DEFINING AN ORDER PARAMETER FOR CONFINEMENT

Adriano Di Giacomo

Pisa University and INFN, Italy.

JULY 20 2017
K.KONISHI RETIREMENT FEST

PLAN OF THE TALK

- A FEW REMARKS ON COLOUR CONFINEMENT.
- ORDER PARAMETERS AND GAUGE INVARIANCE.
- ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY OF QCD VACUUM.

REMARKS ON CONFINEMENT- 1

QUARKS STRICTLY CONFINED IN NATURE

- $\frac{n_q}{n_p} \leq 10^{-27}$ EXPECT $\approx 10^{-12}$
- $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$ EXPECT $O(1)$

NATURAL EXPLANATION : A SYMMETRY.

DECONFINING TRANSITION A CHANGE OF SYMMETRY.

REMARKS ON CONFINEMENT-2

- GAUGE INVARIANCE UNAFFECTED BY DECONFINING TRANSITION.
- D.O.F. INVOLVED LIVE ON THE SURFACE AT SPATIAL INFINITY (TOPOLOGY).
- TOPOLOGICAL EXCITATIONS IN (3+1)d MONOPOLES $[S_2 \to SU(2)]$, IN (2+1)d VORTICES $[S_1 \to U(1)]$
- CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM ['tHooft , Mandelstam 1975]

ORDER PARAMETER AND GAUGE INVARIANCE-1

- HIGGS MODEL

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + D_\mu \phi^\dagger D_\mu \phi - V(\phi^\dagger \phi)$$

$$D_\mu \phi \equiv (\partial_\mu + igA_\mu)\phi \qquad V(\rho^2) = \rho^2(m^2 + \lambda\rho^2),\ A_\mu \equiv T^a A^a{}_\mu$$

CHANGE VARIABLES : $\phi\ ,\ \phi^\dagger \longrightarrow \rho\ ,\ \hat{\phi}$

$$\phi = \rho\hat{\phi} \qquad \rho = \rho^\dagger \geq 0 \qquad \hat{\phi}^\dagger \hat{\phi} = 1, \qquad \phi^\dagger \phi = \rho^2$$

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \partial_\mu \rho \partial_\mu \rho + g^2 \rho^2 \hat{\phi}^\dagger \tilde{A}_\mu \tilde{A}_\mu \hat{\phi} - V(\rho^2)$$

$\tilde{A}_\mu \equiv A_\mu - \frac{i}{g}[\partial_\mu \hat{\phi}]\hat{\phi}^\dagger$ GAUGE COVARIANT

- $SU(2) \times U(1)$: BOSONIC SECTOR STANDARD MODEL.
- $U(1)$: $\hat{\phi} = \exp(-ig\Theta)$

  $\tilde{A}_\mu = A_\mu - \partial_\mu \Theta$ GAUGE INVARIANT.
- $SU(2)$, Φ IN THE FUNDAMENTAL REPRESENTATION : $\hat{\phi}^\dagger \tilde{A}_\mu \tilde{A}_\mu \hat{\phi} = \tilde{A}^a_\mu \tilde{A}^a_\mu$

ORDER PARAMETER AND GAUGE INVARIANCE-2

- EQUATIONS OF MOTION

$U(1)$ $\qquad\qquad$ $SU(2)$, $\phi \in$ FUND.REPR.

$$\partial_\mu F_{\mu\nu} + 2g^2\rho^2\tilde{A}_\nu = 0 \qquad D_\mu G_{\mu\nu} + 2g^2\rho^2\tilde{A}_\nu = 0$$

$$\partial_\mu[\rho^2\tilde{A}_\mu] = 0 \qquad D_\mu[\rho^2\tilde{A}\ _{mu}] = 0$$

$$\Box\rho - g^2\rho\tilde{A}_\mu\tilde{A}_\mu + m^2\rho(1 + \frac{\lambda\rho^2}{m^2}) = 0$$

