



北京大學

9th international workshop: Quantum Phase Transitions in Nuclei  
and Many-body Systems

May 22-25, 2018, Padova

Nuclear multiple chirality in Rh-106: a manifestation of  
triaxial shape coexistence

**Pengwei Zhao** (赵鹏巍)

Peking University

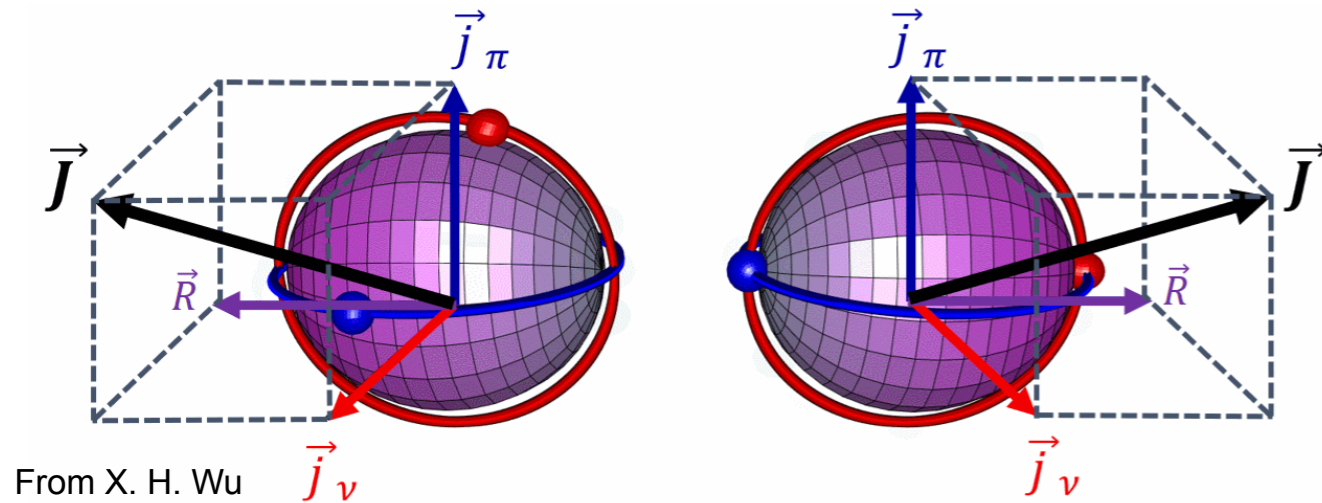
# Outline

- Chirality and multiple chirality in nuclei
- Tilted axis cranking covariant DFT
- Multiple chirality in Rh-106
- Summary

# Nuclear spin-chirality

The **aplanar (3D-) rotation** of a **triaxial nucleus** could present **chiral geometry**.

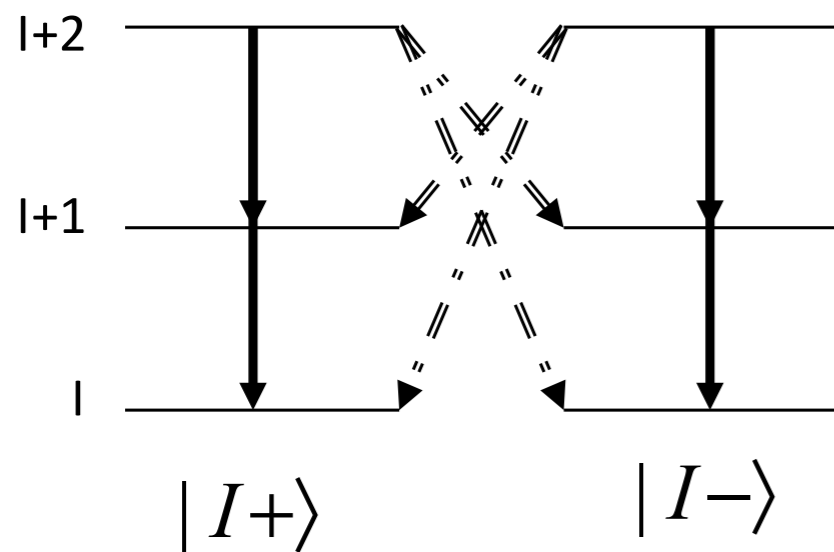
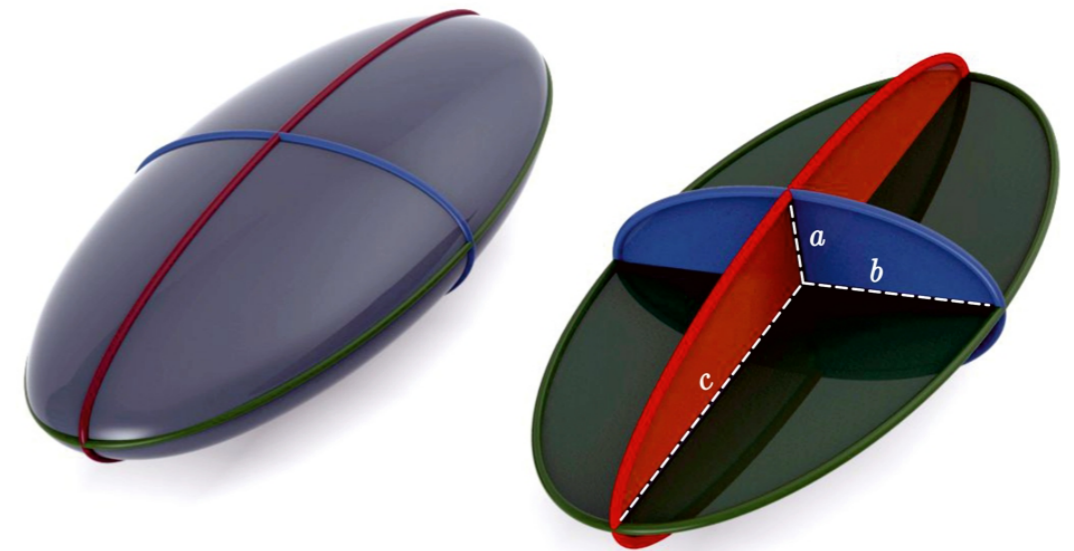
Frauendorf and Meng NPA 1997



From X. H. Wu

Left-handed  $|\mathcal{L}\rangle$

Right-handed  $|\mathcal{R}\rangle$



Lab. frame:

Chiral Symmetry restoration

$$|I+\rangle = \frac{1}{\sqrt{2}}(|\mathcal{L}\rangle + |\mathcal{R}\rangle)$$

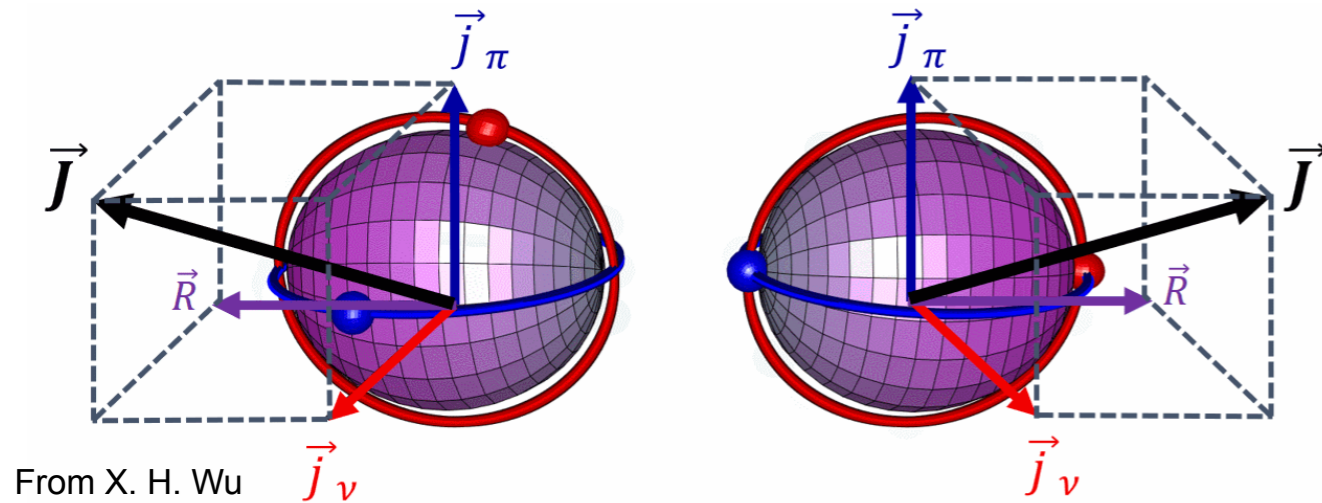
$$|I-\rangle = \frac{i}{\sqrt{2}}(|\mathcal{L}\rangle - |\mathcal{R}\rangle)$$

Exp. signal: **Two near degenerate  $\Delta I = 1$  bands**, called **chiral doublet bands**

# Nuclear spin-chirality

The **aplanar (3D-) rotation** of a **triaxial nucleus** could present **chiral geometry**.

Frauendorf and Meng NPA 1997



From X. H. Wu

Left-handed  $|\mathcal{L}\rangle$

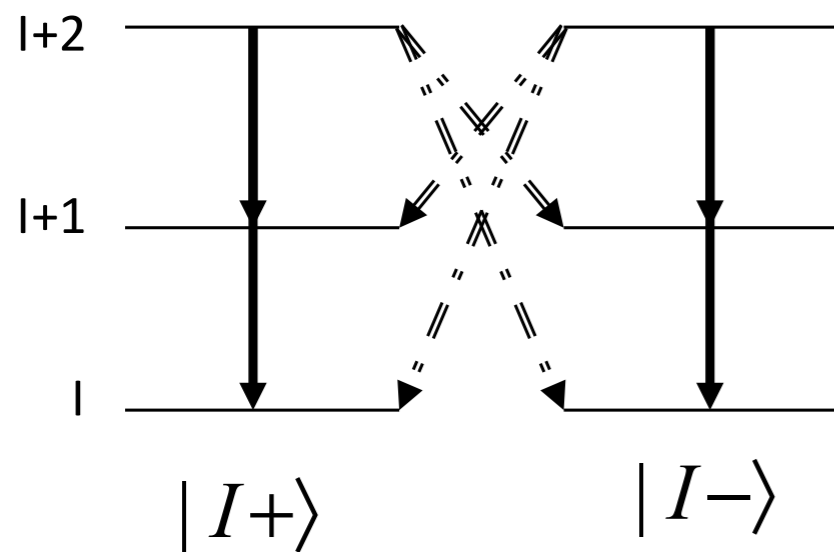
Right-handed  $|\mathcal{R}\rangle$

Intrinsic frame:

Chiral Symmetry breaking

$$\hat{\chi} = \hat{T} \hat{R}_y(\pi)$$

$$\hat{\chi} |\mathcal{L}\rangle = |\mathcal{R}\rangle \quad \hat{\chi} |\mathcal{R}\rangle = |\mathcal{L}\rangle$$



Lab. frame:

