9th Quantum phase transitions in nuclei and many-body systems, Padova

Shape evolutions in Gd isotopes studied with 5DCH based on CDFT

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Zhi Shi, Qibo Chen, Shuangquan Zhang, EPJA 54, 53 (2018)

Outline

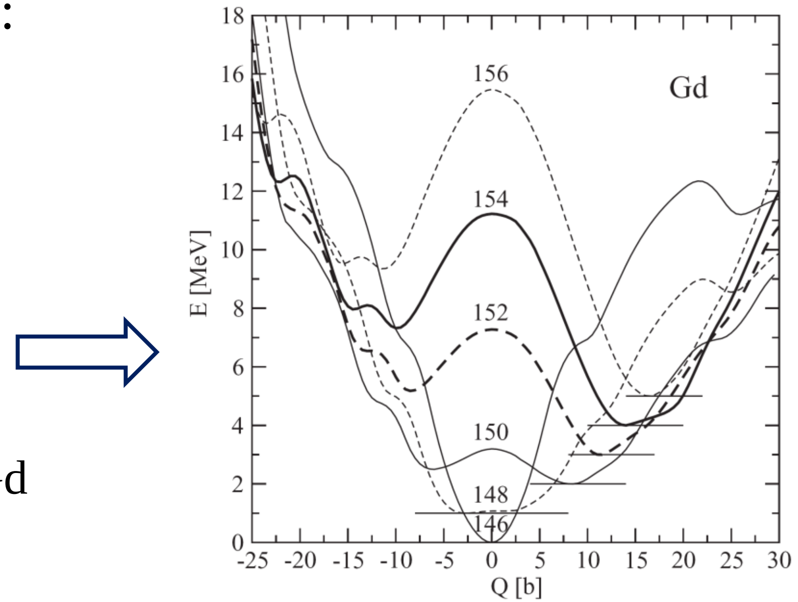
- Introduction
- Theoretical framework
- Numerical details
- Results and discussion
- Summary

Introduction

- The rare-earth region, which possesses many transitional and well-deformed nuclei, is an ideal venue to study the origination of deformation and collective motion. One interesting and important example is the gadolinium isotopes.

- A well-known phenomenon in Gd isotopes is the presence of quantum phase transition, which has been investigated by the potential energy surfaces (PESs) within the framework of mean-field theory:

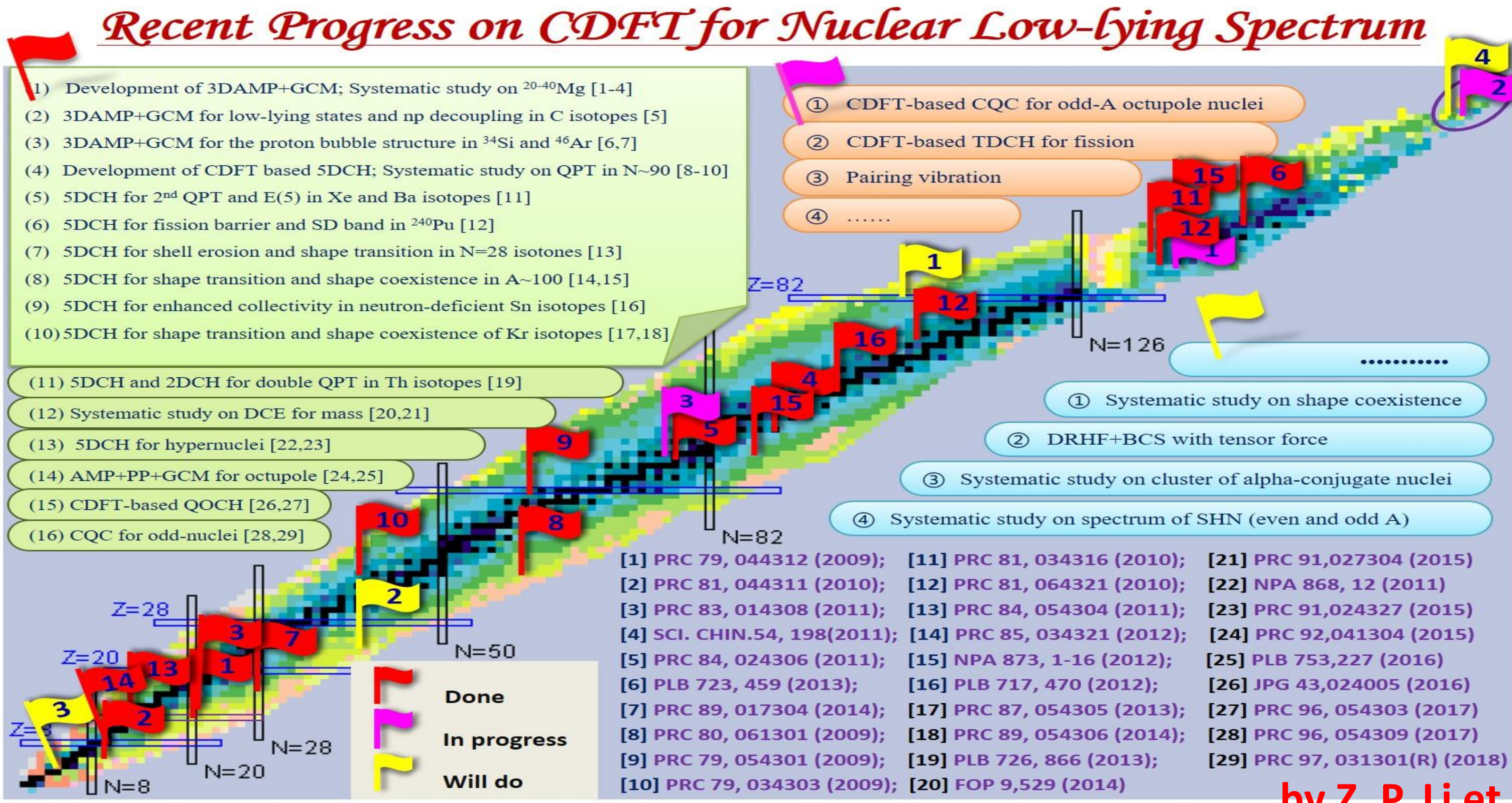
  - Relativistic mean-field: $^{146\text{-}160}$Gd
    Sheng_Guo, MPLA (2005) 20: 2711
  - Relativistic Hartree-Bogoliubov: $^{148\text{-}156}$Gd
    Fossion_Bonatsos_Lalazissis, PRC (2006) 73: 044310
  - Skyrme-Hartree-Fock (1D PES): $^{146\text{-}156}$Gd
    Rodriguez-Guzman_Sarriguren, PRC (2007) 76: 064303
  - Hartree-Fock-Bogoliubov (2D PES): $^{152\text{-}156}$Gd
    Robledo_Rodriguez-Guzman_Sarriguren, PRC (2008) 78: 034314



- The five-dimensional collective Hamiltonian based on the covariant density functional theory (5DCH-CDFT) : $^{152\text{-}160}$Gd, PC-F1 interaction
  Niksic_Li_Vretenar_Prochniak_Meng_Ring, PRC (2009) 79: 034303

Introduction

- In recent years, 5DCH-CDFT has been extensively applied to describe the low-lying spectra:



by Z. P. Li et al

- In this work, we will investigate shape evolution in the $^{148-162}$Gd isotopes with the framework of 5DCH-CDFT (with PC-PK1).