- $m^2 < 0$ STABLE VACUUM : $\rho^2 = \bar{\rho}^2 = -\frac{m^2}{\lambda}$

  VECTOR BOSONS BECOME MASSIVE SPIN 1 PARTICLES $\mu^2 = 2g^2\bar{\rho}^2$: MEISSNER EFFECT AND CONSERVED LONDON CURRENT $J_\mu = \bar{\rho}^2\tilde{A}_\mu$ IN $U(1)$.
- WHAT SYMMETRY IS BROKEN AND WHAT IS THE ORDER PARAMETER (INVARIANT UNDER LOCAL GAUGE TRANSFORMATIONS [Elitzur 1975])?

ORDER PARAMETER AND GAUGE INVARIANCE-3

- WEINBERG Phys.Rev.D7 1068 (1973)
  ORDER PARAMETER $\langle \phi_u \rangle$, $\phi_u \equiv \phi$ IN UNITARY GAUGE.
  $\langle \phi^\dagger \phi \rangle = \langle \phi^\dagger \rangle \langle \phi \rangle$ (CLASSICAL ZERO MODES).
  BROKEN SYMMETRY : GLOBAL GAUGE GROUP.
  Elitzur 1975 SATISFIED. $U(1)$: $\langle \phi_u \rangle = \bar{\rho} = \sqrt{(\frac{-m^2}{\lambda})}$
- $U(1)$ GAUGE INVARIANT CHARGED FIELDS [Dirac 1955]

$$\Phi \equiv \exp(ig \int d^4 y A_\rho(y) c_\rho(x-y))\phi(x)$$
$$\partial_\rho c_\rho(x-y) = \delta^4(x-y) \qquad \Phi^\dagger \Phi = \phi^\dagger \phi = \rho^2$$

$$\Phi_\Lambda \to \Phi \exp(-ig\Lambda + ig \int d^4 y \partial_\rho \Lambda(y) c_\rho(x-y))$$

$$c_\rho^{(4)}(z) = \frac{1}{2\pi^2} \frac{z_\rho}{z^4};$$
$$c_0^{(3)}(z) = 0, \vec{c}^{(3)}(z) = \delta(z_0) \frac{1}{4\pi} \frac{\vec{z}}{z^3};$$
$$c_0^{(2)}(z) = c_3^{(2)}(z) = 0, \vec{c}^{(2)}(z) = \delta(z_0)\delta(z_3) \frac{1}{2\pi} \frac{\vec{\zeta}}{\zeta^2}, \ \vec{\zeta} = (z_1, z_2);$$
$$\vec{c}^{(1)}(z) = 0 \ , \qquad c_0^{(1)}(z) = \delta^3(\vec{z})\theta(z_0)$$

ORDER PARAMETER AND GAUGE INVARIANCE-4

$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \bar{D}_\mu\Phi^\dagger\bar{D}_\mu\Phi - V(\Phi^\dagger\Phi)$
$\bar{D}_\mu\Phi = \partial_\mu + ig(A_\mu - \partial_\mu\int d^4yA_\rho(y)c_\rho(x-y))$
$c_\rho = c_\rho^{(4)} \,:\, A_\mu - \partial_\mu\int d^4yA_\rho(y)c_\rho^{(4)}(x-y) = (\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box})A_\nu)$
$\langle\phi\rangle$ IN THE LANDAU GAUGE IS A GAUGE INVARIANT ORDER PARAMETER. $\langle\Phi\rangle = \bar{\rho}$ UP TO A PHASE.
$c_\rho = c_\rho^{(3)} \,:\, A_m - \partial_m\int d^4yA_r(y)c_3^{(3)}(x-y) = (\delta_{mn} - \frac{\partial_m\partial_n}{\nabla^2})A_n)$
$\langle\phi\rangle$ IN THE COULOMB GAUGE IS A GAUGE INVARIANT ORDER PARAMETER. $\langle\Phi\rangle = \bar{\rho}$ UP TO A PHASE.
FOR ANY GAUGE GROUP AND GAUGE $\langle\phi\rangle$ ORDER PARAMETER UP TO A GLOBAL GAUGE TRANSFORMATION:

$$\int d^4y\partial_\mu^{(y)}[c_\mu^{(4)}(y-x)\phi(y)] = \phi(x) + \int d^4yc_\mu^{(4)}(y-x)\partial_\mu\phi(y)$$