Chiral Symmetry restoration

$$|I+\rangle = \frac{1}{\sqrt{2}}(|\mathcal{L}\rangle + |\mathcal{R}\rangle)$$

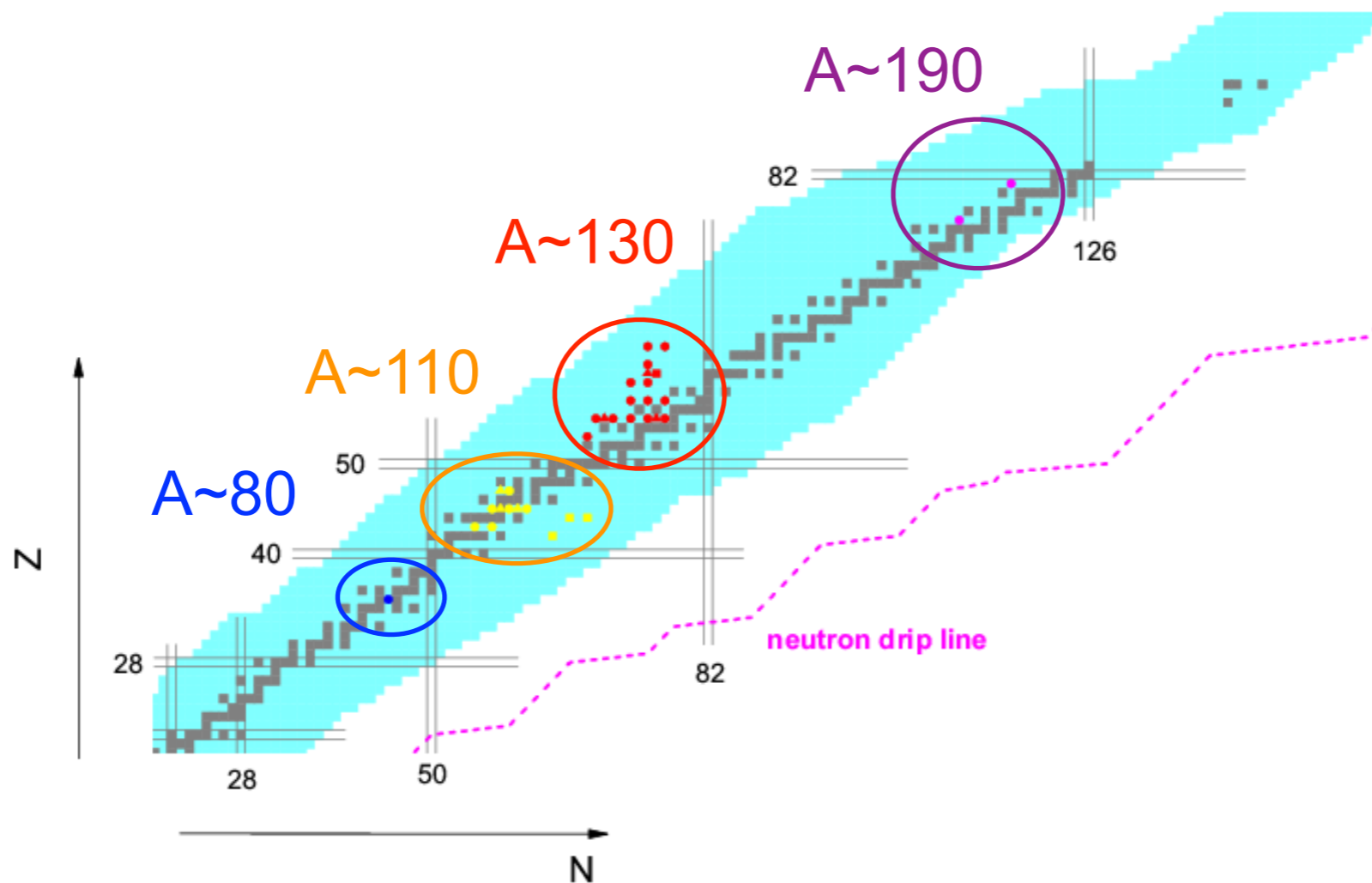
$$|I-\rangle = \frac{i}{\sqrt{2}}(|\mathcal{L}\rangle - |\mathcal{R}\rangle)$$

Exp. signal: **Two near degenerate  $\Delta I = 1$  bands**, called **chiral doublet bands**

# Observed chiral nuclei

More than 45 candidate chiral nuclei have been reported in the  $A \sim 80$ , 100, 130, and 190 mass regions, so far.

Xiong and Wang arXiv:1804.04437



Multiple spin-chirality?

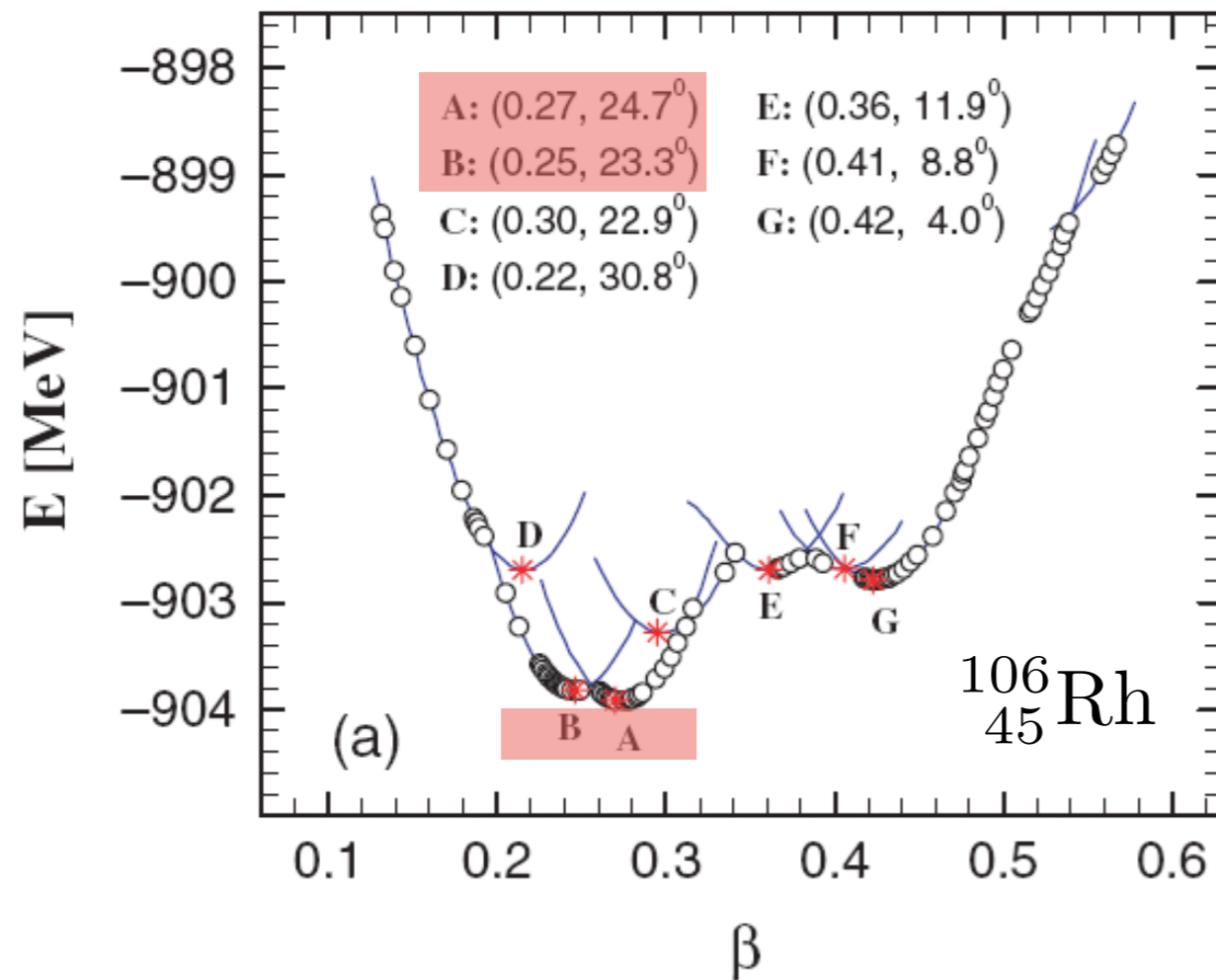
a manifestation of **Triaxial Shape Coexistence**

having more than one pair of chiral bands in a single nucleus ?

# Predictions of multiple spin-chirality

## Covariant DFT on a static mean field: no cranking

Meng, Peng, Zhang, Zhou PRC 2006



Config. A  $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^1$

Config. B  $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2 (gd)^1$

### Two requirements of chirality:

1. strong triaxial deformation
2. high-j particle(s) and hole(s)

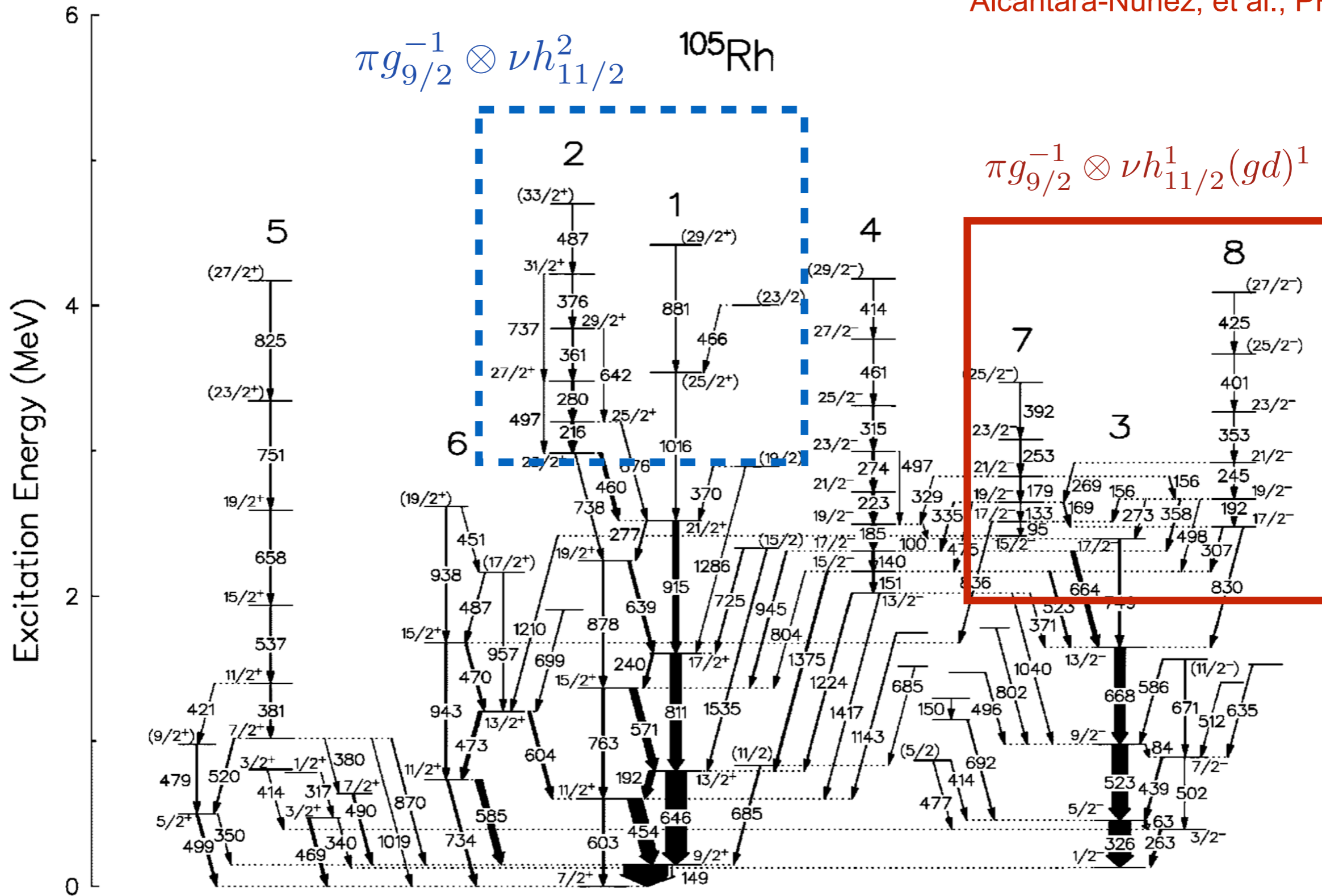
A possible multi-chirality candidate

The investigation followed by:

- Prediction for other odd-odd Rh isotopes: Peng et al., PRC 2008
- Confirmed with time-odd fields included: Yao et al., PRC 2009

