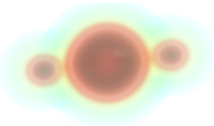 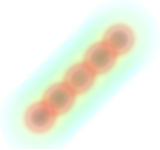 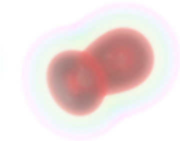

Nuclear clustering with and beyond the RMF framework

P. Marević[1,2], J.-P. Ebran[1], E. Khan[2], T. Nikšić[3], D. Vretenar[3]

[1]CEA, DAM, DIF, France
[2]IPN Orsay, France
[3]University of Zagreb, Croatia

Outline of the talk

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Clustering in atomic nuclei

1

Outline of the talk

- **Clustering in atomic nuclei**
- **Relativistic mean-field**
  - **Theoretical background**
  - **How atomic nuclei cluster**

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Clustering in atomic nuclei

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Clustering in atomic nuclei

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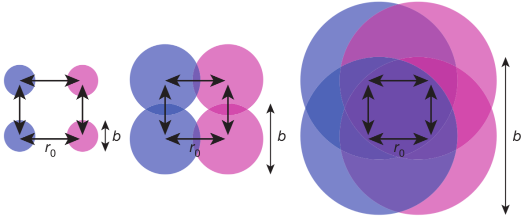
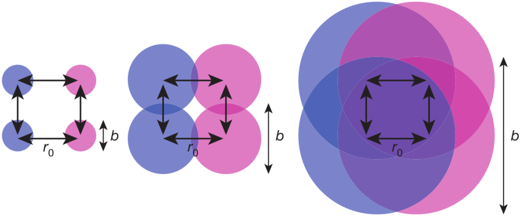


$$\alpha \equiv \frac{b}{r_0}$$

$\alpha < 1$ $\alpha \approx 1$ $\alpha > 1$

J.-P. Ebran *et al.*, Nature 487, 341 (2012).

Clustering in atomic nuclei



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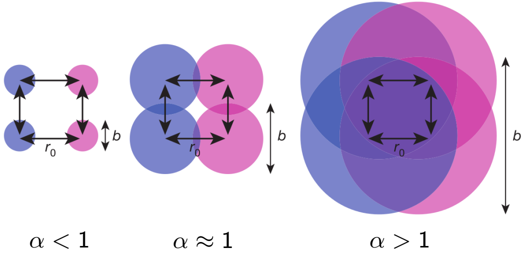

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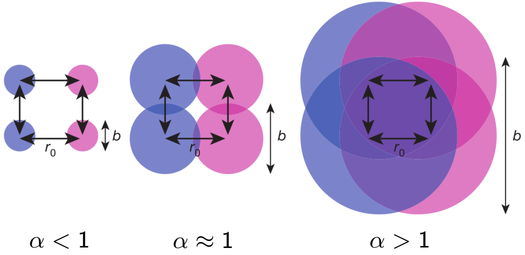

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- transition phenomenon between quantum liquid and crystalline phases
- rich phenomenology (molecular bonds, Hoyle state, radioactivity, ...)
- various theoretical approaches (AMD, FMD, NCSM, MCSM, ...)

Relativistic mean-field

3

Relativistic mean-field
Theoretical background

Relativistic mean-field
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- NEDFs as global theoretical framework

Relativistic mean-field
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$$\mathcal{L} = \bar{\psi}(i\gamma\cdot\partial - m)\psi - \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\psi)$$

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  - TMR separable pairing [Y. Tian *et al.* PLB 676, 44 (2009).]