BY TRANSLATION INVARIANCE $\langle\phi(x)\rangle = \frac{1}{V}\int d^4x\phi(x)$ AND

$$\langle\phi(x)\rangle = \langle\phi(\infty)\rangle - \frac{1}{V}\int d^4z\frac{z_\mu}{z^4}\int d^4y\partial_\mu\phi(y) = \langle\phi(\infty)\rangle$$

GLOBAL SYMMETRY BREAKING.

ORDER PARAMETER AND GAUGE INVARIANCE-5

– $SU(2)$, ϕ FUNDAMENTAL REPRESENTATION ρ FIXED [E. Fradkin, S. Shenker Phys. Rev. D19 3682 (1979)]

SYSTEM IN BROKEN PHASE : NO PHASE TRANSITION EXPECTED.

ONLY UNPHYSICAL LINE OF TRANSITIONS OBSERVED ON LATTICE, DUE TO LATTICE ARTEFACTS [Bonati et al. Nucl.Phys. B828 , 330 (2010)] .

CONSISTENT WITH THE STATEMENTS OF [Fradkin et al].

ORDER PARAMETER FOR CONFINEMENT -1

– DEFINING A GAUGE INVARIANT ELECTRICALLY OR MAGNETICALLY CHARGED OPERATOR IN ABSENCE OF FUNDAMENTAL HIGGS. [LATTICE $U(1)$]

q, p $\quad \exp(-ip\delta)|q\rangle = |q+\delta\rangle \qquad \exp(iq\alpha)|p\rangle = |p+\alpha\rangle$

$$q \leftrightarrow \vec{A}(\vec{x}) \qquad p \leftrightarrow \vec{E}(\vec{x})$$

E- CHARGED FIELD $\quad \epsilon(\vec{x}) = \exp(ig\int d^3y\vec{A}(\vec{y})\vec{c}^{(3)}(\vec{x}-\vec{y}))$

$$\epsilon(\vec{x})|\vec{E}(\vec{y})\rangle = |\vec{E}(\vec{y}) + g\vec{c}^{(3)}(\vec{x}-\vec{y})\rangle$$

$\epsilon(\vec{x}) \rightarrow_\Lambda \epsilon(\vec{x})\exp(ig\Lambda(\vec{x}))$ SINCE $\vec{\nabla}\vec{c}^{(3)}(\vec{x}-\vec{y}) = \delta^3(\vec{x}-\vec{y})$

$\vec{c}^{(3)}(\vec{x}-\vec{y}) \rightarrow \vec{c}(\vec{x}-\vec{y}) = \vec{c}^{(3)}(\vec{x}-\vec{y}) - \vec{c}^{(1)}(\vec{x}-\vec{y})$

$\vec{\nabla}\vec{c}(\vec{x}-\vec{y}) = 0$, ONLY THE TRANSVERSE COMPONENT OF $\vec{A}$ CONTRIBUTES, AND $E(\vec{x}) = \exp(ig\int d^3y\vec{A}(\vec{y})\vec{c}(\vec{x}-\vec{y}))$ IS THE GAUGE INVARIANT DIRAC VERSION OF ϵ LIKE Φ WAS of ϕ.

