

Monte Carlo generators for muon-electron scattering

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Monte Carlo generators for μe scattering

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with M. Alacevich, C.M. Carloni Calame, M. Chiesa, G. Montagna, O. Nicrosini

material presented by C.M. Carloni Calame at the μe scattering th. kickoff workshop, Padova, 4-5 September 2017

- ★ Introduction
- ★ Calculation details
- ★ Numerical results
- ★ Conclusions and outlook

Relevant literature:

- ★ G. Abbiendi *et al.*,
Measuring the leading hadronic contribution to the muon $g-2$ via μe scattering
Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ex]
- ★ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
A new approach to evaluate the leading hadronic corrections to the muon $g-2$
Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]
- G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini,
Matching perturbative and parton shower corrections to Bhabha process at flavour factories
Nucl. Phys. B **758** (2006) 227 - hep-ph/0607181

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (x-1) \bar{\Pi}_{\text{had}}(t(x)) = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}(t(x))$$

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_{-\infty}^0 \frac{dt}{\beta t} \left(\frac{1-\beta}{1+\beta} \right)^2 \bar{\Pi}_{\text{had}}(t) = -\frac{\alpha}{\pi} \int_{-\infty}^0 \frac{dt}{\beta t} \left(\frac{1-\beta}{1+\beta} \right)^2 \Delta\alpha_{\text{had}}(t)$$

where

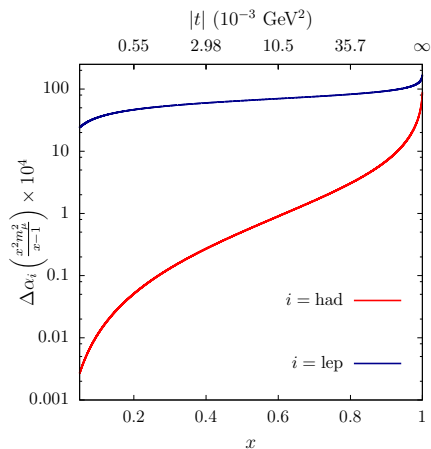
$$t(x) = \frac{x^2 m_\mu^2}{x-1} \quad \beta(t) = \sqrt{1 - \frac{4m_\mu^2}{t}} \quad x(t) = \frac{t(1-\beta(t))}{2m_\mu^2} \quad t = \begin{cases} 0^- & \text{for } x \rightarrow 0^+ \\ -\infty & \text{for } x \rightarrow 1^- \end{cases}$$

$\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of $\alpha_{\text{QED}}(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)}$

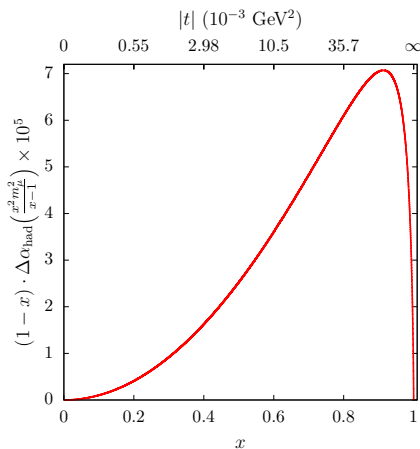
- ★ $\Delta\alpha_{\text{had}}(t)$ in the integrand is evaluated in the space-like region (negative transfer momenta) where it is a smooth function
- ★ we propose to *measure* the running of $\alpha_{\text{QED}}(t < 0)$ to evaluate a_μ^{HLO}
- ★ Roughly, to be competitive with current time-like evaluations, $\Delta\alpha_{\text{had}}(t)$ needs to be known at some % level

see also G.V. Fedotov, CMD-2 Collaboration, Nucl. Phys. Proc. Suppl. 181-182 (2008) 146

similar approach also used for lattice calculations of a_μ^{HLO}



- $\Delta\alpha_{\text{had}}(t(x))$ (red) as a function of x
- $\Delta\alpha_{\text{lep}}(t(x))$ (blue) as a function of x



- integrand function $(1-x)\Delta\alpha_{\text{had}}(t(x))$

$$x_{\text{peak}} \simeq 0.914$$

$$t_{\text{peak}} \simeq -0.108 \text{ GeV}^2$$

$\mu e \rightarrow \mu e$ scattering in a fixed target experiment

→ A 150 GeV high-intensity ($\sim 1.3 \times 10^7 \mu$'s/s) muon beam is available at CERN NA

→ Muon scattering on a low- Z target ($\mu e \rightarrow \mu e$) looks an ideal process

★ it is a pure t -channel process →

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha} \right|^2$$

★ Assuming a 150 GeV incident μ beam we have

$$s \simeq 0.164 \text{ GeV}^2 \quad -0.143 \lesssim t < 0 \text{ GeV}^2 \quad 0 < x \lesssim 0.93 \quad \text{it spans the peak!}$$

★ for detector design and data analysis we need a **high precision event generator:**
th. uncertainty $\delta \leq 10^{-4}$

- extremely challenging, never achieved before for a scattering process
- as reference, at LEP ultimate th. accuracy in Bhabha scattering $\sim 0.5/0.6 \cdot 10^{-3}$
- such a precision is the same that will be required for future high lumi e^+e^- colliders
- **but for $\mu e \rightarrow \mu e$ we need it soon!**