$$\langle k|V^{^1S_0}|k'\rangle = -Gp(k)p(k'), \qquad p(k) = e^{-a^2k^2}$$

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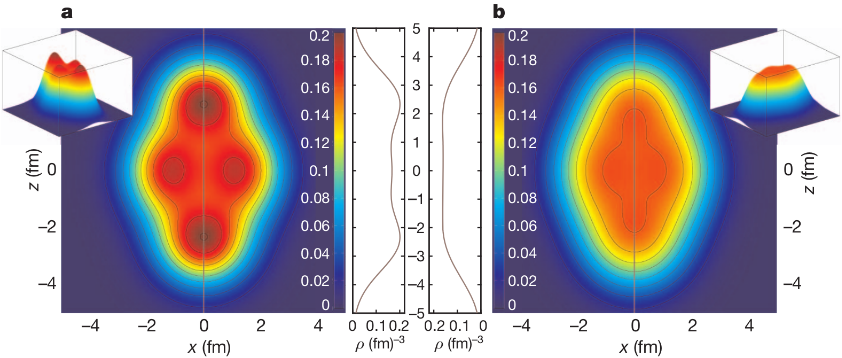
- RHB equations solved by expanding nuclear spinors in HO basis
- dimensionless deformation parameters $\beta_\lambda = \frac{4\pi}{3AR^\lambda}q_{\lambda 0}$
- self-consistent calculation of ground-state properties

4

Relativistic mean-field
How atomic nuclei cluster

Relativistic mean-field
How atomic nuclei cluster



DD-ME2 **Sly4**

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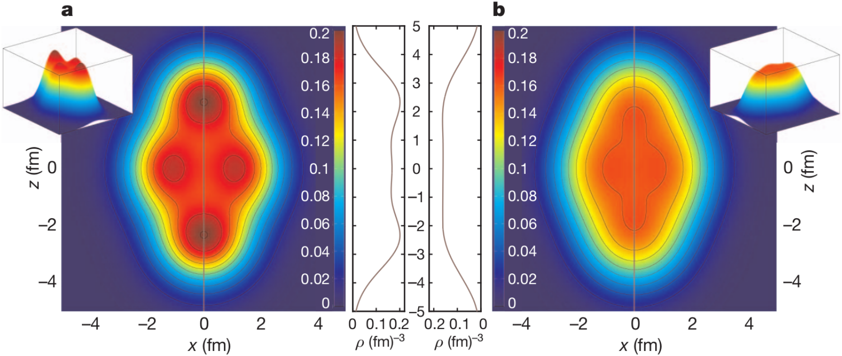

DD-ME2 **Sly4**

$V_0 = -82.4$ **MeV** $V_0 = -72.4$ **MeV**

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Relativistic mean-field
How atomic nuclei cluster

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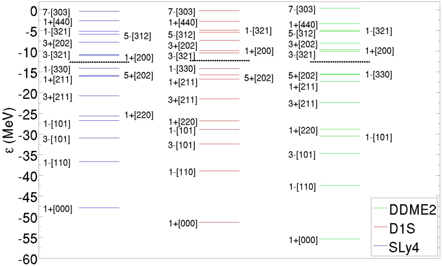

J.-P. Ebran *et al.*, Nature 487, 341 (2012)., PRC 90, 054329 (2014).

Relativistic mean-field
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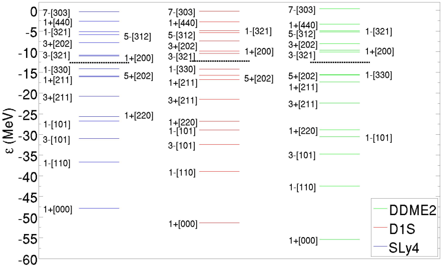

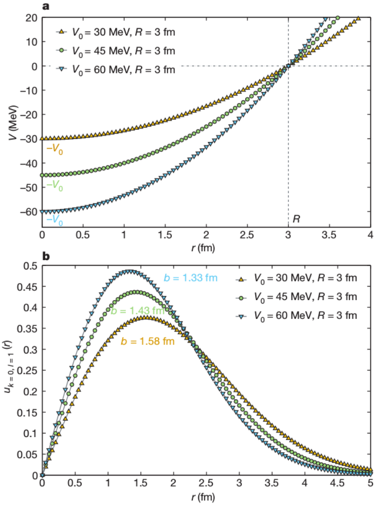

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6

Relativistic mean-field
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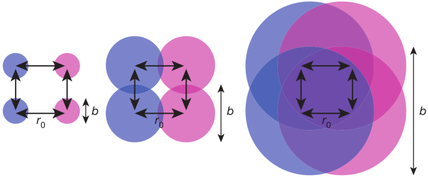
$$\alpha = \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2m V_0 r_0^2)^{1/4}}$$



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J.-P. Ebran *et al.*, PRC 87, 044307 (2013).

