Topological Solitons, Nonperturbative Gauge Dynamics and Confinement

Pisa, 20/07/2017

# Strongly-Coupled Infrared Fixed Points, Confinement and Chiral Symmetry Breaking

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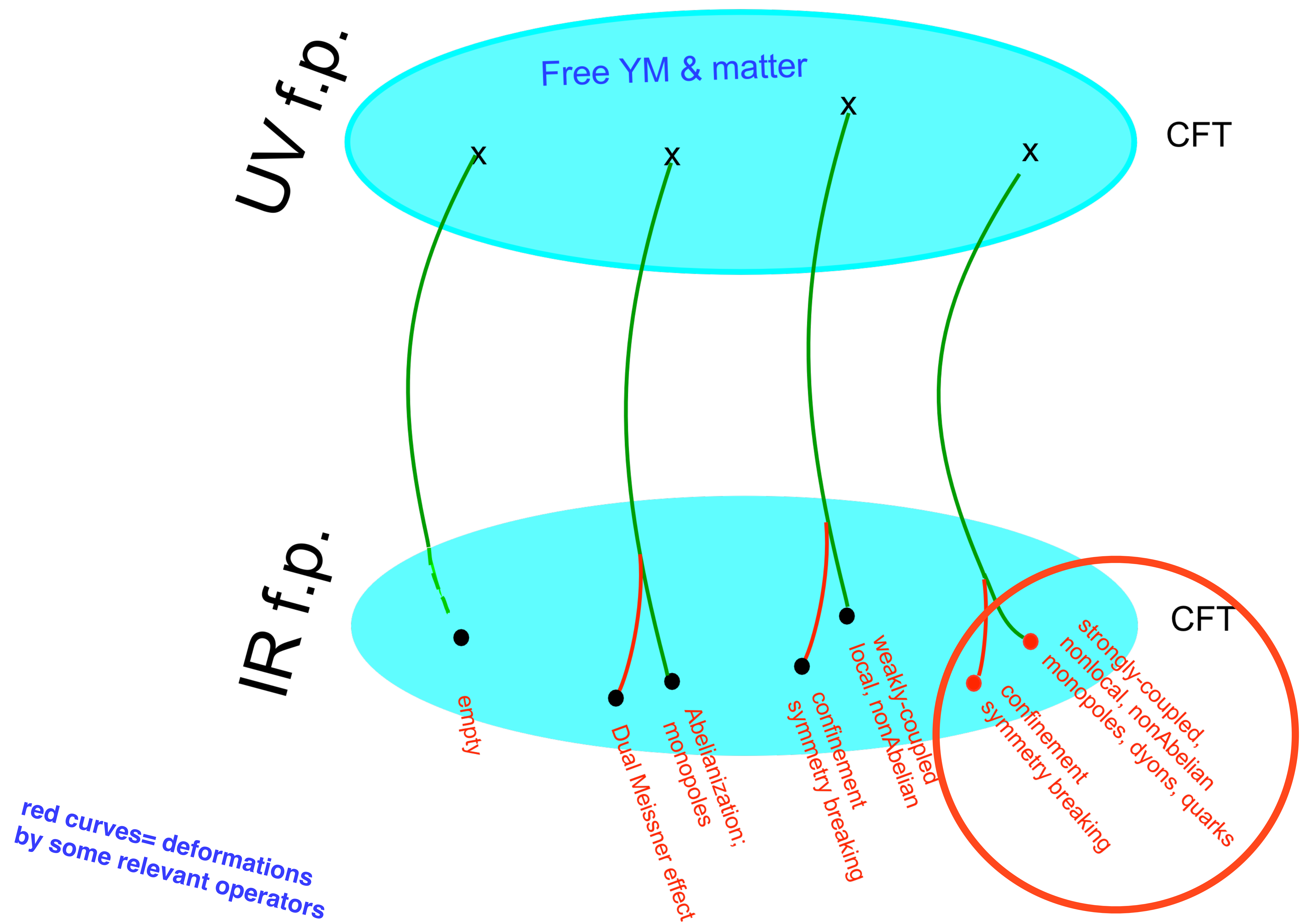
# Confinement and RG flow



# QCD

- Abelian dual superconductor (dynamical Abelianization) ?

't Hooft, Nambu, Mandelstam '80

$$SU(3) \to U(1)^2 \to \mathbf{1}$$

$$\langle M \rangle \neq 0$$

$$\Pi_1(U(1)^2) = \mathbf{Z} \times \mathbf{Z}$$

☞ Doubling of the meson spectrum (*)

- **Non-Abelian monopole condensation ?**

$$SU(3) \to SU(2) \times U(1) \to \mathbf{1}$$

☞ Problems (*) avoided

$$\Pi_1(SU(2) \times U(1)) = \mathbf{Z}$$

But non-Abelian monopoles expected to be strongly coupled (no sign flip of $b_0$)

- **Both the electric (quarks and gluons) and magnetic (monopoles and dyons) d.o.f. become strongly coupled in the Infrared !**

# Can $\mathcal{N}$=2 SQCD teach us anything useful?

- Or more humbly: any analogy between the phenomena occurring in $\mathcal{N}$=2 SQCD and in the real-world QCD?

- **A recent observation:** the most singular ("Argyres-Douglas") SCFT, in $\mathcal{N}$=2 SU(N) QCD with $N_F$ flavors, under an $\mathcal{N}$=1 perturbation,