# Possible candidates in $^{105}\text{Rh}$

Alcántara-Núñez, et al., PRC 2004



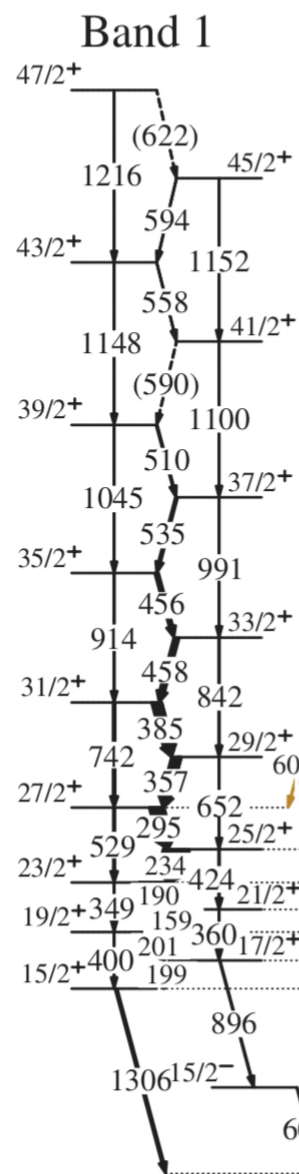




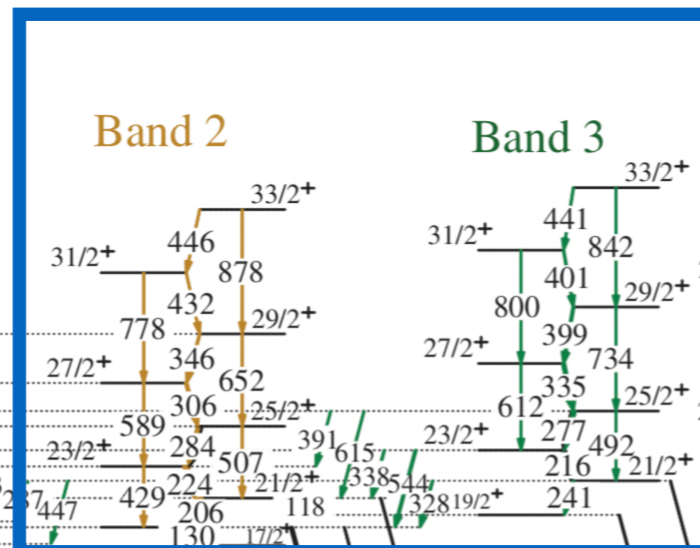
# Multiple chiral doublet bands in $^{133}\text{Ce}$

Two distinct sets of chiral-partner bands have been identified.

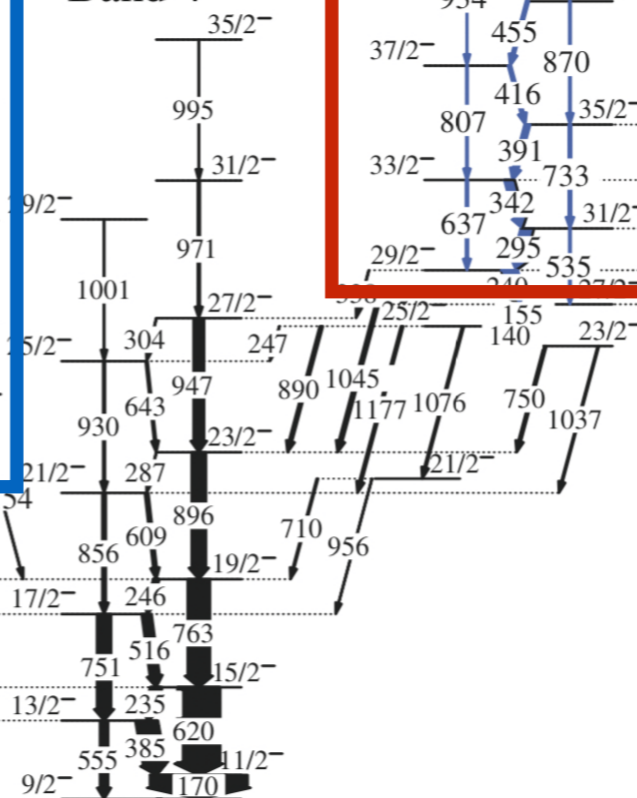
Ayangeakaa et al., PRL 2013



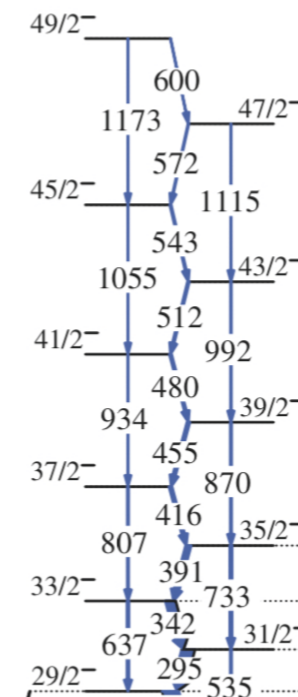
**Experiment: Notre Dame**  
**Theory: Beijing**



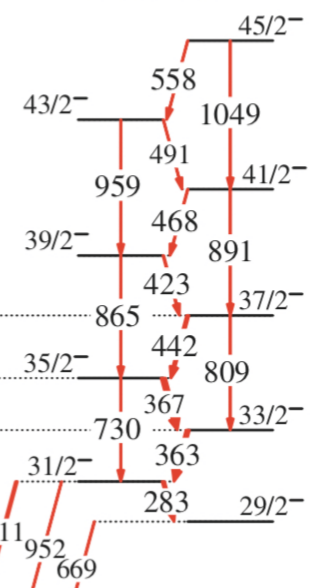
Band 4



Band 5



Band 6

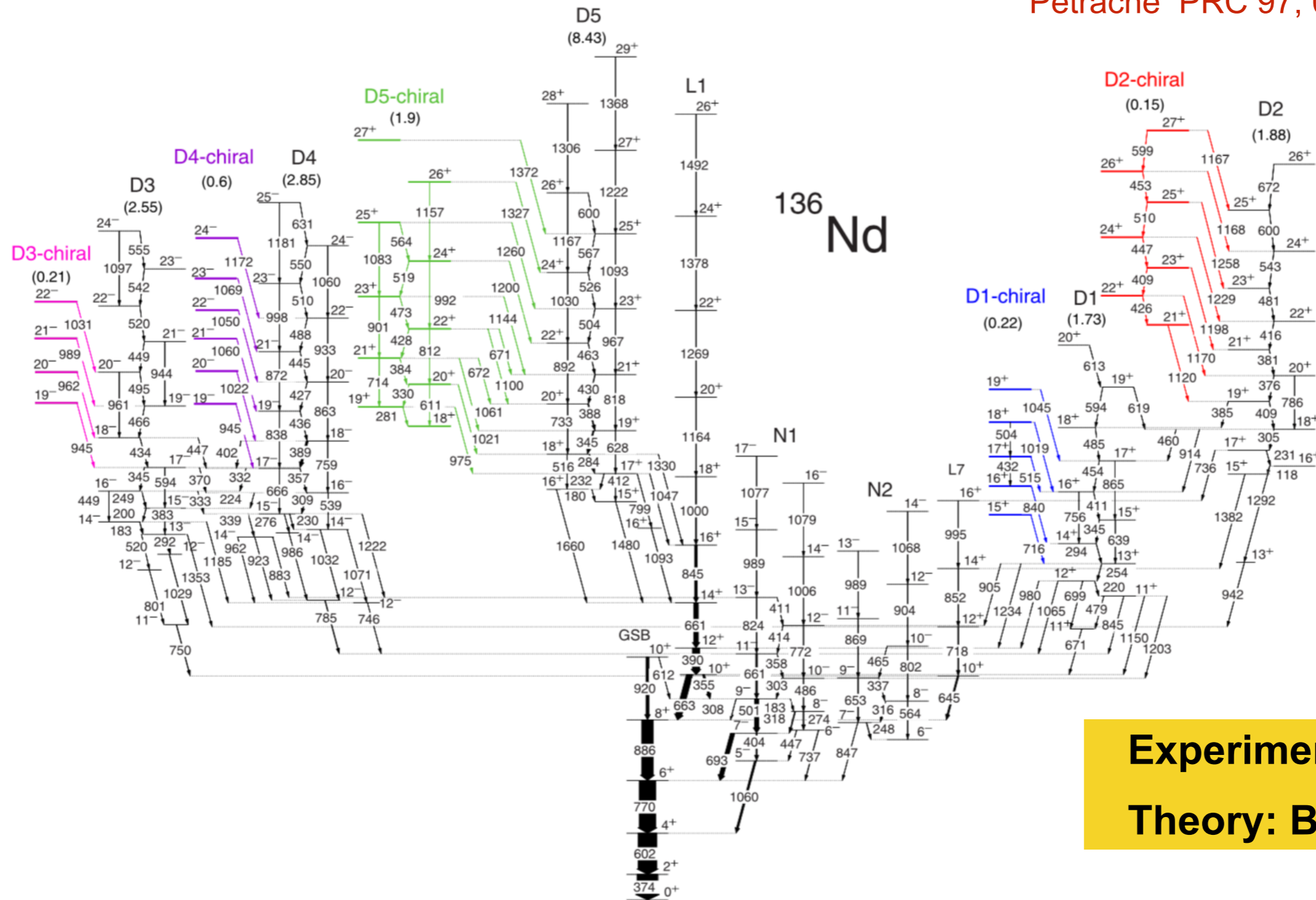


$^{133}\text{Ce}$

Triaxiality can be robust against different configurations

# Multiple chiral doublet bands in $^{136}\text{Nd}$

Petrache PRC 97, 041304(R) (2018)



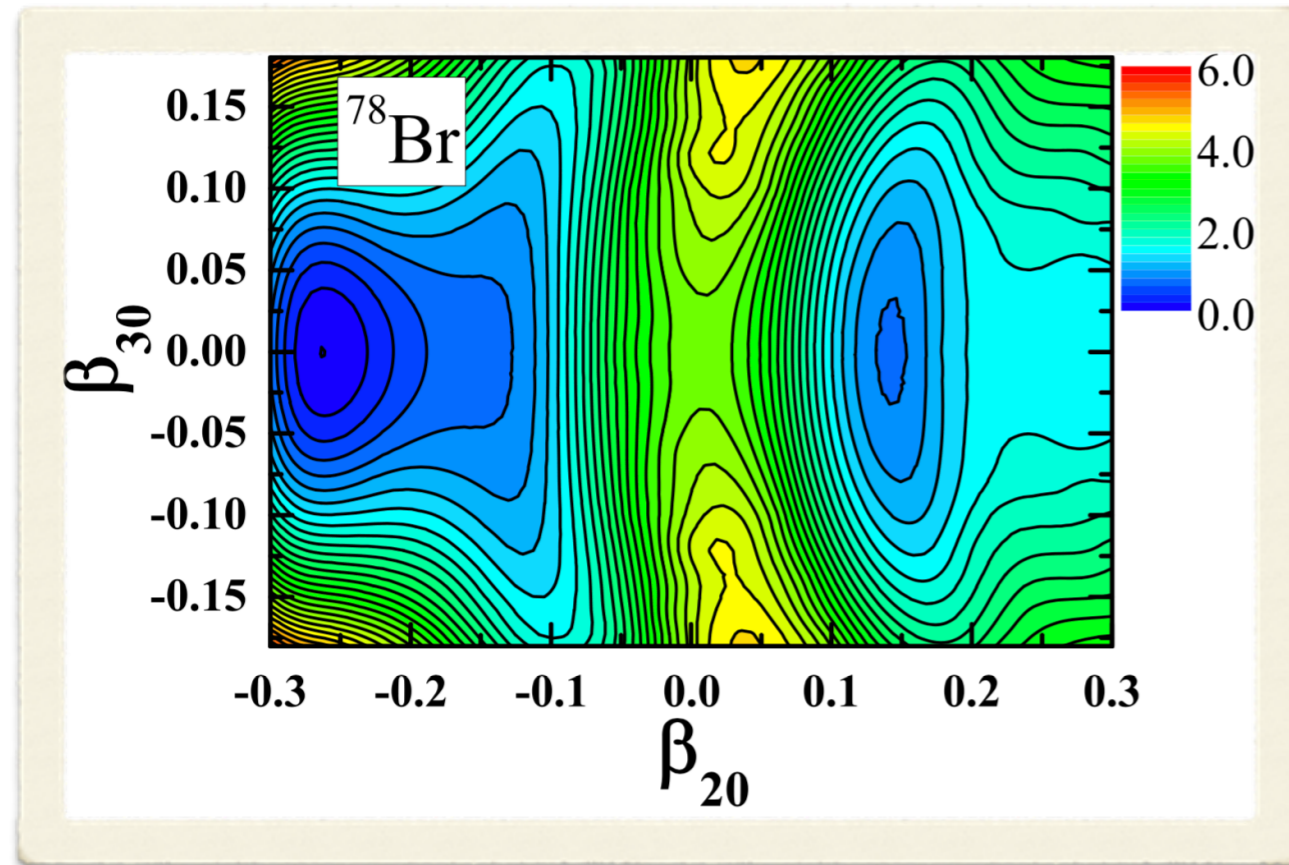
**Experiment: Orsay**  
**Theory: Beijing**

