Signatures of alpha-like quartet condensation in N=Z nuclei

Nicolae Sandulescu

National Institute of Physics and Nuclear Engineering, Bucharest

Outline

- Pairing and alpha-like quartetting in N=Z nuclei
- Quartetting for general (shell model) interactions*
- Quartet phase transition & Penrose-Yang criterion*

*work done with Michelangelo Sambataro (INFN-Catania)

Pairing in N=Z nuclei: main issues

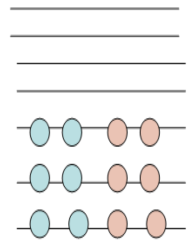
6 types of spin-isospin pairs

S=0, T=1

S=1, T=0

$\nu^+_\uparrow \nu^+_\downarrow$ $\quad$ $\pi^+_\uparrow \pi^+_\downarrow$ $\quad$ $\nu^+_\uparrow \pi^+_\downarrow + \pi^+_\uparrow \nu^+_\downarrow$ $\quad$ $\nu^+_\uparrow \pi^+_\downarrow - \pi^+_\uparrow \nu^+_\downarrow$ $\quad$ $\nu^+_\downarrow \pi^+_\downarrow$ $\quad$ $\nu^+_\uparrow \pi^+_\uparrow$

$\Gamma^+_{\nu\nu} = \sum_i x_i \nu^+_i \nu^+_{\bar{i}}$ $\quad$ pairs $\quad$ $\Gamma^+_{\pi\nu} = \sum_i x_i (\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}})$



$\Delta^+_0 = \sum_i x_i (\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$

$(\Gamma^+_{\nu\nu})^{N/2}$ $\quad$ condensates $\quad$ $(\Gamma^+_{\nu\pi})^{N_{\pi\nu}/2}$ $\quad$ $(\Delta^+_0)^{N_{\pi\nu}/2}$

<u>Long standing questions</u>

there is a "condensate" of pn pairs in nuclei ?

the fingerprints of a pn condensate ?

<u>Theoretical approach</u>

BCS/HFB-type models : - unified descriptions of all types of pairing
- drawback : particle number and isospin are not conserved

beyond BCS ?

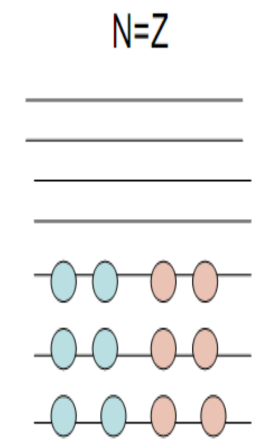
<u>This talk</u>

T=1 and T=0 pairing described by alpha-like quartets not by Cooper pairs

Isovector (T=1) proton-neutron pairing

Isovector proton-neutron pairing in BCS-like models

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{t=-1,0,1} P_{it}^+ P_{jt}$$



collective Cooper pairs

$$\Gamma_{\nu\nu}^+ = \sum_i x_i \nu_i \nu_{\bar{i}} \qquad \Gamma_{\pi\pi}^+ = \sum_i x_i \pi_i \pi_{\bar{i}} \qquad \Gamma_{\pi\nu}^+ = \sum_i x_i (\nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+)$$

"condensates of pairs"

	exact	$(\Gamma_{\nu\upsilon}^+)^{N/2} \Gamma_{\pi\pi}^+)^{Z/2}$	$(\Gamma_{\nu\pi}^+)^{\frac{N+Z}{2}}$
^{44}Ti	5.973	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	9.815 (8.850%)	8.585 (20.273%)

- **large errors & no mixing of pn with nn and pp pairing**

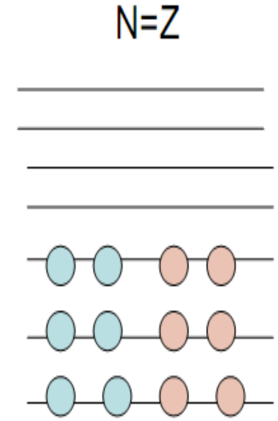
restoration of the isospin symmetry ?