ORDER PARAMETER FOR CONFINEMENT -2

M-CHARGED FIELD $\mu(\vec{x}) = \exp(-i\frac{2\pi}{g}\int d^3y\vec{E}(\vec{y})\vec{A}^{(mon)}(\vec{x}-\vec{y}))$

$$\mu(\vec{x})|\vec{A}(\vec{y})\rangle = |\vec{A}(\vec{y}) + \frac{2\pi}{g}\vec{A}^{(mon)}(\vec{x}-\vec{y})\rangle$$

DUAL VECTOR POTENTIAL $\vec{Z}$: $\vec{E} = \vec{\nabla}\wedge\vec{Z}$

$\mu(\vec{x}) = \exp(-i\frac{2\pi}{g}\int d^3y(\vec{\nabla}\wedge\vec{Z}(\vec{y}))\vec{A}^{(mon)}(\vec{x}-\vec{y})) =$
$\exp(i\frac{2\pi}{g}\int d^3y\vec{Z}(\vec{y})(\vec{\nabla}\wedge\vec{A}^{(mon)})) = \exp(i\frac{2\pi}{g}\int d^3y\vec{Z}\vec{B}^{(mon)}(\vec{x}-\vec{y}))$

DUAL GAUGE INVARIANT IF $\vec{\nabla}\vec{B}^{(mon)} = 0$

— $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(\pm)} = \epsilon_{3mj}\frac{z_j}{z(z\mp z_3)}$ COULOMBIC + STRING ALONG 3-AXIS $\vec{\nabla}\vec{A}_{(\pm)} = 0$, $\vec{\nabla}\vec{B}_{(\pm)} = 0$

— $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(+)}$ $[z_3 < 0]$, $\vec{A}^{(mon)}(\vec{z})_m = \vec{A}_{(-)}$ $[z_3 > 0]$
WU YANG : COULOMBIC + 2d SINGULARITY AT $z_3 = 0$.

ORDER PARAMETER FOR CONFINEMENT -3

TEST CASE : LATTICE $U(1)$ GAUGE THEORY, WILSON ACTION. A FIRST ORDER PHASE TRANSITION at $\beta \equiv \frac{2}{g^2} = \beta_c \approx 1$ FROM A CONFINED PHASE TO A DECONFINED PHASE.

$\langle \mu \rangle \neq 0$ AT $\beta < \beta_c$, $\langle \mu \rangle = 0$ AT $\beta \geq \beta_c$: CONFINING VACUUM IS A DUAL SUPERCONDUCTOR , NORMAL VACUUM IS NOT.

$\langle \mu \rangle = \frac{Z(S+\Delta S)}{Z(S)} \qquad \rho \equiv \partial_\beta \ln(\langle \mu \rangle) = \langle S \rangle_S - \langle (S + \Delta S) \rangle_{(S+\Delta S)}$

$\langle \mu \rangle = \exp(\int_0^\beta \rho(x) dx) \qquad$ SINCE $\qquad \langle \mu \rangle (\beta = 0) = 1$

FINITE SIZE SCALING $\rightarrow$ CRITICAL INDEXES COMPATIBLE WITH FIRST ORDER .

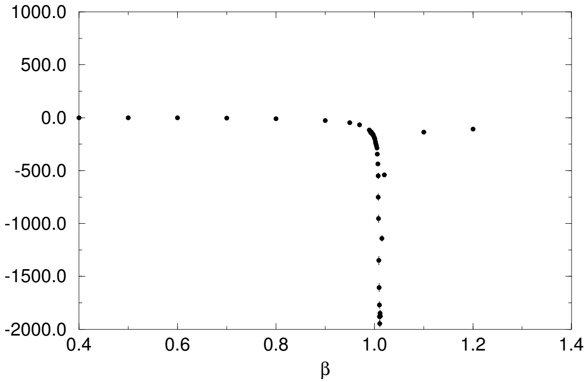

Figure : ρ versus β A.D.G. ,G.Paffuti P.R.D 56,6816 ,1997

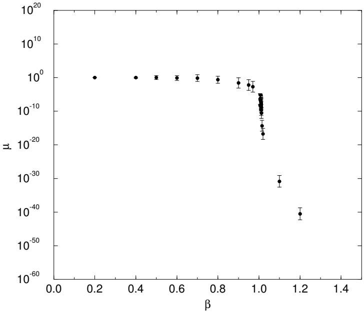

Figure : $\langle\mu\rangle$ for U(1) lattice gauge theory .