Theoretical framework

- The collective Hamiltonian can be written as follows

$$\hat{H}_{\text{coll}}(\beta,\gamma)=\hat{T}_{\text{vib}}(\beta,\gamma)+\hat{T}_{\text{rot}}(\beta,\gamma,\Omega)+V_{\text{coll}}(\beta,\gamma)$$

  - ✓ The vibrational kinetic energy:

$$\hat{T}_{\text{vib}}=-\frac{\hbar^2}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^4 B_{\gamma\gamma}-\frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^3 B_{\beta\gamma}\frac{\partial}{\partial\gamma}\right]\right.$$
$$\left.+\frac{1}{\beta\sin 3\gamma}\left[-\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\frac{\partial}{\partial\beta}+\frac{1}{\beta}\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\frac{\partial}{\partial\gamma}\right]\right\},$$

  - ✓ The rotational kinetic energy:

$$\hat{T}_{\text{rot}}=\frac{1}{2}\sum_{k=1}^{3}\frac{\hat{J}_k^2}{\mathcal{I}_k},\quad k=1,\ 2,\ 3,$$

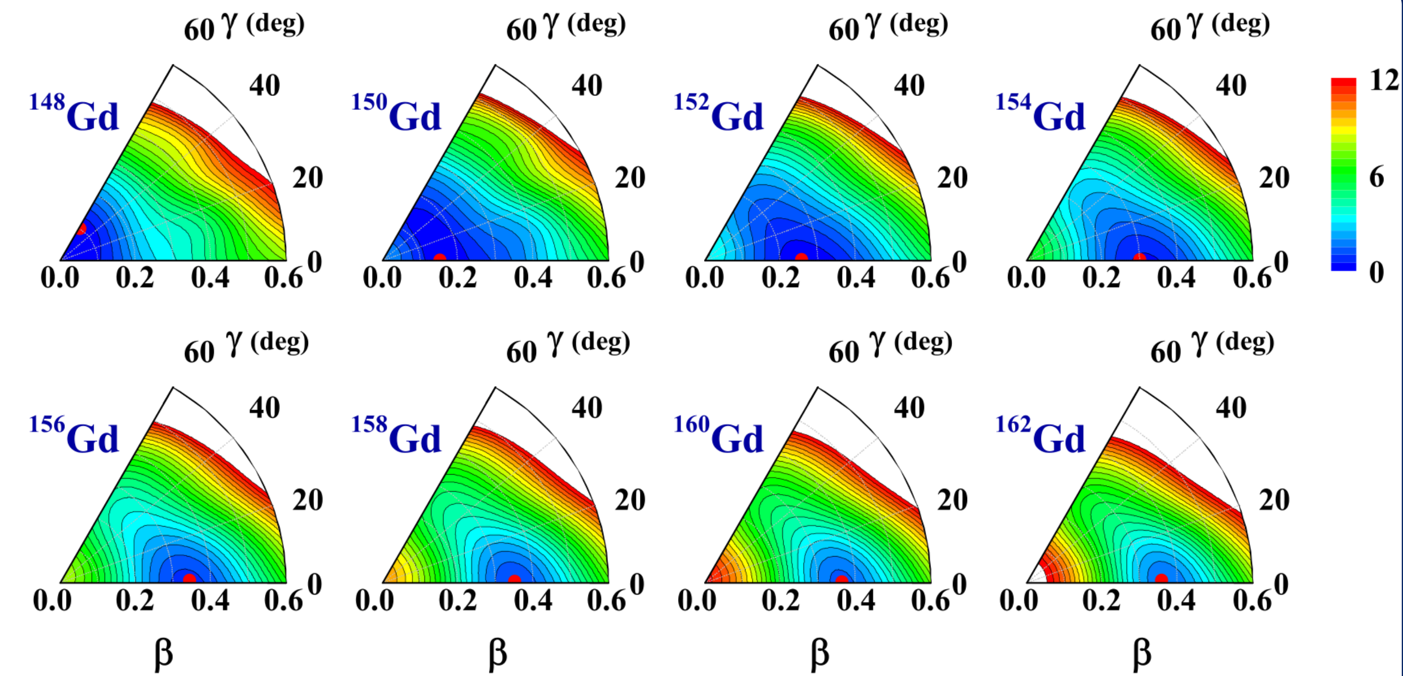
  - ✓ V_{coll} is the collective potential.

$B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}, \mathcal{I}_k,$ and V_{coll} are the collective parameters determined by CDFT.

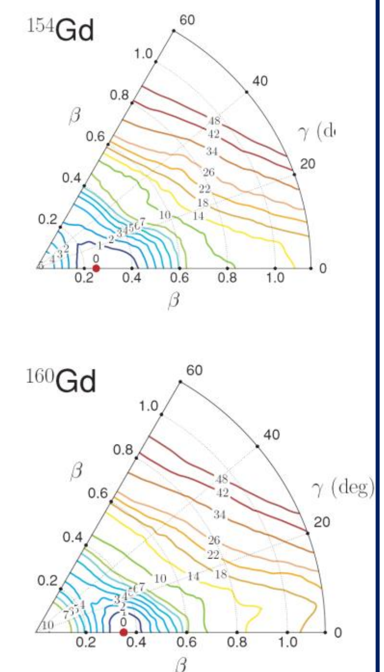
Numerical detail

- Nuclei: $^{148\text{-}162}$Gd;
- Number of harmonic oscillator shells: 14;
- Effective interaction: PC-PK1; Zhao_Li_Yao_Meng, PRC (2010) 82: 054319
- Pairing force: density-independent δ-force;
- Deformation range: $0 \leq \beta \leq 0.6$, $0° \leq \gamma \leq 60°$;
- Grid point: $\Delta\beta = 0.05$, $\Delta\gamma = 6°$