$|PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$ still large erros !

Isovector pairing in terms of alpha-like quartets

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^+ P_{j,\tau}$$

$$P_{i1}^+ \propto \nu_i^+ \nu_{\bar{i}}^+ \qquad P_{i-1}^+ \propto \pi_i^+ \pi_{\bar{i}}^+ \qquad P_{i0}^+ \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+$$



collective quartet

$$Q^+ = \sum_{ij\tau\tau'} x_{ij} [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \propto \sum_{ij\tau\tau'} x_{ij} (P_{\nu\nu,i}^+ P_{\pi\pi,j}^+ + P_{\pi\pi,i}^+ P_{\nu\nu,j}^+ - P_{\nu\pi,i}^+ P_{\nu\pi,j}^+)$$

quartet condensate

$$|QCM> = Q^{+n_q} |->$$ (has T=0, J=0)

(x_{ij} determined variationally)

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Quartet condensation versus pair condensation

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_t \; P_{it}^{+} P_{jt}$$

pairing forces extracted from SM interactions

	SM	QCM $(Q^{+})^{n_q}$	PBCS1 $(\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+})^{n_q}$	PBCS0 $(\Gamma_{\nu\pi}^{+2})^{n_q}$
^{20}Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
^{24}Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
^{28}Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
^{32}S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
^{44}Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
^{104}Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
^{108}Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
^{112}Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

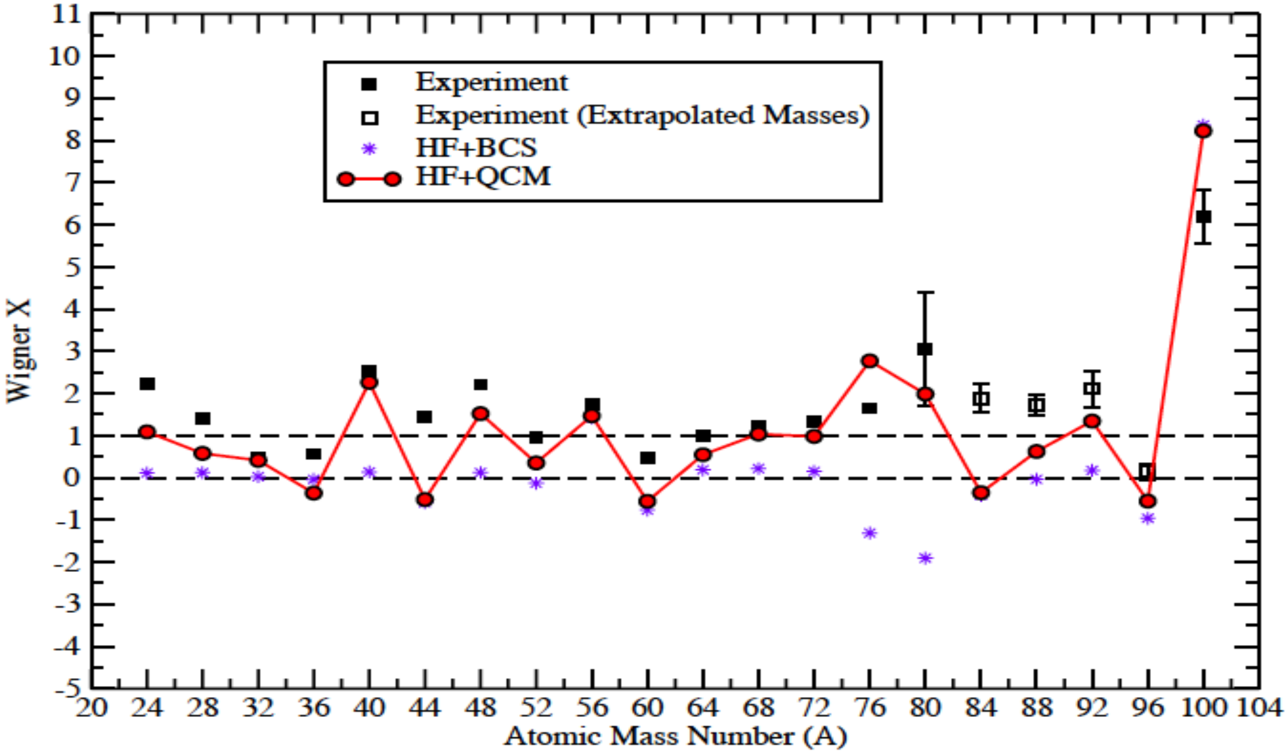
Conclusion: *T=1 pairing is accurately described by quartets, not by pairs*

