

Spring Institute: Challenging the Standard Model after
the Higgs discovery

Bounding the trilinear Higgs self coupling by means of precision electroweak measurements



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based on JHEP 1704 (2017) 155 (arXiv:1702.01737) in collaboration with:

G. Degrassi, P.P. Giardino

Summary

- *Higgs boson self couplings after Run 1:
double Higgs production*
- *Defining an alternative strategy*
- *Interpreting the anomalous coupling*
- *Results*

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Standard Model Lagrangian

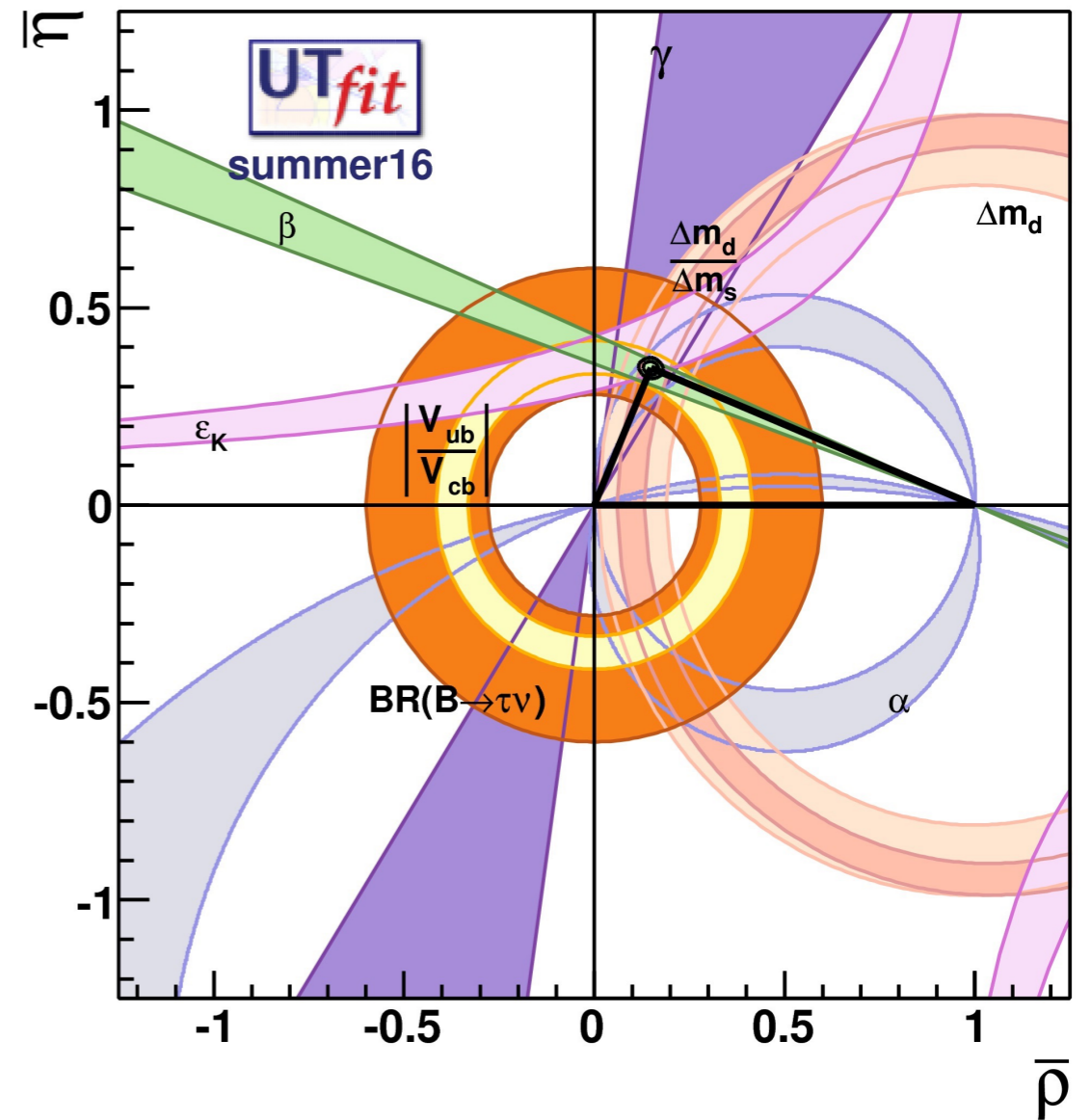
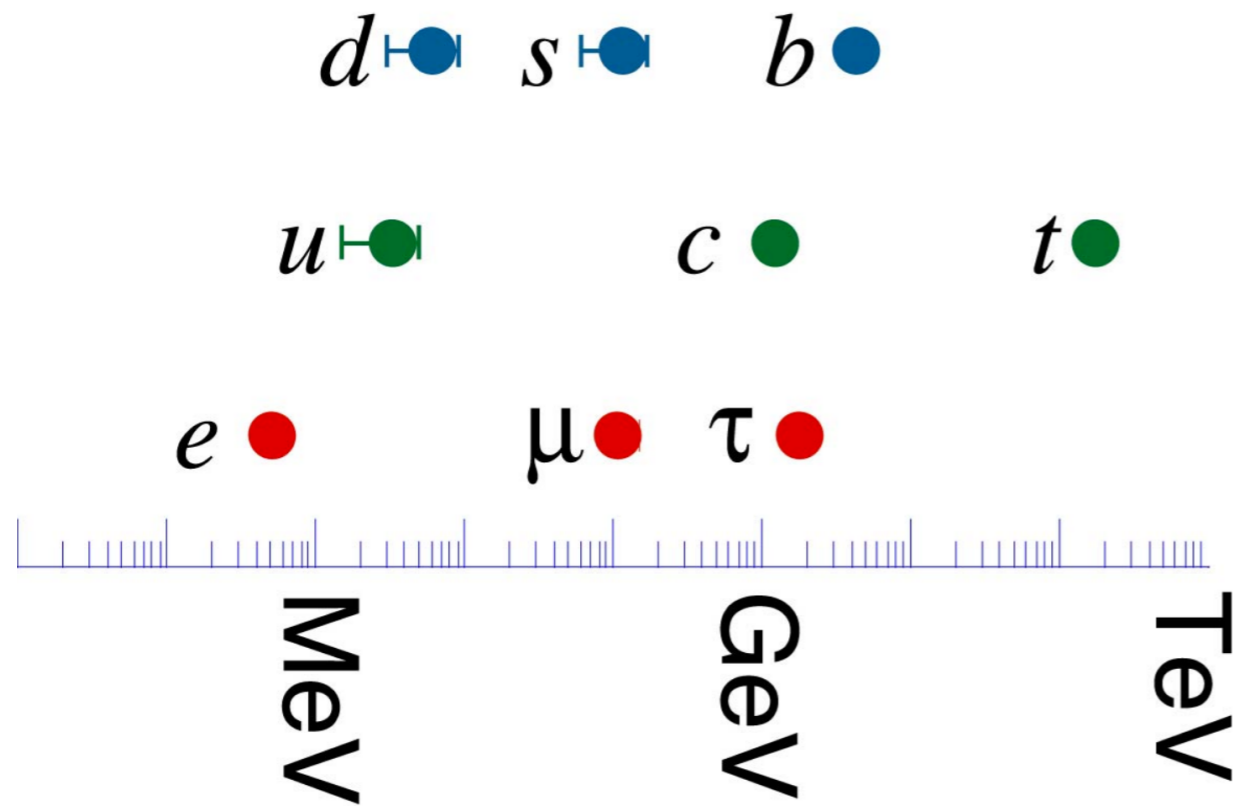
The world of particle physics is described extremely well by the beautiful Standard Model Lagrangian

$$\mathcal{L}^{\text{SM}} = -\frac{1}{4}V^{\mu\nu}V_{\mu\nu} + i\bar{f}\gamma^\mu D_\mu f - \lambda_{ij}\bar{f}^i\phi f^j + D^\mu\phi^\dagger D_\mu\phi + V(\phi)$$

It predicted at an astonishing level basically all the particle physics phenomena experimentally observed in the last several decades!

However, we know that there must be something else beyond (DM, neutrino masses, ...): which sector of this Lagrangian could accommodate for NP effects?