μe scattering kinematics for leading order ($2 \rightarrow 2$, elastic process)

p_1, p_2 initial state μ and e

p_3, p_4 final state μ and e

In the lab

$$\begin{aligned}p_1 &= (E_\mu^{beam}, 0, 0, p) \\p_2 &= (m_e, 0, 0, 0) \\p_3 &= p_1 + p_2 - p_4 \\p_4 &= (E_e, p_e \sin \theta_e, 0, p_e \cos \theta_e)\end{aligned}$$

In the center of mass

$$\begin{aligned}p_1 &= (E_{CM}^\mu, 0, 0, p_{CM}) \\p_2 &= (E_{CM}^e, 0, 0, -p_{CM}) \\p_3 &= (E_{CM}^\mu, p_{CM} \sin \theta, 0, p_{CM} \cos \theta) \\p_4 &= (E_{CM}^e, -p_{CM} \sin \theta, 0, -p_{CM} \cos \theta)\end{aligned}$$

Invariants:

$$\begin{aligned}s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\&= m_e^2 + m_\mu^2 + 2E_{CM}^\mu E_{CM}^e + 2p_{CM}^2 \\&= m_e^2 + m_\mu^2 + 2E_\mu^{beam} m_e\end{aligned}$$

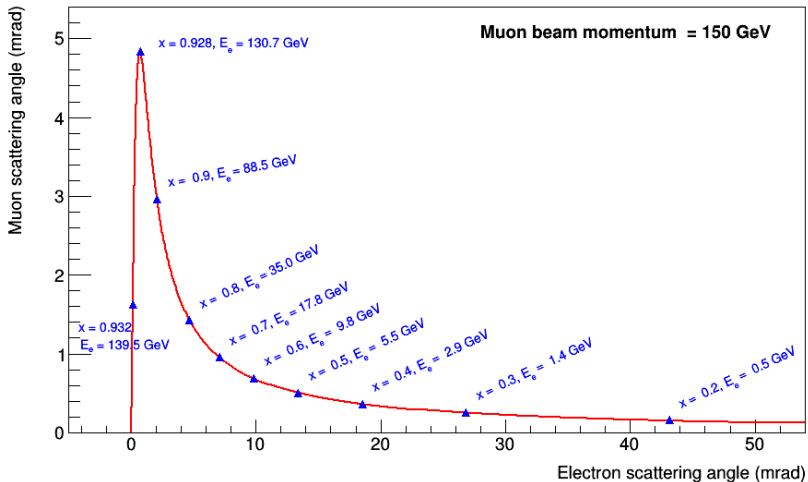
$$\begin{aligned}t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\&= -2p_{CM}^2(1 - \cos \theta) \\&= 2m_e^2 - 2E_e m_e\end{aligned}$$

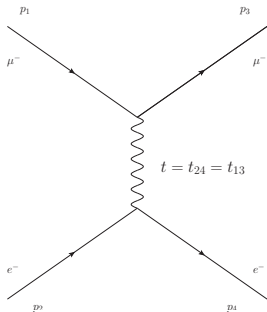
$$p_{CM} = \frac{1}{2} \sqrt{\frac{\lambda(s, m_\mu^2, m_e^2)}{s}}$$

$$t = m_\mu^2 \frac{x^2}{x-1} \propto E_e$$

$$E_e = m_e \frac{1 + r^2 \cos^2 \theta_e}{1 - r^2 \cos^2 \theta_e}$$

$$r \equiv \frac{\sqrt{(E_\mu^{beam})^2 - m_\mu^2}}{E_\mu^{beam} + m_e}$$





- analytical expression

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

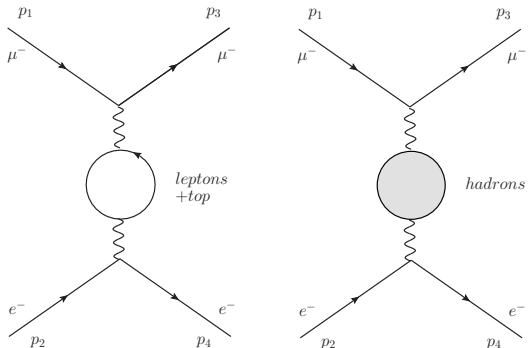
- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \rightarrow \alpha(t)$**

- Studies up to now based on an improved LO event generator
- next steps (in the near future)
 - event generation reaching QED NLOPS accuracy (part of NNLO included)**
 - quantify how large are the NNLO corrections not included in the above formulation through comparison with NNLO fixed order exact calculations

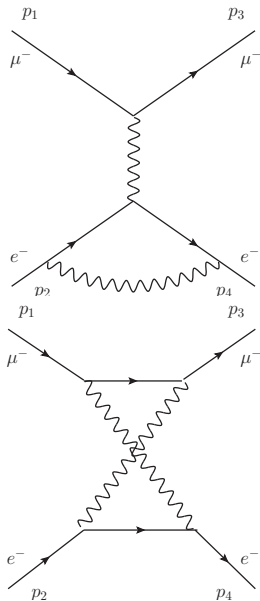
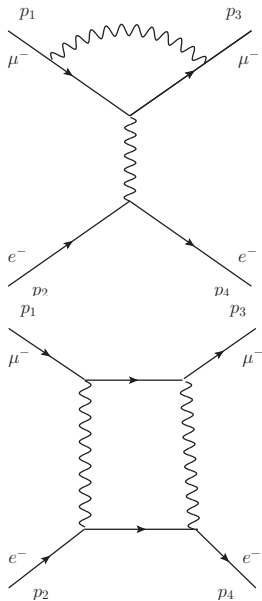
⇒ talk by P. Mastrolia