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Isovector pairing and Wigner energy

$$E(N,Z) = E(N=Z) + a_s \frac{(N-Z)^2}{A} + a_W \frac{|N-Z|}{A} + \delta E_{shell} + \delta E_P$$

$$E(N,Z) = E(N=Z) + \frac{T_z(T_z + X)}{2\Theta} \qquad T_z = 0,2,4$$

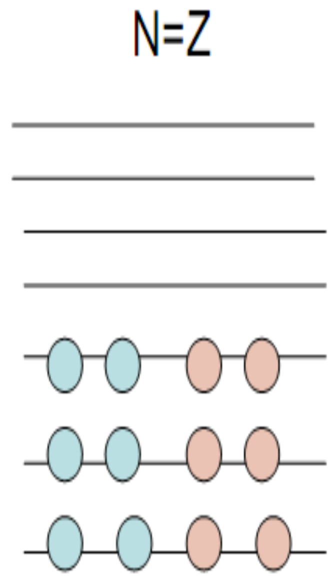


Conclusion: T=1 pairing, when treated by QCM, is able to describe well the Wigner !

D. Negrea and N. S, PRC90 (2014)

Isoscalar and isovector pairing in N=Z nuclei

$$P^{+}_{i,T_z} = [a_i^{+} a_i^{+}]^{T=1,J=0}_{T_z}$$



$$D^{+}_{ij,J_z} = [a_i^{+} a_j^{+}]^{J=1,T=0}_{J_z}$$

Quartetting for isovector (J=0) and isoscalar (J=1) pairing

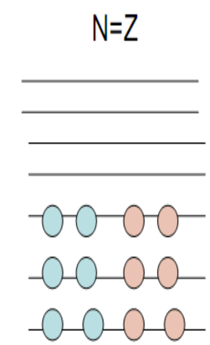
$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

isovector

$$P_{i,T_z}^{+} = [a_i^{+} a_i^{+}]_{T_z}^{T=1,J=0}$$

isoscalar

$$D_{ij,J_z}^{+} = [a_i^{+} a_j^{+}]_{J_z}^{J=1,T=0}$$



collective quartets

$$Q_{\nu}^{+(iv)} = \sum_{i,j} x_{ij}^{(\nu)} [P_i^{+} P_j^{+}]^{T=0}$$

$$Q_{\nu}^{+(is)} = \sum_{ij,kl} y_{ij,kl}^{(\nu)} [D_{ij}^{+} D_{kl}^{+}]^{J=0}$$

generalised quartet

$$Q_{\nu}^{+} = Q_{\nu}^{+(iv)} + Q_{\nu}^{+(is)}$$

Quartet condensate

$$|QCM> = Q^{+n_q} |->$$

superposition of T=0 and T=1 quartets

M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Quartet condensation for isovector & isoscalar pairing

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

$(Q^+)^{n_q} |->$ $(\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} |->$ $(\Gamma_{\nu\pi}^+)^{2n_q} |->$ $(\Delta_0^+)^{2n_q} |0\rangle$

	QCM	PBC1	$PBCS0_{iv}$	$PBCS0_{is}$
^{20}Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

Conclusions

- quartet condensation wins over Cooper pair condensates
- T=1 and T=0 pairing correlations always coexist in quartets