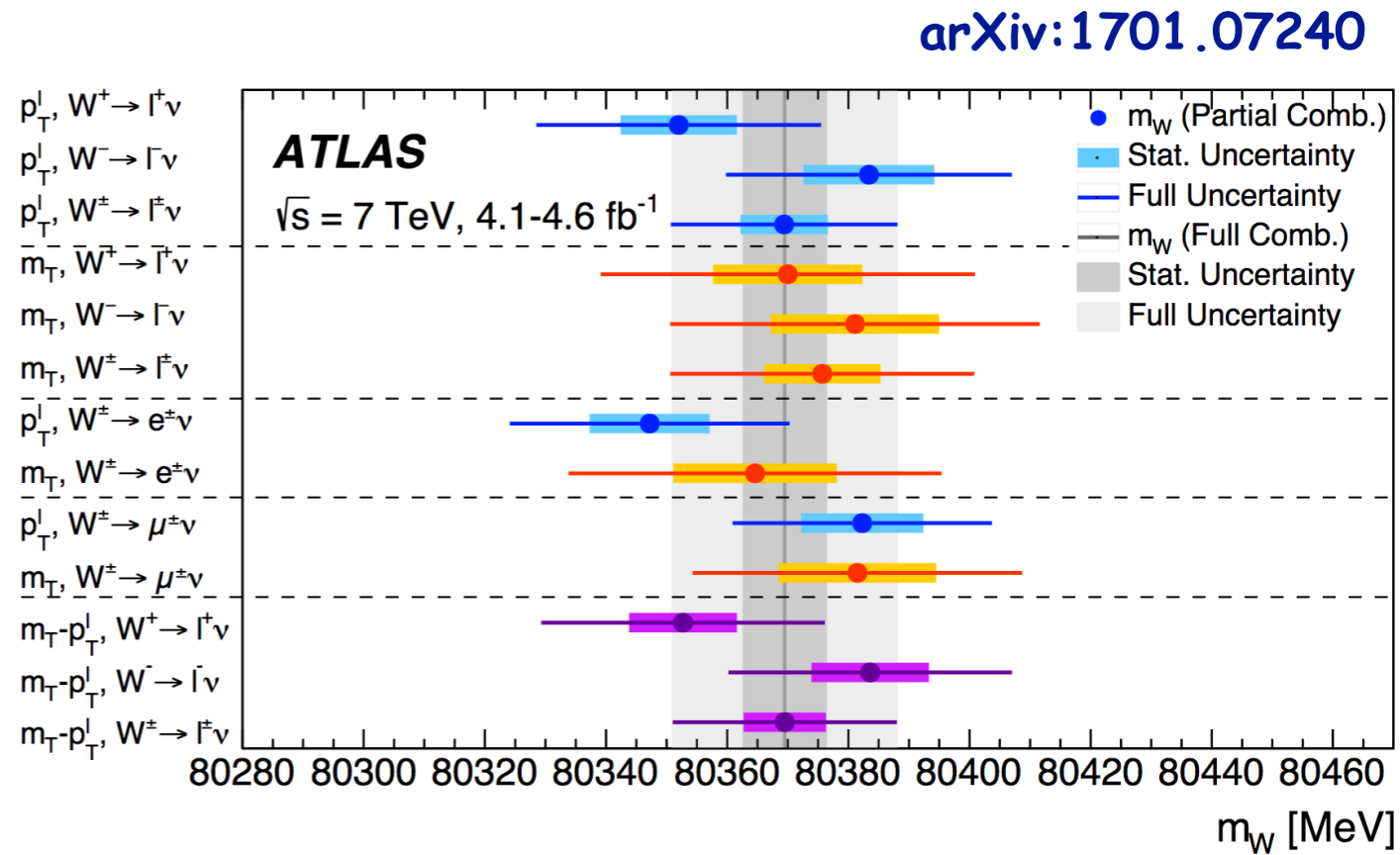
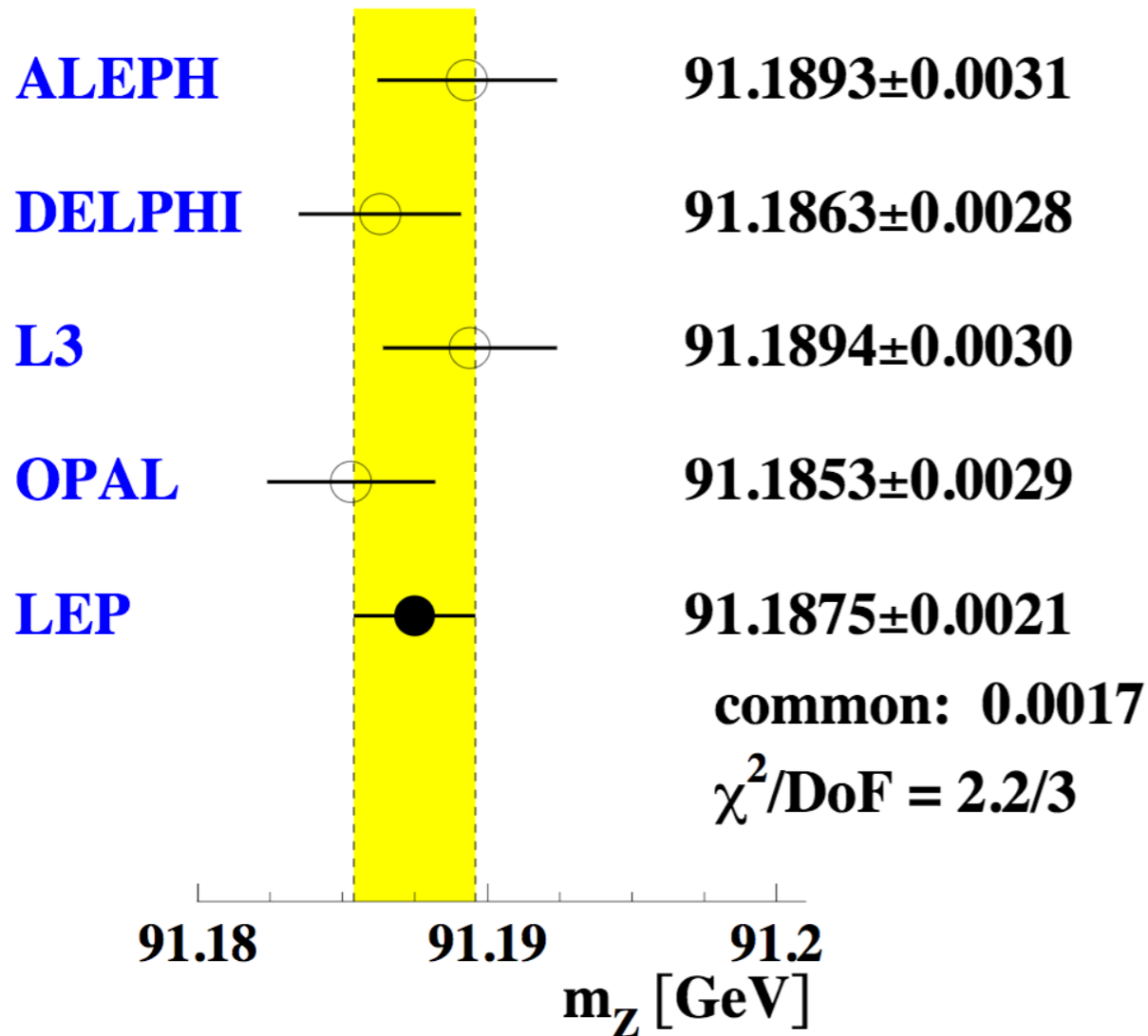
The Fermions



- All massive SM fermions have been observed and their masses have been experimentally determined with good precisions
- Flavour-changing weak decay are described by means of the CKM matrix, with all exp. data pointing at a SM behaviour

Small room for NP

The Vector Bosons



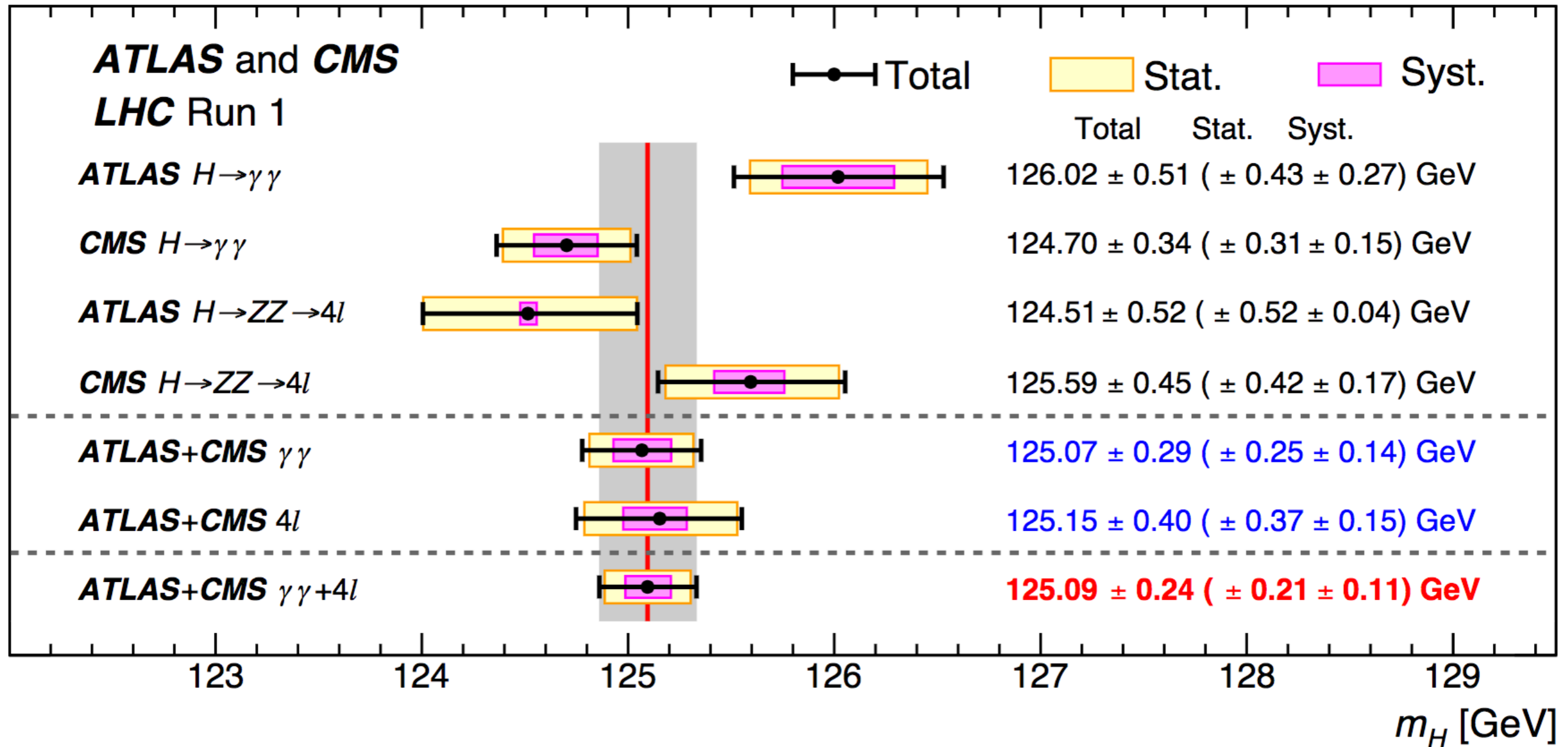
- The Z mass is measured extremely well since LEP
- The latest W mass measurement are in good agreement with the latest SM prediction $m_W = 80.357 \pm 0.009 \pm 0.003$