- eventually build a generator reaching QED NNLOPS accuracy**
- check what is the contribution of resummed terms beyond the LL accuracy of PS
- quantify the effects of tree-level Z exchange and weak corrections

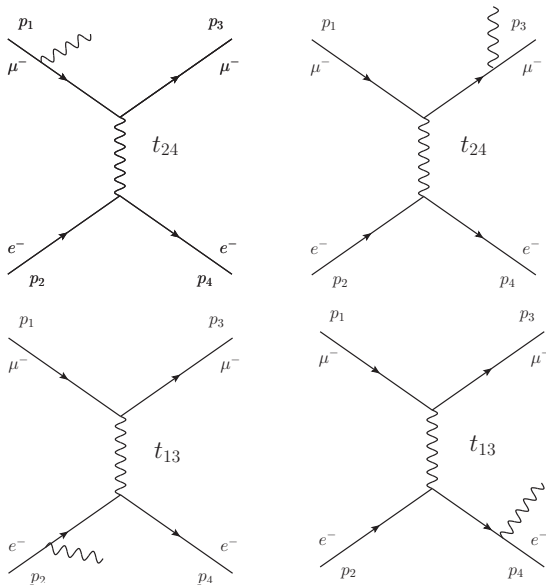
NLO amplitudes: vacuum polarization



NLO QED virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)



+ counterterms



$$\sigma_{NLO} = \sigma_{2 \rightarrow 2} + \sigma_{2 \rightarrow 3} = \sigma_{\mu e \rightarrow \mu e} + \sigma_{\mu e \rightarrow \mu e \gamma}$$

- IR singularities are regularized with a vanishingly small photon mass λ
- $[2 \rightarrow 2]/[2 \rightarrow 3]$ phase space splitting at an arbitrarily small γ -energy cutoff ω_s
 - $\mu e \rightarrow \mu e$

$$\sigma_{2 \rightarrow 2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F} \int d\Phi_2 (|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^* \times \mathcal{A}_{NLO}^{virtual}(\lambda)])$$

- $\mu e \rightarrow \mu e \gamma$

$$\begin{aligned} \sigma_{2 \rightarrow 3} &= \frac{1}{F} \int_{\omega > \lambda} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F} \left(\int_{\lambda < \omega < \omega_s} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega > \omega_s} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 \right) \\ &= \Delta_s(\lambda, \omega_s) \int d\sigma_{LO} + \frac{1}{F} \int_{\omega > \omega_s} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 \end{aligned}$$

- the integration over the 2/3-particles phase space is done with MC techniques and fully-exclusive events are generated

- Calculation performed in the on-shell renormalization scheme
- Full mass dependency kept everywhere, fermions' helicities kept explicit
- Diagrams manipulated with the help of **FORM**, independently by at least two of us
[perfect agreement]
J. Vermaseren, <https://www.nikhef.nl/~form>
- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with **LoopTools** and **Collier** libraries
[perfect agreement]
T. Hahn, <http://www.feynarts.de/looptools>
A. Denner, S. Dittmaier, L. Hofer, <https://collier.hepforge.org>
- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations
[perfect agreement]
- Comparison with past (and present) independent results [to be done]
P. Van Nieuwenhuizen, Nucl. Phys. B **28** (1971) 429
T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725
D. Y. Bardin and L. Kalinovskaya, DESY-97-230, [hep-ph/9712310](https://arxiv.org/abs/hep-ph/9712310)
N. Kaiser, J. Phys. G **37** (2010) 115005
Fael, Passera *et al.*, work in progress
G. D'Ambrosio, Lettere al Nuovo Cimento, Vol. 38, n. 18, 1983

Further cross-checks and (simple) simulation setup

- Good agreement with NLO Bhabha (only t -channel diagrams) if $m_\mu \rightarrow m_e$ as implemented in **BabaYaga@NLO** (within known approximations)
- Everything calculated in the center-of-mass frame, and boosted in the lab if needed
- Implemented in a MC event generator (*temporary* extension of **BabaYaga@NLO**)
- ω_s independence

$$\sigma_{LO} = 245.039062 \mu\text{b}$$

$$\sigma_{NLO}(\omega_s = 5 \cdot 10^{-5} \times \sqrt{s} \equiv \omega_1) = 244.3957 \pm .0005 \mu\text{b}$$

$$\sigma_{NLO}(\omega_s = 1 \cdot 10^{-6} \times \sqrt{s} = \omega_1/50) = 244.3948 \pm .0007 \mu\text{b}$$