$$\mu\Phi^2|_F = \mu\,\psi\psi + \ldots$$

Giacomelli, '15, Bolognesi, Giacomelli, KK '15

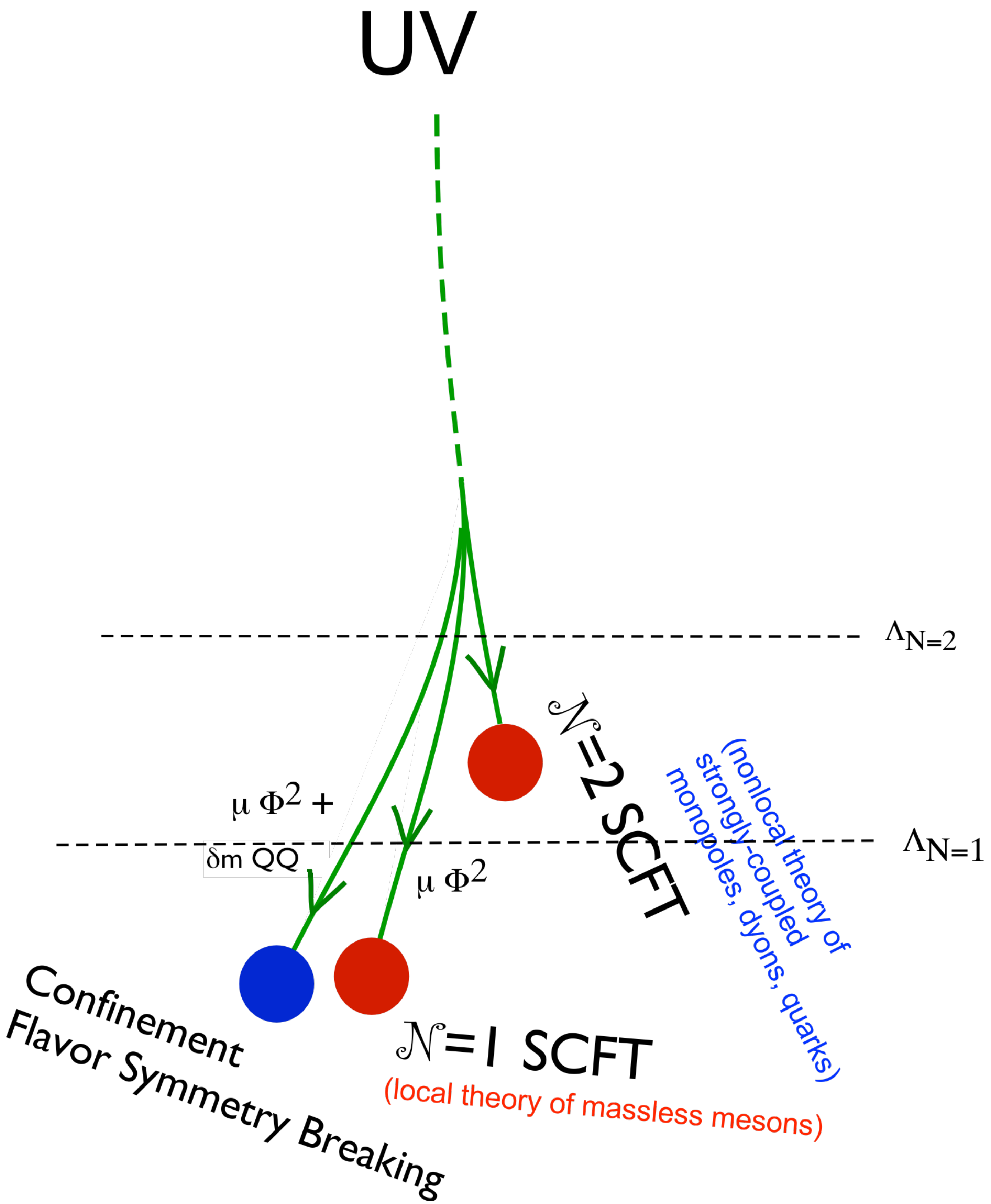
flows down (RG) towards an infrared-fixed-point theory described by massless mesons M in the adjoint representation of $G_F$

- Further relevant deformations (shift of bare mass parameters)

☞ Confinement and flavor symmetry breaking:

M (or a part of it) make metamorphosis to massless Nambu-Goldstone particles

# Blown-up RG flow



# Why remarkable:

- $\mathcal{N}$=2 SCFT is a complicated, nonlocal theory of strongly interacting massless monopoles, dyons and quarks *
- $\mathcal{N}$=1 SCFT is a theory of weak coupled local theory of mesons M
- In the nearby $\mathcal{N}$=1 confining vacuum, M ~ NG bosons of symmetry breaking **
- Analogous to the real-world QCD * **

# Massless mesons in the adjoint representation of $G_F$ in the IR

## Tools:

Giacomelli, '15, Bolognesi, Giacomelli, KK '15

- Seiberg-Witten curves for $\mathcal{N}$=2 gauge theories
  - ☞ SCFT by appropriate tuning of the vacuum parameters (VEVs) and m's

Tachikawa. Lecture Notes In Physics '15

- Trace anomalies (any theory)

$$\langle T^{\mu}_{\mu} \rangle = \frac{1}{16\pi^2} \left[ c\,(R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{R^2}{3}) - a\,(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2) \right]$$

(Weyl)^2 Euler density

- For any $\mathcal{N}$=1 susy theory ( R= $U_R$ (1) charge )

Tr= sum over Weyl fermions

$$a = \frac{3}{32}(3\mathrm{Tr}\, R^3 - \mathrm{Tr}\, R)\ ; \qquad c = \frac{1}{32}(9\mathrm{Tr}\, R^3 - 5\mathrm{Tr}\, R)\ ,$$

Anselmi, Freedman, Grisaru, Johansen ('98)

- For $\mathcal{N}$=2 susy fields

$$\mathrm{Tr}\, R^3_{\mathcal{N}=2} = \mathrm{Tr}\, R_{\mathcal{N}=2} = 48(a-c)\ ; \qquad \mathrm{Tr}\, R_{\mathcal{N}=2} I_a I_b = \delta_{ab}(4a-2c)\ .$$

Any $\mathcal{N}$=2 theory has global R symmetries

$$SU_R(2) \times U_R(1); \qquad \mathcal{R}_{\mathcal{N}=2} \equiv U_R(1)\ \text{ charge}, \quad I_3 \subset SU_R(2)$$

# Flowing down from $\mathcal{N}$=2 SCFT to $\mathcal{N}$=1 SCFT

$$\mu \Phi^2|_F = \mu\, \psi\psi + \ldots$$

Dijkgraaf, Vafa '03
Cachazo-Seiberg-Witten '03
Bonelli, Giacomelli, Maruyoshi, Tanzini ('13)
Giacomelli, '15

- $\mathcal{N}$=1 curves ($\mathcal{N}$=2 SW curves + factorization condition)
- ☞relations

$$\{R_{\mathcal{N}=2}\ , I_3\} \leftrightarrow R_{\mathcal{N}=1}$$

e.g., for SU(2), $N_F$ =1, $R_{\mathcal{N}=1} = \frac{5}{6} R_{\mathcal{N}=2} + \frac{1}{3} I_3$

for SU(N), $N_F$ =2N-1, $R_{\mathcal{N}=1} = \frac{2}{3} R_{\mathcal{N}=2} + \frac{2}{3} I_3$