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Small room for NP

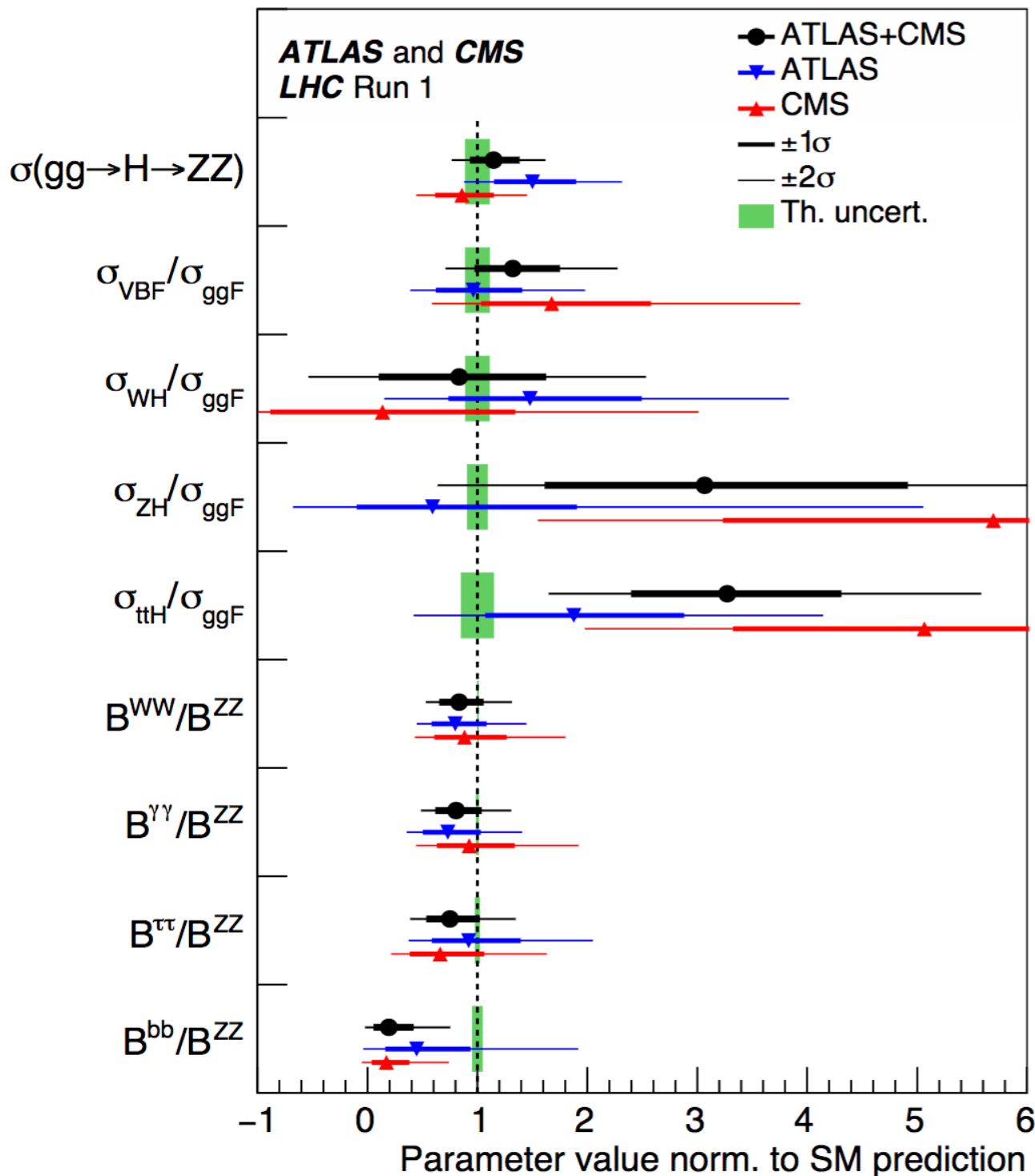
The Higgs Boson

Phys. Rev. Lett. 114, 191803 (2015)



A ~ 125 GeV boson was discovered in 2012 by ATLAS and CMS, with properties consistent with the SM Higgs boson

Higgs boson couplings



JHEP 1608 (2016) 045

- Couplings with the vector bosons compatible with SM predictions within a **$\sim 10\%$ uncertainty**

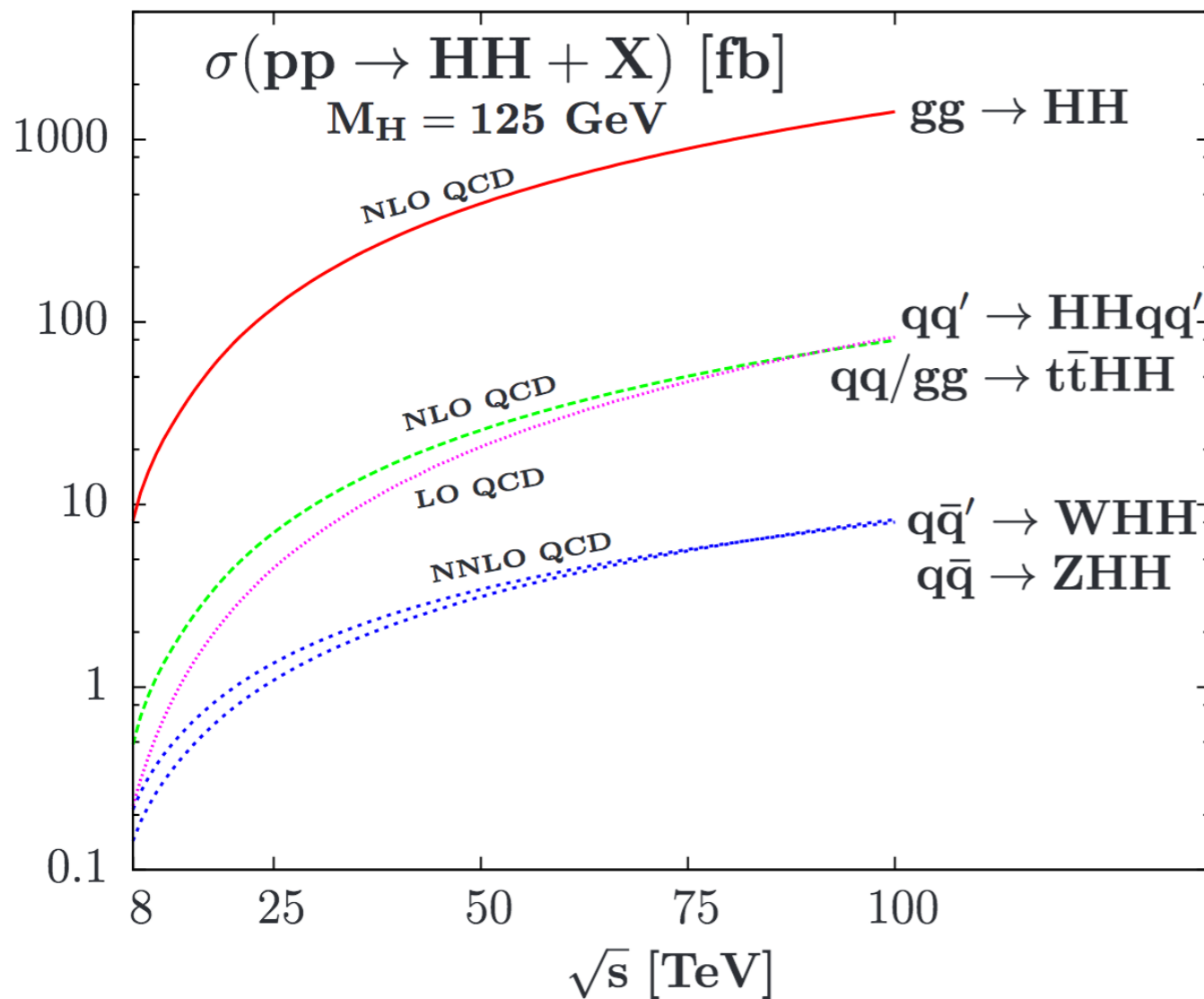
- Couplings with the heaviest fermions compatible with SM predictions within a **$\sim 15 - 20\%$ uncertainty**

The 125 GeV Higgs boson displays couplings with other particles that behave in a quite “Standard” manner

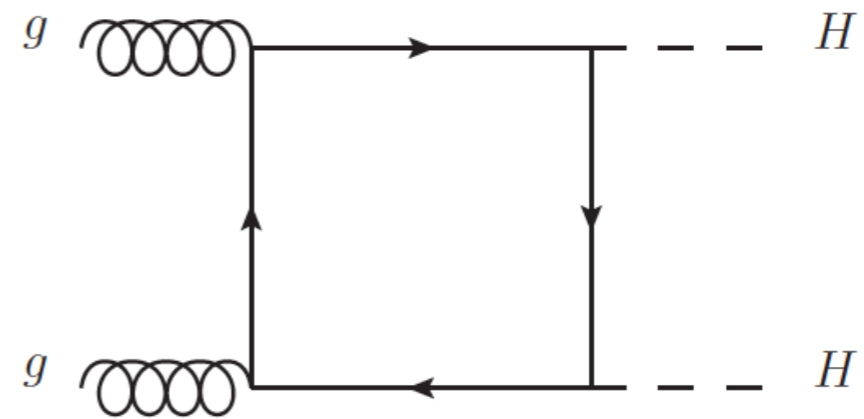
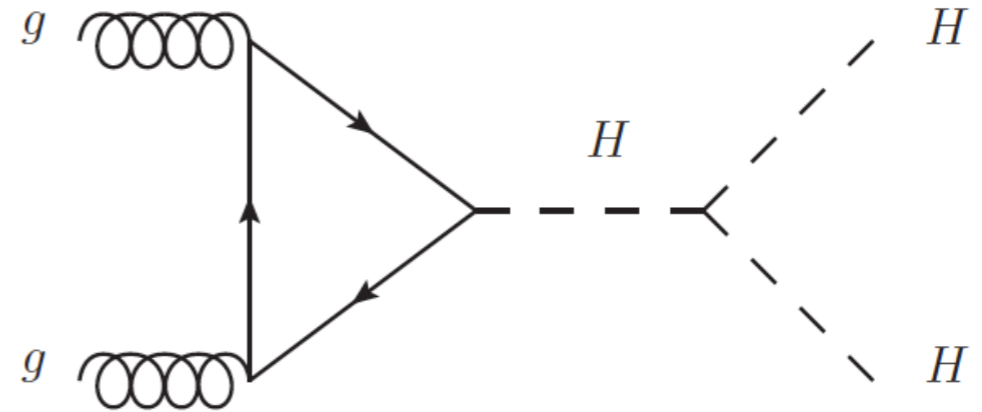
Small room for NP

Higgs boson self coupling - I

However, the study of the Higgs self interactions by means of double Higgs production is in a completely different status



JHEP 1304 (2013) 151



- heavy final states
- destructive interference

Definitely room for NP!

Higgs boson potential

The Higgs potential reads

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \quad \phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2) \end{pmatrix}$$

↓ EWSB, unitary gauge

$$V(\phi_1) = \frac{m_H^2}{2}\phi_1^2 + \lambda_3 v\phi_1^3 + \frac{\lambda_4}{4}\phi_1^4$$