correction to the integrated cross section at the few % level

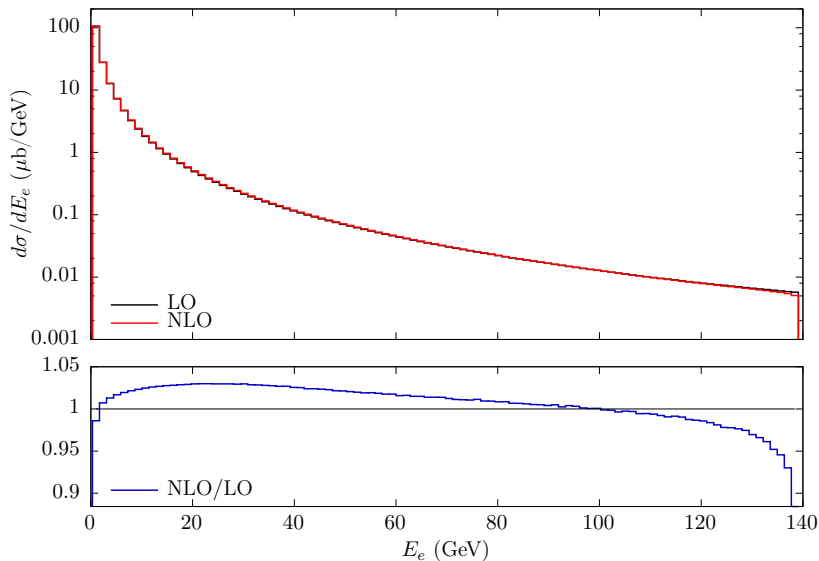
★ Simulation setup for $\mu^- e^- \rightarrow \mu^- e^- (\gamma)$

- $E_\mu^{beam} = 150 \text{ GeV} \rightarrow \sqrt{s} \simeq 0.4055 \text{ GeV}$
- $t_{min} = -\lambda(s, m_\mu^2, m_e^2)/s \simeq -0.1429 \text{ GeV}^2$
- $t_{max} \simeq -1.021 \cdot 10^{-3} \text{ GeV}^2$ (i.e. $E_e > 1 \text{ GeV}$ in the lab)

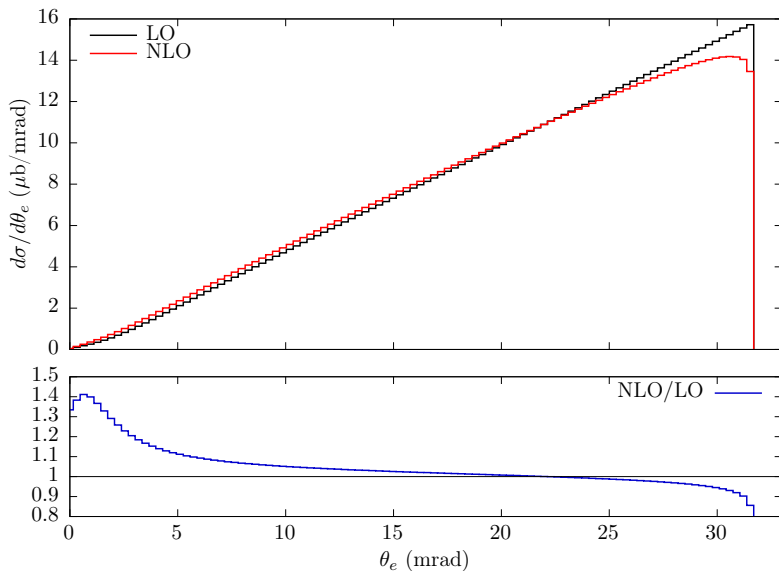
$$\rightarrow \text{Cuts: } t_{min} \leq t_{24} \leq t_{max} \quad t_{min} \leq t_{13} \leq t_{max}$$

preliminary sample results

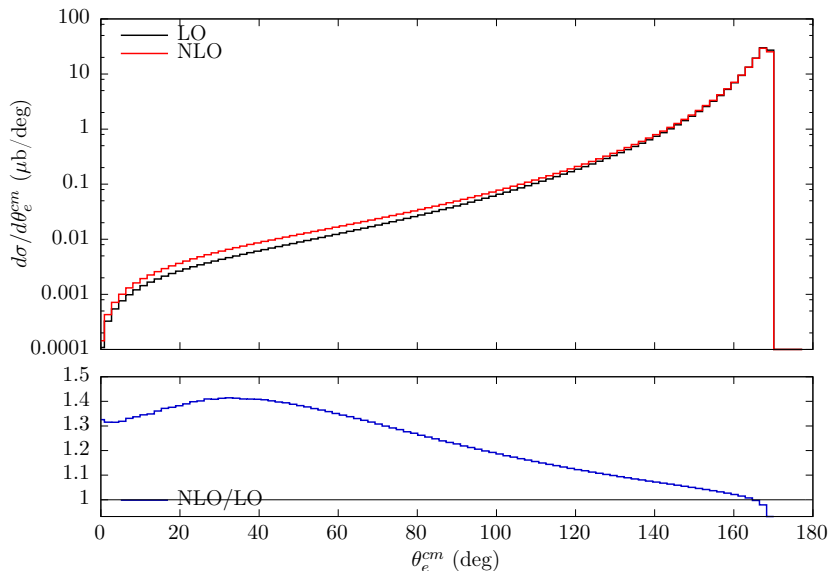
E_e distribution and corrections in the lab