Potential energy surface



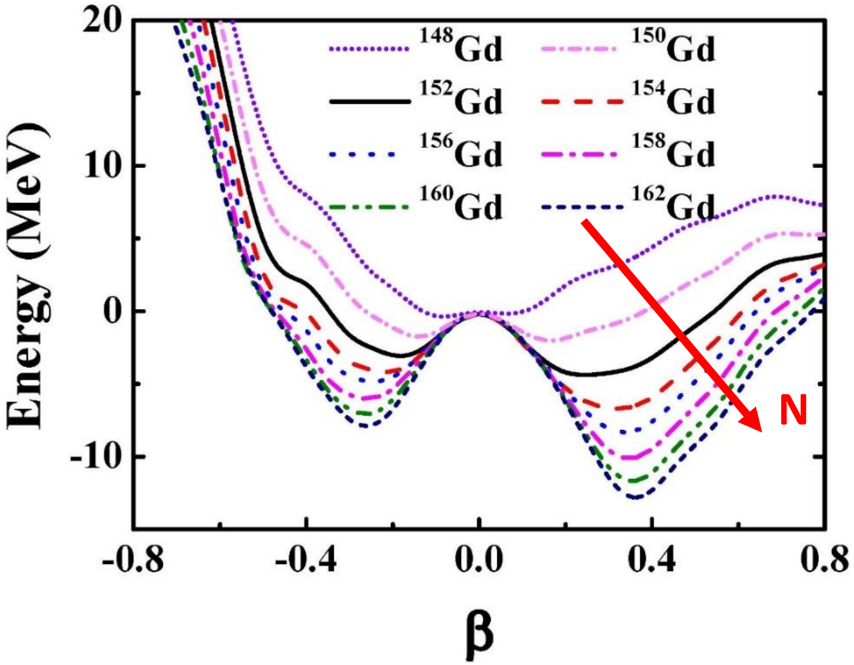
PC-PK1

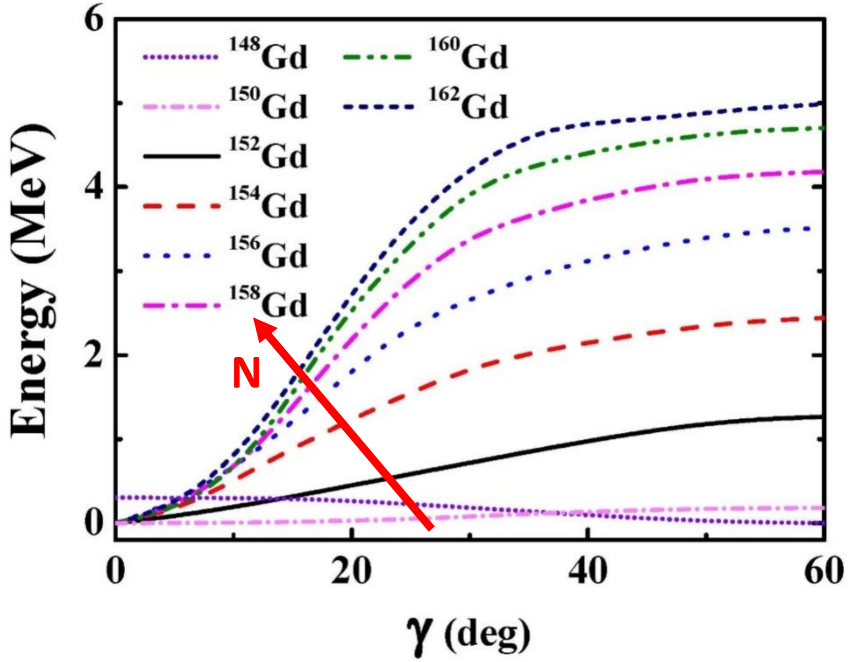


PC-F1

- A shape evolution from weakly deformed ^{148,150}Gd to soft prolate ^{152,154}Gd and to the well deformed prolate $^{156-162}$Gd for the even-even $^{148-162}$Gd isotopes is presented.
- Similar properties are seen in the PC-F1 calculations. Niksic2009PRC