Five candidates for chiral bands are identified

# Octupole Correlations in Multiple Chiral Doublet Bands

Chiral bands in octupole soft nuclei

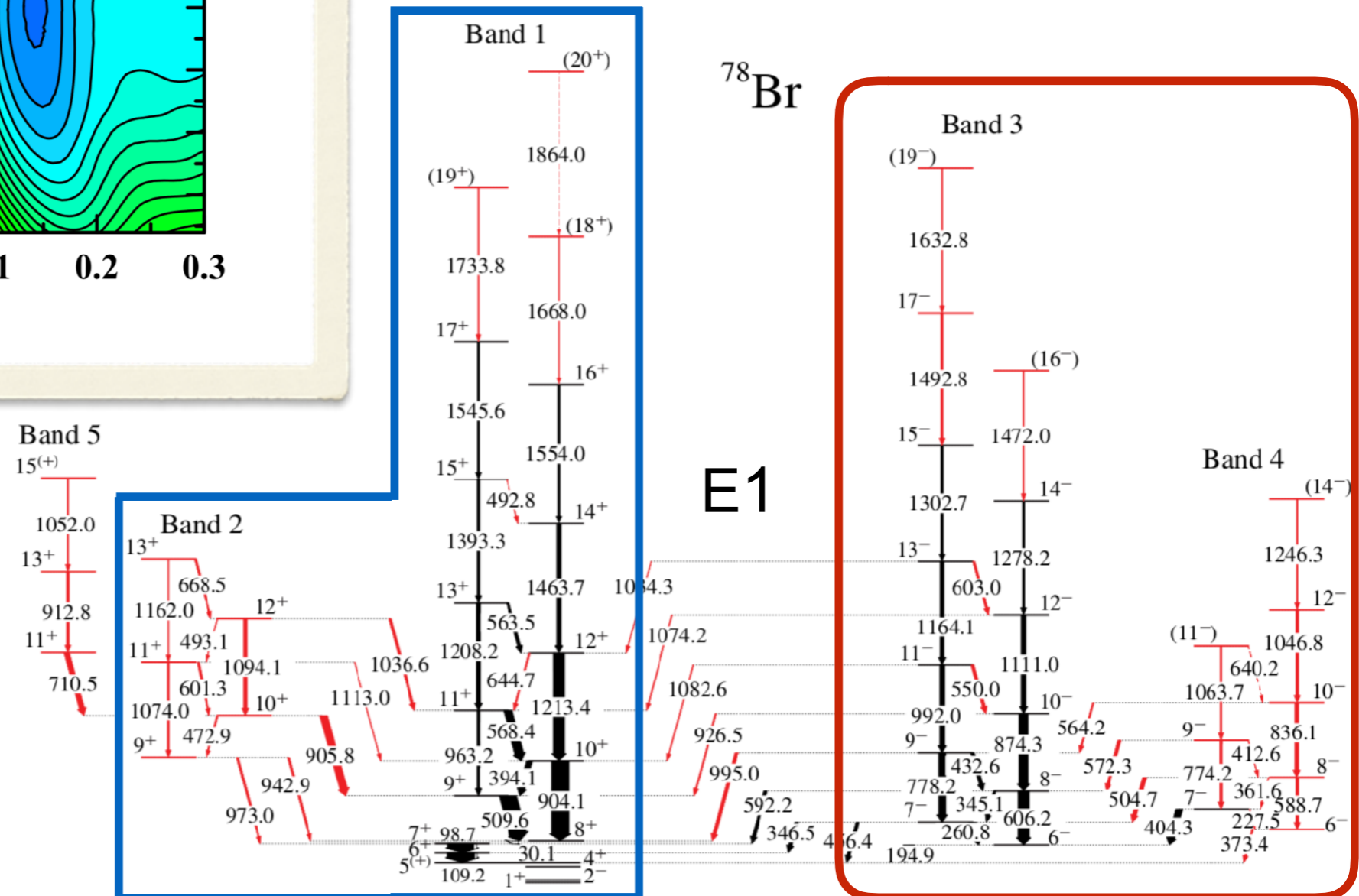
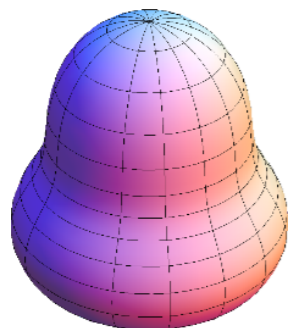
Liu et al, PRL 2016



**Experiment: iThemba LABS**  
**Theory: Beijing**

Multi-dimensional CDFT

Lv, Zhao, Zhou, PRC (2012)



# Theoretical tools for nuclear chirality

- **Particle Core Coupling**

*(lab frame, phenomenological, quantal, with quantum tunneling)*

- ▶ **Triaxial Particle Rotor Model (TPRM)**

Frauendorf and Meng NPA(1997); Peng et al PRC(2003); Koike et al PRL(2004), Zhang et al PRC(2007); Lawrie et al PRC (2008); Qi et al PLB(2009)

- ▶ Core-quasiparticle coupling model Starosta et al PRC(2002); Koike et al PRC(2003)

- ▶ Interacting Boson Fermion Fermion Model S. Brant et al PRC (2004), PRC (2008), Tonev et al PRL(2006)

- **Tilted axis cranking mean-field**

*(intrinsic frame, microscopic, semi-classical, no quantum tunneling)*

- ▶ Single-j model Frauendorf and Meng NPA(1997)

- ▶ Hybrid Woods-Saxon and Nilsson model Dimitrov et al PRL(2000)

- ▶ Non-relativistic Skyrme DFT Olbratowski et al PRL(2004), PRC(2006)

- ▶ **Covariant DFT** Madokoro et al PRC(2000); PWZ PLB (2017)

- **Shell-model**

- ▶ Pair Truncated Shell Model K. Higashiyama et al, PRC(2005)

- ▶ Projected Shell Model F. Q. Chen et al., PRC (2017)

- ▶ Shell-model N. Shimizu' talk

# TPRM for multi-chirality

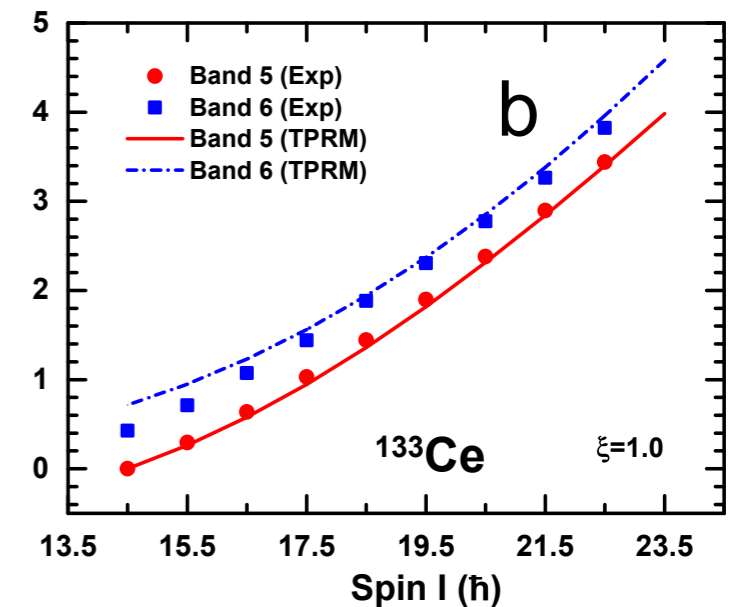
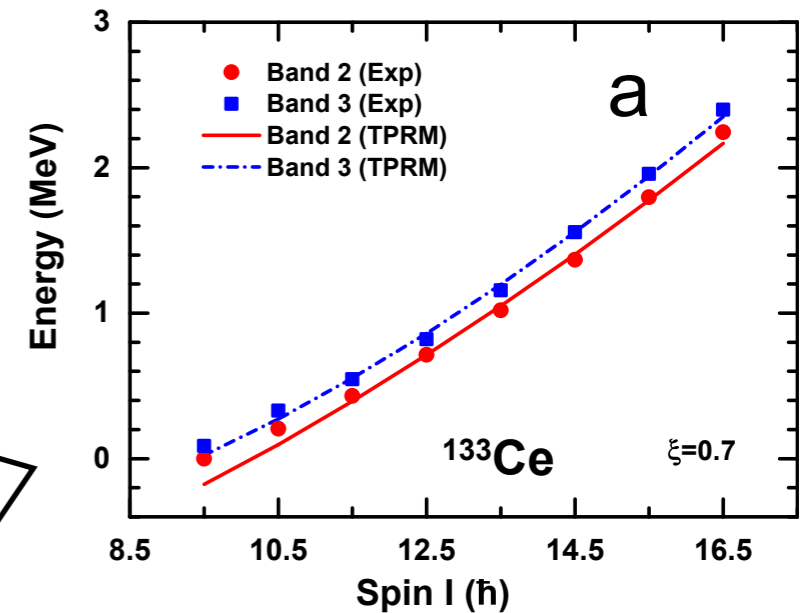
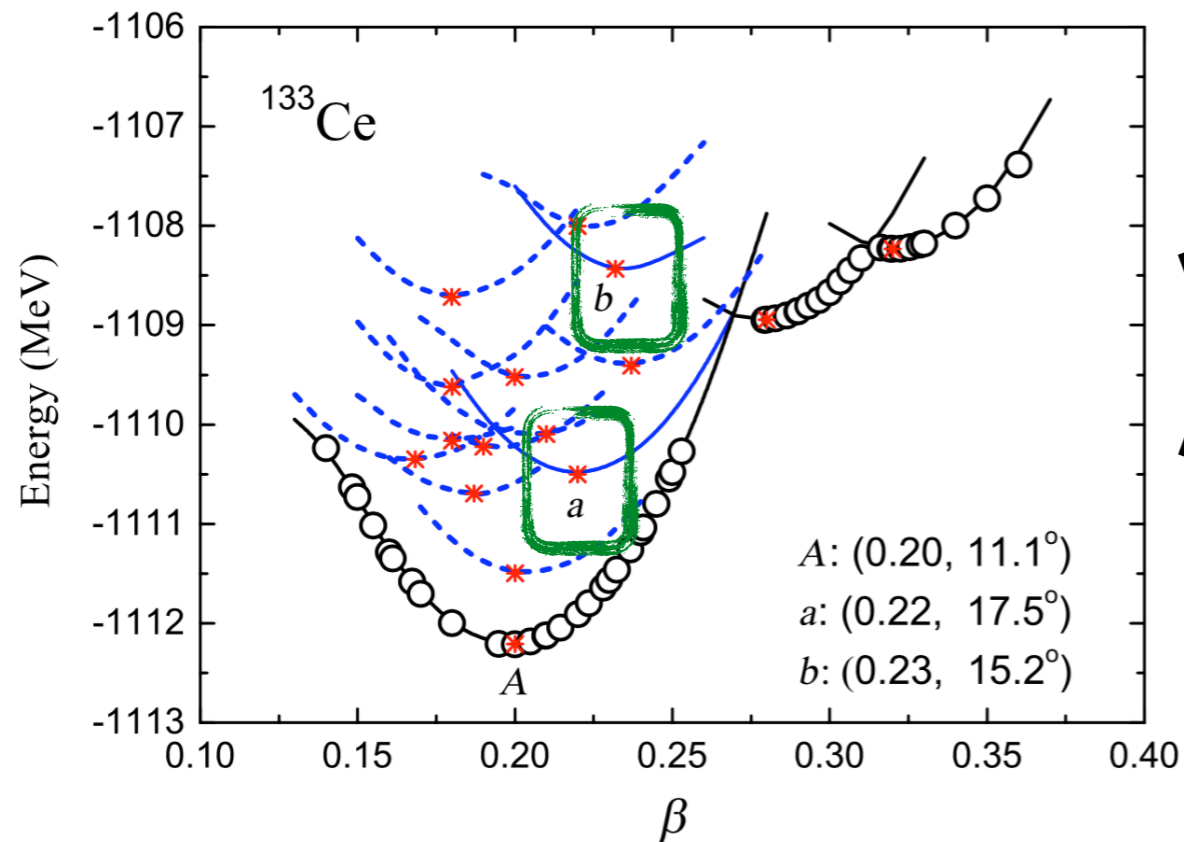
Calculations based on a combination of the triaxial CDFT and the PRM