Relativistic mean-field
How atomic nuclei cluster

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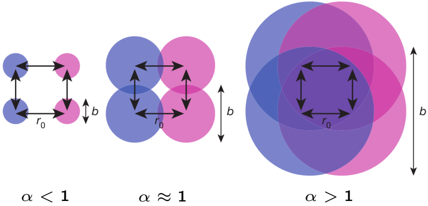

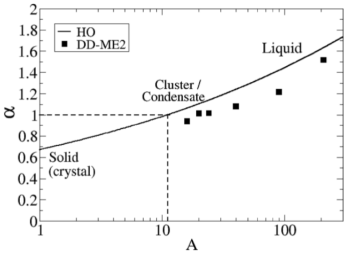

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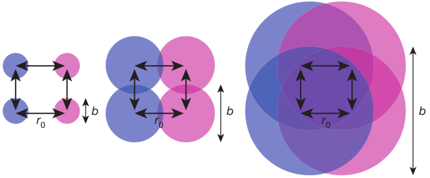
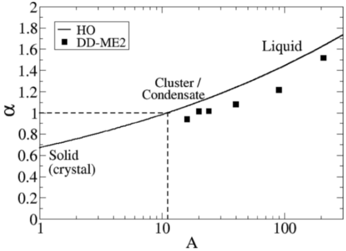


	Self-consistent	
	SLy4	DDME2
^{20}Ne	0.99	0.97
^{24}Mg	1.00	0.95
^{28}Si	0.99	0.96
^{32}S	0.99	0.96
^{208}Pb	1.28	1.31



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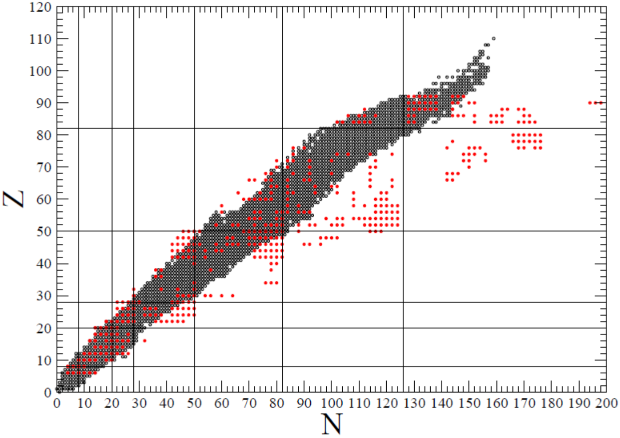

J.-P. Ebran *et al.*, arXiv:1805.05099 [nth]

... and beyond

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Beyond relativistic mean-field
Symmetry restoration and configuration mixing

Beyond relativistic mean-field
Symmetry restoration and configuration mixing

- constrained RHB solutions as BMF input: $|\phi(q_j)\rangle$, $q \equiv (\beta_2, \beta_3)$

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- configuration mixing of symmetry-restored states:

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$$\sum_j \underbrace{\mathcal{H}^{J\pi}(q_i,q_j)}_{\text{Hamiltonian kernel}} \overbrace{g_\alpha^{J\pi}(q_j)}^{\text{coll. w. f.}} = \underbrace{E_\alpha^{J\pi}}_{\text{exc. spectra}} g_\alpha^{J\pi}(q_i)$$

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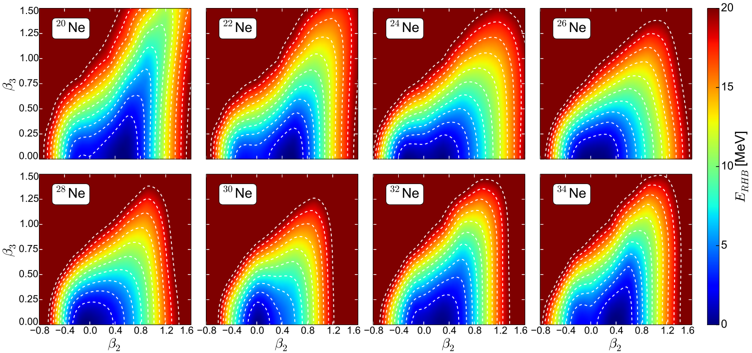
- solving the HWG equation gives collective spectra and wave functions
- calculation of various observables (Q_λ^{spec}, $B(E\lambda)$, $F_L(q)$, ...)