where all the parameters are linked in the SM by the relation

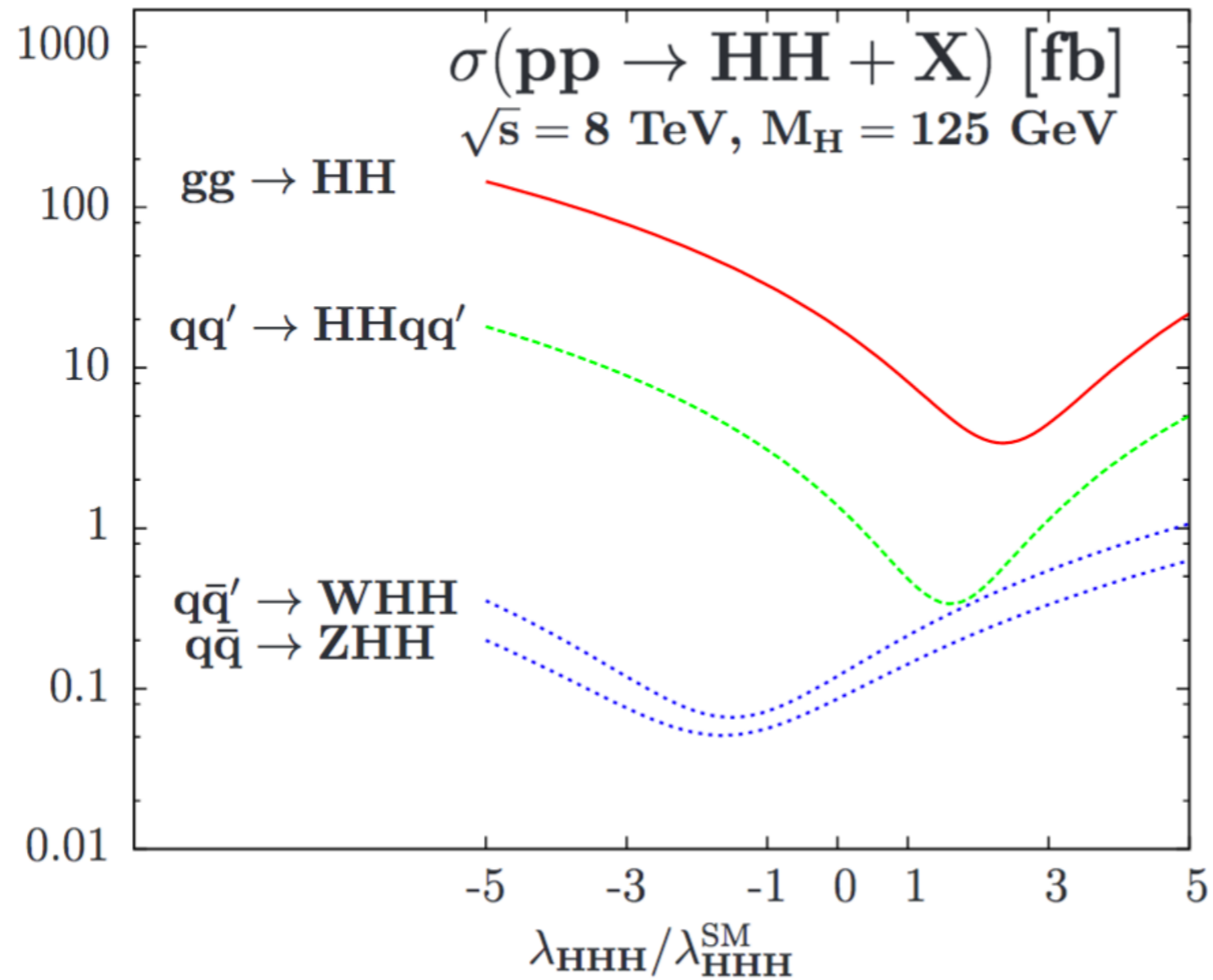
$$\lambda_3^{\text{SM}} = \lambda_4^{\text{SM}} = \lambda = \frac{m_H^2}{2v^2}$$

However, what happens if one allows for an anomalous Higgs trilinear self coupling, due to BSM effects?

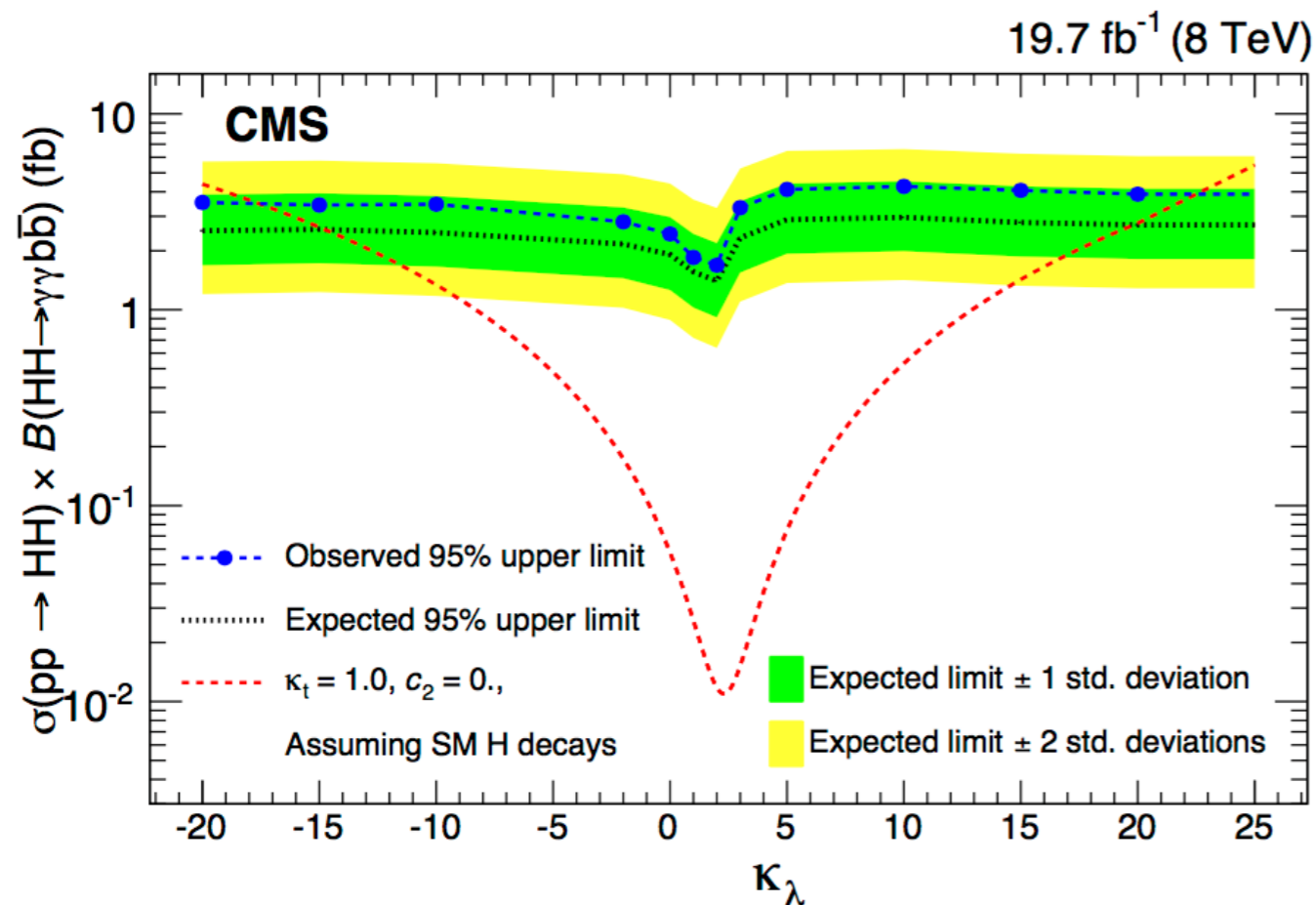
$$V_{\phi_1^3} = \lambda_3 v\phi_1^3 = \kappa_\lambda \lambda_3^{\text{SM}} v\phi_1^3$$

Higgs boson self coupling - 2

The Cross Section is heavily affected by the size of the parameter κ_λ !



Bounds on the self coupling



Phys. Rev. D 94, 052012

Results from Run 1

allow to constrain λ_3

within $O(\pm(15 - 20)\lambda_3^{\text{SM}})$

Assuming an integrated luminosity of 3000 fb⁻¹, it will be possible to constrain κ_λ at the LHC only in the range (-1.3, 8.7)

A complementary strategy could help alleviate the situation!

Loop corrections

Study the effect of the anomalous trilinear coupling in better measured channels (e.g. single Higgs production, or precision electroweak measurements), where Higgs boson self interaction arises at loop level!

- *Re-compute the observables for the chosen channels in terms of a SM part plus an anomalous part proportional to powers of κ_λ*
- *Vary the value of the anomalous coupling compatibly with the experimental informations on such observables*
- *Extract indirectly bounds on κ_λ*

We will focus on the W boson mass, m_W , and the effective sine, $\sin^2 \theta_{\text{eff}}^{\text{lep}}$

Summary

- *Higgs boson self couplings after Run 1:
double Higgs production*
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Definitions of observables

The \overline{MS} formulation of the radiative corrections to m_W and $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ can be expressed in terms of the following physical quantities

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi \hat{\alpha}(m_Z)}{2m_W^2 \hat{s}^2} (1 + \Delta \hat{r}_W) \quad \hat{\alpha}(m_Z) = \frac{\alpha}{1 - \Delta \hat{\alpha}(m_Z)}$$

$$\hat{\rho} \equiv \frac{m_W^2}{m_Z^2 \hat{c}^2} = \frac{1}{1 - Y_{\overline{MS}}}$$

where $\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(m_Z)$ and $\hat{c}^2 = 1 - \hat{s}^2$

$$m_W^2 = \frac{\hat{\rho} m_Z^2}{2} \left\{ 1 + \left[1 - \frac{4\hat{A}^2}{m_Z^2 \hat{\rho}} (1 + \Delta \hat{r}_W) \right]^{1/2} \right\}$$

with $\hat{A} = (\pi \hat{\alpha}(m_Z) / (\sqrt{2} G_\mu))^{1/2}$

Definitions of observables

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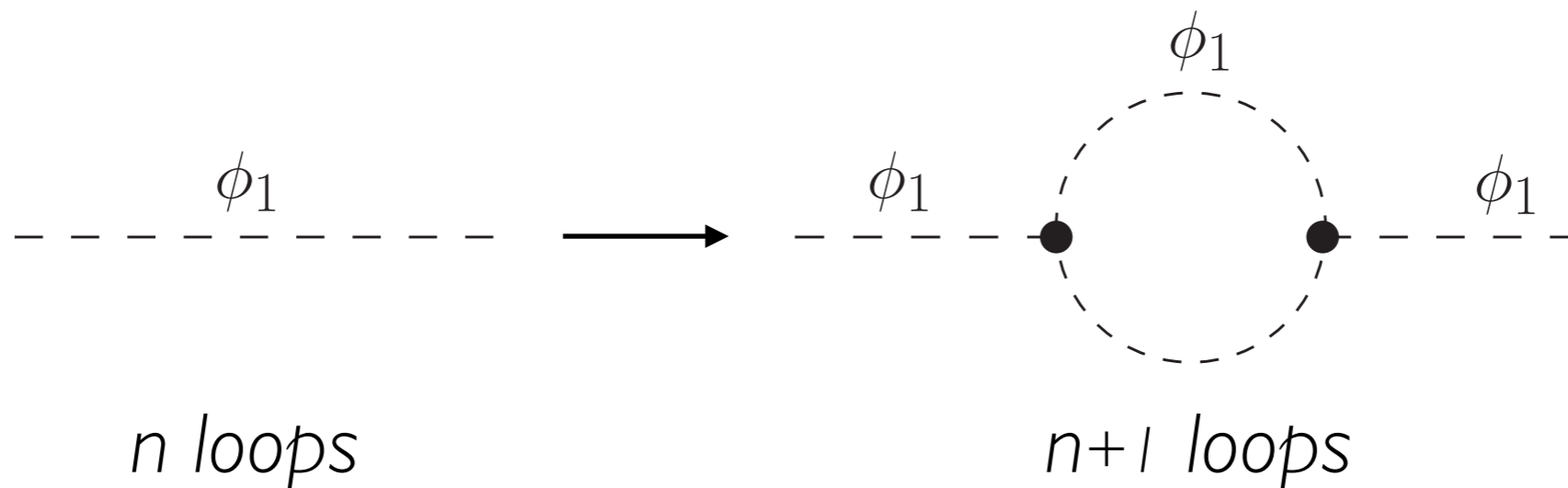
where $\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(m_Z)$ and $\hat{c}^2 = 1 - \hat{s}^2$

$$\sin^2 \theta_{\text{eff}}^{\text{lep}} \simeq \frac{1}{2} \left\{ 1 - \left[1 - \frac{4\hat{A}^2}{m_Z^2 \hat{\rho}} (1 + \Delta \hat{r}_W) \right]^{1/2} \right\}$$

with $\hat{A} = (\pi \hat{\alpha}(m_Z) / (\sqrt{2} G_\mu))^{1/2}$

Loop level

In order to obtain an effect induced by the anomalous coupling, we first need to have an internal Higgs propagator, and then we need to go to the following order inserting e.g. a wave-function contribution