- Known $\{R_{\mathcal{N}=2}\ , I_3\}$ of $\mathcal{N}$=2 SCFT $\Longrightarrow$ $R_{\mathcal{N}=1}$

- <u>'t Hooft anomaly matching conditions</u>
  Tr $R^3$ , Tr R , Tr $R(G_F)^2$

**Very nontrivial check / powerful info on the massless d.o.f.**

a and c

- Known Tr $R^3$ , Tr R of the IR theory - > a, c of the IR theory

e.g., for SU(2), $N_F$ =1, $a = \frac{1}{48}$ , $c = \frac{1}{24}$ ,

for SU(2), $N_F$ =3, $a = \frac{1}{6}$ , $c = \frac{1}{3}$ ,

**This is $3^2$ -1= 8 massless meson chiral fields !!**

$$W_{eff} = \mu\, Tr M^3 + \cdots$$

Bolognesi, Giacomelli, KK '15

- Works for $\mathcal{N}$=2 SCFT with SU(N) color, any $N_F$

- The result has been checked by following different RG paths

New route

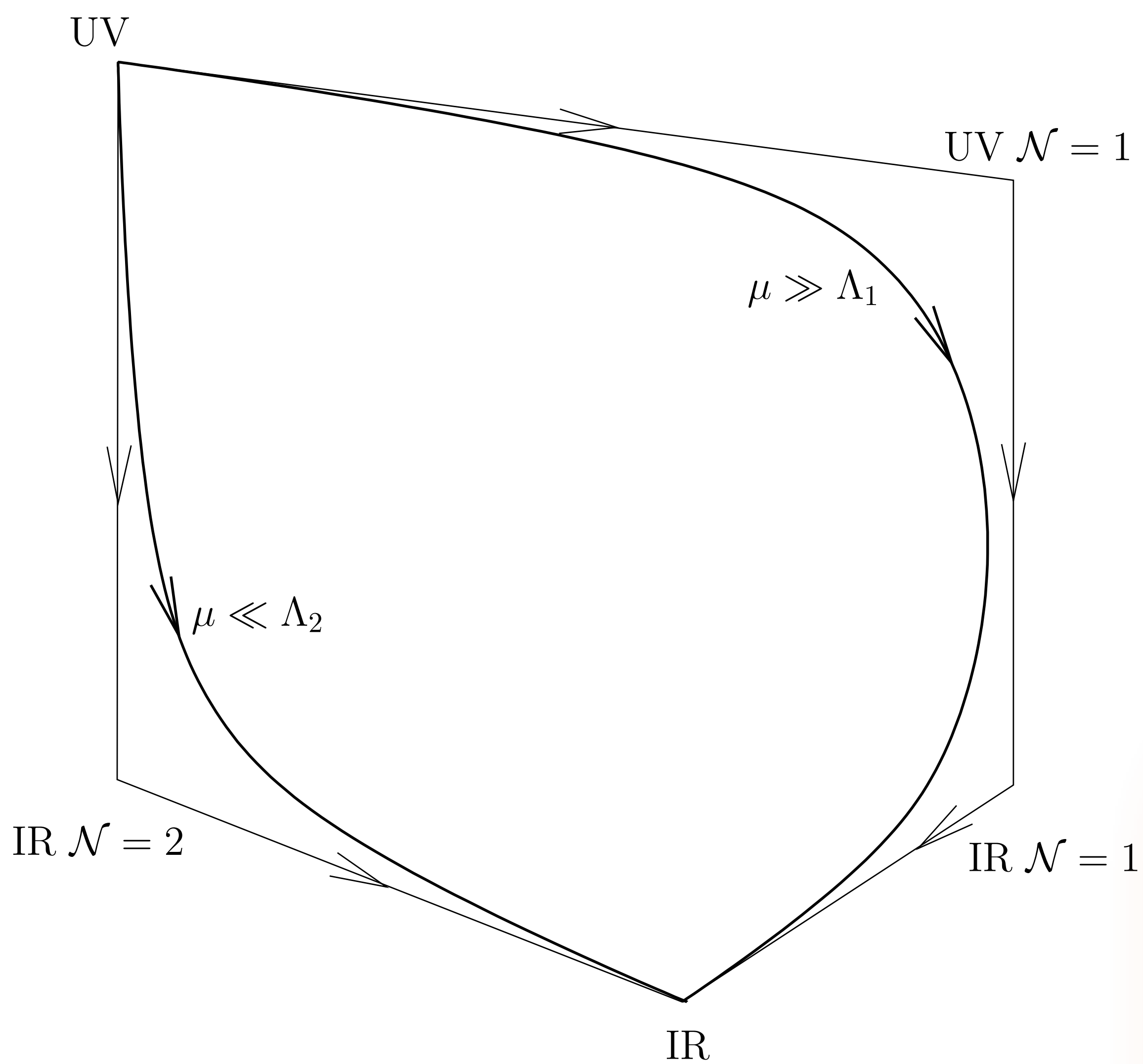



Figure 1: RG flow for various values of $\mu$.

$\mathcal{N}=1$ $L_{eff}$ of SQCD

(M, B, q, ...)

Seiberg, '94

Carlino, K.K. Murayama '00

Di Pietro, Giacomelli '12

BTW: our mesons M

~ Meson M in the

Seiberg's $\mathcal{N}=1$ duality

SU(N), $N_F$ Q's

< —- >

SU($N_F$ - N), $N_F$ q's, M

# A puzzle ?

Weak GST duals for SU(N) $N_F$ = 2n Infrared theory

Gaiotto, Seiberg, Tachikawa '11

- $\mathcal{N}$=1 perturbation —> confinement and XSB

Giacomelli, Konishi '14 ,

in simplest cases: USp(2N), $N_F$ =4; SU(3), $N_F$ =4; SU(4), $N_F$ =4;

SO(2N), $N_F$ =2.