ORDER PARAMETER FOR CONFINEMENT -QCD- 1

- MONOPOLES DEFINED AS ABELIAN MONOPOLES IN $U(1)$ SUBGROUPS OF THE GAUGE GROUP (ABELIAN PROJECTIONS). ABELIAN PROJECTIONS PHYSICALLY EQUIVALENT.['tHooft 1981].
- MONOPOLE CREATION OPERATOR IN A GIVEN ABELIAN PROJECTION (GAUGE CHOICE) [$SU(2)$]:

$$\mu(\vec{x}, x_0) = \exp(-i\frac{2\pi}{g}\int d^3y \vec{E}_3(\vec{y}, x_0) T^3 \vec{A}^{(mon)}(\vec{x}-\vec{y})) \quad (1)$$

$$\frac{\partial \ln(\langle\mu\rangle)}{\partial\beta} \equiv \rho = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{(S+\Delta S)} \quad (2)$$

- IF ABELIAN PROJECTIONS ARE EQUIVALENT EQ (1) WITH NO GAUGE FIXING IS THE ORDER PARAMETER. $\langle\mu\rangle \neq 0 \rightarrow$ CONFINEMENT, $\langle\mu\rangle = 0$ DECONFINEMENT.

ORDER PARAMETER FOR CONFINEMENT -QCD- 2

- CHECK WITH PURE GAUGE $SU(2)$, $SU(3)$ WHERE CONFINED -DECONFINED ARE WELL DEFINED IN TERMS OF WILSON LOOPS. EXPECT A BEHAVIOR SIMILAR TO $U(1)$.
- SEEMED TO WORK AT THE BEGINNING WHEN VOLUMES WERE RATHER SMALL AND THE GROUPS $SU(2)$, $SU(3)$.(PISA GROUP , BARI GROUP 1996-2004).
- PROBLEMS WITH $G(2)$ GAUGE GROUP [Cossu, D'Elia , Di Giacomo, Lucini (2007)] . EXPLORE $SU(2)$, $SU(3)$ AT LARGER VOLUMES.
- DOES NOT WORK! . ρ DIVERGES AT LARGE VOLUMES $\rightarrow \langle \mu \rangle = 0$ ALSO IN THE CONFINED PHASE.
  AN INFRARED PROBLEM.

ORDER PARAMETER FOR CONFINEMENT -QCD- 3

– THE PROBLEM COULD BE IN THE DEFINITION OF $\langle\mu\rangle$:
A CHARGED OPERATOR HAS INFRARED PROBLEMS.
BETTER CREATE A MONOPOLE-ANTIMONOPOLE PAIR ,
SEND THEIR DISTANCE LARGE AND USE CLUSTER
PROPERTY TO DEFINE $\langle\mu\rangle$.

– STRONG COUPLING EXPANSION A GOOD DESCRIPTION
IN THE DEEPLY CONFINED PHASE:
USE IT TO UNDERSTAND INFRARED PROBLEMS.

ORDER PARAMETER FOR CONFINEMENT -QCD- 4

— DEFINING μ BY CLUSTER PROPERTY DOES NOT SOLVE THE PROBLEM.