M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Quartetting for general (shell model) forces

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

<u>ansatz for the ground state</u>

$$|QCM\rangle = Q^{+n_q}|-\rangle \qquad Q^+ = \sum_{ii',jj',JT} x_{ii',jj'} [A^+_{ii'JT} A^+_{jj'JT}]^{0,0}$$

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$E_{corr}(QM)$	$\langle SM\|QCM\rangle$
^{20}Ne	24.77	24.77	24.77	1
^{24}Mg	55.70	53.04 (4.77%)	53.24 (4.41%)	0.85
^{28}Si	88.75	86.52 (2.52%)	87.12 (1.84%)	0.86
^{32}S	122.51	122.02 (0.40%)	122.29 (0.18%)	0.98

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q-1)}{2} \times V(n_q),$$

the interaction between the quartets is small compared to their “binding” energies

<u>Conclusion:</u> quartets acts as weakly interacting building blocks

M. Sambataro and N. S., EPJ A53 (2017) 43

How to identify the transition to a quartet condensate ?

Penrose & Yang criterion:

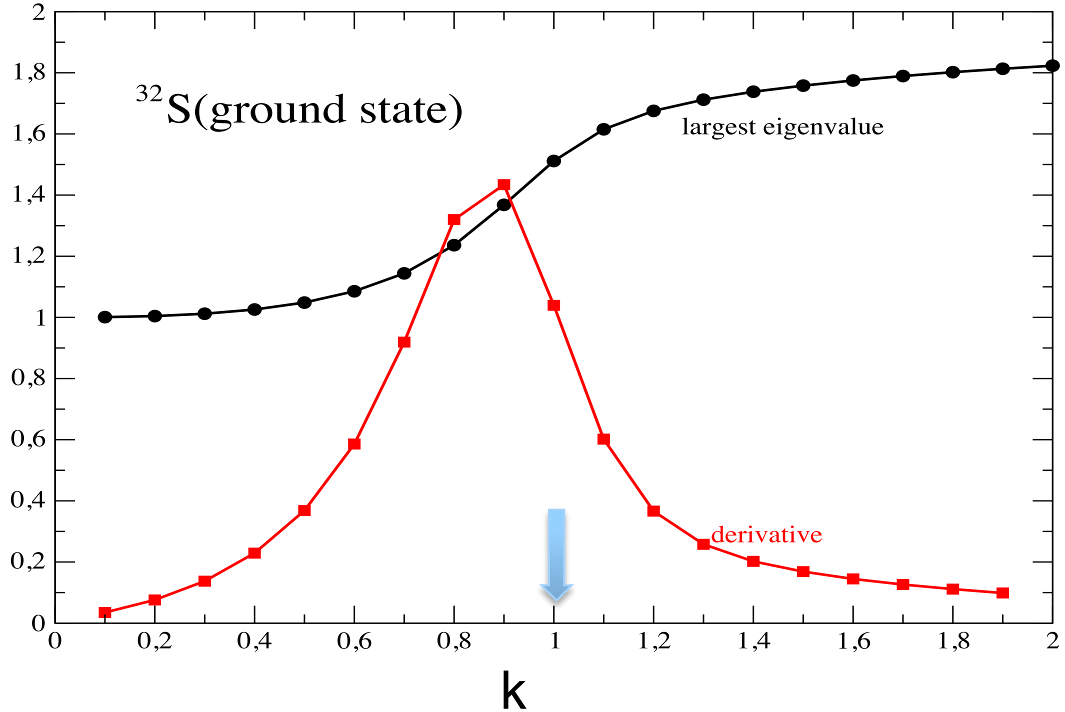
n-body long-range correlations ⟺ a large eigenvalue of n-body density

Transition to a quartet condensate phase in ^{32}S

$$H(k) = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \mathrm{k} \sum_{all} V_{JT}(ii'; jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$H(k) = (1-k)\, H(0) + \mathrm{k\, H(1)}$$

4-body density matrix $\rho^{(4)}_{i,j}(k) = < SM(k) | q_i^+ q_j | SM(k) >$ $q_i^+ = (a^+_{i_1} a^+_{i_2} a^+_{i_3} a^+_{i4})^{T=0}$



Conclusion: a fast transition towards a quartet condensate !