- In all cases the GST dual description correctly realize XSB;
- In simplest cases, d.o.f ~ monopoles carrying flavor q.n.'s
- **SU(N), $N_F$ = 2n, GST dual looked more difficult**

$$\underbrace{\boxed{S_{N-n+1}}}_{AD_{N_f=2}(SU(N-n+1))} - SU(2) - \boxed{R_n} \ ,$$

Nonlocal SCFT of monopoles, dyons and quarks with flavor group SU(2) x SU(2n)

- Actually $\mathcal{N}$=1 deformation can be worked out also in this case

Giacomelli, Konishi '17

☞ Free mesons in adjoint representation of SU(2n)

(Tools; Conformal anomalies, 't Hooft anomaly matching conditions)

# To conclude:

UV

Real-world QCD

$\mathcal{N}$=0 SCFT

$\mathcal{N}$=2 SCFT

SU(N), $N_F$ =2N-1

$$a_{UV} = \frac{11 N_f N_c}{360} + \frac{31}{180}(N_c^2 - 1)$$

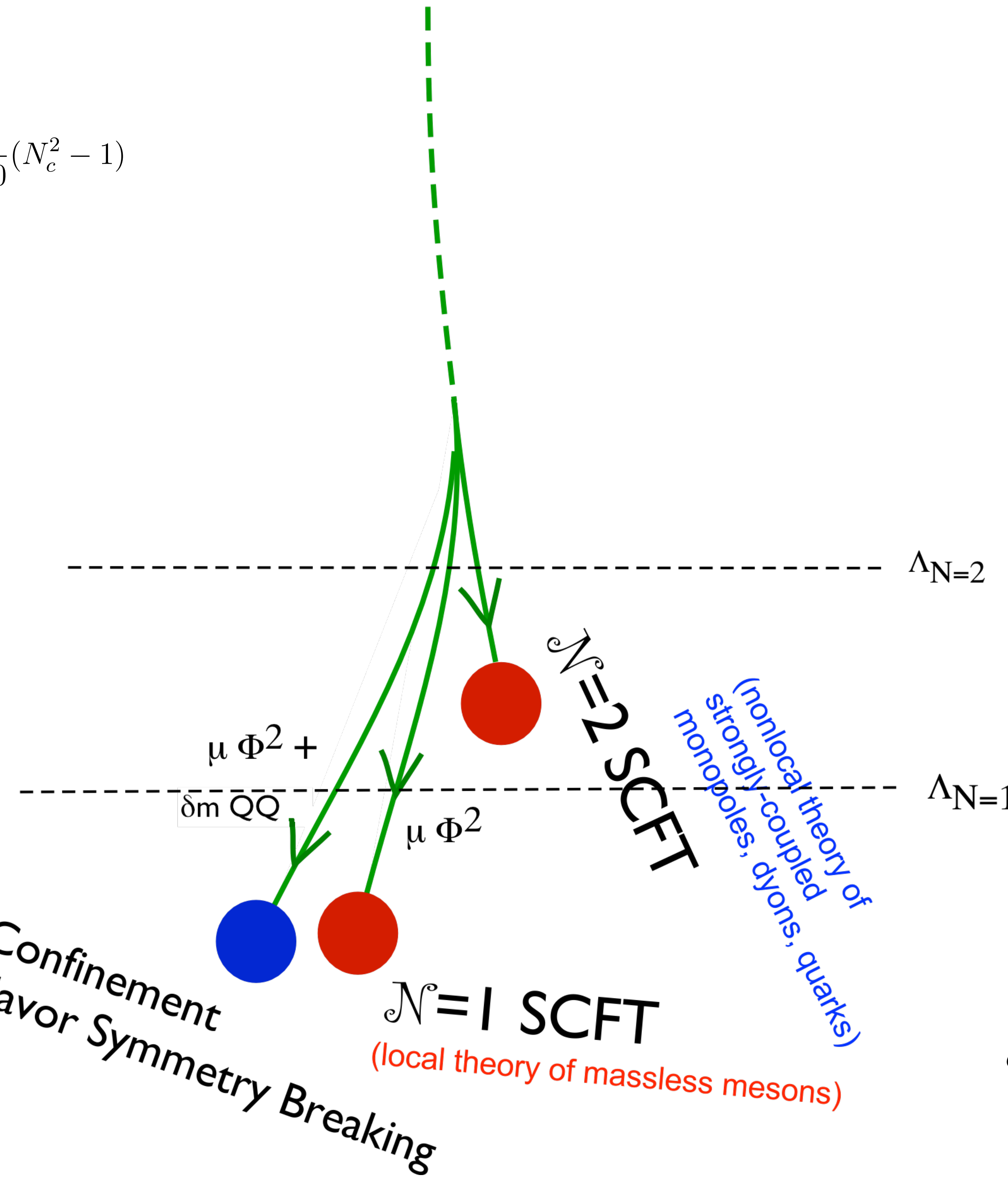
$$a_{UV} = \frac{7N^2 - N - 5}{24}$$

$\vee$

$$a_{UV} > a_{IR} \quad if \quad N_f < \frac{11}{2} N_c$$

$$a_{N=2SCFT} = \frac{7N(N-1)}{24}$$

$\vee$




$$a_{IR} = \frac{N_f^2 - 1}{360}$$

$$a_{IR} = \frac{(2N-1)^2 - 1}{48}$$

# Thank you, all
# Grazie a tutti
# ありがとう
# (arigatou)

……

"古稀"

ありがとう (ARIGATOU)

＝　ありがたく (ARIGATAKU)

=　Rare, difficult to have, precious（luck, etc）

“ひとの生をうくるはかたく、死すべきものの、生命あるも**ありがたし**”

“That a man is given life in this universe, where death is normal, **is an incredibly rare and precious thing** ”

(from ”Dhammapada”, ”法句経”)

# Thank you, all
# Grazie a tutti
# ありがとう

## ’t Hooft anomaly matching conditions between $\mathcal{N}$=2 SCFT and $\mathcal{N}$=1 SCFT

SU(N), $N_F$ =2N-1   AD vacuum

$\mathcal{N}$=2 SCFT  (“UV”)   Input :

$$a' = \frac{7}{24}N(N-1), \quad c' = \frac{1}{3}N(N-1), \quad k_{SU(2N-1)} = 2N-1$$

Shapers, Tachikawa ‘08
Cecotti, Del Zotto, Giacomelli ‘15

$$R_{\mathcal{N}=1} = \frac{2}{3}R_{\mathcal{N}=2} + \frac{2}{3}I_3$$

$$\Longrightarrow \quad Tr\,R_{\mathcal{N}=1} = \frac{2}{3}Tr\,R_{\mathcal{N}=2} = -\frac{4}{3}N(N-1)$$

$$Tr\,R^3_{\mathcal{N}=1} = \frac{8}{27}\left[Tr\,R^3_{\mathcal{N}=2} + 3\,Tr\,R_{\mathcal{N}=2}I_3^2\right] = -\frac{4}{27}N(N-1)$$

$$Tr\,R_{\mathcal{N}=1}SU(2N-1)^2 = \frac{2}{3}\,Tr\,R_{\mathcal{N}=2}SU(2N-1)^2 = \frac{1-2N}{3}$$