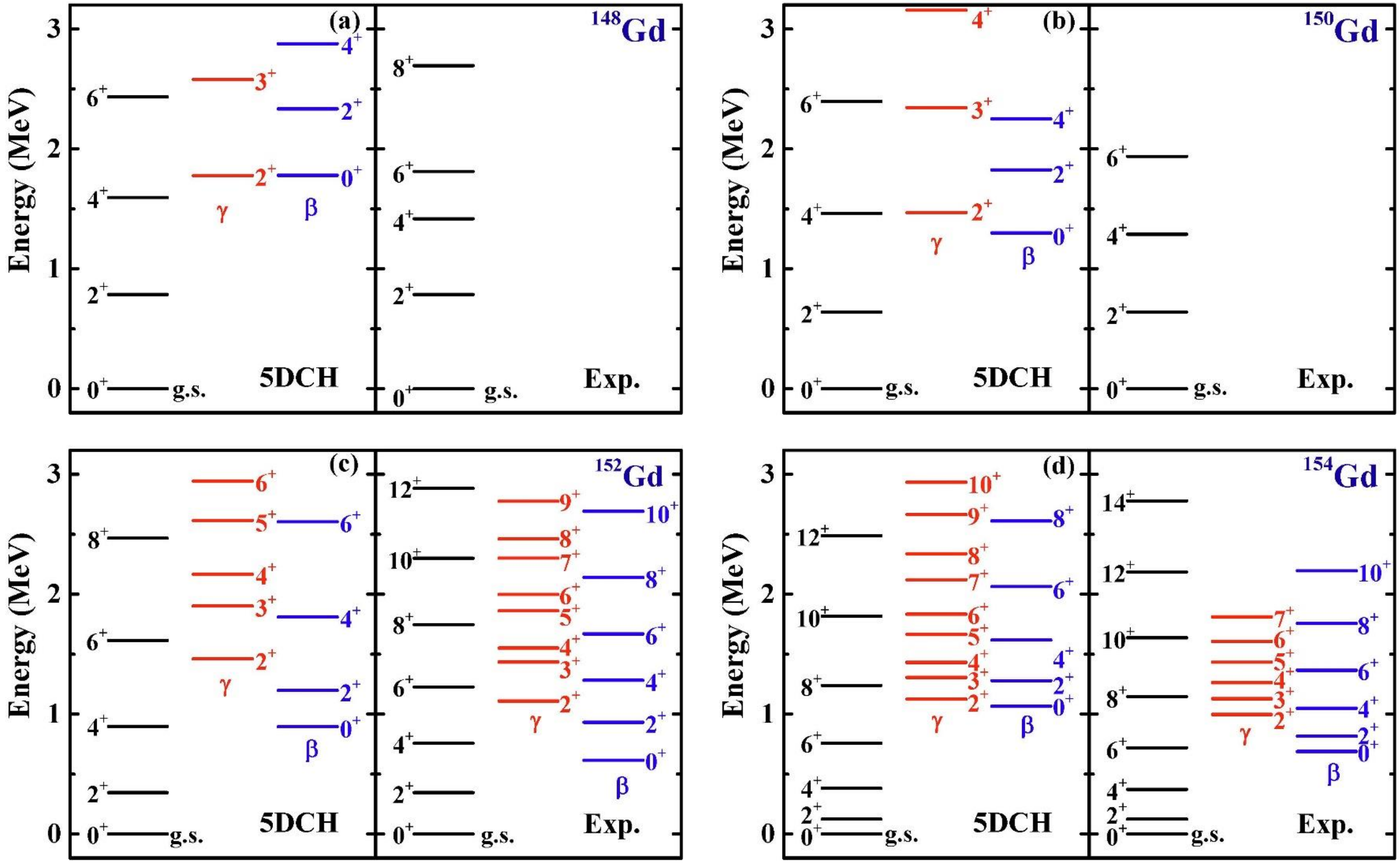
Potential energy curve





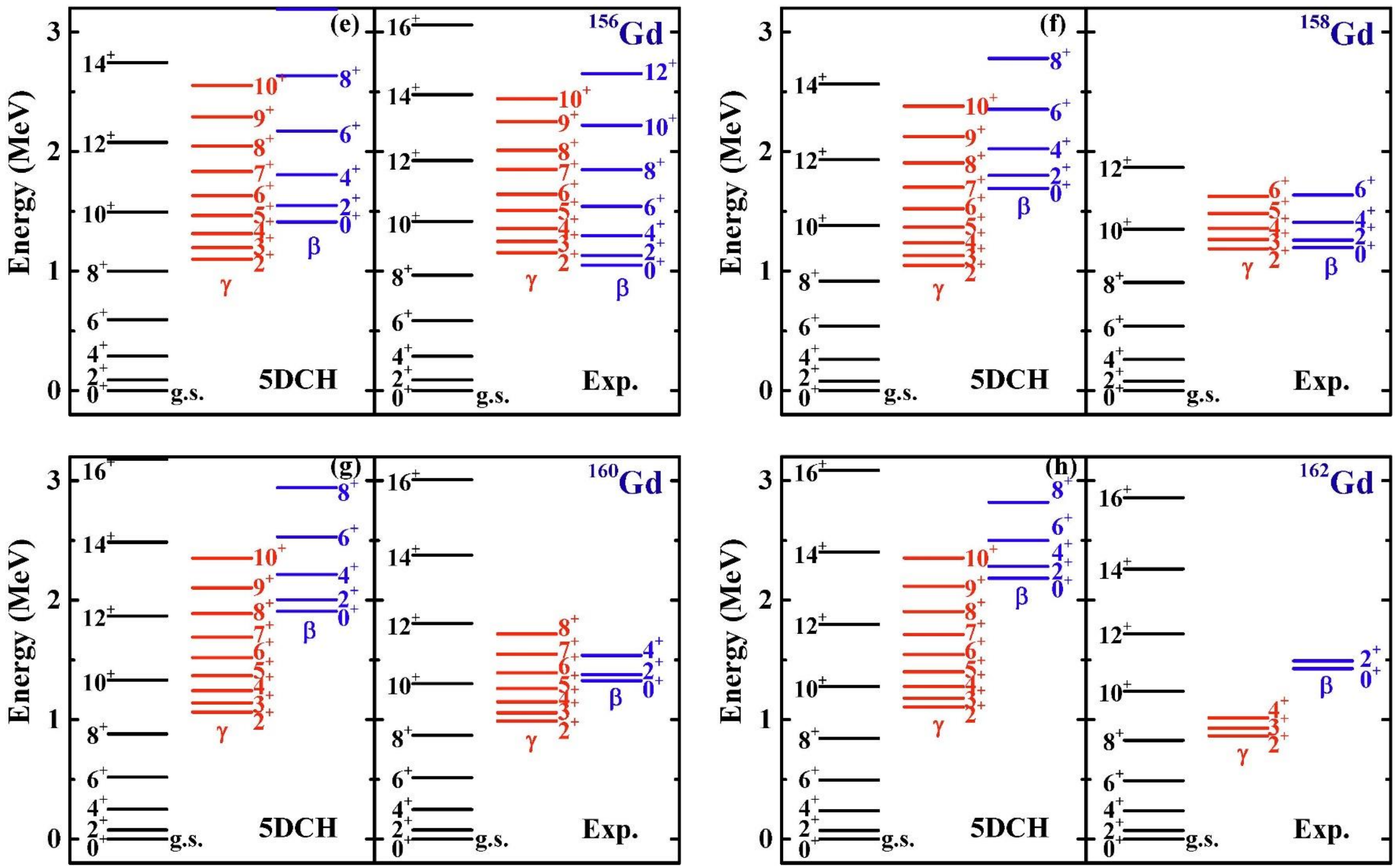
- From the potential energy curve, it is easy to find that the ground state energy got deeper and deeper with the increase of the mass number.
- For the right panel, the energy shows as functions of γ. The energy curve in γ direction become more and more steep.

Energy spectra



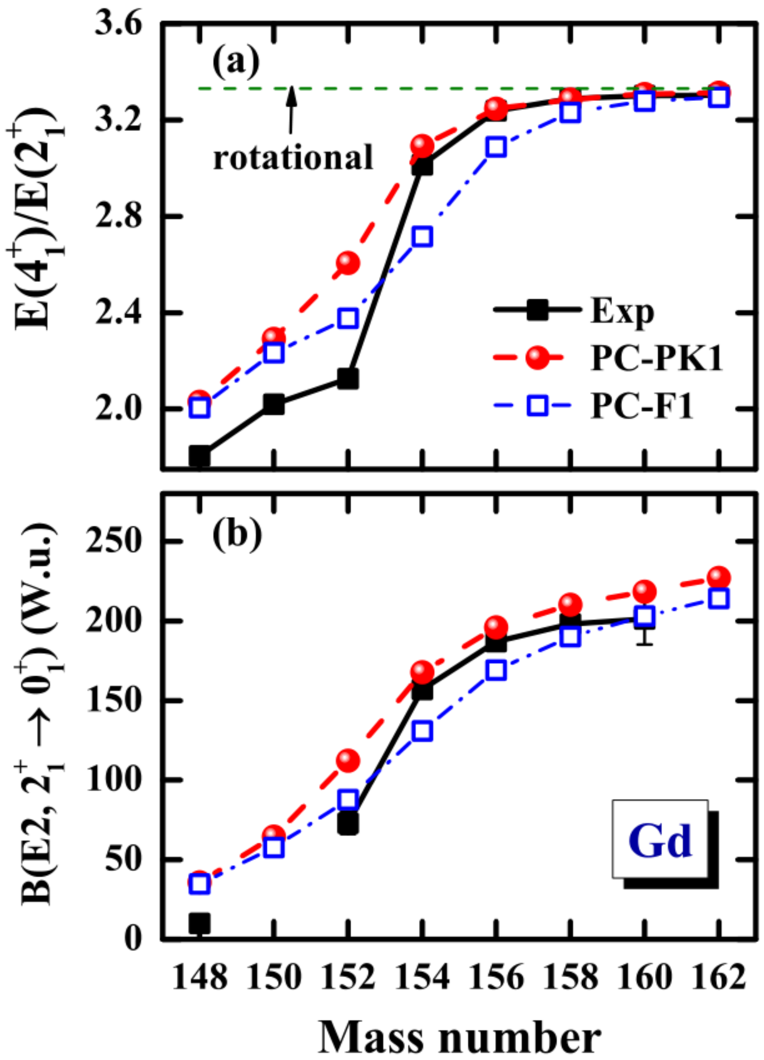
- The energy spectra of the ground-state, γ and β bands in $^{148\text{-}162}$Gd are reproduced by the 5DCH-CDFT calculations.

Energy spectra



- The energy spectra of the ground-state, γ and β bands in $^{148\text{-}162}$Gd are reproduced by the 5DCH-CDFT calculations.