M. Sambataro and N.S., in preparation

Summary and Conclusions

Main message: *isovector and isoscalar pairing are accurately described by alpha-like quartets, not by Cooper pairs*

- *T=1 and T=0 pairing correlations always coexist, through the alpha-like quartets*
- *proton-neutron pairing correlations are still significant away of N=Z line*
- *T=1 proton-neutron pairing is providing a good description of Wigner energy*
- *alpha-like quartets are relevant degrees of freedom for general two-body forces*
- *a fast transition to a quartet condensate phase in ^{32}S*

some open issues

- *testing the quartet condensation by alpha transfer reactions ?*
- *unified microscopic treatment of quartetting and clustering ?*

Thanks for your attention !

Quartet condensation in excited states of N=Z nuclei ?

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$|0_n^+; QCM\rangle = (Q_n^+)^{n_q}|-\rangle \qquad Q_n^+ = \sum_{ii',jj',JT} x^{(n)}_{ii',jj'} [A^+_{ii'JT} A^+_{jj'JT}]^{0,0}$$

First excited 0^+

	$E_{0_1^+}(SM)$	$E_{0_1^+}(QCM)$	$\langle SM\|QCM\rangle$
^{20}Ne	−33.77 (6.7)	−33.77 (6.7)	1
^{24}Mg	−79.76 (7.34)	−76.97 (7.47)	0.70
^{28}Si	−131.00 (4.84)	−126.91 (6.71)	0.65
^{32}S	−178.98 (3.46)	−178.04 (3.92)	0.95

← SM is a QCM state !?

Second excited 0^+

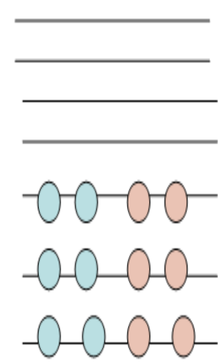
	$E_{0_2^+}(SM)$	$E_{0_2^+}(QCM)$
^{20}Ne	−28.56 (11.91)	−28.56 (11.91)
^{24}Mg	−77.43 (9.67)	−70.85 (13.59)
^{28}Si	−128.51 (7.33)	−120.64 (12.99)
^{32}S	−175.04 (7.4)	−170.84 (11.12)

superposition of many shell-model states: **cluster- type excitations** ?

M. Sambataro and N. S., EPJ A53 (2017) 43

Isoscalar and isovector proton-neutron pairing in time-reversed states

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$



isovector

isoscalar

$$P^{+}_{i,0} = (\nu^{+}_{i} \pi^{+}_{\bar{i}} + \pi^{+}_{i} \nu^{+}_{\bar{i}})/\sqrt{2}.$$

$$D^{+}_{i,0} = (\nu^{+}_{i} \pi^{+}_{\bar{i}} - \pi^{+}_{i} \nu^{+}_{\bar{i}})/\sqrt{2}$$

$$P^{+}_{i1} = \nu^{+}_{i} \nu^{+}_{\bar{i}} \qquad P^{+}_{i-1} = \pi^{+}_{i} \pi^{+}_{\bar{i}}$$

$$Q^{+}_{T=1} = \sum_{ij} x_i x_j [P^{+}_{i\tau} P^{+}_{j\tau'}]^{T=0}$$

$$\Delta^{+}_{0} = \sum y_i D^{+}_{i,0}$$

ansatz for ground state

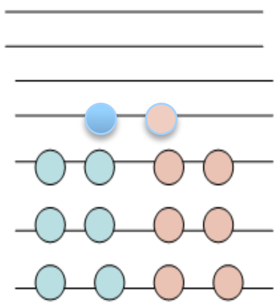
$$|\Psi\rangle = (Q^{+}_{T=1} + \Delta^{+2}_{0})^{n_q} |-\rangle$$