☞  $\mathcal{N}$=1 SCFT  (“IR”)

Massless mesons in the adjoint representation of SU($N_F$) !!!

$$a = \frac{(2N-1)^2-1}{48}, \qquad c = \frac{(2N-1)^2-1}{24}$$

$$\Longrightarrow \quad TrR_{\mathcal{N}=1} = 16(a-c) = -\frac{4}{3}N(N-1), \qquad TrR^3_{\mathcal{N}=1} = \frac{16}{9}(5a-3c) = -\frac{4}{27}N(N-1)$$

$$\Longrightarrow \quad TrR_{\mathcal{N}=1}SU(2N-1)^2 = (R_{\mathcal{N}=1}(M)-1)(2N-1) = \frac{1-2N}{3}$$

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# Complex Structure of Susy 4D Gauge Theories

- **Chiral superfields**

$$\Phi(x,\theta,\bar\theta) = \phi(y) + \sqrt{2}\,\theta\,\psi(x) + \theta\theta F(y), \qquad y = x + i\theta\sigma\bar\theta$$

- **Verctor superfields** $V^\dagger = V$,

$$W_\alpha = -\frac{1}{4}\bar D^2 e^{-V} D_\alpha e^V = -i\lambda + \frac{\mu}{2}\,(\sigma^\mu\,\bar\sigma^\nu)_\alpha^\beta\,F_{\mu\nu}\,\theta_\beta + \ \ldots$$

- **Supersymmetric Lagrangian** ($\int d\theta_1\,\theta_1 = 1$**, etc)**

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\,\tau_{cl}\left[\int d^4\theta\,\Phi^\dagger e^V\Phi + \int d^2\theta\,\frac{1}{2}\,W_\alpha\,W^\alpha\right] + \int d^2\theta\ W(\Phi)$$

- $W(\Phi) =$ **superpotential;** $\qquad \tau_{cl} = \frac{\theta}{2\pi} + \frac{4\,\pi\,i}{g^2}$

- **Potential (F term and D term)**

$$V_{sc} = \sum_{mat}\left|\frac{\partial W}{\partial\phi}\right|^2 + \frac{1}{2}\sum_a\left|\sum_{mat}\phi^* t^a\phi\right|^2$$

**Vacuum Degeneracy (Space of Vacua)**

- **Non-renormalization Theorem (perturbative) (***cfr.* **Anomaly)**

- **Q: Superpotential Dynamically Generated ?**

$\mathcal{N} = 2$ if

$\Phi \sim adj$ and

$W(\Phi) = 0 \qquad (W = M\Phi\tilde M$ in SQCD)

# Seiberg-Witten solution in N=2, SU(2) susy gauge theories

'94

- Fields: $W = (A_\mu, \lambda), \quad \Phi = (\phi, \psi)$

$SU_R(2)$

Moduli of vacua (degeneracy):

$$\langle \phi \rangle = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \qquad u = Tr\langle \phi^2 \rangle$$

SU(2)/U(1): monopoles

$$F_{\mu\nu}F^{\mu\nu} + i\,\bar\lambda\,\gamma^\mu \mathcal{D}^\mu \lambda + iF_{\mu\nu}\tilde F^{\mu\nu} + \ldots$$

F(A)= prepotential
holomorphic

- $L_{eff}$ :

$$L_{eff} = \mathrm{Im}[\int d^4\theta\, \bar A \underbrace{\frac{\partial F(A)}{\partial A}}_{= A_D} + \int d^2\theta \frac{\partial^2 F(A)}{\partial A^2} W^\alpha W_\alpha]$$

$$\tau = F''(A) = \frac{dA_D}{dA} = \frac{\theta_{eff}}{2\pi} + \frac{4\pi i}{g^2_{eff}}$$

- Duality: L=$L_{eff}$ formally inv under SL(2,Z) (ad-bc=1) ⊃ EM duality

$$\begin{pmatrix} A_D \\ A \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A_D \\ A \end{pmatrix} \qquad \begin{pmatrix} \delta L/\delta F^+_{\mu\nu} \\ F^+_{\mu\nu} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \delta L/\delta F^+_{\mu\nu} \\ F^+_{\mu\nu} \end{pmatrix}$$

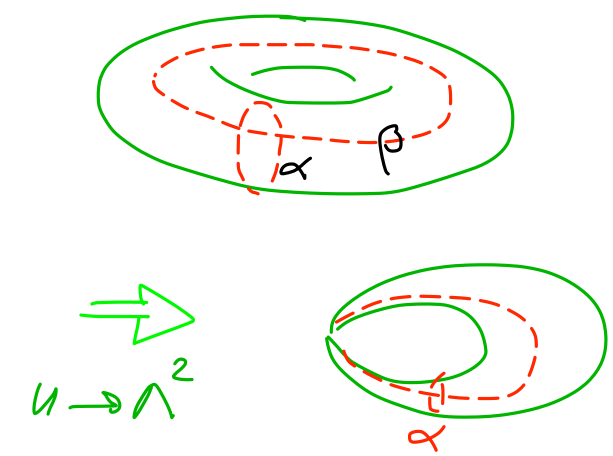
$A_D = \partial F/\partial A$

- Which description? Depends on u !

- Assume:

massless monopoles at u= ± $\Lambda^2$ $\longrightarrow$ F(A) !