Beyond relativistic mean-field
Cluster structures in Ne isotopes



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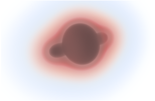

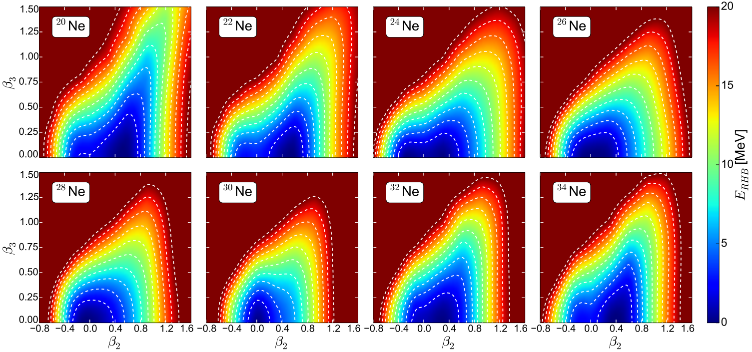

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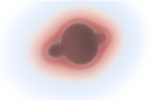

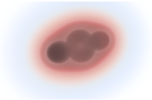

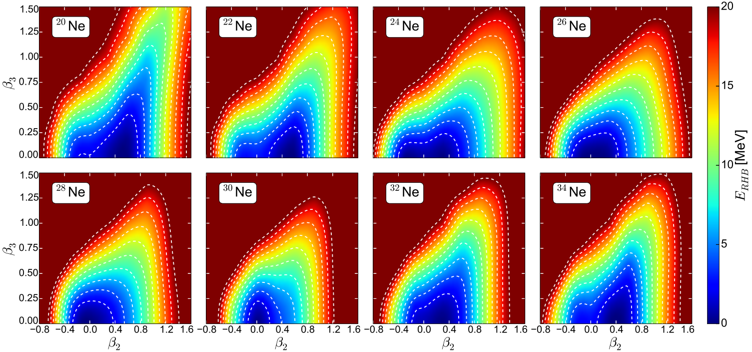

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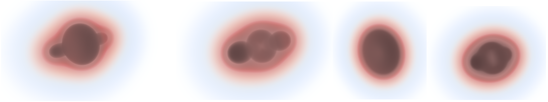

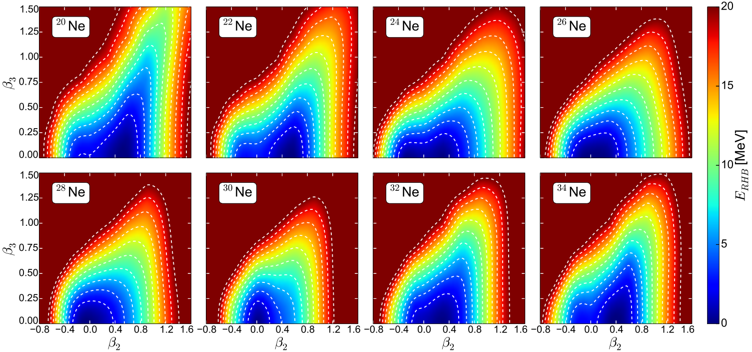

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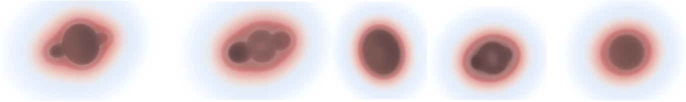