Ayangeakaa et al., PRL 2013

*quantal, no self-consistency*



**Experiment: Notre Dame**  
**Theory: Beijing**



# TAC for multi-chirality

Calculations based on the tilted axis cranking CDFT and beyond...

*An on going project ...*

TAC-CDFT

Deformations  
Configurations  
Energy Spectra  
EM transitions

Beyond the rotating mean-field

2D Collective Hamiltonian

*Self-consistent*

*Quantum fluctuations*

# TAC for multi-chirality

Calculations based on the tilted axis cranking CDFT and beyond...

*An on going project ...*

TAC-CDFT

Deformations  
Configurations  
Energy Spectra  
EM transitions

Beyond the rotating mean-field

2D Collective Hamiltonian

*Self-consistent*

In the present talk

PWZ, PLB 773, 1 (2017)

*Quantum fluctuations*

# TAC for multi-chirality

Calculations based on the tilted axis cranking CDFT and beyond...

*An on going project ...*

TAC-CDFT

Deformations  
Configurations  
Energy Spectra  
EM transitions

*Self-consistent*

In the present talk

PWZ, PLB 773, 1 (2017)

Beyond the rotating mean-field

2D Collective Hamiltonian

*Quantum fluctuations*

available based on a single j model

Chen, Zhang, PWZ, Jolos, Meng,  
PRC 87, 024314 (2013)



# Outline

- Chirality and multiple chirality in nuclei
- Tilted axis cranking covariant DFT
- Multiple chirality in Rh-106
- Summary

# Tilted axis cranking CDFT

The cranking mean-field model has been very successful for rotations

## Meson exchange version:

3-D Cranking: *Madokoro, Meng, Matsuzaki, Yamaji, PRC 62, 061301 (2000)*

2-D Cranking: *Peng, Meng, Ring, Zhang, PRC 78, 024313 (2008)*

## **Point-coupling version:**

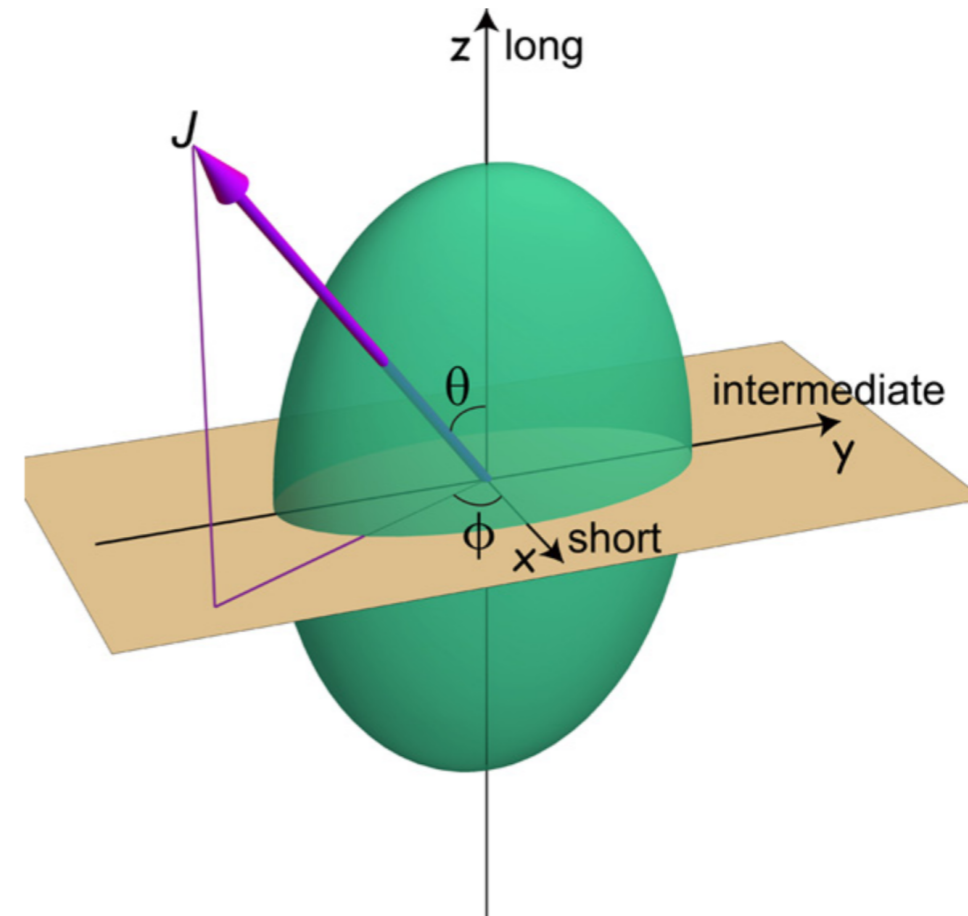
Simple and more suitable for systematic investigations

2-D Cranking: *PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)*

2-D Cranking + Pairing: *PWZ, Zhang, Meng, PRC 92, 034319 (2015)*

3-D Cranking: *PWZ, PLB 773, 1 (2017)*

3-D Cranking + Pairing: *PWZ, in preparation*



## **Self-consistent and microscopic investigations**

- full account of polarization effects
- self-consistent treatment of the nuclear currents
- no additional parameter beyond a well-determined functional

# Cranking Relativistic Kohn-Sham Equation

## Dirac Equation

$$\begin{pmatrix} m + V + S - \boldsymbol{\omega} \cdot \mathbf{J} & \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} \\ \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} & -m + V - S - \boldsymbol{\omega} \cdot \mathbf{J} \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

Consistent treatment for time-odd fields from nuclear currents

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

# Cranking Relativistic Kohn-Sham Equation

## Dirac Equation

Coriolis term

Time-odd mean fields

$$\begin{pmatrix} m + V + S - \omega \cdot J & \sigma \cdot p - \sigma \cdot V \\ \sigma \cdot p - \sigma \cdot V & -m + V - S - \omega \cdot J \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

Consistent treatment for time-odd fields from nuclear currents

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

# Cranking Relativistic Kohn-Sham Equation

## Dirac Equation

Coriolis term

Time-odd mean fields

$$\begin{pmatrix} m + V + S - \omega \cdot J & \sigma \cdot p - \sigma \cdot V \\ \sigma \cdot p - \sigma \cdot V & -m + V - S - \omega \cdot J \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

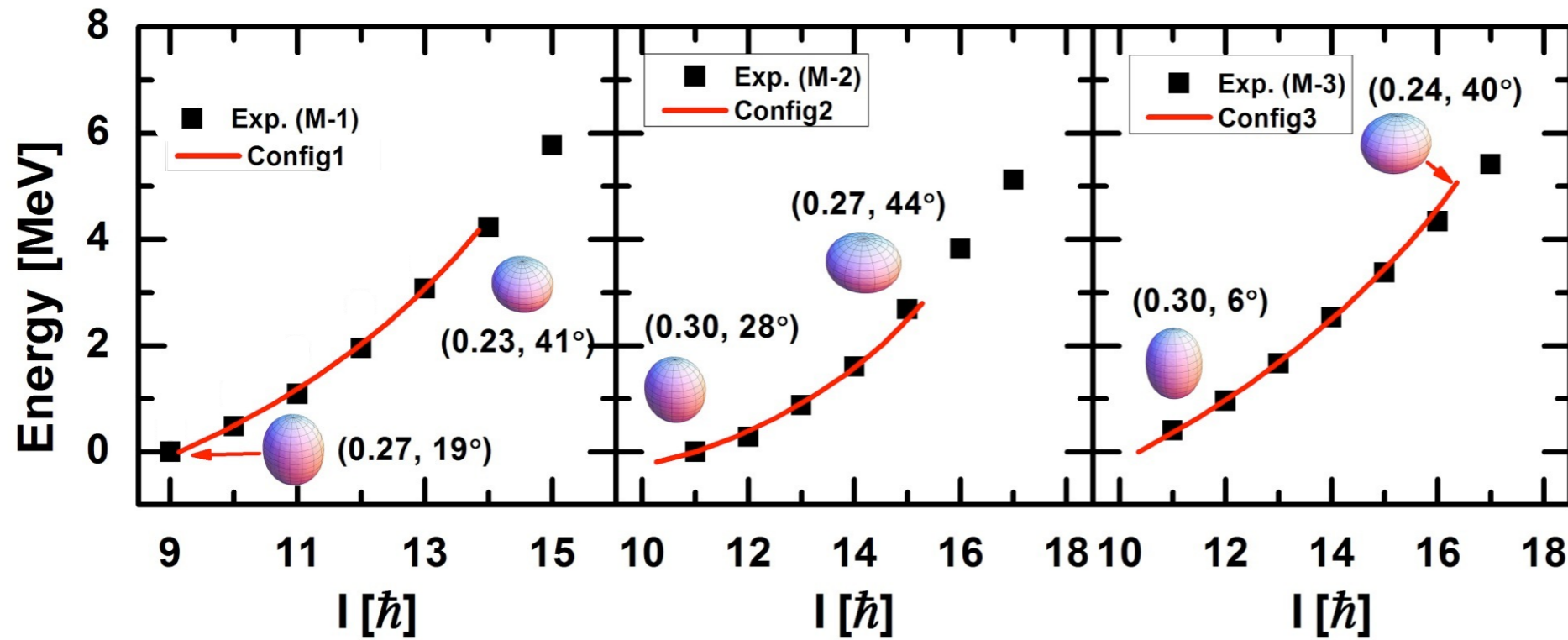
$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