superposition of T=1 quartet condensates and T=0 pair condensates

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B751 (2015) 348

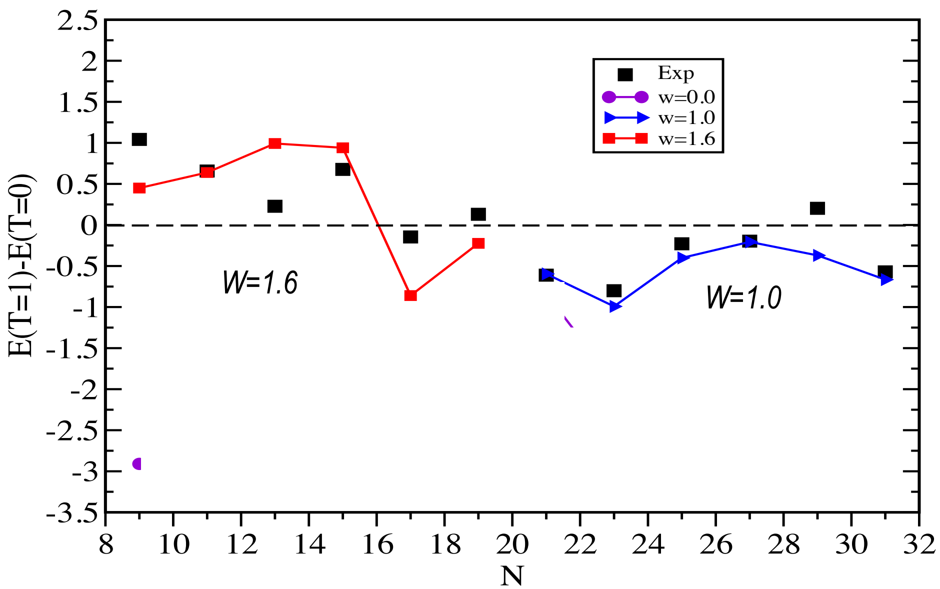
Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$



T=1 state $\quad | iv; QCM >= \tilde{\Gamma}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} | - >$

T=0 state $\quad | is; QCM >= \tilde{\Delta}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} | - >$



$$V^{T=\{0,1\}}_{paring} = V^{T=\{0,1\}}_{0} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}} \qquad w = \frac{V^{T=0}_{0}}{V^{T=1}_{0}}$$

what we can learn about
the structure of the states ?

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$(\Delta^{+}_{\nu\pi})^{2n_q+1}$	$\tilde{\Delta}^{+}_{\nu\pi}(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

		Exact	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(\Delta^{+2}_{\nu\pi})^{n_q}$	$(\Gamma^{+}_{\nu\pi})^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