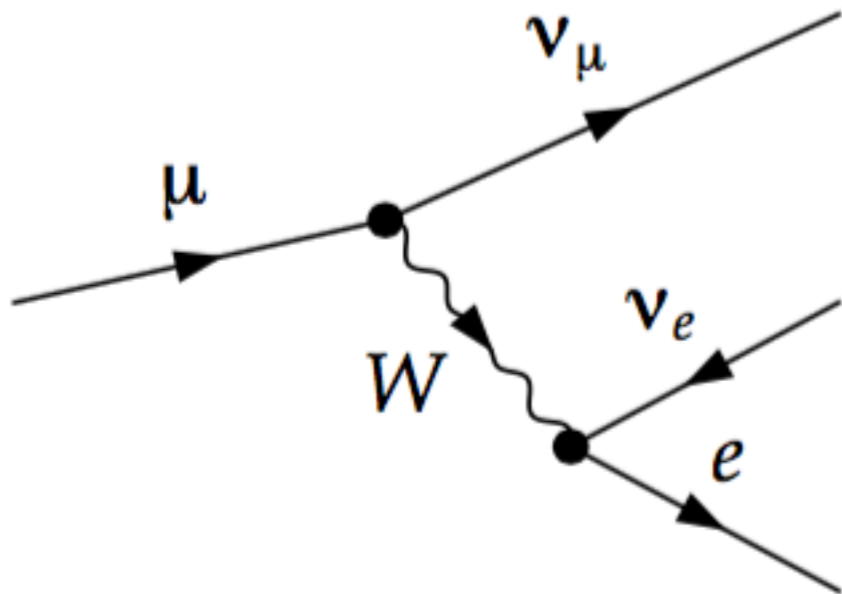
Following this prescription, $\Delta\hat{r}_W$ and $Y_{\overline{MS}}$ will be affected already at two loops, while $\Delta\hat{\alpha}$ will get contributions only starting from tree loop: we will focus only on the first two, since we performed a two-loop computation

$$\Delta \hat{r}_W$$

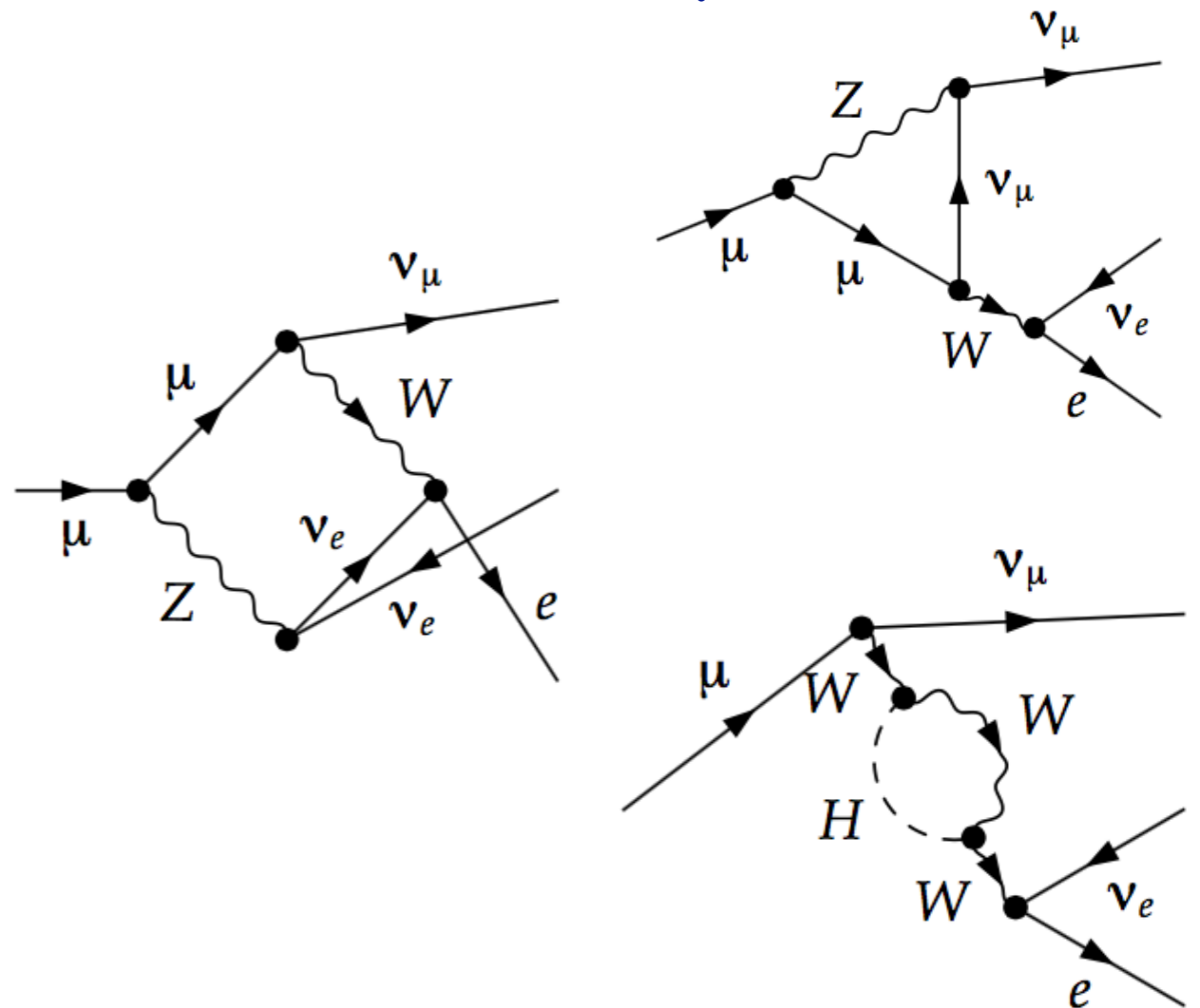
$\Delta \hat{r}_W$ describes the radiative corrections to $\frac{G_\mu}{\sqrt{2}} = \frac{\pi \hat{\alpha}(m_Z)}{2m_W^2 \hat{s}^2} (1 + \Delta \hat{r}_W)$

\Rightarrow related to muon decay

tree level



one loop

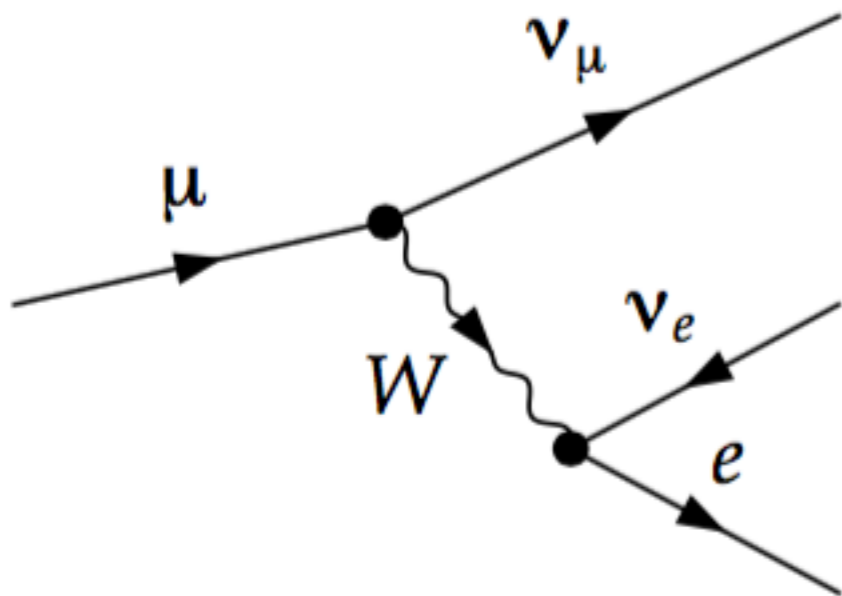


$$\Delta \hat{r}_W$$

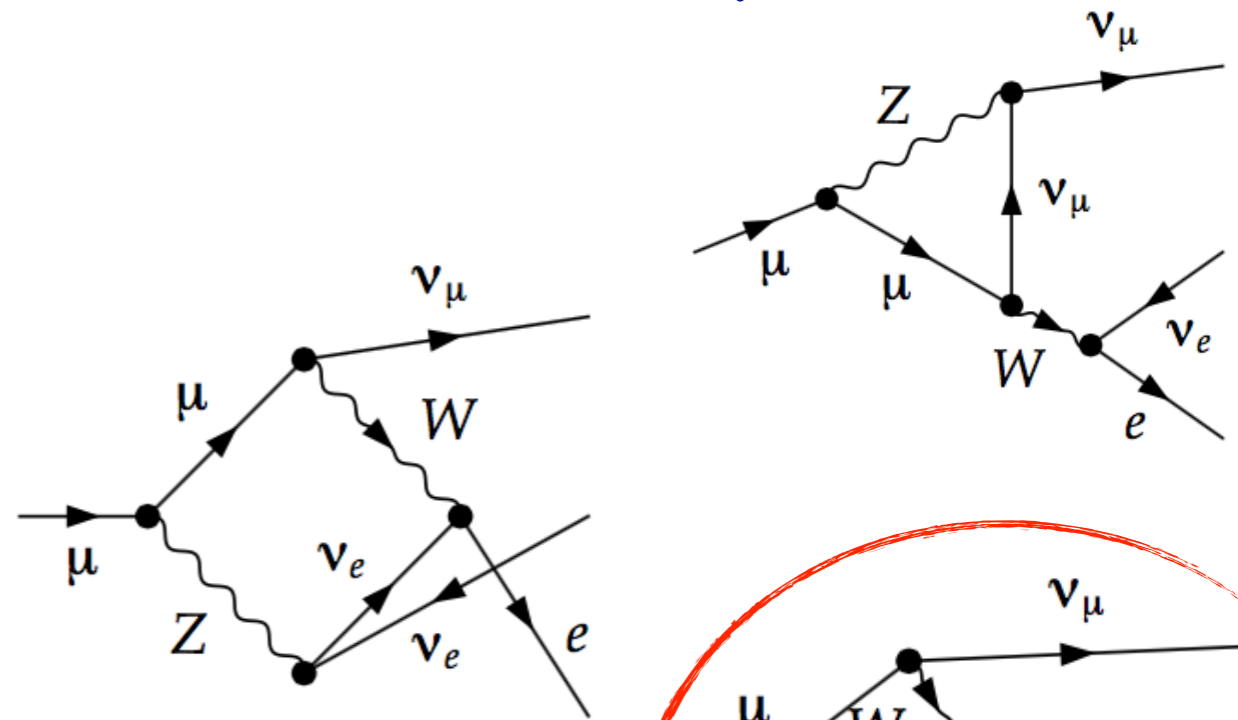
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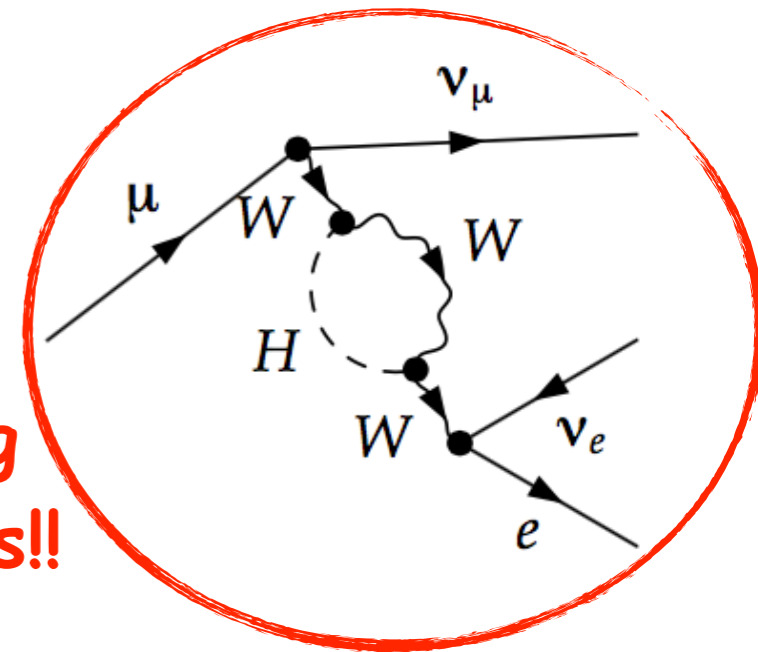
tree level