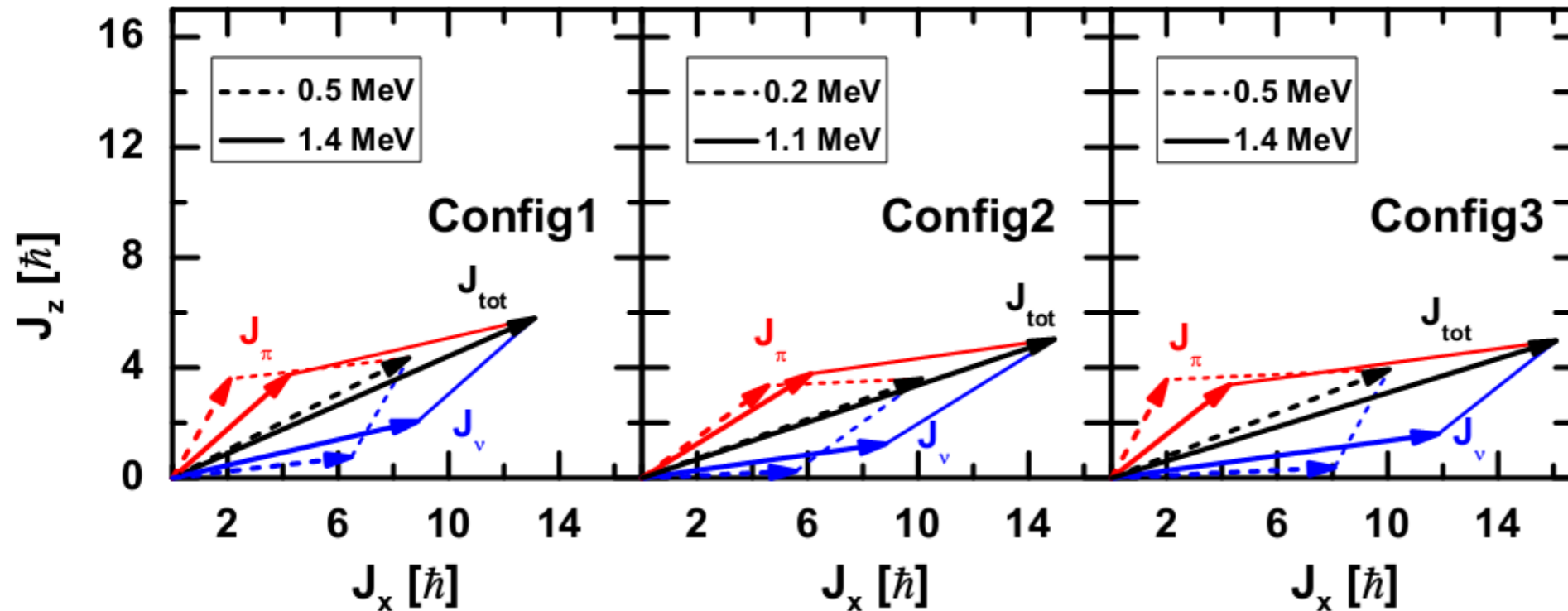
Consistent treatment for time-odd fields from nuclear currents

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

# Magnetic Rotation in $^{60}\text{Ni}$



Energy Spectra  
Shape evolution



Shears  
mechanism

# Outline

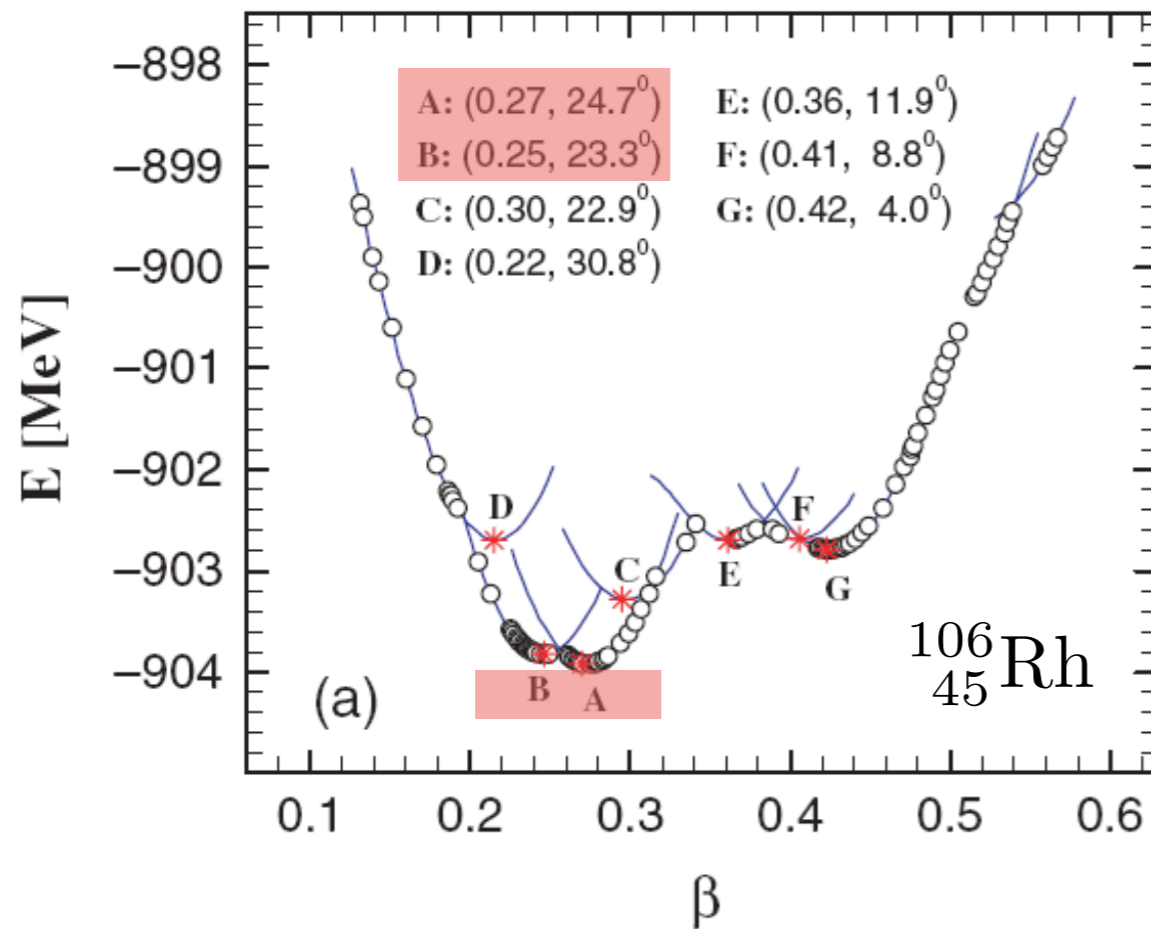
- Chirality and multiple chirality in nuclei
- Tilted axis cranking covariant DFT
- Multiple chirality in Rh-106
- Summary

# Configurations

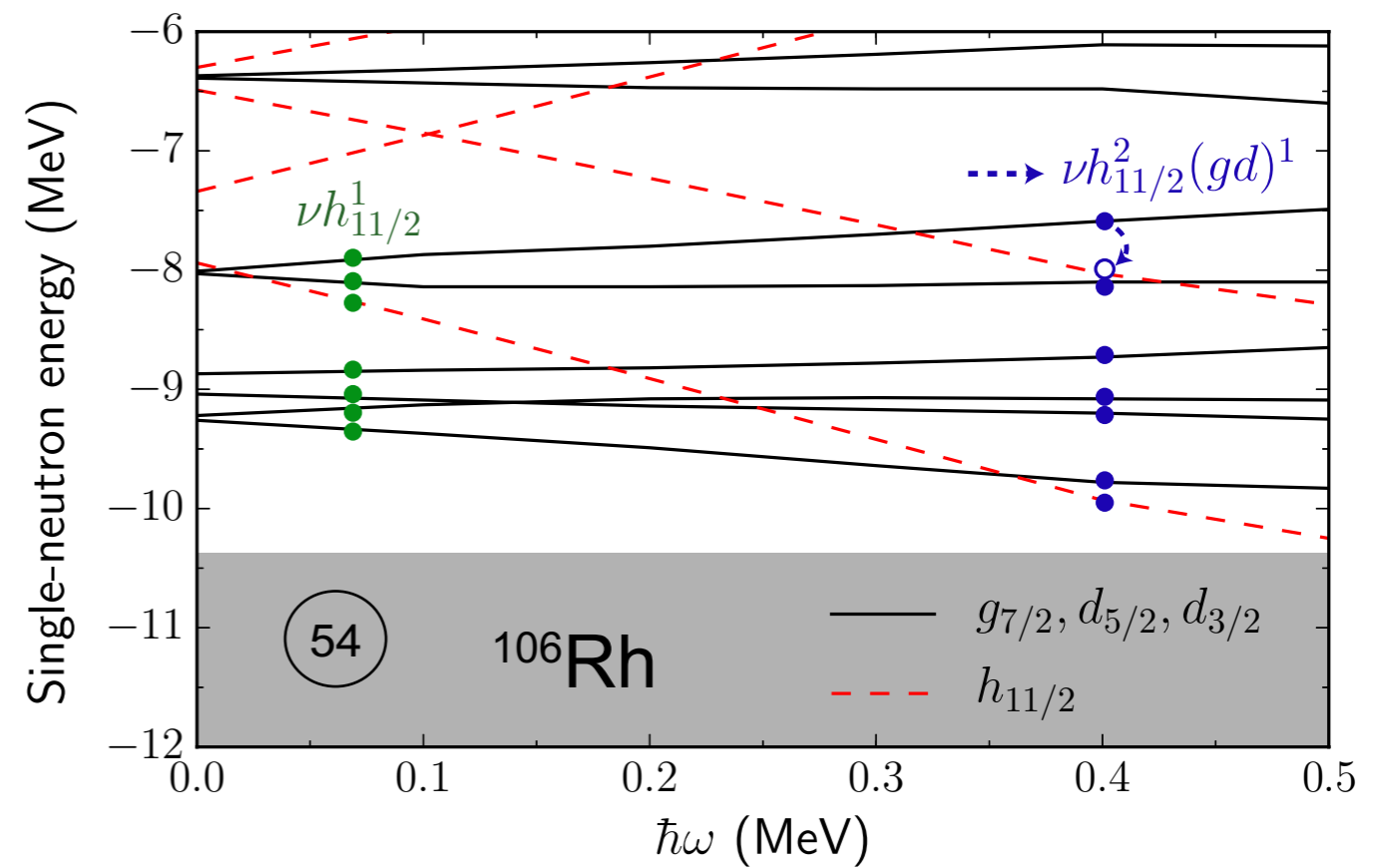
Config. A  $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^1$