Seiberg-Witten curve (SU(2) YM)




$$\boxed{y^2 = (x-u)(x-\Lambda^2)(x+\Lambda^2)} \quad ※$$

- solves the theory

$$\frac{dA_D}{du} = \oint_\alpha \frac{dx}{y}, \qquad \frac{dA}{du} = \oint_\beta \frac{dx}{y}, \qquad \text{➡ F(A)}$$

$$Im \frac{\oint_\alpha \frac{dx}{y}}{\oint_\beta \frac{dx}{y}} > 0$$

$$M_{n_m,n_e} = |n_m A_D + n_e A|, \qquad A_D = \oint_\alpha \lambda, \quad A = \oint_\beta \lambda,$$

- Perturbative and nonperturbative quantum effects (instantons) fully encoded in ※

- Effective theory near u= $\Lambda^2$

$$L_{eff}(A_D, F_D^{\mu\nu}, \ldots) + \int d^4\theta\, \bar{M} e^{V_D} M + (M \to \tilde{M}) + \sqrt{2}\int d^2\theta\, M A_D \tilde{M}$$

Magnetic monopole coupled minimally to the dual gauge field

Seiberg-Witten curves for general gauge groups

• SU(N) with $N_F$ quarks

$$y^2 = \prod_{i=1}^{N}(x-\phi_i)^2 - \Lambda^{2N-N_F}\prod_{a=1}^{N_F}(x+m_a)$$

• SO(N) with $N_F$ quarks in vector representation

$$y^2 = x\prod_{i=1}^{[N/2]}(x-\phi_i^2)^2 - 4\Lambda^{2(N-N_F-N_F)}x^{2+\epsilon}\prod_{a=1}^{N_F}(x+m_a^2)$$

• etc.

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## Argyres-Seiberg's S duality

Argyres, Seiberg '07

- SU(3) with $N_F = 6$ hypermultiplets ( $Q_i, \tilde{Q}_i$ ' s ) at infinite coupling

Minahan-Nemeschansky '96

$$SU(3)\ w/\ (6 \cdot \mathbf{3} \oplus \bar{\mathbf{3}}) \quad = \quad SU(2)\ w/\ (2 \cdot \mathbf{2} \oplus \mathrm{SCFT}_{E_6})$$

g = ∞ g = 0

$$SU(2) \times SU(6) \subset E_6$$

Flavor symmetry ~ SU(6) x U(1)

"$\mathcal{N}$=2 dualities"

D. Gaiotto '07

Explosive developments in studies of $\mathcal{N}$=2 SCFT

'07-'17

- $U(1)^{N-n}$ gauge multiplets

Gaiotto-Seiberg-Tachikawa '11

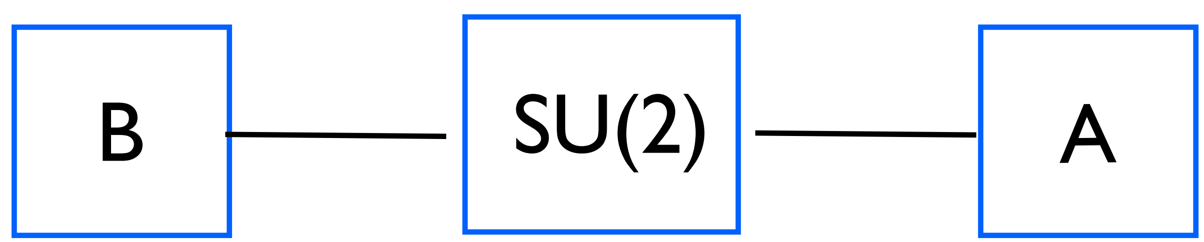
- SU(2) gauge field coupled to the SU(2) flavor symmetry of the SCFT A & B

$$b_0 = \frac{N-n}{N-n+2}$$

- The A sector: the SCFT entering in the Argyres-Seiberg dual of SU(n), $N_F$ =2n with $G_F$ = SU(2)x SU(2n), known as Rn

- The B sector: the singular SCFT of the SU(N-n+1) theory with two flavors

Argyres-Plesser-Seiberg-Witten '95




- GST solved the problem of wrong dimensions for the masses in EHIY
- GST eliminated a counter example (Shapere-Tachikawa) to the "a" theorem
- GST dualities generalized to SO(N), USp(2N) theories

Giacomelli '12

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# Gaiotto-Seiberg-Tachikawa (GST) 2011

- Apply the basic idea of Argyres-Seiberg duality to the IR f.p. SCFT

- SU(N) with $N_F = 2n$ :

$$y^2 = (x^N + u_1 x^{N-1} + u_2 x^{N-2} + \cdots + u_N)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x + m_i)$$

At $u = m = 0$,* $y^2 \sim x^{N+n}$ (EHIY point)

Note

* except for one u

relatively non-local
massless monopoles and dyons

Eguchi-Hori-Ito-Yang '96

- Straightforward treatment of fluctuations
  $\Longrightarrow$ wrong scaling laws for the masses

❀ To keep the correct dimensions of masses, introduce two different scalings:

$$u_{N-n+2} \sim O(\epsilon_A^2), \quad u_{N-n+3} \sim O(\epsilon_A^3), \quad \ldots, \quad u_N \sim O(\epsilon_A^n).$$

$$u_1 \sim O(\epsilon_B), \quad u_2 \sim O(\epsilon_B^2), \quad \ldots, \quad u_{N-n+2} \sim O(\epsilon_B^{N-n+2}).$$

$$\epsilon_A^2 = \epsilon_B^{N-n+2} \qquad \Longrightarrow$$

$$a_i = \oint_{\alpha_i} \lambda, \quad a_{D\,i} = \oint_{\beta_i} \lambda$$

$$\lambda \sim dx\, y/x^n$$

$$m_{(n_m, n_e, n_i)} = \sqrt{2}\, |n_m a_D + n_e a + n_i m_i|$$

## GST dual for the singular point of USp(2N) (also SO(N))

Giacomelli '12

$$y^2 \sim x^{2n} \quad \approx \quad \boxed{B} - \boxed{SU(2)} - \boxed{A}$$

- $U(1)^{N-n}$ gauge multiplets

- The A sector: a (in general) non-Lagrangian SCFT having SU(2)xSO(4n) flavor symmetry

- The B sector: a free doublet (coupled to U(1) gauge boson)

