Ions: Mode Coupling Instability

R. Wanzenberg, 20.07.2013

Main parameters and settings, 240 Bunches, Mode 68

In[1]:=
```
(* Parameters in SI units*)

Clear["Global`*"];

c = 2.99792458 × 10^8;
eq = 1.602177 × 10^-19;
rp  = 1.534686 × 10^-18;
re  = 2.8179 × 10^-15;
(* PETRA III Parameter *)
Nb  = 240;
Itot  = 85.73 × 10^-3;
CP3  = 2304 ;
γP3  = 6.0 × 10^9 / (0.551 × 10^6) ;
f0  = c / CP3;
ω0  = 2 Pi f0;
N0  = Itot /  (Nb × f0 × eq) ;
Ntot  = Itot /  (f0 × eq) ;
(* Electron  Parameter *)
λe  = Ntot / CP3;
(* Beam Parameter *)
dp  = 1 × 10^-3; (* energy spread *)
νx  = 36.127;
νy = 30.296;
βx = CP3 / (2 Pi νx ) ;
βy = CP3 / (2 Pi νy ) ;
Dx  = 0.5;
ϵx = 1.0 × 10^-9;
ϵy = 0.01 × ϵx;
sx = Sqrt[ϵx βx] + Dx * dp;
sy = Sqrt[ϵy βy] ;
(* x: 800 mu m, y: 11 mu m *)
σx =  800.0 × 10^-6;
σy =  10.0 × 10^-6;
(* Ion Parameter O2 A=32  *)
Ai  = 32;
λi  = 20.0 / 0.01;
(*  600/12 A = 28,  10 none,  14 ... 50 m = 76, 2000 m = 75,76,77
           A = 32   20 m = 73
    800/11 A =32 m =69
```

```
800/10 A =32 20 m =71*)
```

$$\omega i = \sqrt{\lambda e \times \frac{rp\, c^2}{Ai} \times \frac{1}{\sigma y\ (\sigma y + \sigma x)}};$$

$$\omega e = \sqrt{\lambda i \times \frac{re\, c^2}{\gamma P3} \times \frac{1}{\sigma y\ (\sigma y + \sigma x)}};$$

```
Print["PETRA 3"]
Print["I / mA            ", Itot / 10^-3]
Print["f0 / kHz          ", f0 / 10^3]
Print["γP3               ", γP3]
Print["N0                ", N0]
Print["Ntot              ", Ntot]
Print["horz Tune / kHz   ", (νx - 36) f0 / 10^3]
Print["vert Tune / kHz   ", (νy - 30) f0 / 10^3]
Print["horz. beta/ m     ", βx,      "   vert. beta/ m     ", βy]
Print["sx/ μm            ", sx / 10^-6, "  sy/ μm            ", sy / 10^-6]
Print["Beam size x,y/ μm ", σx / 10^-6, "  /  ", σy / 10^-6, " Para ", 1/(σy (σy + σx))]
Print["Ntot rp CP3        ", Ntot rp CP3]
Print["Ai                ", Ai]
Print["ωi / MHz          ", ωi / 10^6]
Print["ωi / ω0           ", ωi / ω0]
Print["ωe / MHz          ", ωe / 10^6]
Print["ωe / ω0           ", ωe / ω0]
Print["ω0 / MHz          ", ω0 / 10^6]
```

```
PETRA 3
I / mA            85.73
f0 / kHz          130.118
γP3               10889.3
N0                1.71346×10^10
Ntot              4.11229×10^12
horz Tune / kHz   16.525
vert Tune / kHz   38.515
horz. beta/ m     10.1501   vert. beta/ m     12.1037
sx/ μm            600.748  sy/ μm            11.0017
Beam size x,y/ μm 800.  /  10. Para 1.23457×10^8
Ntot rp CP3        0.0145407
Ai                32
ωi / MHz          30.8187
ωi / ω0           37.696
ωe / MHz          0.0757803
```

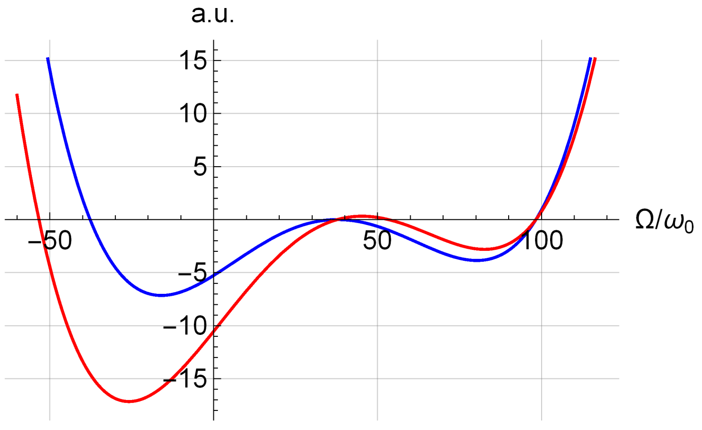
```
ωe / ω0          0.0926911

ω0 / MHz         0.817557
```