Config. B  $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2 (gd)^1$

CDFT: no cranking



With cranking



Meng, Peng, Zhang, Zhou PRC 2006

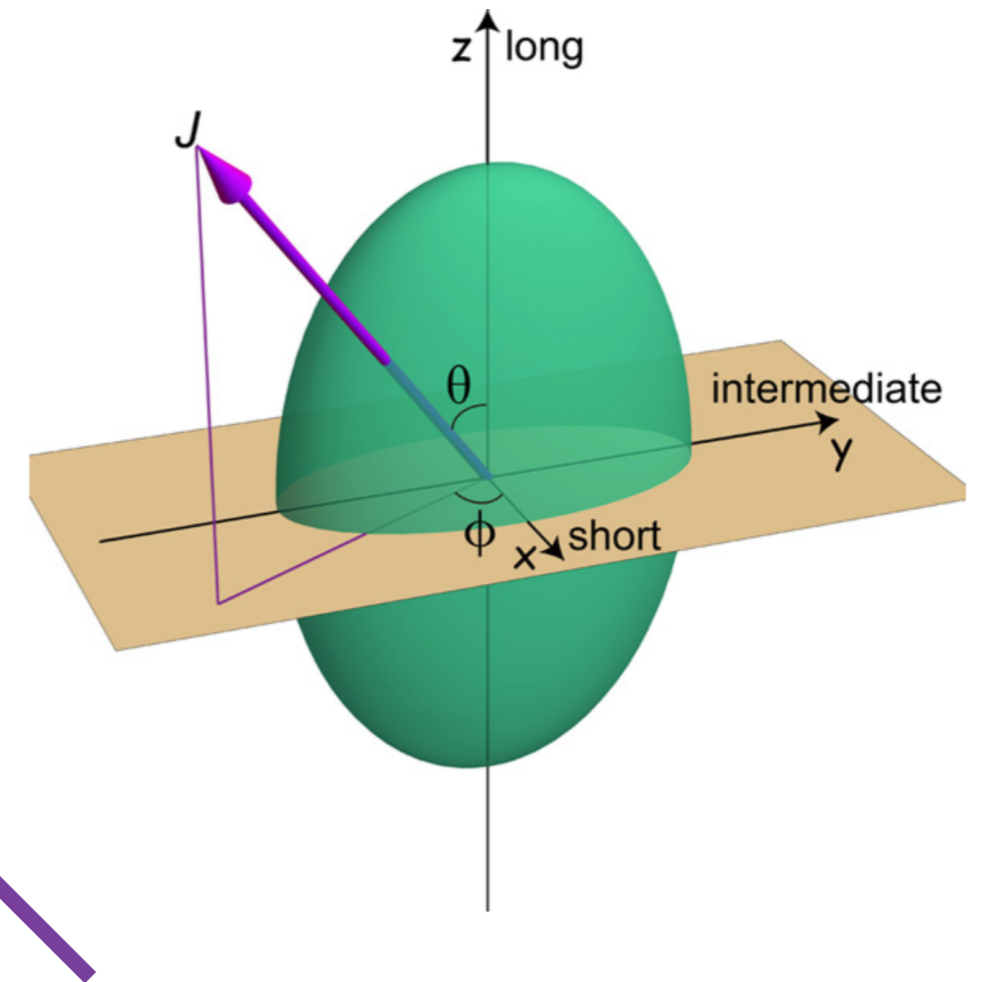
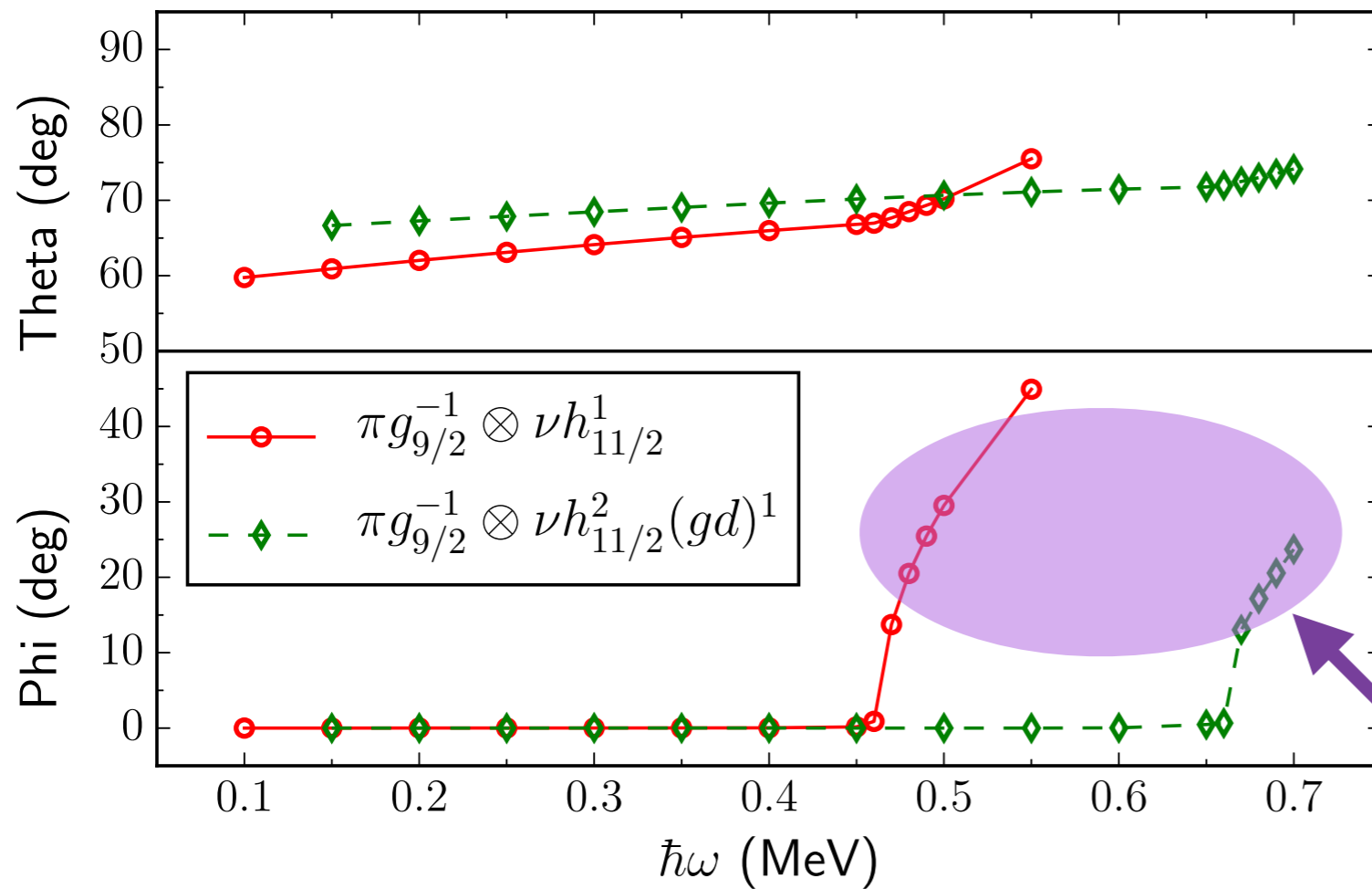
PWZ PLB 773 (2017) 1-5



# Aplanar rotation in Rh-106

PWZ PLB 773 (2017) 1-5

Rotational Angles (Theta, Phi)



Chiral geometry appears!

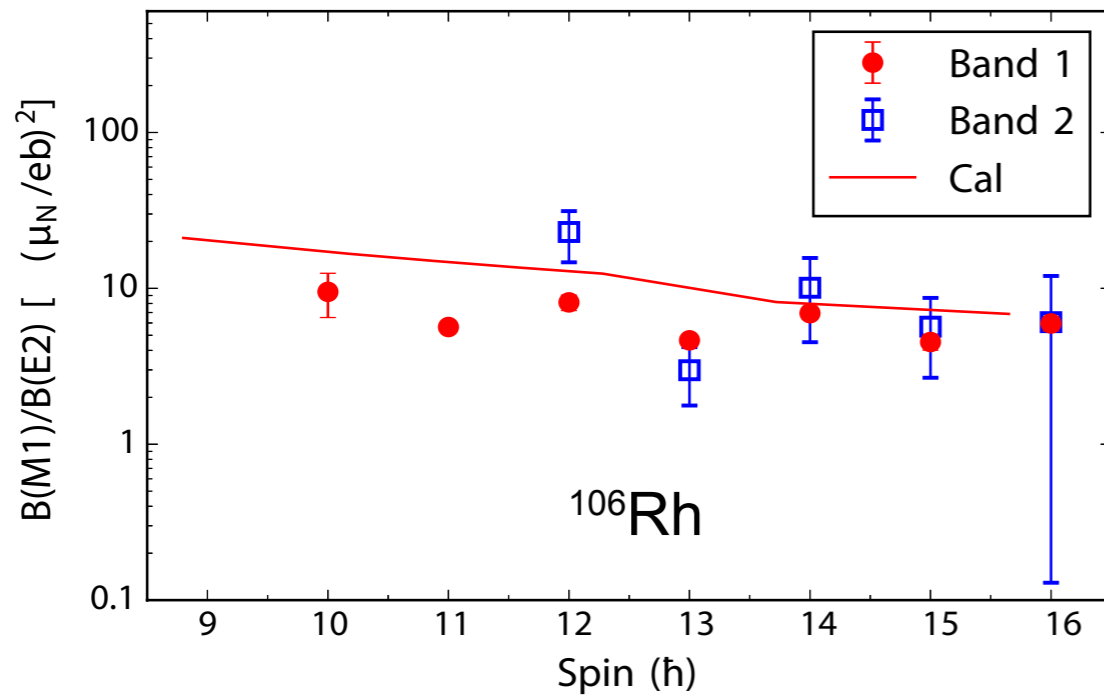
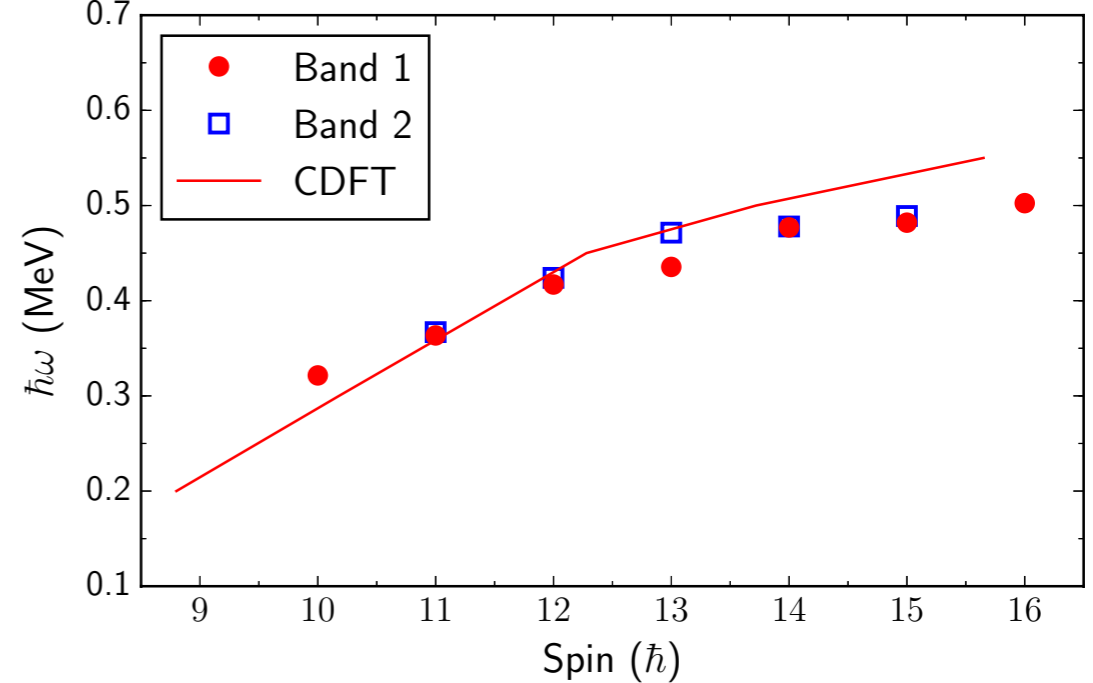
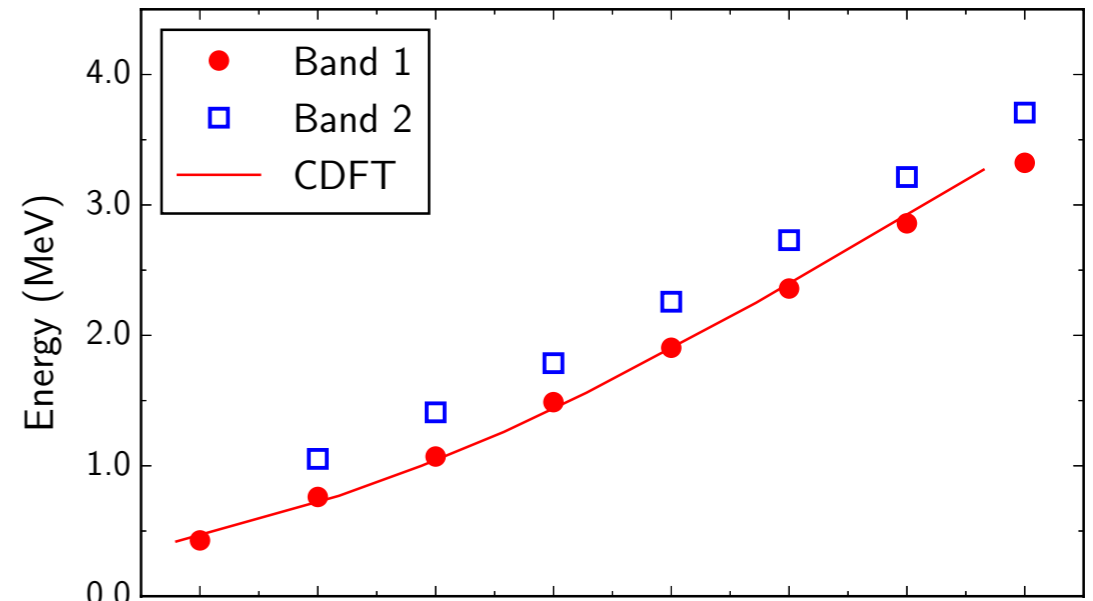
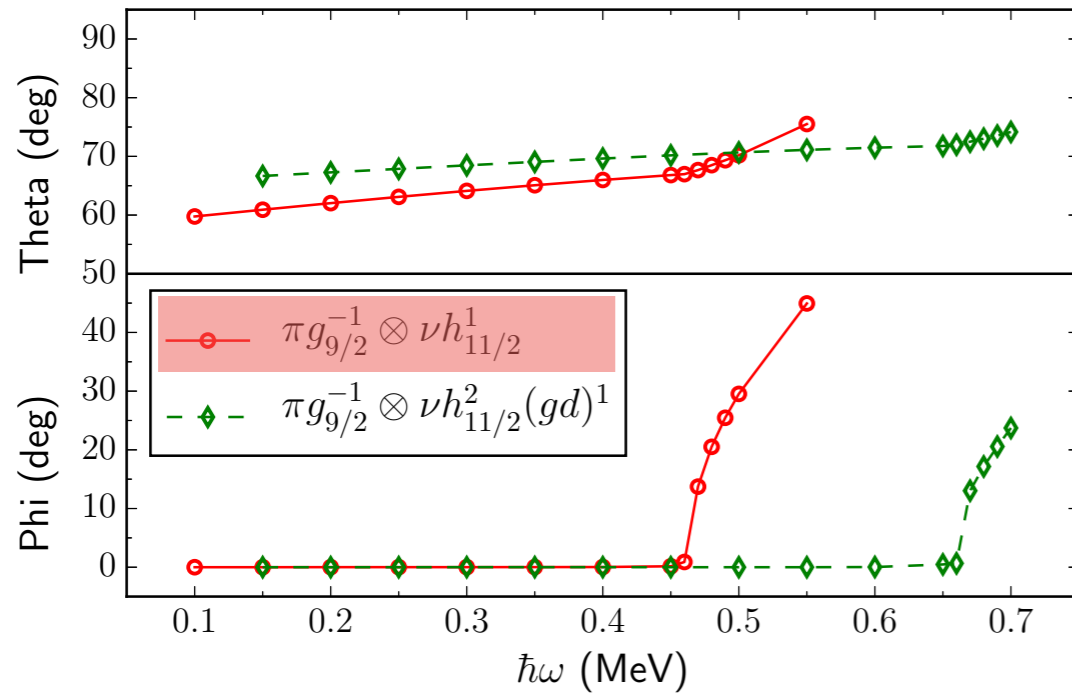
Critical frequency in nuclear chiral rotation