R_{42} and B(E2)

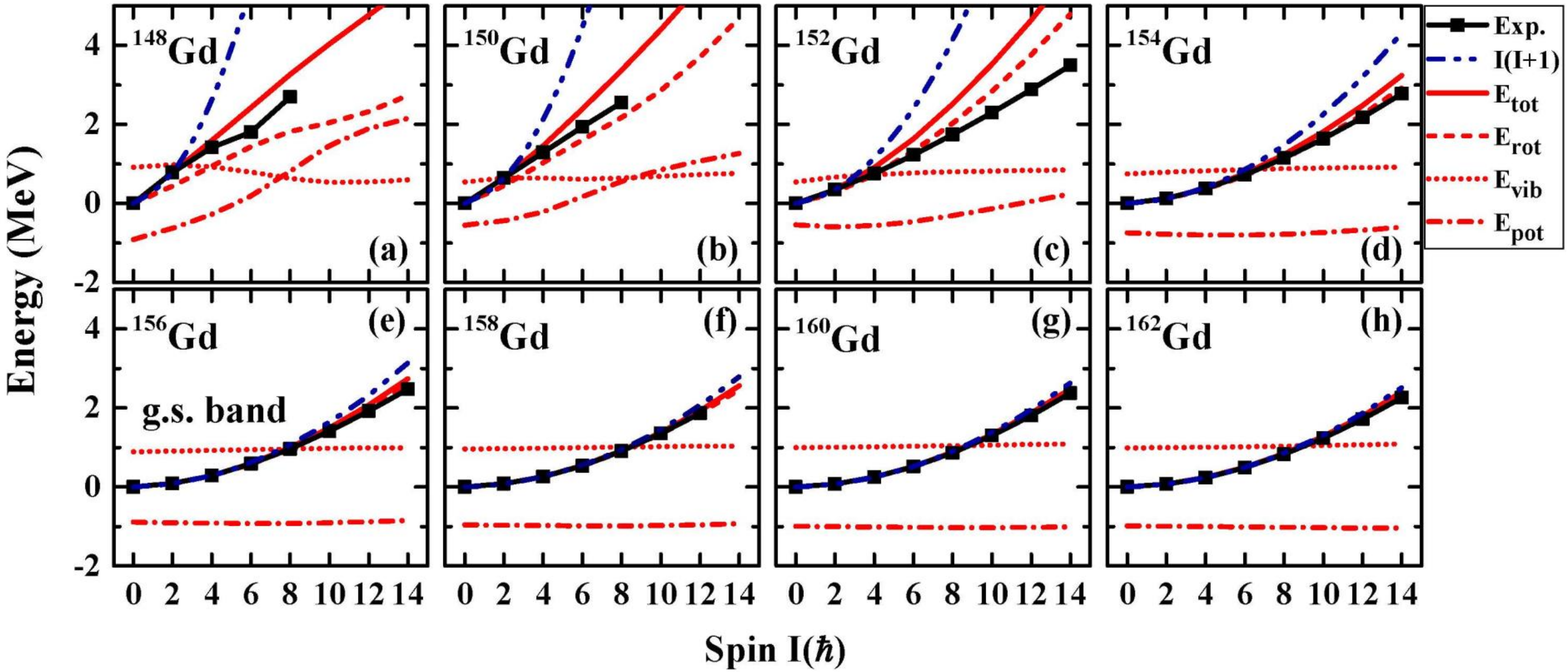


- The overall trend of the R42 ratio for the Gd isotopes is reproduced.
- With the increasing neutron numbers, the rotational behavior become more and more obvious. For $^{158\text{-}162}$Gd, the R42 ratio arrive at the rotational limit 3.33.
- The results obtained from PC-F1 are smaller than those from PC-PK1 due to the softer PESs of PC-F1.

- The 5DCH calculations reproduce the data well;
- The B(E2) values increase with increasing neutron number, and are consistent with the increasing quadrupole deformation;
- The results obtained from PC-F1 are smaller than those from PC-PK1.

Energy contribution: g.s. band

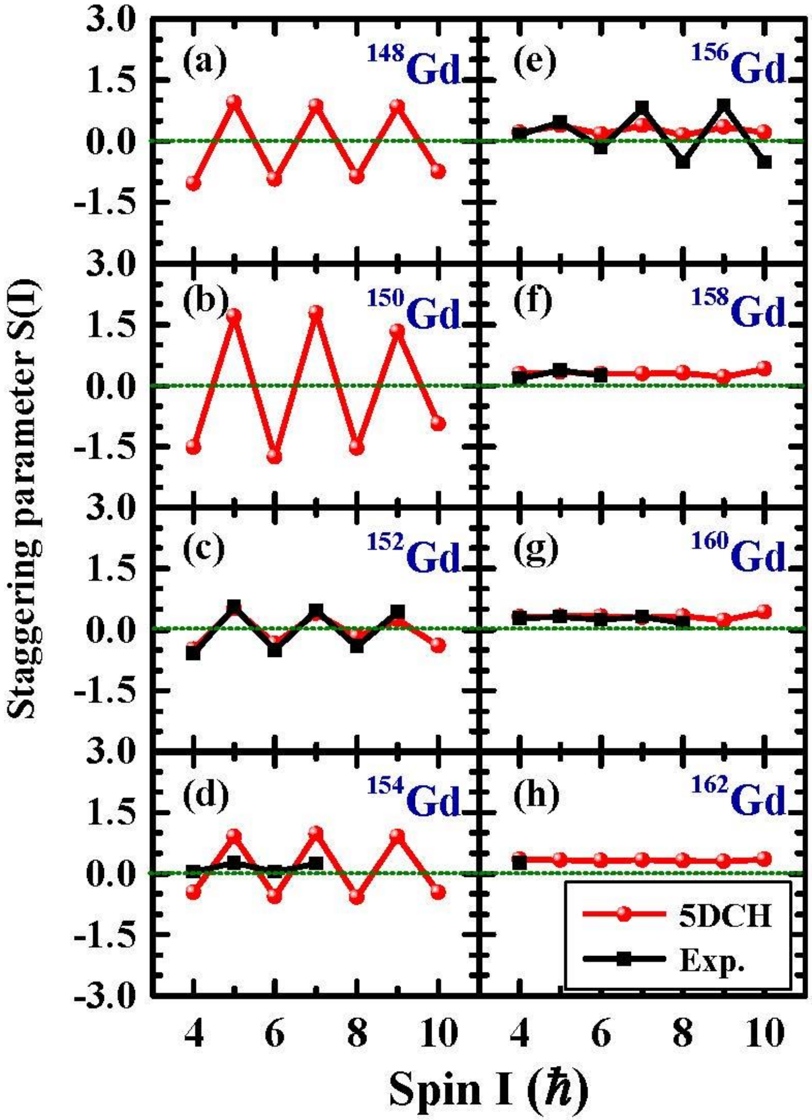
$$\hat{H}_{\text{coll}}(\beta,\gamma) = \hat{T}_{\text{rot}}(\beta,\gamma,\Omega) + \hat{T}_{\text{vib}}(\beta,\gamma) + V_{\text{coll}}(\beta,\gamma)$$



- In $^{148\text{-}154}$Gd, with the increase of neutron number, the changes of E_{pot} with the spin becomes smaller, and E_{tot} get closer to I(I+1).
- In $^{156\text{-}162}$Gd, $E_{vib}+E_{pot}$~0 and E_{tot}/E_{rot}~I(I+1).