For $N_F = 2n = 4$, A sector ~ 4 free doublets

## Argyres-Douglas

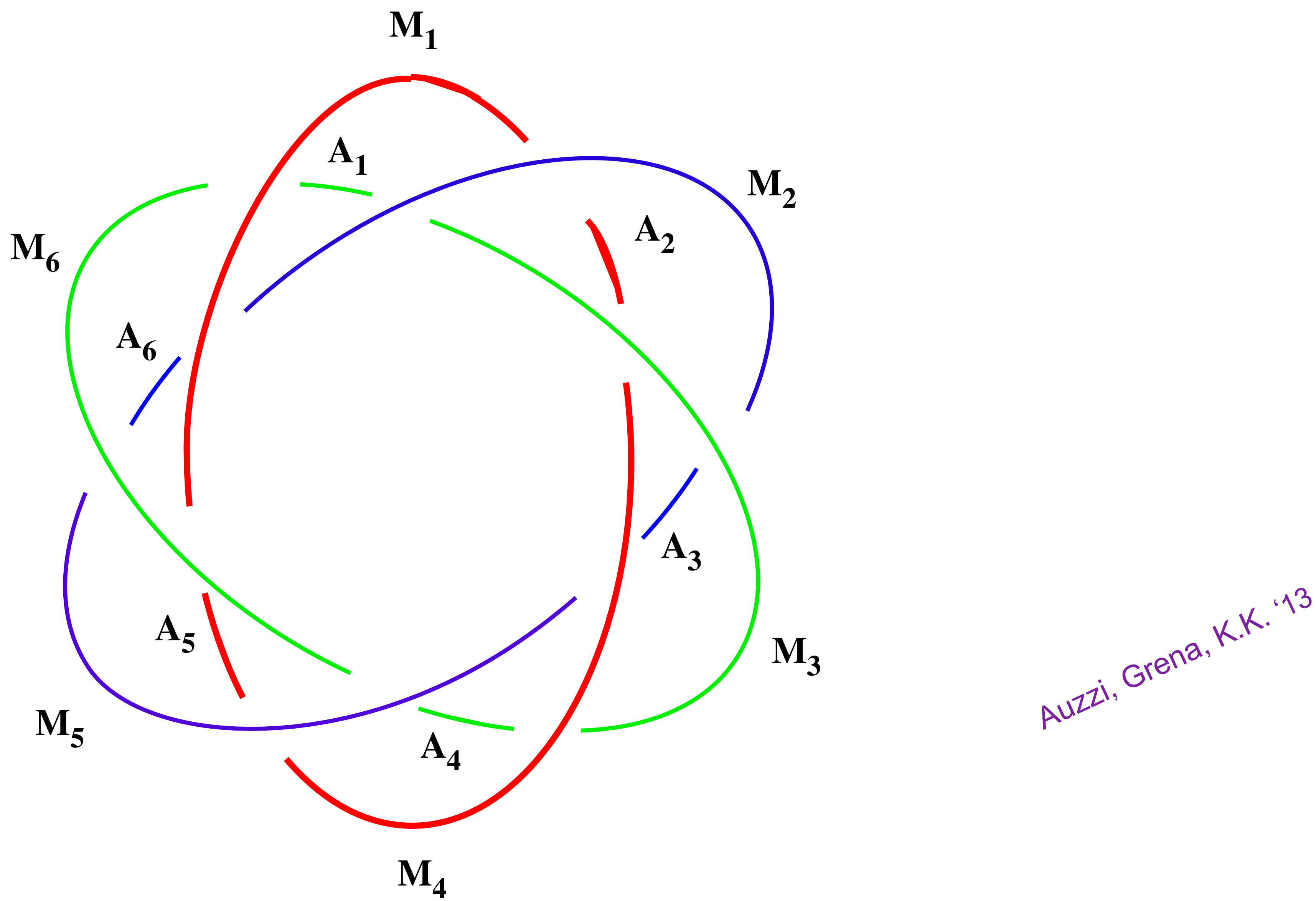
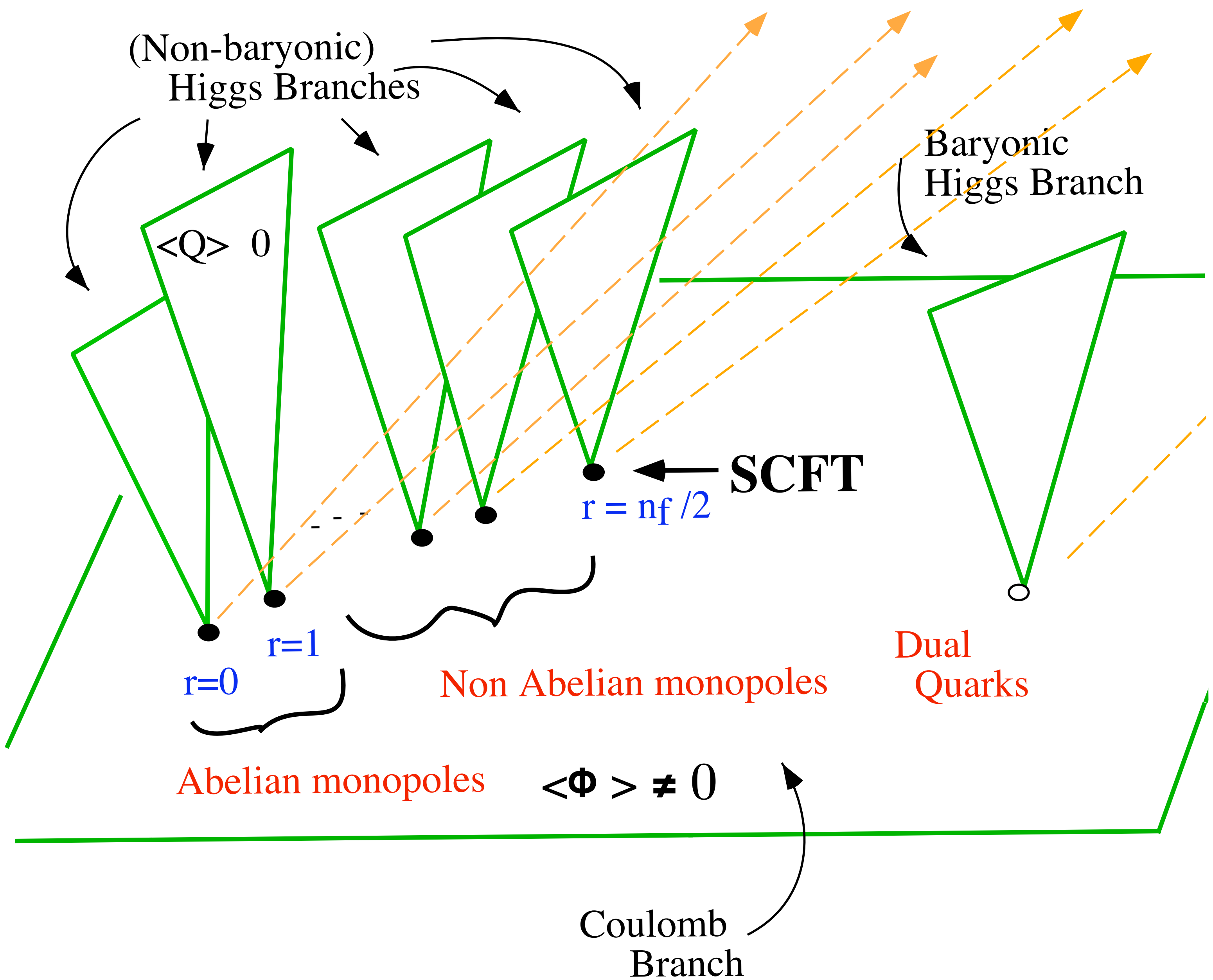



Figure 3: Zero loci of the discriminant of the curve of $\mathcal{N}=2$, $SU(3)$, $n_f=4$ theory at small $m$.