one loop



interesting
at two loops!!

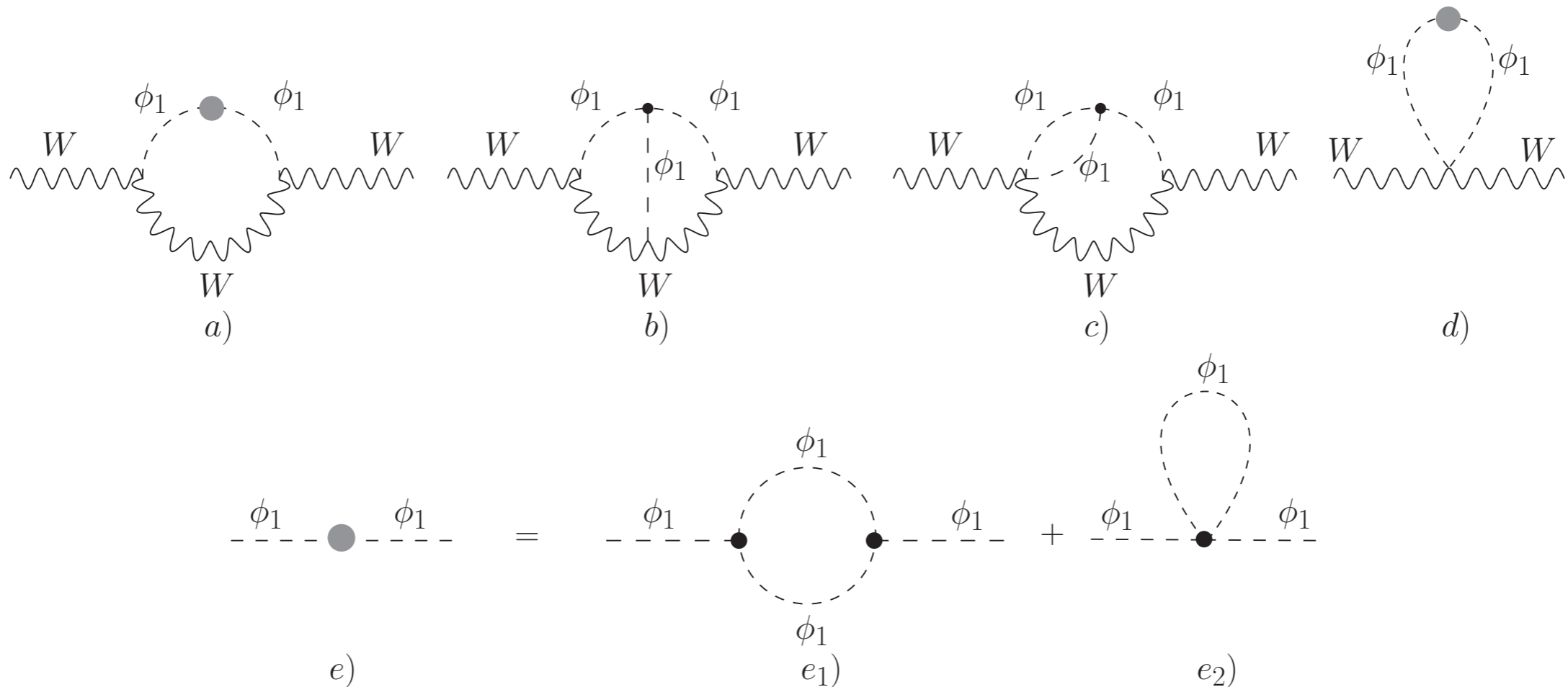


$$\Delta \hat{r}_W$$

The piece affected at two loops level by the anomalous coupling reads

$$\Delta \hat{r}_W^{(2, \kappa_\lambda)} = \frac{\text{Re} A_{WW}^{(2)}(m_W^2)}{m_W^2} - \frac{A_{WW}^{(2)}(0)}{m_W^2}$$

We need to compute only the following two-loop W self-energy diagrams

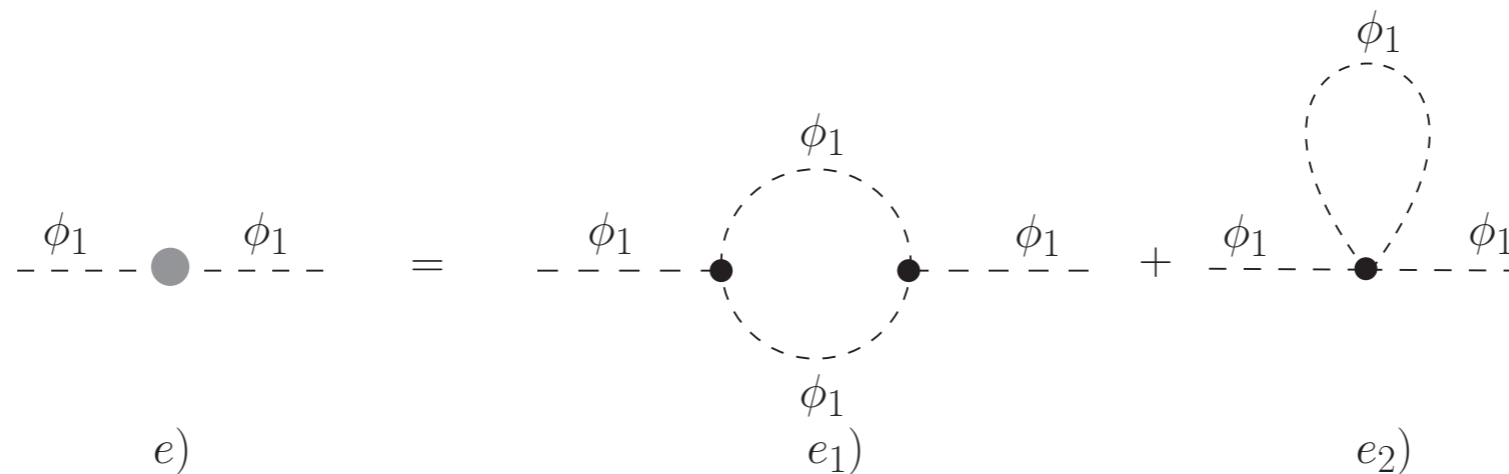
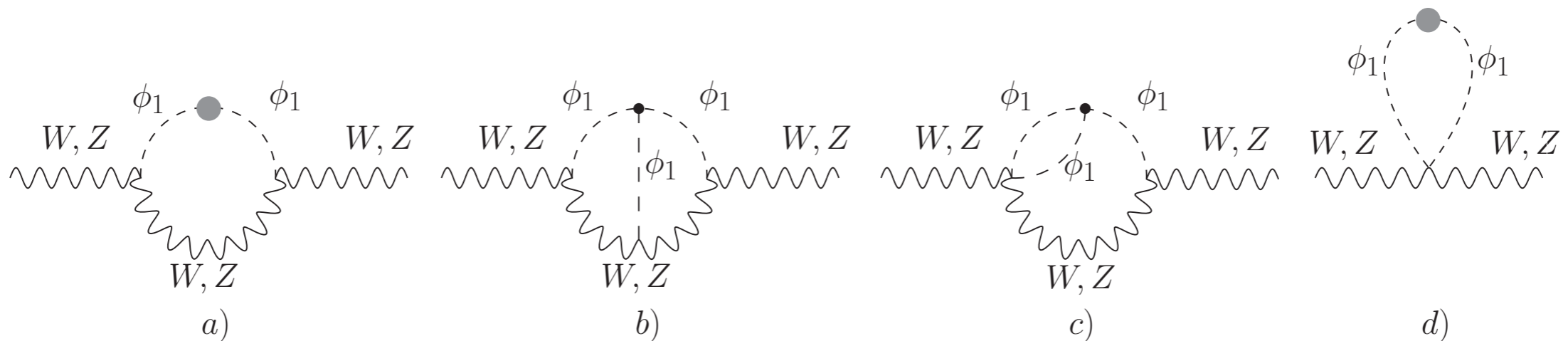