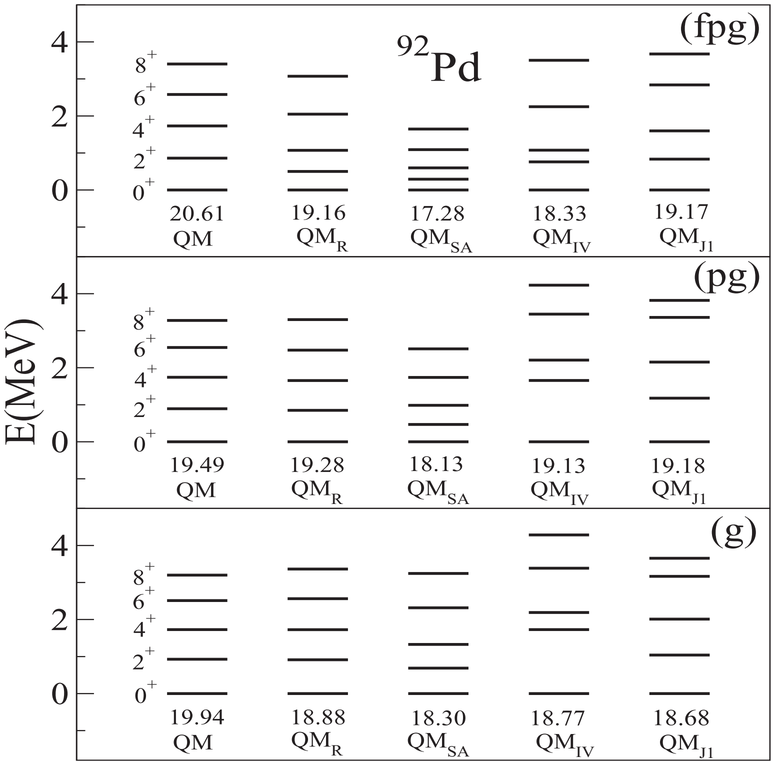
conclusion

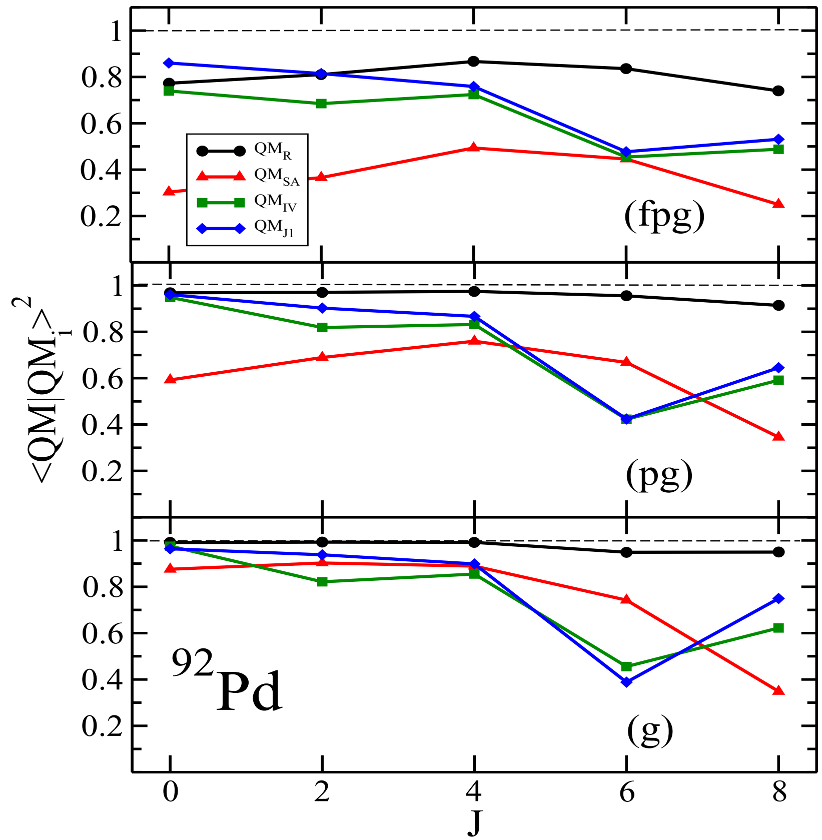
isovector correlations are stronger in both T=0 and T=1 low-lying states

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

Role of spin-aligned pairs in ^{92}Pd

$$Q^{+}_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times \left[[a^{+}_{i_1} a^{+}_{j_1}]^{J_1 T_1} [a^{+}_{i_2} a^{+}_{j_2}]^{J_2 T_2} \right]^{JT}_{MT_z}, \qquad \left[Q^{+}_{\alpha_1, J', T'} \otimes Q^{+}_{\alpha_2, J'', T''} \right]^{J,T},$$





<u>Conclusions</u>

the structure of ^{92}Pd **is not** dominated by J=9 pairs
ground state is mainly built by J=0 and J=1 pairs

M. Sambataro and N. S, PRC91 (2015)