> Two-particle transfer reactions: a key tool for the study of phase transitions in nuclei

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Basic point to discuss: how the nuclear behavior of the pairing degree of freedom can provide an additional and complementary clear-cut signature of the occurrence and on the nature of the phase transition in nuclear systems.

This dynamical source of information should be complementary (but as important) to the one associated to other properties (as for example, in the case of even-even nuclei, the energy of the first $2+$ state, the ratio E4/E2 and
the magnitude of the electromagnetic E2 transition connecting ground state and the first excited 2+ state)

How to single out phase transitions?


The main road to use dynamics to study pairing effects along phase transitions is clearly provided by the study of those processes where a pair of particles is involved, e.g. transferred from/to another nucleus (two-particle transfer) or ejected into the continuum (two-particle break-up or
two-particle knock-out). Clearly the probabilities for such processes must be influenced by the particle-particle correlations, but these will depend on the specific "shape phase" of the system. So they will be sensitive to any change in the status of the system, for example along an isotope chain.

The essential quantity to characterize the system from the pairing point of view is given by the "pairing response", namely all the $T_{0}$ values of the square of the matrix element of the pair creation (or removal) operator

$$
P^{+}=\sum_{j}\left[a_{j}^{+} a^{+}{ }_{j}\right]_{00} \quad \text { (and similarly for } P^{-} \text {) }
$$

connecting the ground state of nucleus N with all $0+$ states of nucleus $A+2$ (or $A-2$ ). It is often assumed that the cross section for two-particle transfer just scale with $T_{0}$.
The traditional way to define and measure the collectivity of pairing modes is to compare with single-particle pair transition densities and matrix elements to define some "pairing" single-particle units and therefore "pairing" enhancement factors.

Obs: We discuss here monopole $T=1$ pairing modes, i.e. $0+$ states, but similar arguments would apply to $\mathrm{T}=0$ neutron-proton pairs.

Typical "pairing" response

208 Pb
Addition modes



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Sofia, Dasso, Vitturi

The pairing response is characterized by the pairing phase (normal or superfluid) and by the shape phase (e.g. spherical or deformed). Therefore it will be a clear signature of phase transitions (in addition to the standard signatures, as $E_{4} / E_{2}, B(E 2)$, etc) in both the
shape degree of freedom
pairing degree of freedom

| Shape Transitions | Pairing Transitions |
| :---: | :---: |
| $\mathcal{R}(\theta)=\exp (-\mathrm{iI} \theta)$ | $G(\phi)=\exp (-i \mathcal{N} \phi)$ |
| Angular Momentum, I | Particle Number, $\mathcal{N}$ |
| $Z \mathcal{R}(\theta)$ | $Z \sim G(\phi)$ |
| $\beta, \gamma$, Euler angles $\theta$ | Pair deformation, $\alpha$ Gauge angle, $\phi$ |
| Violation of spherical symmetry | Violation of particle number |
| Physical space | Abstract "gauge" space |

Phase transition from "normal" to "superfluid" phases: characteristic behavior of the pair transfer matrix element


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Pair-transfer probability in open- and closed-shell Sn isotopes
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An example of a "superfluid" nucleus (pairing rotations), which shows a characteristic pairing response


Practically all pairing strength goes to the ground state

In a similar way pair-transfer probabilities show characteristic behaviors in correspondence of shape phase transitions

For simplicity we move within the framework of the Interacting Boson Model, but the results are similar within other microscopic models


The IBM does not explicitly use the fermion degrees of freedom. From mapping procedure the "form" of the two-particle addition operator is simply assumed as $s^{+}$, neglecting higher-order terms, as $s^{+} s^{+} s$ or $\left[d^{+} d^{+}\right]_{0} s$ or $\left[d^{+} s^{+} d\right]_{0}$ etc ......