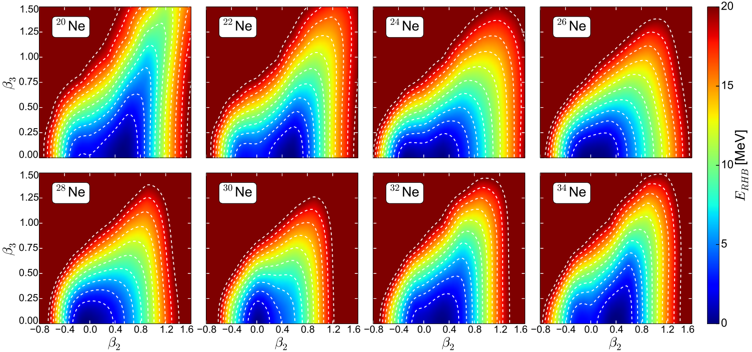

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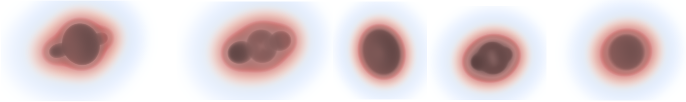

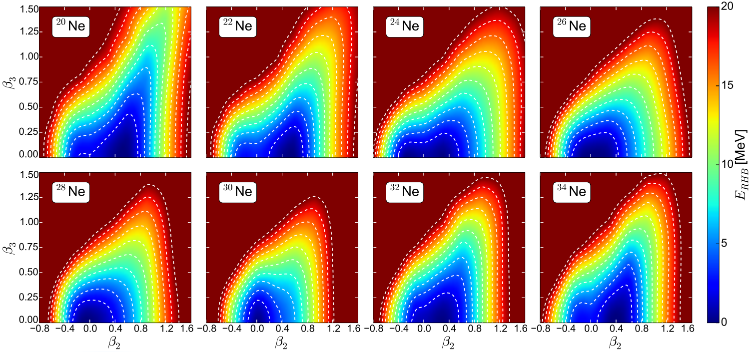

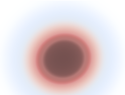

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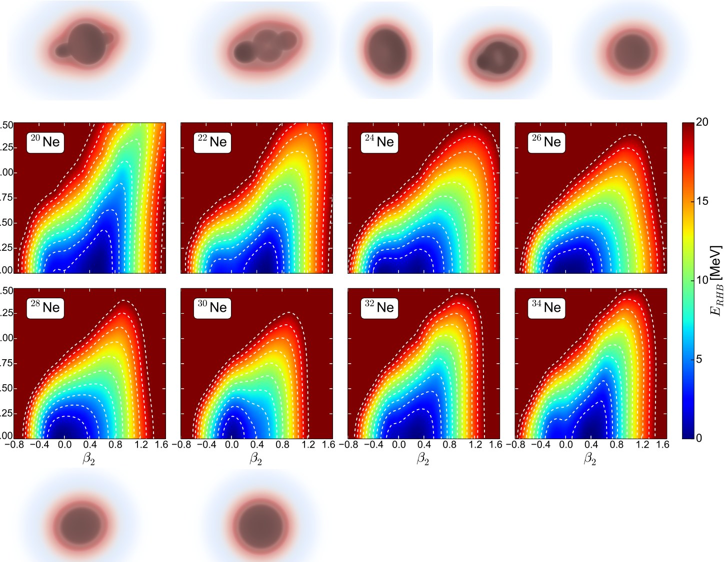

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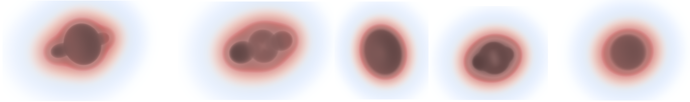