Olbratowski PRL 2004

# The 1st pair of chiral bands

PWZ PLB 773 (2017) 1-5

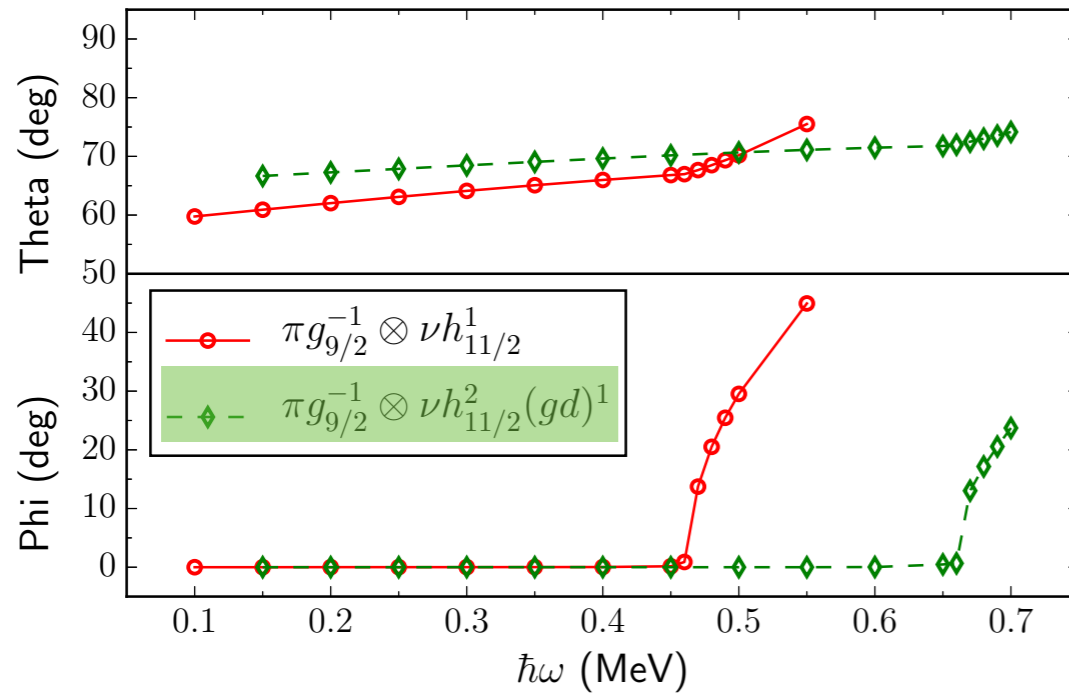
Good agreement with the data !



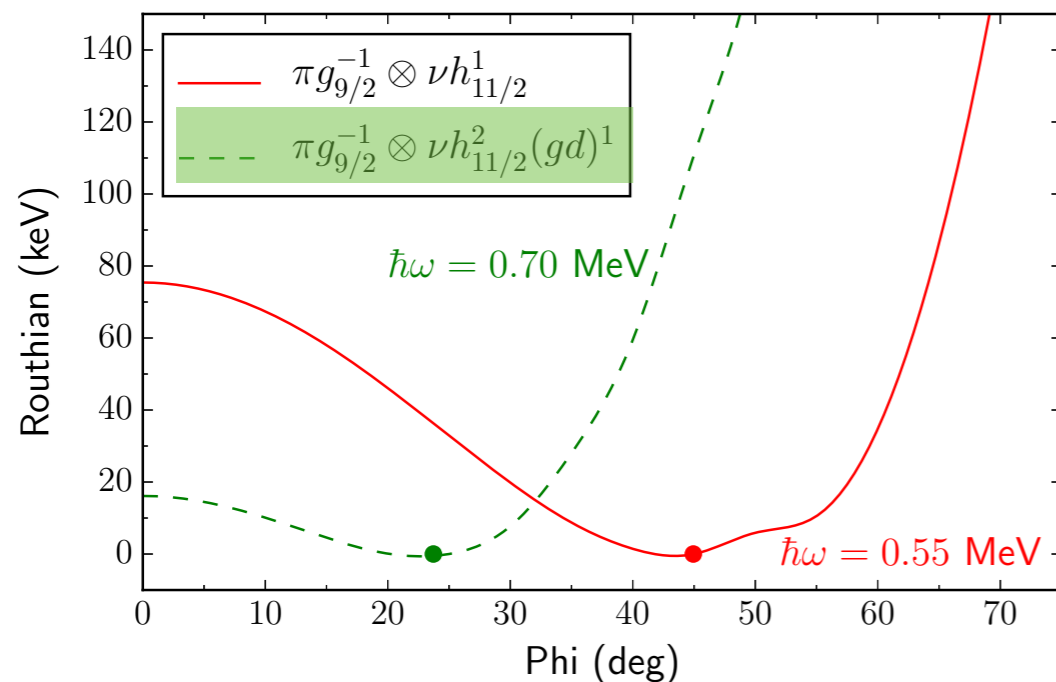
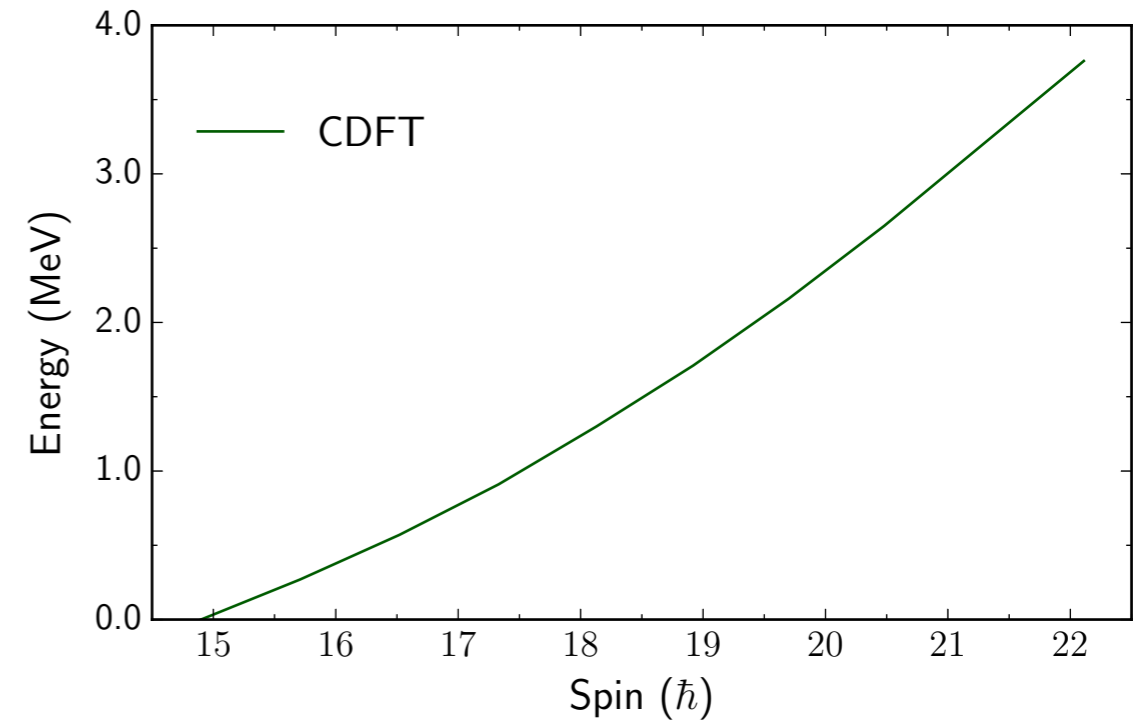
Data from Joshi et al., PLB 595 (2004) 135

# The 2nd pair of chiral bands?

PWZ PLB 773 (2017) 1-5



waiting for data...



The tunneling between the left and the right-handed sectors could be substantial.

A strong degeneracy of the chiral twin bands is not expected.

# Outline

- Chirality and multiple chirality in nuclei
- Tilted axis cranking covariant DFT
- Multiple chirality in Rh-106
- Summary

# Summary

Covariant density functional theory has been extended for describing nuclear multiple chirality.

- The first DFT description of the multiple chiral bands has been presented.
- Two distinct sets of chiral solutions have been uncovered in the nucleus  $^{106}\text{Rh}$ , and a transition between planar and chiral rotation has been found for both configurations.
- The calculated energy spectrum and  $B(M1)/B(E2)$  ratios for the negative-parity band are in good agreement with the corresponding experimental data.
- Experiments on the other predicted positive-parity chiral bands are welcome, and a strong degeneracy is not expected due to the soft Routhians.

# Collaborations

## Beijing

Qibo Chen

Jie Meng

Shuangquan Zhang

## Munich

Peter Ring

## Notre Dame

Umesh Garg

...

and many other experimentalists...

# Thank you for your attention!