Staggering parameter: γ band

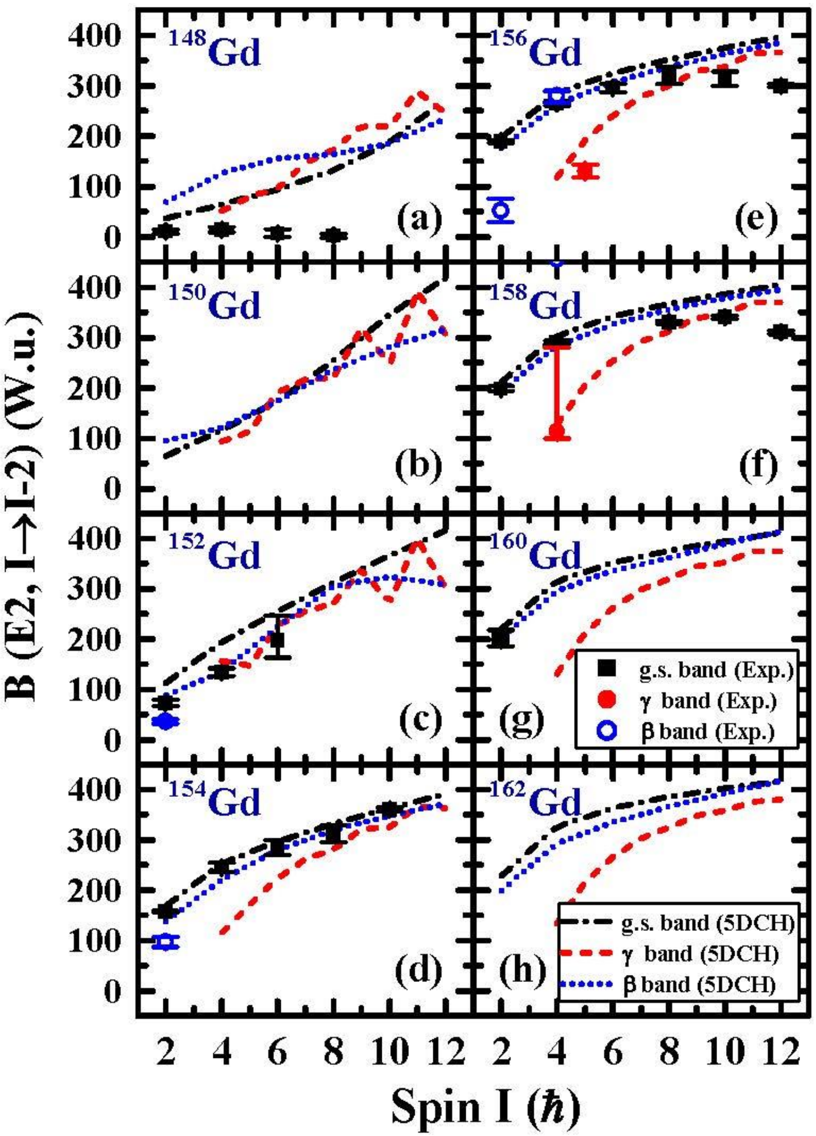
Zamfir_Casten, PLB (1991) 260: 265



$$S(I) = \frac{[E(I) - E(I-1)] - [E(I-1) - E(I-2)]}{E(2_1^+)}$$

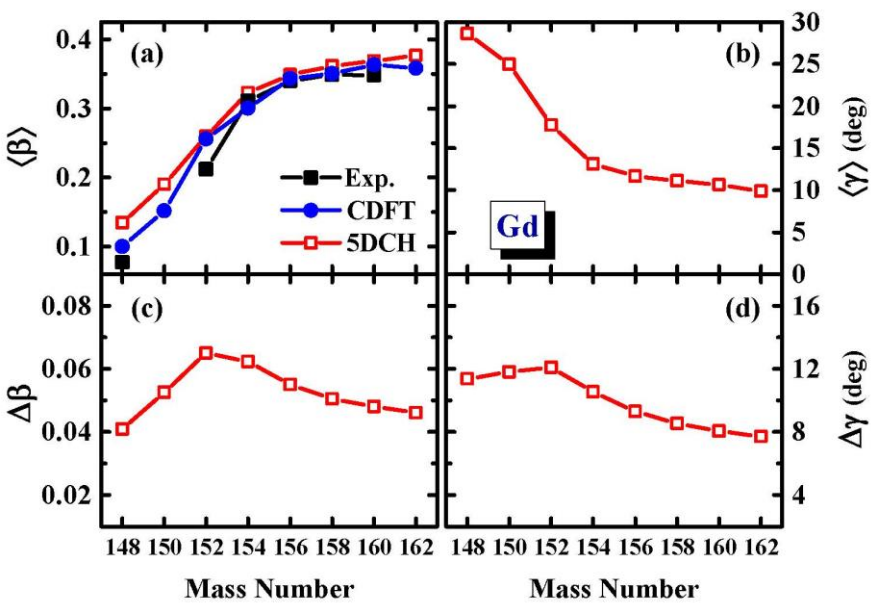
- For a vibration, S(I) oscillates between negative values for even-spin states and positive values for odd-spin states, with |S(I)=1|: ^{148}Gd
- For a γ-soft nucleus, S(I) are negative for even-spin states and positive for odd-spin states: ^{150,152,154}Gd.
- For a axial rotor, S(I) parameters are close to a constant value of 0.33: $^{156-162}$Gd.
- For a γ-rigid nucleus, S(I) are positive for even-spin states and negative for odd-spin states.

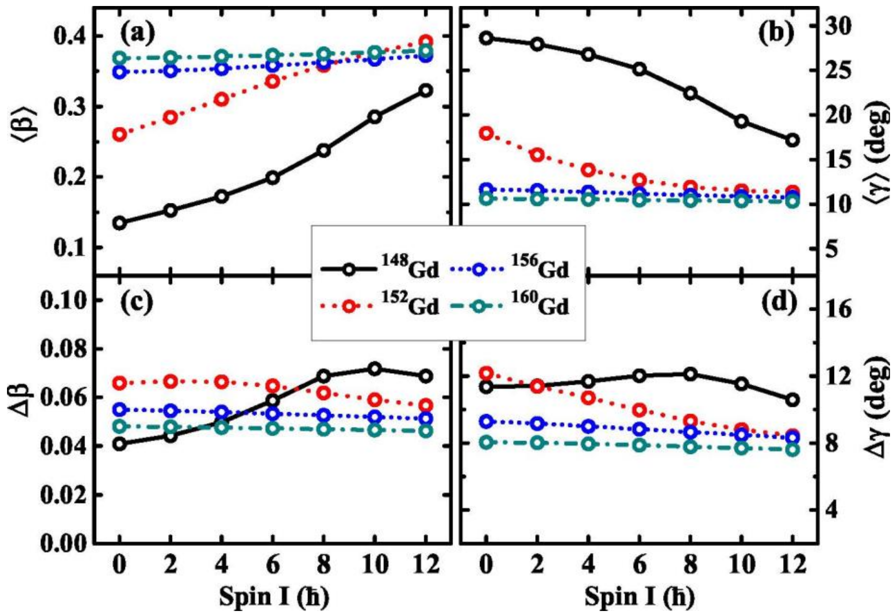
B(E2)



- For ^{148}Gd, the 5DCH calculation overestimates the data significantly. This may be because that the excitation mode in ^{148}Gd does not belong to the collective excitation.
- The available experimental data in $^{152-162}$Gd are well reproduced by the 5DCH calculations. It is also found that in $^{154-162}$Gd, the B(E2) transitions of the g.s. and β bands are similar, and the B(E2) values in the γ bands are smaller than those of the g.s. and β bands.
- The increasing behaviors for the B(E2) of each band in $^{156-162}$Gd are very similar, indicating similar structures in these four nuclei.

