,\lambda^e_{kn})\,[-12\,y_t^2\lambda^u_{33}C_3]$

Table 2: Operators Q_i and coefficients ξ_i for the purely leptonic part of the effective Lagrangian $\mathcal{L}^{EW}_{eff}$. We set $\sin^2\theta_W \equiv s^2_\theta$.

Effective Lagrangian at low energy

$$\delta\mathcal{L}_{eff}^{QED} = \frac{1}{16\pi^2\Lambda^2}\log\frac{m_{EW}}{\mu}\sum_i \delta\xi_i\, Q_i$$

Q_i	$\delta\xi_i$
$(\bar\nu_{iL}\gamma_\mu\nu_{jL})\,(\bar\nu_{kL}\gamma^\mu\nu_{nL})$	0
$(\bar\nu_{iL}\gamma_\mu\nu_{jL})\,(\bar e_k\gamma^\mu e_n)$	$\lambda^e_{ij}\,\delta_{kn}\cdot\frac{4}{3}e^2\Big[(C_1+3C_3)-2(C_1+C_3)(\hat\lambda^u_{33}\log\frac{m_t}{\mu}+\hat\lambda^u_{22}\log\frac{m_c}{\mu})$ $+(C_1-C_3)\hat\lambda^d_{33}\log\frac{m_b}{\mu}\Big]$
$(\bar e_{iL}\gamma_\mu e_{jL})\,(\bar e_k\gamma^\mu e_n)$	$\lambda^e_{ij}\,\delta_{kn}\cdot\frac{4}{3}e^2\Big[(C_1-3C_3)-2(C_1-C_3)(\hat\lambda^u_{33}\log\frac{m_t}{\mu}+\hat\lambda^u_{22}\log\frac{m_c}{\mu})$ $+(C_1+C_3)\hat\lambda^d_{33}\log\frac{m_b}{\mu}\Big]$

Table 6: Operators Q_i and coefficients $\delta\xi_i$ for the purely leptonic part of the effective Lagrangian $\delta\mathcal{L}_{eff}^{QED}$. We set $\hat\lambda^{u,d}_{ii} = \lambda^{u,d}_{ii}/\log\frac{m_{EW}}{\mu}$.

tree-level mediators of $O_{lq}^{(1,3)}$

Field	Spin	Quantum Numbers	Operator	C_1	C_3
A_μ	1	$(1,1,0)$	$\bar{q}'_L\gamma^\mu q'_L\ \bar{\ell}'_L\gamma_\mu\ell'_L$	-1	0
A^a_μ	1	$(1,3,0)$	$\bar{q}'_L\gamma^\mu\tau^a q'_L\ \bar{\ell}'_L\gamma_\mu\tau^a\ell'_L$	0	-1
U_μ	1	$(3,1,+2/3)$	$\bar{q}'_L\gamma^\mu\ell'_L\ \bar{\ell}'_L\gamma_\mu q'_L$	$-\frac{1}{2}$	$-\frac{1}{2}$
U^a_μ	1	$(3,3,+2/3)$	$\bar{q}'_L\gamma^\mu\tau^a\ell'_L\ \bar{\ell}'_L\gamma_\mu\tau^a q'_L$	$-\frac{3}{2}$	$+\frac{1}{2}$
S	0	$(3,1,-1/3)$	$\bar{q}'_L i\sigma^2\ell'^c_L\ \overline{i\sigma^2\ell'^c_L}q'_L$	$+\frac{1}{4}$	$-\frac{1}{4}$
S^a	0	$(3,3,-1/3)$	$\bar{q}'_L\tau^a i\sigma^2\ell'^c_L\ \overline{i\sigma^2\ell'^c_L}\tau^a q'_L$	$+\frac{3}{4}$	$+\frac{1}{4}$

Table 11: Spin zero and spin one mediators contributing, at tree-level, to the Lagrangian $\mathcal{L}^0_{NP}(\Lambda)$ of eq. (7). Also shown are the operators they give rise to and the contribution to the coefficients C_1 and C_3 of the Lagrangian $\mathcal{L}^0_{NP}(\Lambda)$, when a single fermion generation is involved.