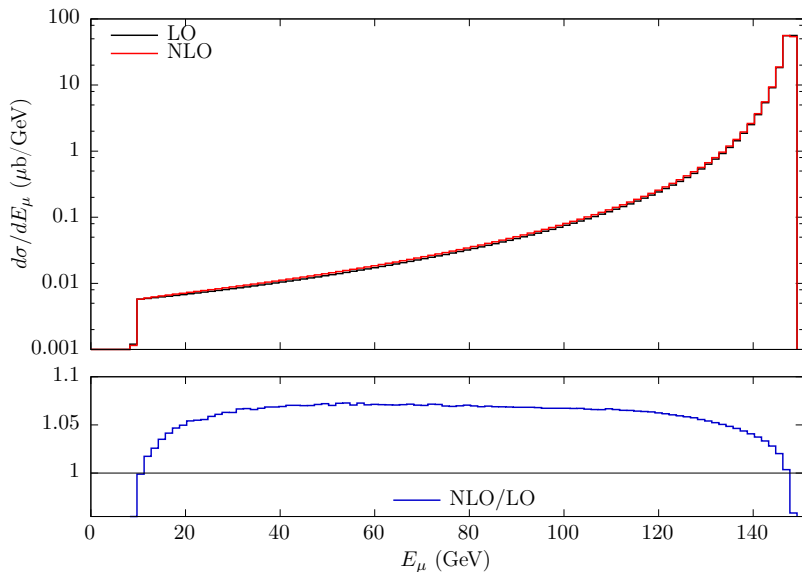
θ_e distribution and corrections in the lab



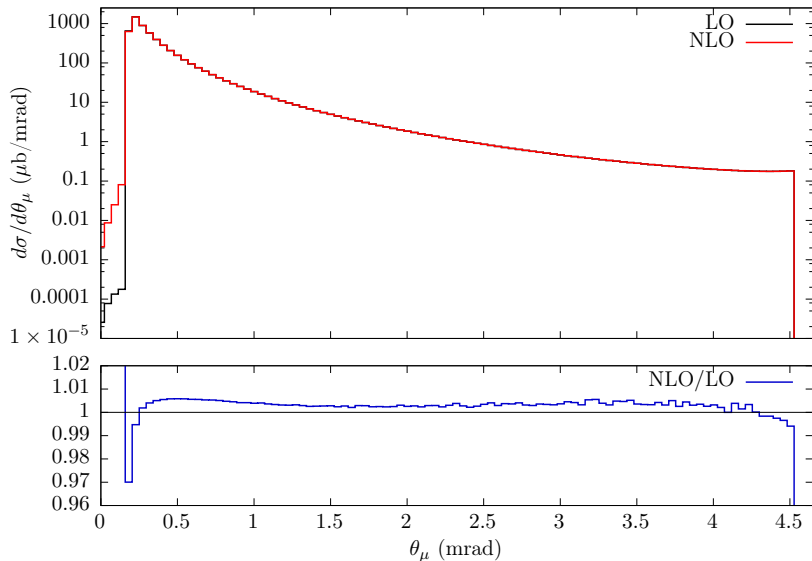
θ_e distribution and corrections in the c.m.s.



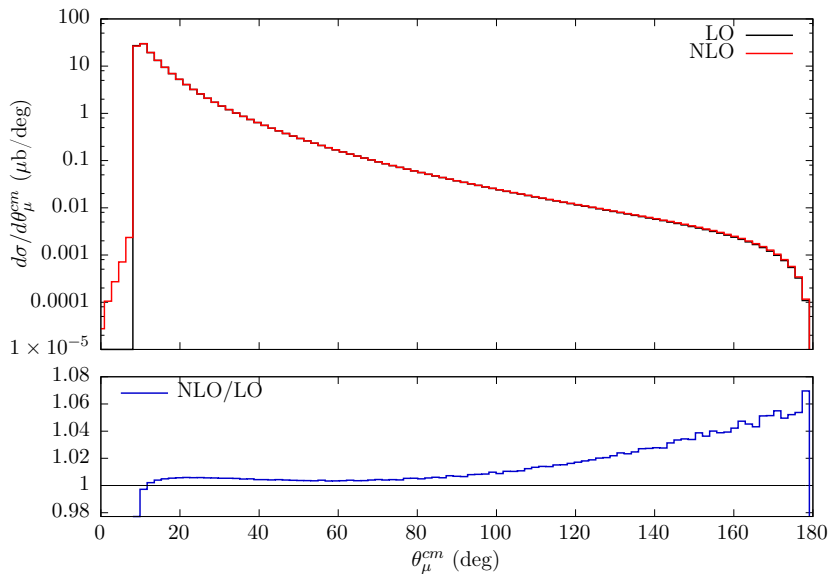
E_μ distribution and corrections in the lab