Y_{MS}

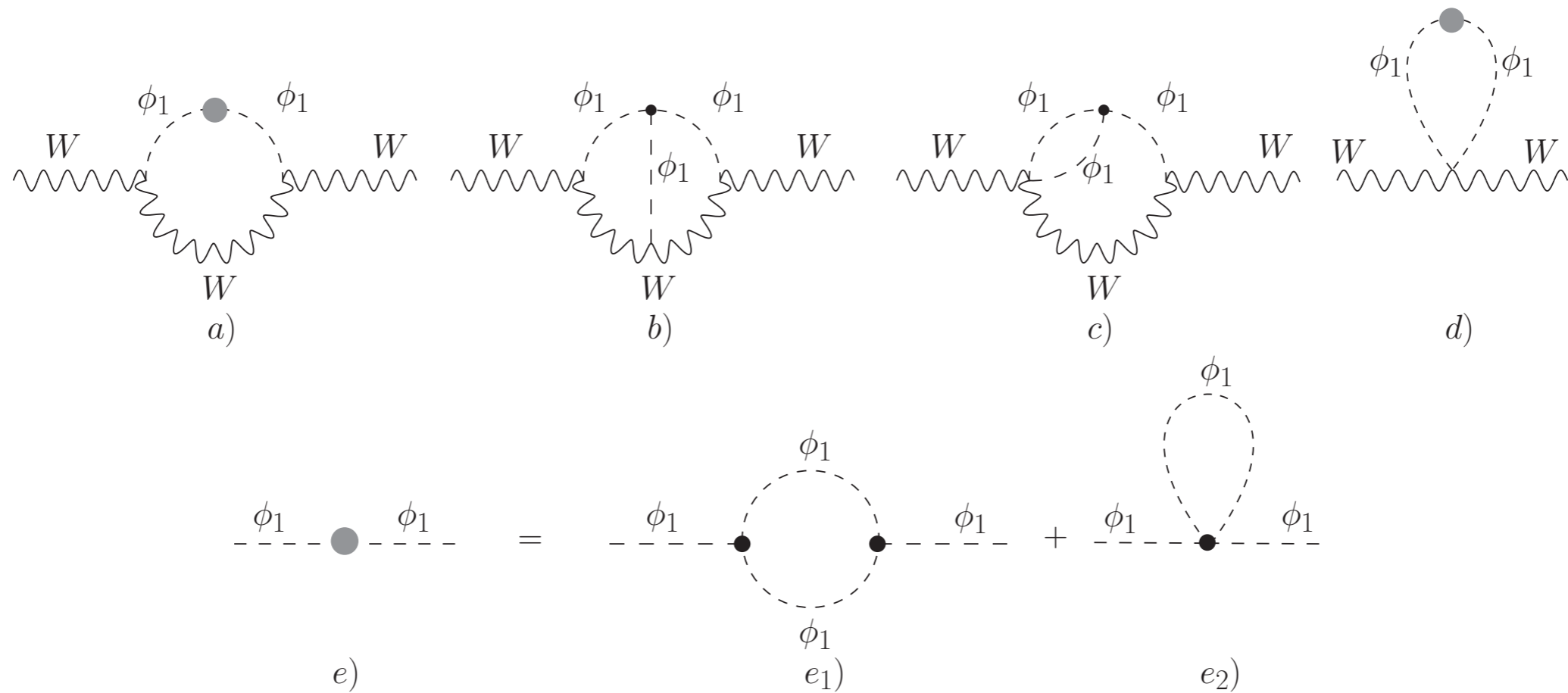
Y_{MS} describes the radiative corrections to $\hat{\rho} \equiv \frac{m_W^2}{m_Z^2 \hat{c}^2} = \frac{1}{1 - Y_{MS}}$

\Rightarrow related to wave functions difference

$$Y_{MS}^{(2, \kappa_\lambda)} = \text{Re} \left[\frac{A_{WW}^{(2)}(m_W^2)}{m_W^2} - \frac{A_{ZZ}^{(2)}(m_Z^2)}{m_Z^2} \right]$$



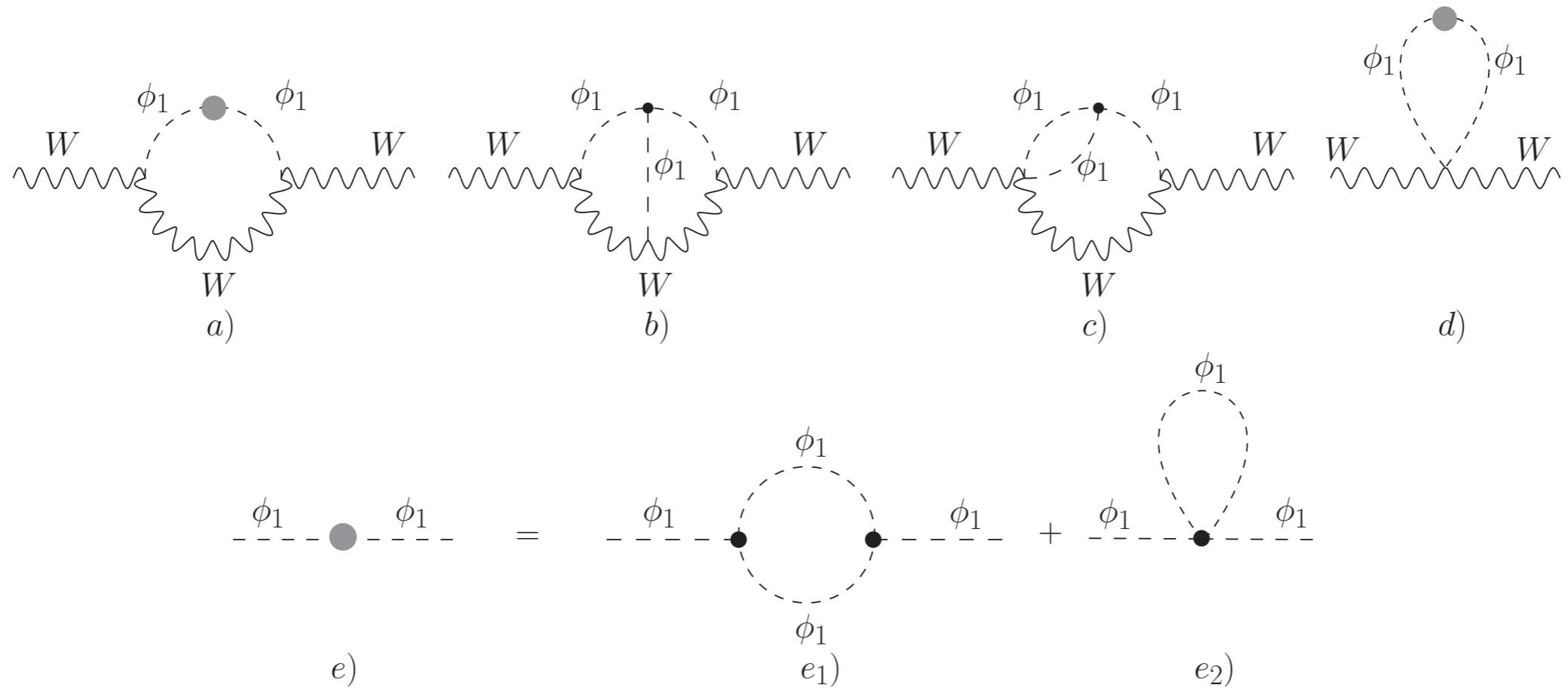
A few observations - I



- A generic modified potential could contain self interactions between more than 4 fields, however they don't contribute at this level

- Higgs cactus diagram contribution is exactly canceled by the mass counterterm diagram, hence it's not possible to probe the quartic coupling with this approach

A few observations - 2



In principle, the anomalous coupling breaks the renormalizability of the theory, yielding results proportional to Λ .

However our results are finite, since the tree level diagrams do not depend on λ_3 , and hence we don't have to renormalize it.

Summary

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κ_λ coming from EFT

Where does this anomalous coupling come from?

$$\mathcal{L} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda} \cancel{\mathcal{L}^{\text{dim5}}} + \frac{1}{\Lambda^2} \mathcal{L}^{\text{dim6}} + \frac{1}{\Lambda^3} \cancel{\mathcal{L}^{\text{dim7}}} + \dots$$

Build an Effective Field Theory using only SM fields and preserving the SM Gauge Symmetries: SMEFT

- We want to preserve B-L conservation
- We assume that higher dimension operators give smaller contributions

κ_λ coming from EFT

This is in general a good bottom-up approach, when you look for small deviations from SM. Unfortunately, this does not hold in our case...

$$V^{\text{dim6}}(\phi) = V^{\text{SM}}(\phi) + \frac{c_6}{v^2} (\phi^\dagger \phi)^3 \quad \Rightarrow \quad \kappa_\lambda = 1 + \frac{2c_6 v^2}{m_H^2}$$

However, we need to impose that $\phi = \frac{v}{\sqrt{2}}$ is still a global minimum!

$$V^{\text{dim6}}\left(\frac{v}{\sqrt{2}}\right) = \frac{c_6 v^4 - m_H^2 v^2}{8} < 0 = V^{\text{dim6}}(0) \quad \Rightarrow \quad \kappa_\lambda < 3$$

Not interesting from a phenomenological point of view...!

κ_λ coming from a tower of operators

An alternative explanation consists into interpreting the anomalous coupling as the effect of an (in)finite tower of operators

$$V^{\text{SM+NP}}(\phi) = \sum_{n=1}^N c_{2n} (\phi^\dagger \phi)^n$$

- *We don't impose any constraints on the coefficients, apart from the requirement that the series is convergent*
- *We do not assume an EFT scaling on the coefficients: $c_{2n+2} \not\propto \frac{c_{2n}}{\Lambda^2}$*

κ_λ coming from a tower of operators

Defining $\xi = \phi^- \phi^+ + \frac{1}{2} \phi_2^2$ the potential, up to 4 fields, reads

$$V_{4\phi}^{\text{NP}} = \frac{m_H^2}{2v^2} \xi^2 + \left(\frac{m_H^2}{2v^2} + d\lambda_4 \right) \frac{1}{4} \phi_1^4 + \left(\frac{m_H^2}{2v^2} + 3d\lambda_3 \right) \xi \phi_1^2 \\ + \left(\frac{m_H^2}{2v} + v d\lambda_3 \right) \phi_1^3 + \frac{m_H^2}{v} \xi \phi_1 + \frac{1}{2} m_H^2 \phi_1^2$$

With

$$d\lambda_3 = \frac{1}{3} \sum_{n=3}^N c_{2n} n(n-1)(n-2) \left(\frac{v^2}{2} \right)^{n-2}$$

$$d\lambda_4 = \frac{2}{3} \sum_{n=3}^N c_{2n} n^2(n-1)(n-2) \left(\frac{v^2}{2} \right)^{n-2}$$

$$\kappa_\lambda = 1 + \frac{2v^2}{m_H^2} d\lambda_3$$

κ_λ coming from a tower of operators

$$V_{4\phi}^{\text{NP}} = \frac{m_H^2}{2v^2} \xi^2 + \left(\frac{m_H^2}{2v^2} + d\lambda_4 \right) \frac{1}{4} \phi_1^4 + \left(\frac{m_H^2}{2v^2} + 3d\lambda_3 \right) \xi \phi_1^2 \\ + \left(\frac{m_H^2}{2v} + v d\lambda_3 \right) \phi_1^3 + \frac{m_H^2}{v} \xi \phi_1 + \frac{1}{2} m_H^2 \phi_1^2$$

At the order of our computation, the only anomalous term affecting our results is the one coming from the physical Higgs trilinear coupling

\Rightarrow *We can work in Unitary Gauge, reproducing the same results of a generic R_ξ Gauge!*

Caveat: *this theory cannot work at arbitrary scale, otherwise perturbative unitarity is lost in processes like e.g. $V_L V_L \rightarrow n\phi_1$*

A preliminary study indicates that $\Lambda \sim 1 - 3 \text{ TeV}$, $|\kappa_\lambda| \lesssim 20$