OBS: See OAI mapping

Example: $L=0$ pair transfer in a phase transition from spherical to axial deformation (from $U(5)$ to $S U(3)$ in algebraic language)


There is a clear signal at the phase transition

Obs: fragmentation of the pairing strength in correspondence to phase transitions along an isotope chain (in this case chosen to take place at $\mathrm{N}=8$ )
Increasing number of valence pairs

fragmentation of the pairing strength


Another scenario of phase transition: shape co-existence, for example of a spherical and a deformed state within the same nucleus

$$
\begin{aligned}
& \left.\left|0^{+}{ }_{g}, N\right\rangle=\alpha\left|N>_{U(5)}+\beta\right| N+2\right\rangle_{S U(3)} \\
& \left|0^{+}{ }_{\text {exc }, ~}, N\right\rangle=-\beta|N\rangle(5)+\alpha|N+2\rangle_{s U(3)}
\end{aligned}
$$

Mixing of two configurations, with mixture changing along the isotope chain

N particle pairs
0 hole pairs
$U(5)$ hamiltonian (spherical)
 $\mathrm{N}+1$ particle
pairs, 1 hole pair (2p-2h exc): total $N+2$ pairs
$|N\rangle_{U(5)}$
$|N+2\rangle_{S U(3)} \quad \begin{gathered}\text { sU(3) } \\ \text { hailt }\end{gathered}$
hamiltonian (deformed)

A simple model: along the isotope chain a sharp inversion of the structure



Transfer operator in now more complex: $S^{+}+S$ (one can create a particle pair or destroy a hole pair)


There is a clear signal at the phase transition
deformed
spherical


As in the previous situation a clear discontinuity appears at the critical point. However, at variance with the previous case, the pair strength is always practically concentrated in a single state, without the fragmentation illustrated in the previous case

Another case: shape-coexistence with a smoother transition
Mixing


OBS: Cf. EO transitions between the two $0+$ states

Transfer operator for pair removal : $S+\mathbf{S}^{+}$(one can destroy a particle pair or create a hole pair)


So far we have considered matrix elements of the pair operator: but what about pair transfer cross sections?

Unfortunately, at variance, for example, from low-energy one-step Coulomb excitation, where the excitation probability is directly proportional to the $B(E \lambda)$ values, the reaction mechanism associated with pair transfer is rather complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious. The situation is actually more complicated even with respect to other processes (as inelastic nuclear excitation) that may need to be treated microscopically, but where the reaction mechanism is somehow well established.

We expect an correlation between cross sections and square of the pair operator. But if the qualitative behavior may be clear, the quantitative aspects require a proper treatment of the reaction mechanism. All approaches, ranging from macroscopic to semi-microscopic and to fully microscopic, try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles), to more tractable frameworks.

Two models are most popular:
A, Successive single-particle transfer
B. Cluster transfer

## A

Sequential two-step process: each step transfers one particle
Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in ( $a-1$ ) and ( $A$ $+1)$ nuclei, due to the correlations in initial and final wave functions

Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions
Expansion to second-order in the transfer potential


## Effect of kinematical conditions

The transfer probabilities vary strongly with the involved orbital. In addition whether the final wave function only involves a "pure" orbital, or whether it is correlated


OBS: The shape of the angular distribution is the same, being associated with the L=0 transfer

But the dependence on the microscopy also arises from the reaction mechanism associated with each specific two-particle transfer process

If we consider the same case as before, i.e. the transfer of two neutrons from ${ }^{110} \mathrm{Sn}$ to ${ }^{112} \mathrm{Sn}(0+;$ gs), but using different reactions, e.g.
$(14 C, 12 C)$ or $(180,160)$ the ranking of the cross sections associated to the different orbitals changes.