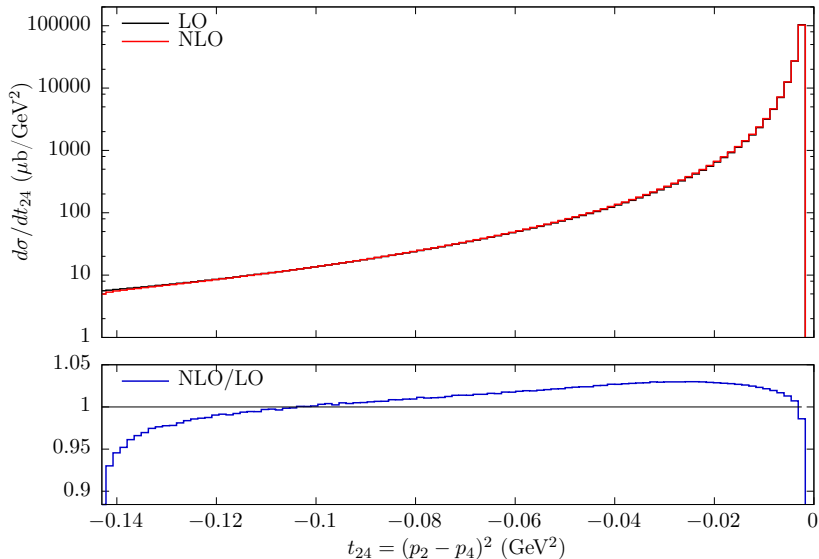
θ_μ distribution and corrections in the lab



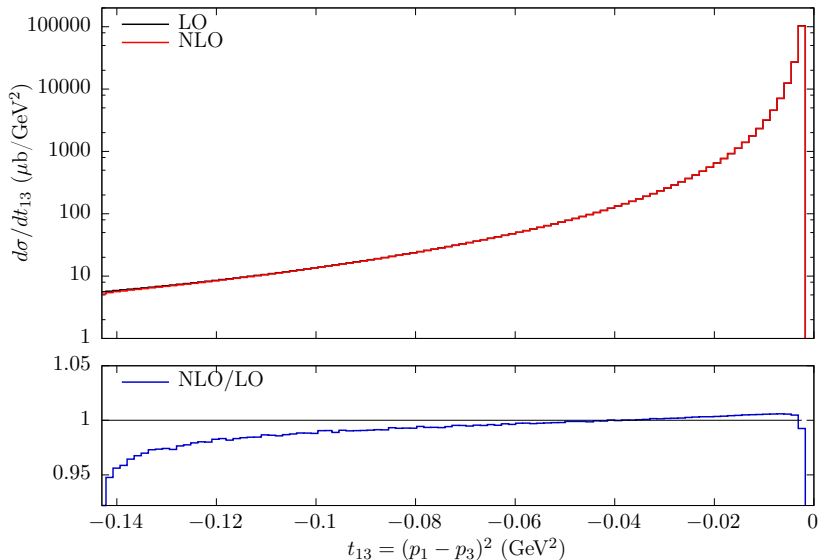
θ_μ distribution and corrections in the c.m.s.