(R. Rattazzi, A. Falkowski, in preparation)

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Working Assumptions

- *We suppose that the only sizable BSM effects arise in the Higgs trilinear self coupling*
- *We perform our computation in Unitary Gauge, but no gauge-dependent terms are induced by this choice at this order*
- *We do not assume any size for the coefficients of the operators, contrary to the EFT approach*
- *The validity of our approach holds only for $|\kappa_\lambda| \lesssim 20$, up to a scale of $\Lambda \sim 1 - 3 \text{ TeV}$*

Outline of the computation

- *Amplitudes generated by FEYNARTS*
- *Tensor integrals reduced to scalar Master Integrals by means of private codes, FeynCalc and Tarcer*
- *Two-loop vacuum integrals evaluated analytically*
- *Two-loop vacuum self energies at external momentum different from zero evaluated numerically using TSIL*

Writing the result

We can now combine the results obtained for the anomalous diagrams with the SM ones ([JHEP 1505 \(2015\) 154](#))

$$O = O^{\text{SM}} \left[1 + (\kappa_\lambda - 1)C_1 + (\kappa_\lambda^2 - 1)C_2 \right]$$

where the SM result is obtained for a non-anomalous coupling ($\kappa_\lambda = 1$)

	C_1	C_2
m_W	6.27×10^{-6}	-1.72×10^{-6}
$\sin^2 \theta_{\text{eff}}^{\text{lep}}$	-1.56×10^{-5}	4.55×10^{-6}

Fitting bounds on κ_λ

In order to set limits on the anomalous coupling from the analysis of precision observables, we perform a simplified fit

$$\chi^2(\kappa_\lambda) \equiv \sum \frac{(O_{\text{exp}} - O_{\text{the}})^2}{(\delta)^2}$$

In order to ascertain the goodness of our fit, we also compute the p -value as a function of the anomalous coupling

$$p\text{-value}(\kappa_\lambda) = 1 - F_{\chi^2_{(n)}}(\chi^2(\kappa_\lambda))$$

Experimental informations - I

The experimental data employed in our fit is

arXiv:1701.07240

- *Latest result by ATLAS for W mass $m_W = 80.370 \pm 0.019$ GeV*

largest uncertainties wrt world average, but closer to SM prediction $m_W = 80.357 \pm 0.009 \pm 0.003$

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PDG

- *CDF and D0 combination for eff. sine $\sin^2 \theta_{\text{eff}}^{\text{lep}} = 0.23185 \pm 0.00035$*

to confront against the SM prediction $\sin^2 \theta_{\text{eff}}^{\text{lep}} = 0.23145 \pm 0.00012 \pm 0.00005$

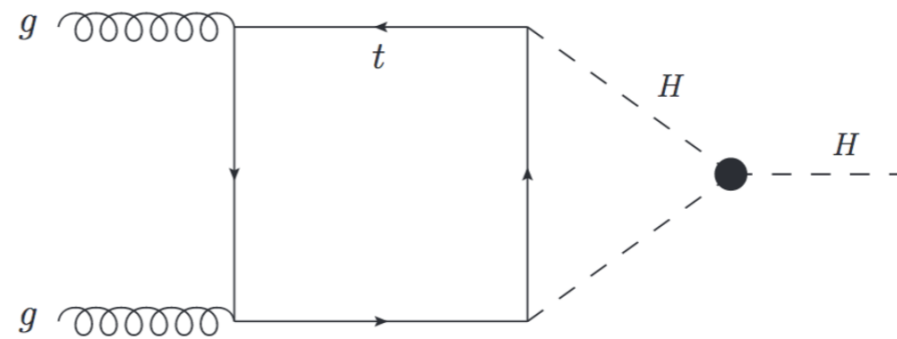
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Experimental informations - 2

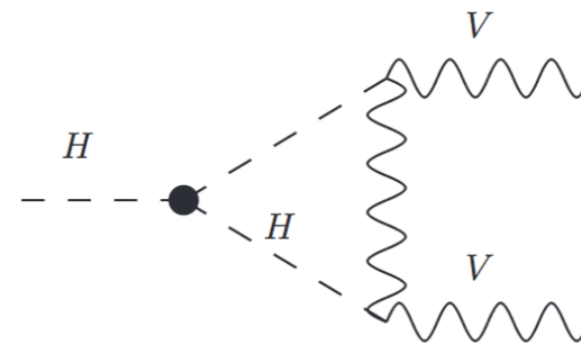
Moreover, we also include the results obtained from a previous study performed in an analogous manner (anomalous coupling affecting single Higgs production channels by means of electroweak correction)

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- gluon gluon fusion (ggF)

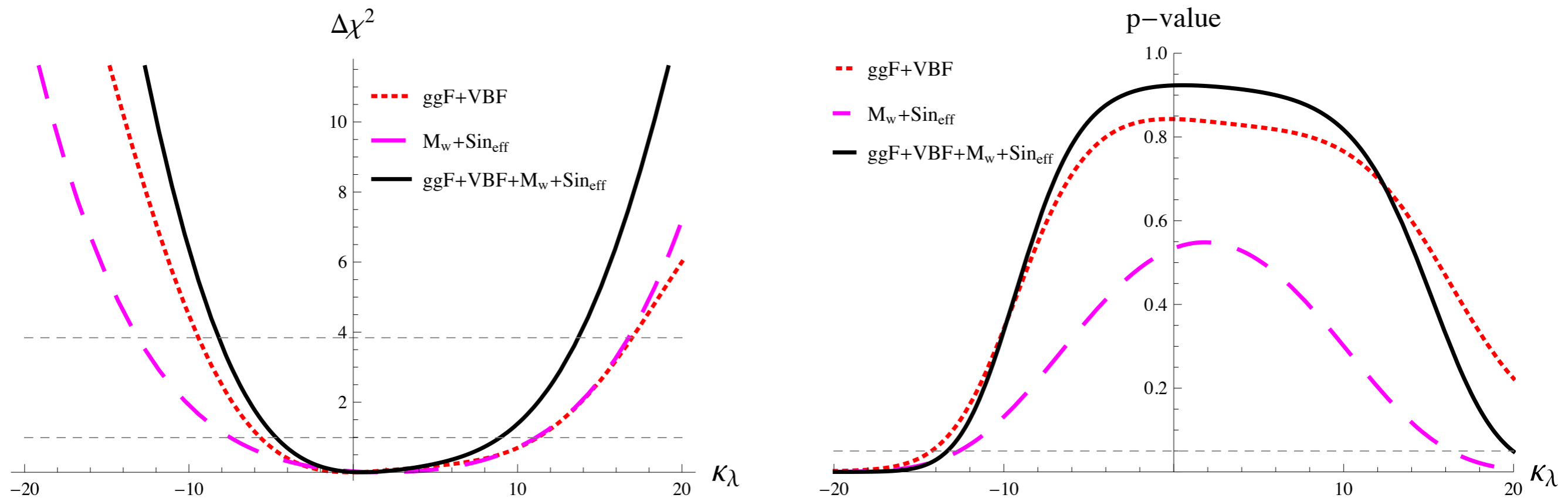


- vector boson fusion (VBF)



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Fit results

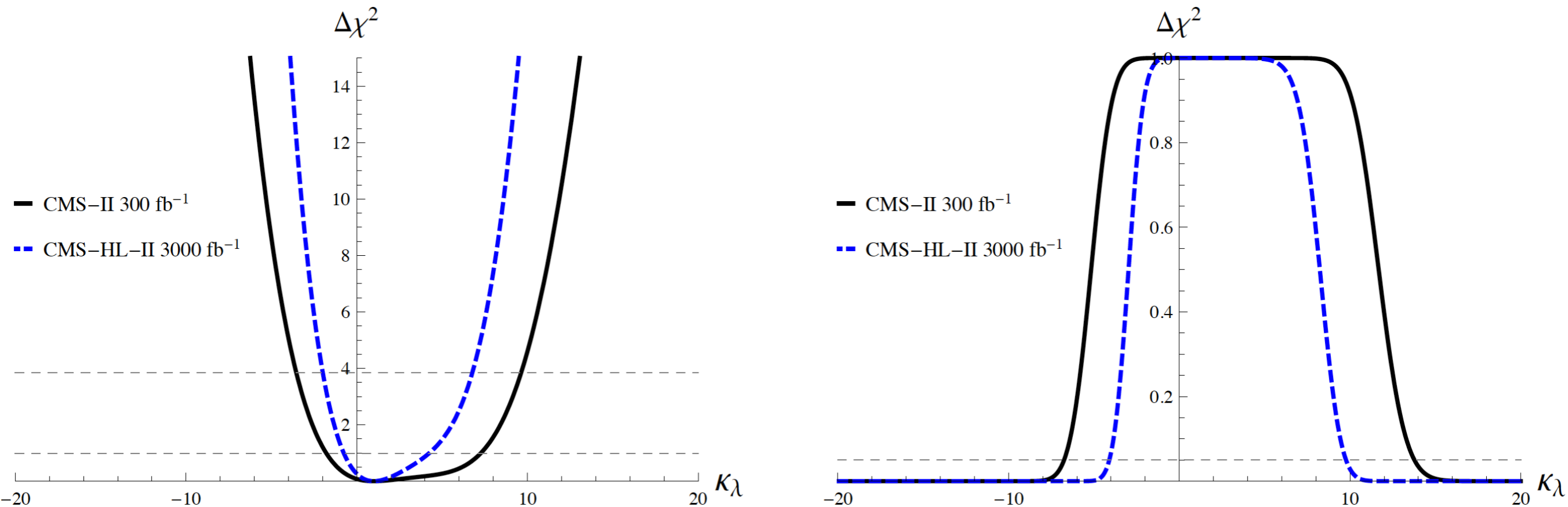


$$\kappa_\lambda^{\text{best}} = 0.5, \quad \kappa_\lambda^{1\sigma} = [-4.7, 8.9], \quad \kappa_\lambda^{2\sigma} = [-8.2, 13.7]$$

Models with κ_λ in the region $\kappa_\lambda < -13.3$ and $\kappa_\lambda > 20.0$ are excluded at more than 2σ

Competitive with direct search, where $\kappa_\lambda = O(\pm(15 - 20))$

Future perspective



Assuming we measure SM, treating uncertainties according to [arXiv:1312.4974](#)

$$\kappa_{\lambda}^{1\sigma} = [-0.75, 4.23], \quad \kappa_{\lambda}^{2\sigma} = [-1.99, 6.77], \quad \kappa_{\lambda}^{p>0.05} = [-4.10, 9.77]$$

Competitive with direct search, where we expect $\kappa_{\lambda} = [-1.3, 8.7]$

Conclusions

- *No direct measurement of Higgs self couplings so far*
- *Trilinear coupling can be investigated by means of radiative corrections to precision electroweak measurements*
- *Anomalous coupling can be interpreted as the effect of an (in)finite tower of BSM operators*
- *Compared to double Higgs production, the bounds obtained are competitive and complementary*
- *Precision physics can help constrain the allowed region!*