$U = \mathrm{Tr}\,\Phi^2 \simeq 3m^2$ ; $\qquad V = \mathrm{Tr}\,\Phi^3 \simeq 2m^3$

# QMS of N=2 SQCD (SU(n) with $n_f$ quarks)




● N=1 Confining vacua (with $\Phi^2$ perturbation)

○ N=1 vacua (with $\Phi^2$ perturbation) in free magnetic pha

$m = m^{cr}$

next
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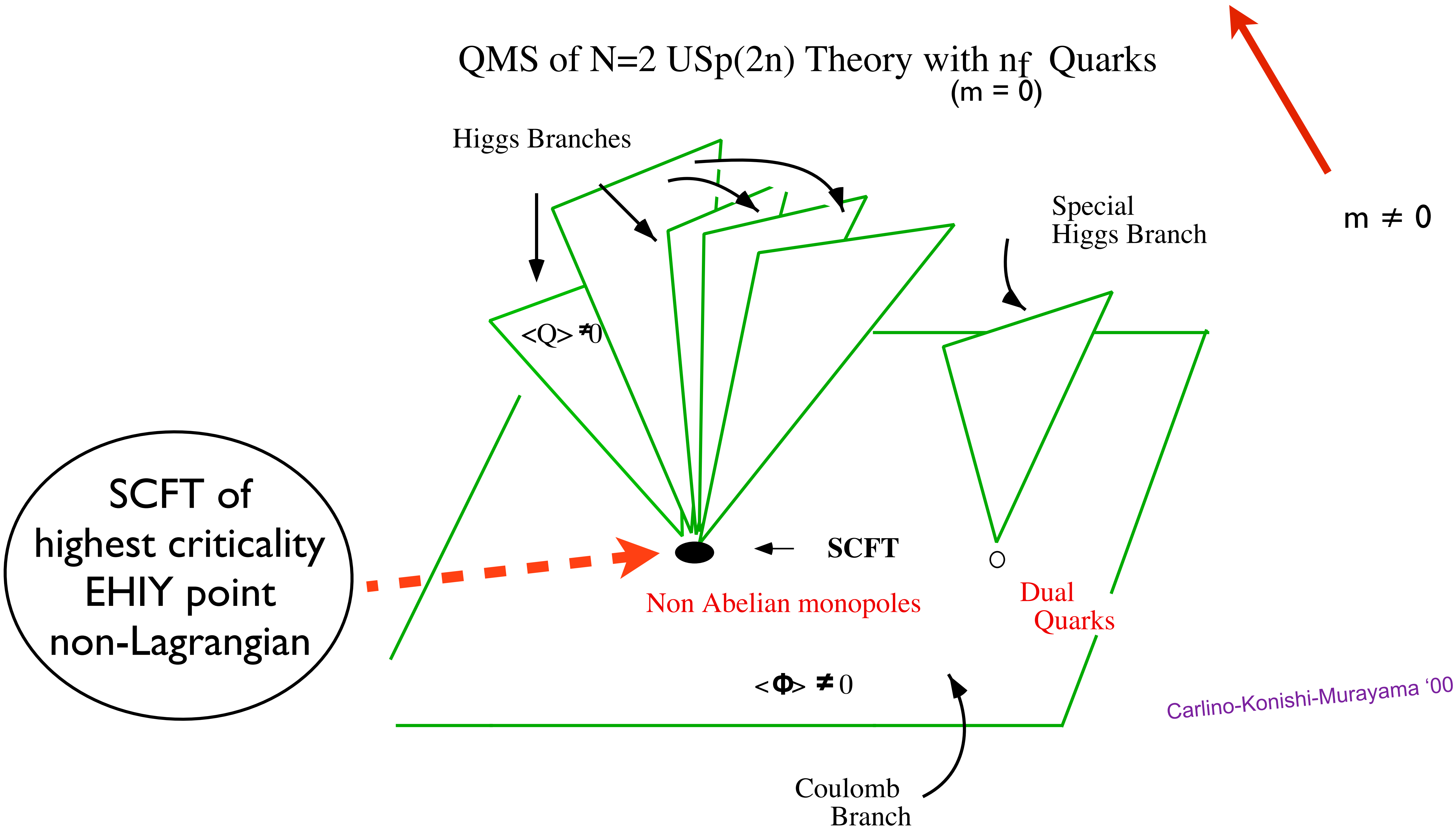
Di Pietro, Giacomelli '11

# Quantum space of vacua in $\mathcal{N}$=2 SQCD with critical mass

previous slide

(Universality)




● N=1 Confining vacua (with $\Phi^2$ perturbation)

○ N=1 vacua (with $\Phi^2$ perturbation) in free magnetic pha

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# a and c coefficients for free particles

$$c = \frac{1}{120}(N_S + 6\,N_F + 12\,N_V)\ ; \qquad a = \frac{1}{360}(N_S + 11\,N_F + 62\,N_V)$$

☞ For N=2 hypermultiplet (pair of chiral multiplets Q and Q bar):

$$a = \frac{1}{360}(4+11) = \frac{1}{24}\ ; \qquad c = \frac{1}{120}(4+6) = \frac{1}{12}\ .$$

☞ For N=2 vector multiplet:

$$a = \frac{1}{360}(2+11+62) = \frac{5}{24}\ ; \qquad c = \frac{1}{120}(2+6+12) = \frac{1}{6}\ .$$

☞ For real-world QCD:

$$c_{UV} = \frac{1}{20}N_f N_c + \frac{N_c^2-1}{10}\ ; \qquad a_{UV} = \frac{11 N_f N_c}{360} + \frac{31}{180}(N_c^2-1)\ ,$$

$$c_{IR} = \frac{N_f^2-1}{120}\ ; \qquad a_{IR} = \frac{N_f^2-1}{360}\ .$$

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