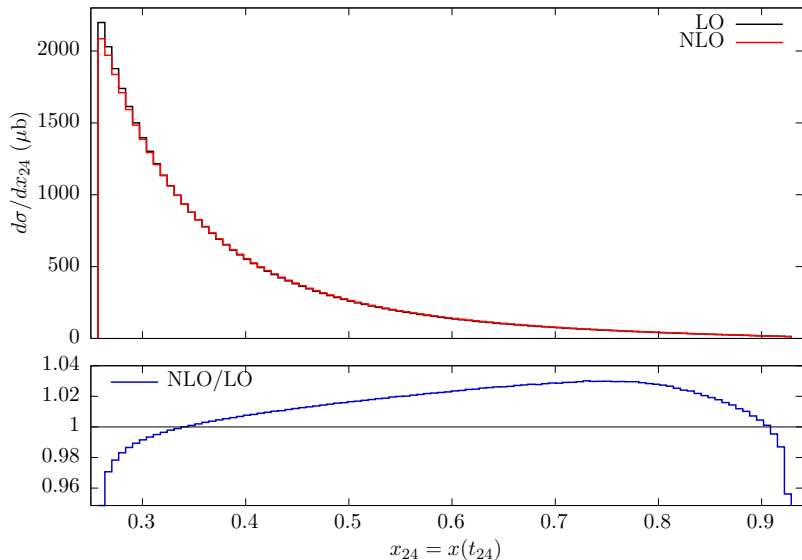


t_{24} (t on the “electron line”) distribution and corrections

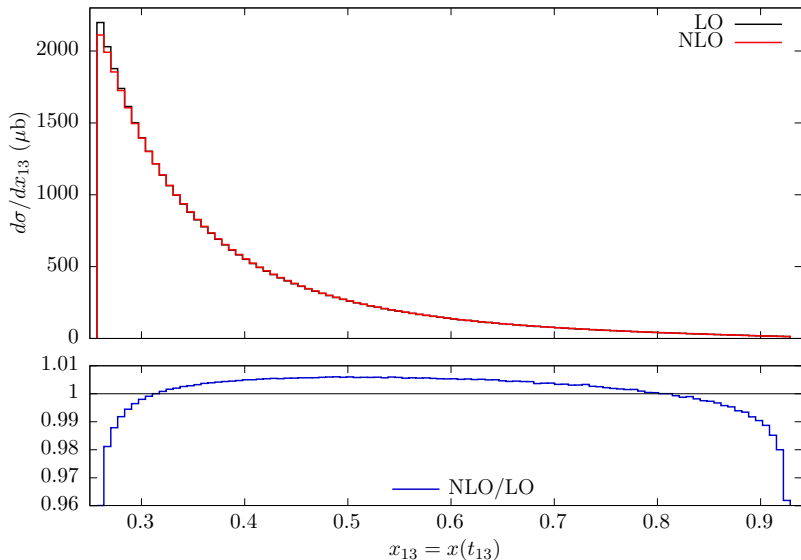


t_{13} (t on the “muon line”) distribution and corrections

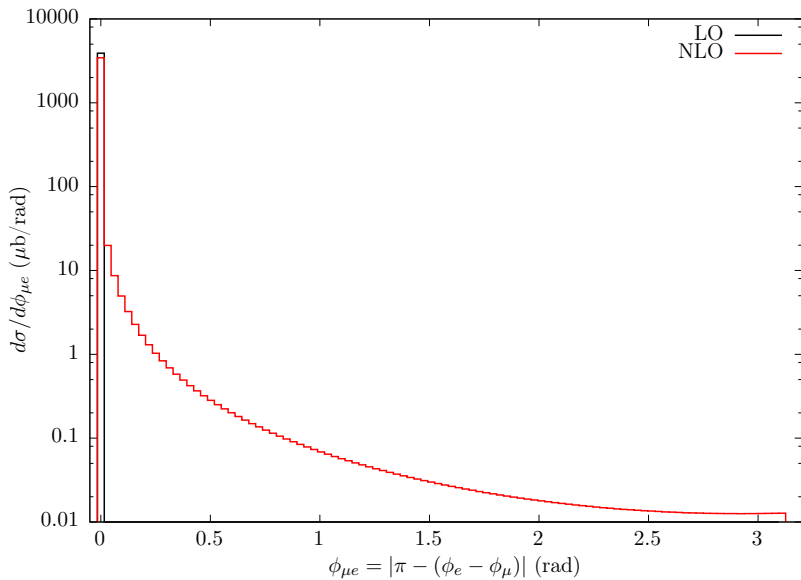




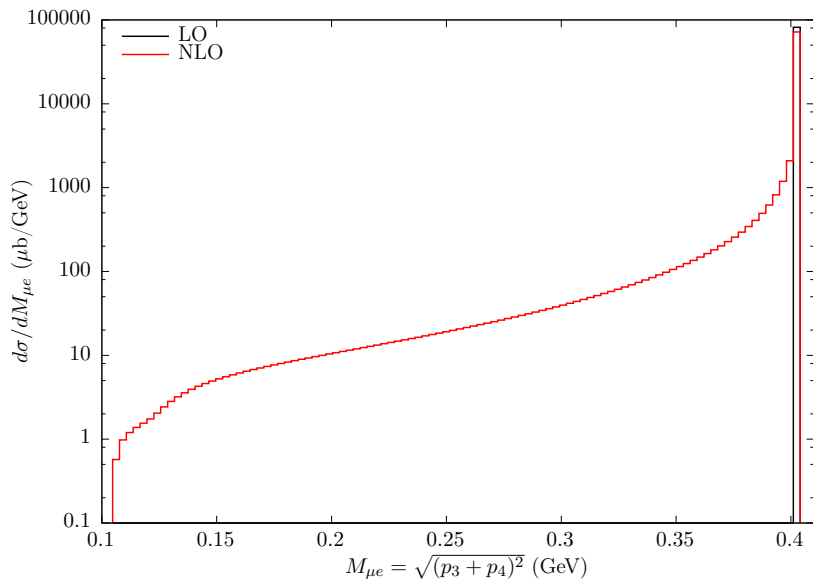
x_{13} distribution and corrections



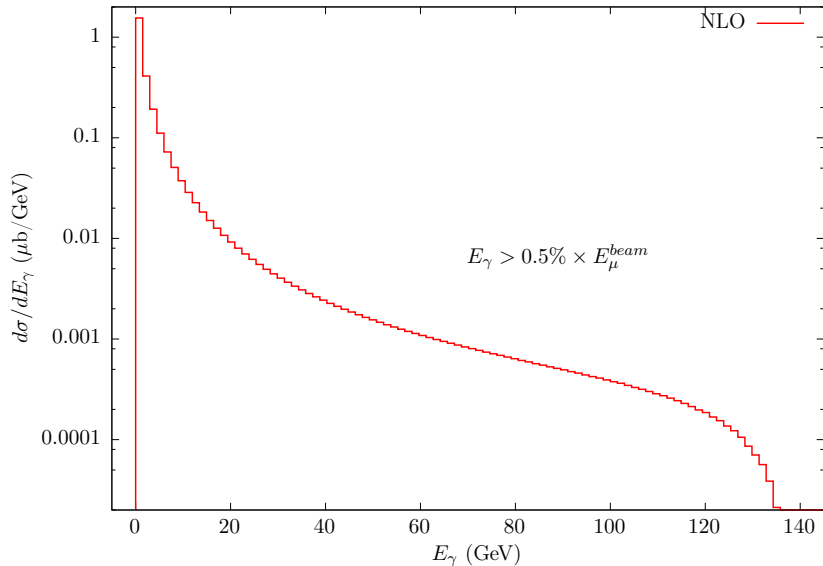
Acoplanarity distribution in the lab



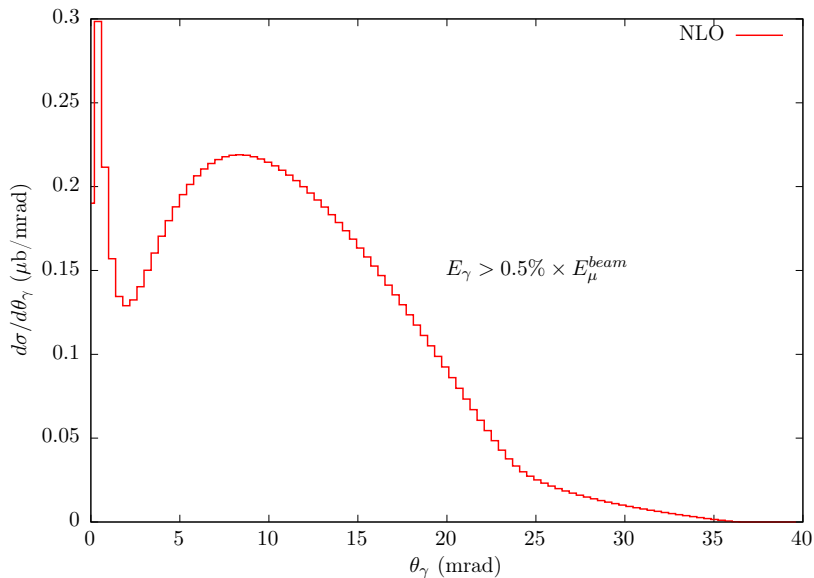
Final state μe invariant mass distribution



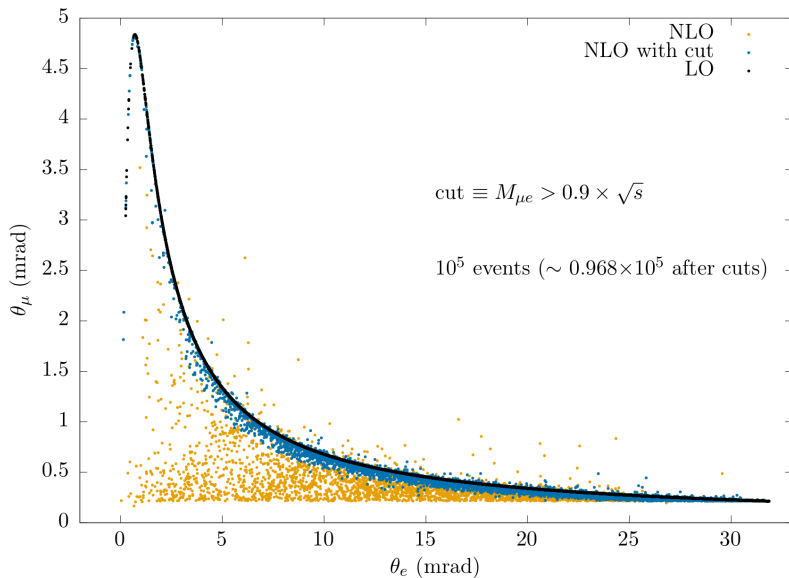
Photon energy distribution in the lab (radiative events)



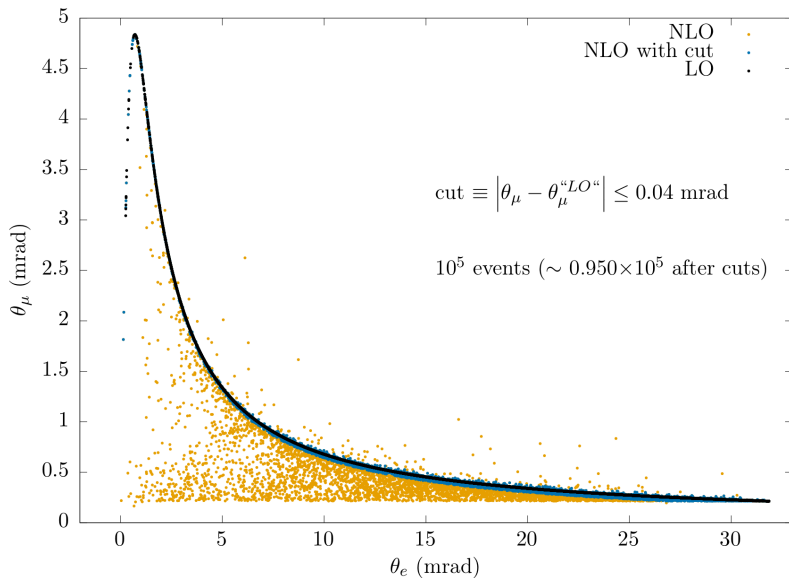
Photon angle distribution in the lab (radiative events)



μ - e angle correlation in the lab



μ - e angle correlation in the lab



- ★ (At least) one working QED NLO calculation is available in an event generator for our μe proposal
- ★ The size of QED NLO RCs on the considered observables lies in the 1-5% range (*except in the low θ_e and θ_μ distributions*), **within the loose cuts applied here**
- ★ The “Bhabha experience” at LEP & flavour factories compels to include also higher-orders (h.o., beyond NLO) RCs to reach high theoretical accuracy
- ★ Exact NNLO corrections are needed to reduce the theoretical uncertainty at the required level
- ★ A QED Parton Shower approach (matched to NLO) could be used to resum h.o. (*multiple-photon emission effects*) preserving fully exclusive generation
 - needs to be re-thought for the inclusion of (muon) mass effects
 - needs to be extended to be matched to exact NNLO corrections