Calculate coefficiens

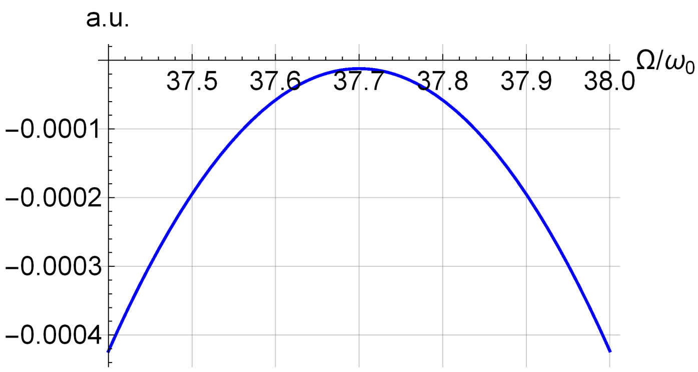
In[49]:=
```
ai = ωi^2/ω0^2;
b = ωi^2/ω0^2 × ωe^2/ω0^2;
ae = ωe^2/ω0^2;
m0 = 68;

Print["Coefficiens"]
Print["ai                  ", ai]
Print["ae                  ", ae]
Print["b                   ", b]
Print["Mode                ", m0]
Print["vert Tune / kHz     ", (νy - 30) f0 / 10^3]
(* Define Function *)
DF0[x_, m_, aai_, aae_] := (x^2 - aai) ((x - m)^2 - νy^2 - aae) - aai * aae; (* Plot function *)
Plot[{DF0[x, m0, ai, ae] × 10^-6, DF0[x, m0, ai * 2, ae] × 10^-6},
 {x, -60, 120}, PlotStyle -> {Blue, Red}, GridLines → Automatic,
 AxesLabel → {"Ω/ω0", "a.u."}, LabelStyle → {Directive[Larger]}]
Plot[{DF0[x, m0, ai, ae] × 10^-6}, {x, 37.4, 38}, PlotStyle -> {Blue},
 GridLines → Automatic, AxesLabel → {"Ω/ω0", "a.u."}, LabelStyle → {Directive[Larger]}]
```

```
Coefficiens

ai                 1420.99

ae                 0.00859164

b                  12.2086

Mode               68

vert Tune / kHz    38.515
```

Out[60]=



Out[61]=



In[62]:=
```
Solve[(x^2 - a0) ((x - m0)^2 - νy^2) == 0, {x}]
```

Out[62]=
```
{{x → 37.704}, {x → 98.296}, {x → -√a0}, {x → √a0}}
```

Calculate Roots

In[63]:=
```
sol = Solve[DF0[xx, m0, ai, ae] == 0, {xx}];
res = xx /. sol
ires = Im[res]
ires ω0
(* define Funktion according to the previous results *)
```

Out[64]=
```
{-37.6961, 37.6999 - 0.0515455 i, 37.6999 + 0.0515455 i, 98.2962}
```

Out[65]=
```
{0, -0.0515455, 0.0515455, 0}
```

Out[66]=
```
{0., -42141.3, 42141.3, 0.}
```

In[67]:=
```
count = {};
For[ir = 1, ir < 241, ir++,
  sol = Solve[DF0[xx, ir, ai, ae] == 0, {xx}];
  ires = Im[xx /. sol];
  sum = Abs[ires[[1]]] + Abs[ires[[2]]] + Abs[ires[[3]]] + Abs[ires[[4]]];
  count = If[sum > 0, Join[count, {ir}], count];
  (* Print["Mode ",ir,"  ",sum,"  ",ires] *)
 ];
```

In[69]:=
```
Print["Mode List ", count]
```

```
Mode List {68}
```

Moore Roots

In[70]:=

```
sol = Solve[DF0[xx, 68, ai, ae] == 0, {xx}];
res = xx /. sol
Sqrt[ai]
ires = Im[res]
ires * ω0
(* define Funktion according to the previous results *)
```

Out[71]= {-37.6961, 37.6999 - 0.0515455 ⅈ, 37.6999 + 0.0515455 ⅈ, 98.2962}

Out[72]= 37.696

Out[73]= {0, -0.0515455, 0.0515455, 0}

Out[74]= {0., -42141.3, 42141.3, 0.}