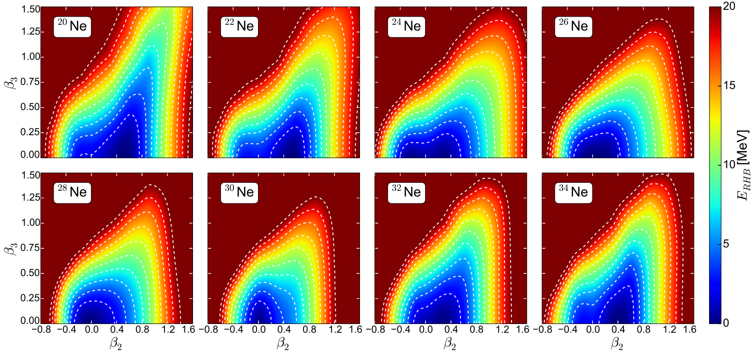

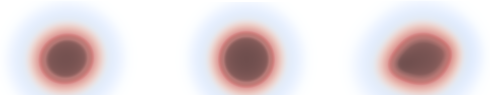

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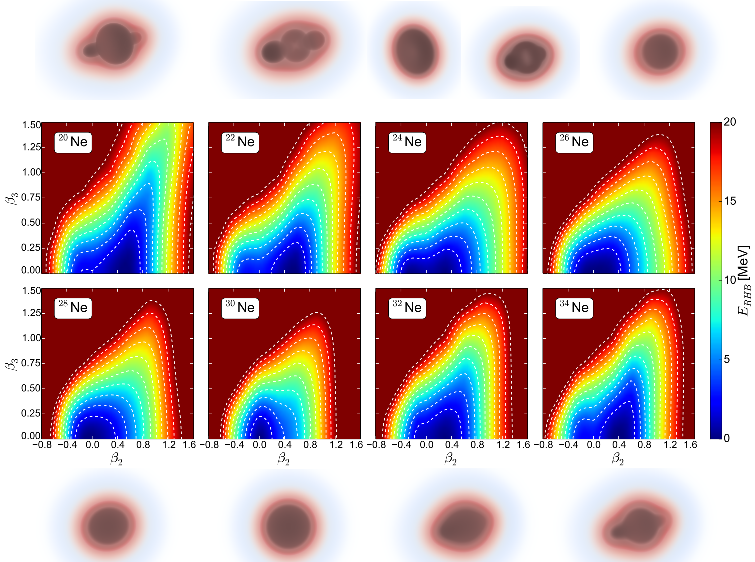

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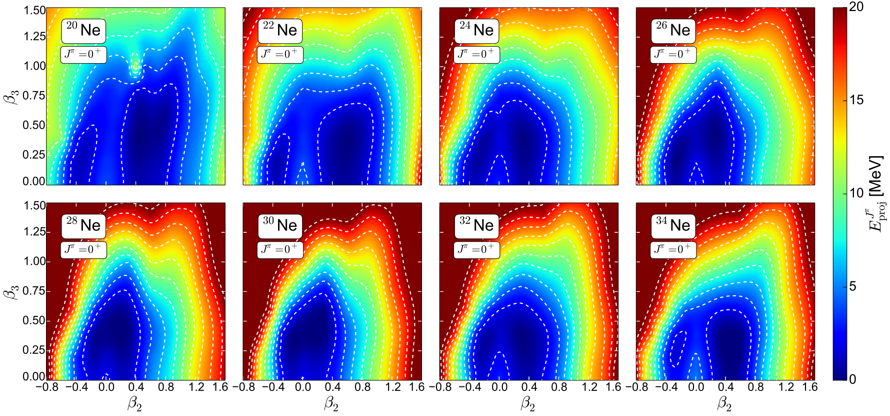
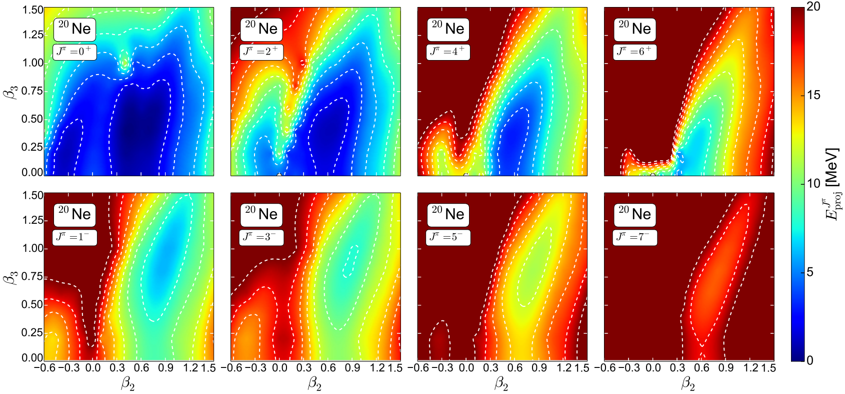