## projectile

|  | ${ }^{112} \mathrm{Sn}$ | (t,p) | $\left({ }^{14} \mathrm{C},{ }^{12} \mathrm{C}\right)$ |  |  |  | $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\left(0 p_{1 / 2}\right)^{2}$ | $\left(1 s_{1 / 2}\right)^{2}$ | $\left(0 p_{3 / 2}\right)^{2}$ | $\left(0 d_{5 / 2}\right)^{2}$ | $\left(0 d_{5 / 2}\right)^{2}$ | $\left(1 s_{1 / 2}\right)^{2}$ | Conf |
|  | $\left(0 g_{7 / 2}\right)^{2}$ | 2.80E-5 | 1.73E-5 | 1.19E-4 | $7.09 \mathrm{E}-4$ | $9.00 \mathrm{E}-4$ | $1.19 \mathrm{E}-3$ | $2.01 \mathrm{E}-4$ | $1.24 \mathrm{E}-3$ |
|  | $\left(1 d_{5 / 2}\right)^{2}$ | 1.13E-3 | $3.00 \mathrm{E}-2$ | $4.71 \mathrm{E}-3$ | $5.54 \mathrm{E}-3$ | $1.18 \mathrm{E}-3$ | $3.55 \mathrm{E}-3$ | 1.13E-2 | $1.19 \mathrm{E}-2$ |
|  | $\left(2 s_{1 / 2}\right)^{2}$ | $1.08 \mathrm{E}-3$ | $1.53 \mathrm{E}-2$ | $5.38 \mathrm{E}-3$ | $7.05 \mathrm{E}-3$ | $1.16 \mathrm{E}-3$ | $5.02 \mathrm{E}-3$ | $1.42 \mathrm{E}-2$ | $1.59 \mathrm{E}-2$ |
|  | $\left(1 d_{3 / 2}\right)^{2}$ | $4.73 \mathrm{E}-4$ | $1.34 \mathrm{E}-3$ | $2.79 \mathrm{E}-3$ | $9.87 \mathrm{E}-3$ | $4.14 \mathrm{E}-3$ | $1.26 \mathrm{E}-2$ | $6.62 \mathrm{E}-3$ | $1.83 \mathrm{E}-2$ |
|  | $\left(0 h_{11 / 2}\right)^{2}$ | $7.50 \mathrm{E}-5$ | $7.77 \mathrm{E}-4$ | $5.29 \mathrm{E}-5$ | $1.05 \mathrm{E}-4$ | $7.65 \mathrm{E}-5$ | 1.10E-4 | $9.06 \mathrm{E}-5$ | $1.88 \mathrm{E}-4$ |
|  | $\mathrm{Conf}_{A}$ | $2.54 \mathrm{E}-3$ | $8.77 \mathrm{E}-2$ | $2.26 \mathrm{E}-2$ | $3.77 \mathrm{E}-2$ | $1.21 \mathrm{E}-2$ | $8.60 \mathrm{E}-3$ | $1.95 \mathrm{E}-2$ | $7.53 \mathrm{E}-2$ |

Cluster-transfer model (suggested by the close radial correlation of the pairs)


Initial and final cluster wave functions are obtained by taking the overlap between the two-particle wave functions and a Os wave function for the relative motion

Also in this case the resulting cross section depends on the specific single-particle orbitals (via the Talmi-Moshinsky brackets), but the dependence is different from the one associated with the sequential transfer (!!!)

The preference to either model may depend on the colliding systems and on kinematical conditions.

The proper approach will depend on the competition between the two colliding single-particle mean fields and the residual two-body interaction (for relatively weak interaction the mean fields will prevail, while in the other extreme of infinite pairing correlation the cluster structure will take over).

One case in more details (with full microscopic wave functions):
Shape phase transition in Zr isotopes

Other examples: Mg isotopes, Ni isotopes

First example: Shape phase transition in Zr isotopes between $\mathrm{N}=58$ and 60


Quantum Phase Transition in the Shape of $\mathbf{Z r}$ isotopes
Tomoaki Togashi ${ }^{1}$, Yusuke Tsunoda ${ }^{1}$, Takaharu Otsuka ${ }^{1,2,3,4}$ and Noritaka Shimizu ${ }^{1}$
relevant 2-particle spectroscopic amplitudes

|  | 90>92gs | 92>94gs | 94>96gs | 96>98gs | 98>100gs | $98>100\left(0+{ }_{4}\right)$ | 100>102gs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| d5/2 | 0.74 | 0.86 | 0.86 | 0.13 | 0.0 | 0.16 | 0.08 |
| S1/2 | 0.10 | 0.08 | 0.10 | 0.90 | 0.0 | 0.16 | 0.05 |
| d3/2 | 0.13 | 0.18 | 0.16 | 0.07 | 0.0 | 0.90 | 0.04 |
| h11/2 | 0.22 | 0.20 | 0.19 | 0.08 | 0.0 | 0.14 | 0.55 |
|  |  |  |  |  |  |  |  |

Cross sections for pure configurations


Calculation of two-particle transfer reactions using: sequential model for the reaction mechanism one- and two-particle spectroscopic amplitudes from the Tokyo group


## Conclusions:

Pairing response (tested in two-particle transfer reactions but also in other dynamical processes involving pairs of particles) gives strong constrains on nuclear wave functions. The effect is amplified in correspondence of critical situations associated with shape phase transitions, with "abnormal" population of excited $0+$ states and weakening of the ground state transition.

Further data on two-particle transfer reactions are definitely needed

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