— COMPUTE ρ IN STRONG COUPLING EXPANSION [C. Bonati, G. Cossu, M. DElia, A. Di Giacomo Phys Rev D85 065001 (2011)] $\langle\mu\rangle = \langle\mu|0\rangle$ IMPROVE TO $\langle\mu\rangle_i$

$$\langle\mu\rangle_i = \frac{\langle\mu|0\rangle}{\sqrt(\langle 0|0\rangle)\sqrt(\langle\mu|\mu\rangle)} \tag{3}$$

AT ORDER β^5 $\rho \approx -\beta^5 V^{\frac{1}{3}}$, $\langle\mu|0\rangle \approx \exp(-k\beta^5 V^{\frac{1}{3}})$, BUT $\sqrt(\langle\mu|\mu\rangle) \approx \exp(-k\beta^5 V^{\frac{1}{3}})$ AND $\langle\mu\rangle_i > 0$.
IN THE POLYAKOV ABELIAN PROJECTION $k = 0$.
MONOPOLE FIELD COMMUTES WITH TIME LINKS.

—$\langle\mu\rangle_i$ SOLVES THE PROBLEM $SU(2)$ $\langle\mu\rangle_i \neq 0$ $T < T_c$, $\langle\mu\rangle_i = 0$ $T > T_c$ CLUSTER PROPERTY OBEYED, CORRECT F.S.S. . $SU(3)$ $G2$ IN PROGRESS C. Bonati

– STABLE MONOPOLES NEED HIGGS FIELD, NOT PRESENT IN QCD. $A_4 = iA_0$ CAN PLAY THE ROLE OF HIGGS FIELD. [B. Julia, A. Zee Phys.Rev. D11 2227 (1975)]

$$D_j \vec{G}_{ij} = g\vec{\Phi} \wedge D_i\vec{\Phi} - g\vec{A}_0 \wedge D_i\vec{A}_0$$

$$D_i D_i \vec{A}_0 = g\vec{\Phi} \wedge D_0\vec{\Phi}$$

$$D_\mu D_\mu \vec{\Phi} + \lambda\vec{\Phi}(\Phi^2 + \frac{m^2}{\lambda}) = 0$$

IN THE GAUGE $\vec{A}_0 = 0$, ['tHooft, Polyakov (1974)]

$$D_j \vec{G}_{ij} = g\vec{\Phi} \wedge D_i\vec{\Phi}$$

$$D_i D_i \vec{\Phi} + \lambda\vec{\Phi}(\Phi^2 + \frac{m^2}{\lambda}) = 0$$

IN ABSENCE OF HIGGS FIELD (QCD)

$$D_j \vec{G}_{ij} = g\vec{A}_4 \wedge D_i\vec{A}_4$$

$$D_i D_i \vec{A}_4 = 0$$

SOLUTIONS : B.P.S. MONOPOLES OR DYONS [D. Diakonov (2008)]

POLYAKOV LINE $L(\vec{x}) = \exp(ig \int_0^{\frac{1}{T}} \vec{A}_4(\vec{x}, t)dt)$

IN THE GAUGE $\partial_4 A_4 = 0$: $L(\vec{x}) = \exp(i\frac{gA_4(\vec{x})}{T})$

THE ABELIAN PROJECTION OF THE MONOPOLES IS THE ONE IN WHICH THE POLYAKOV LINE IS DIAGONAL.

CONCLUSIONS AND OUTLOOK

- ORDER PARAMETERS WELL DEFINED IN GAUGE THEORIES: TRANSFORM UNDER GLOBAL GAUGE GROUP.
- LATTICE $U(1)$ CONFINES ELECTRIC CHARGES BY DUAL SUPERCONDUCTIVITY OF VACUUM. ORDER PARAMETER THE *vev* OF A MAGNETICALLY CHARGED OPERATOR.
- EXTENDING NAIVELY THE CONSTRUCTION TO N.A.G.T. HAS INFRARED PROBLEMS. IMPROVEMENT $\langle\mu\rangle \to \langle\mu\rangle_i$ SOLVES THEM (WORK IN PROGRESS)
- CONFINEMENT BY DUAL SUPERCONDUCTIVITY DEMONSTRATED. ASSUMPTION : ALL ABELIAN PROJECTIONS PHYSICALLY EQUIVALENT. ['tHooft 1981]