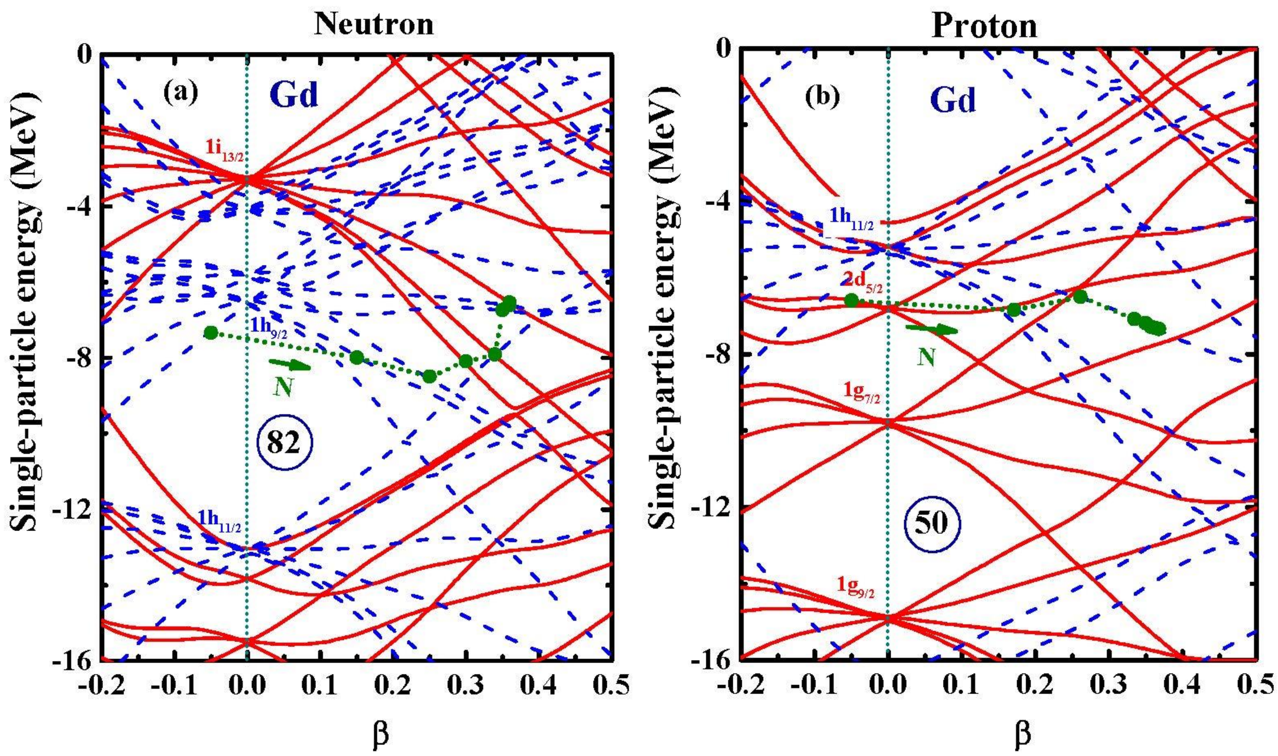
Quadrupole deformation





- With increasing mass number, the ⟨β⟩ becomes larger and larger. The trend of ⟨γ⟩ is opposite to that of ⟨β⟩, i.e., decreases with mass number.
- The trends of Δβ and Δγ for Gd isotopes are similar. The shape fluctuations indicate that ^{152}Gd locates at a critical point in the even-even $^{148–162}$Gd isotopes.
- The strong dependence on the spin of the deformations in ^{148,152}Gd is associated with the soft behavior in the PESs.
- For ^{156,160}Gd, all of the four quantities ⟨β⟩, ⟨γ⟩, Δβ, and Δγ are almost independent of spin. This indicates that the shapes of ^{156,160}Gd are so stable that their deformations do not change with spin.

Single particle levels



- The occupied $i_{13/2}$ levels in the ground states of Gd isotopes make the deformations so stable that they even do not change with the increase of angular momentum.

Summary

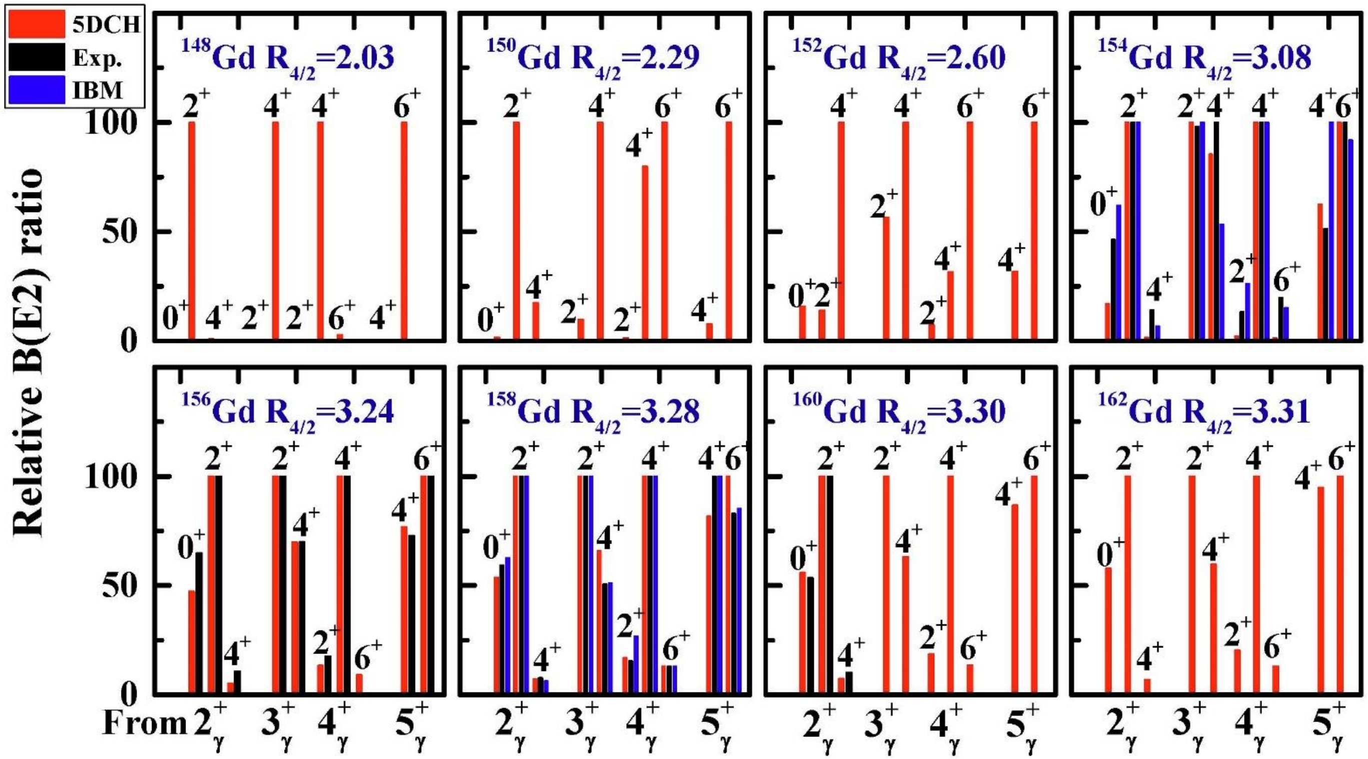
- The low-lying states for the even-even $^{148\text{–}162}$Gd isotopes have been investigated in the framework of 5DCH-CDFT.

- A clear shape evolution from weakly deformed ^{148,150}Gd to γ-soft ^{152,154}Gd to well deformed prolate $^{156\text{–}162}$Gd has been presented. The shapes of $^{156\text{–}162}$Gd are all well-deformed prolate with the minima located at $\beta \sim 0.35$, and these deformations are almost independent on the angular momentum.

- The available experimental data are reproduced by the calculations, including the energy spectra, γ band staggering and intraband B(E2) transition

- The occupations of neutron $i_{13/2}$ orbitals, which lead to the stable PESs of $^{156\text{–}162}$Gd, are essential for these isotopes.

Relative B(E2) ratio



IBM: Casten_Cakirli_Blaum_Couture, PRL (2014) 113: 112501

Q: How can we discuss the PDS/QDS in the framework of collective Hamiltonian?

Theoretical framework

- Wave function

The eigenvalue problem of the collective Hamiltonian is solved using an expansion of eigenfunctions in terms of a complete set of basis functions. The obtained collective wave functions is

$$\Psi_{\alpha}^{IM}(\beta,\gamma,\Omega) = \sum_{K\in\Delta I} \psi_{\alpha K}^{I}(\beta,\gamma)\Phi_{MK}^{I}(\Omega).$$

- Observables and expectation values

  - Reduced *E2* transition

$$B(E2;\alpha I \to \alpha' I') = \frac{1}{2I+1}|\langle\alpha' I'||\hat{M}(E2)||\alpha I\rangle|^2,$$

  - The average and fluctuation of β and γ

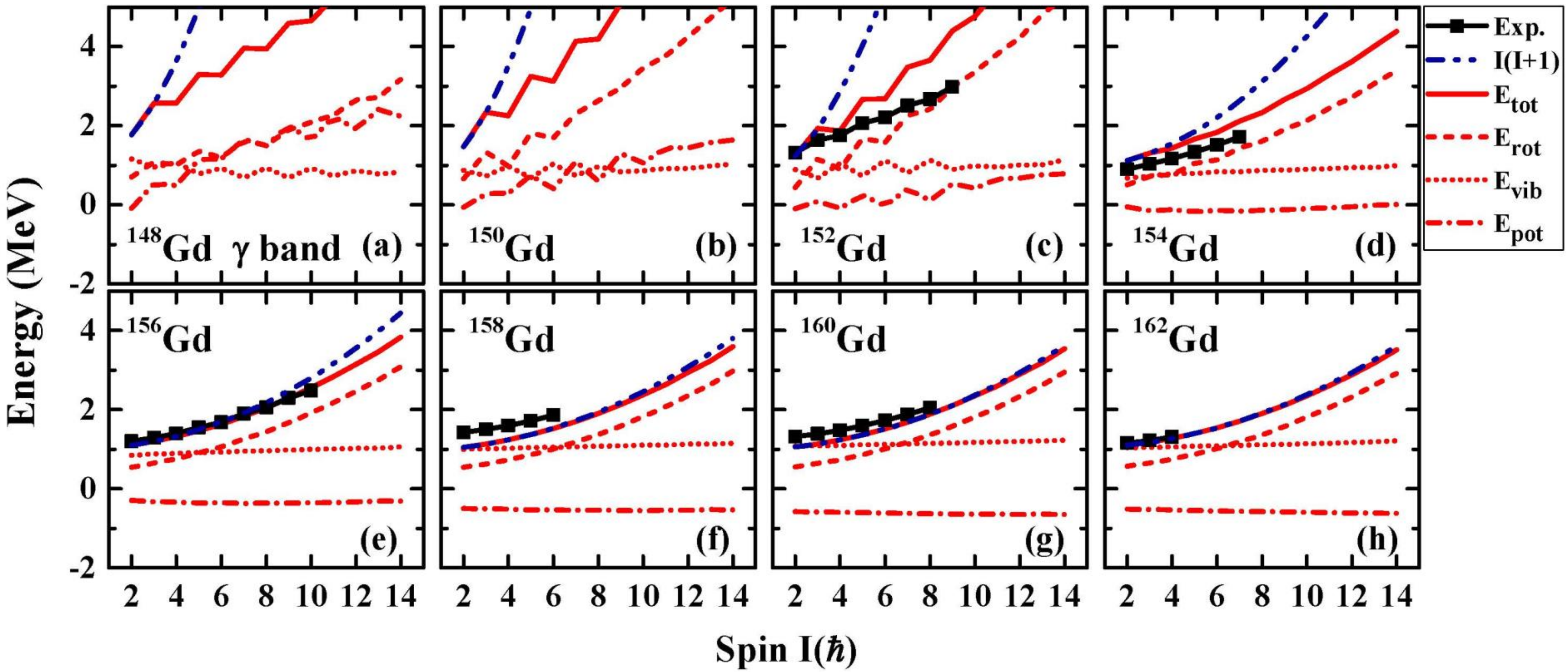
$$\langle\beta\rangle = \sqrt{\langle\beta^2\rangle},$$

$$\langle\gamma\rangle = \arccos(\langle\beta^3\cos 3\gamma\rangle/\sqrt{\langle\beta^4\rangle\langle\beta^2\rangle})/3,$$

$$\Delta\beta = \frac{\sqrt{\langle\beta^4\rangle - \langle\beta^2\rangle^2}}{2\langle\beta\rangle},$$

$$\Delta\gamma = \frac{1}{3\sin 3\langle\gamma\rangle}\sqrt{\frac{\langle\beta^6\cos^2 3\gamma\rangle}{\langle\beta^6\rangle} - \frac{\langle\beta^3\cos 3\gamma\rangle^2}{\langle\beta^4\rangle\langle\beta^2\rangle}}.$$

Energy spectra



Energy spectra