P. Marević *et al.*, PRC 97, 024334 (2018).

Beyond relativistic mean-field
Cluster structures in Ne isotopes

Projected energy surfaces:



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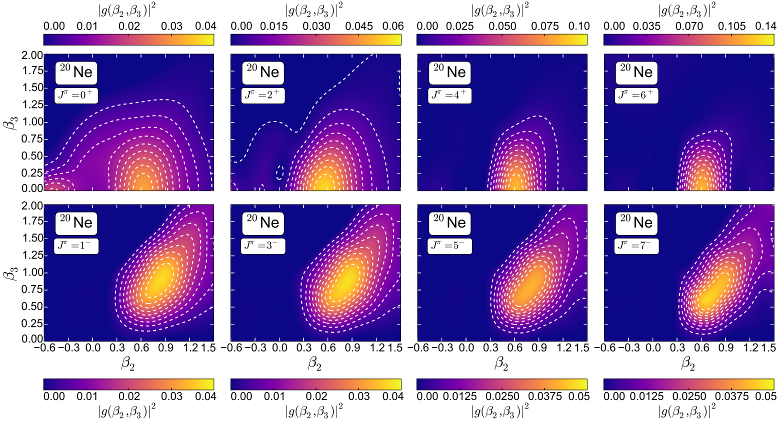
Collective wave functions:



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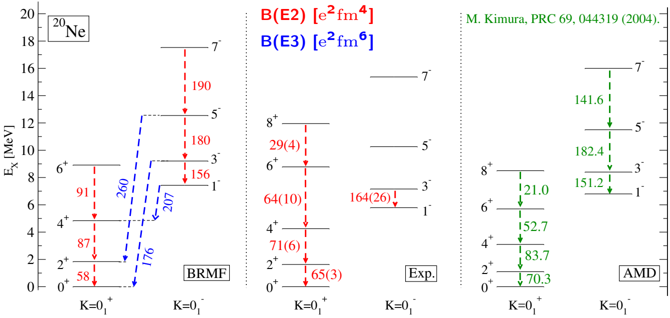

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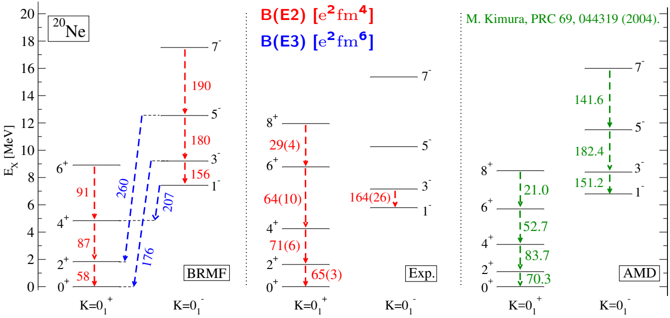

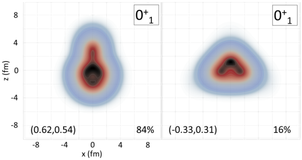

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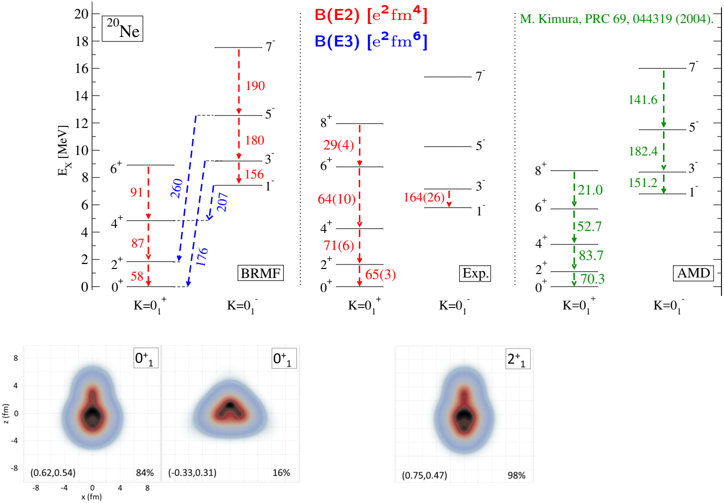

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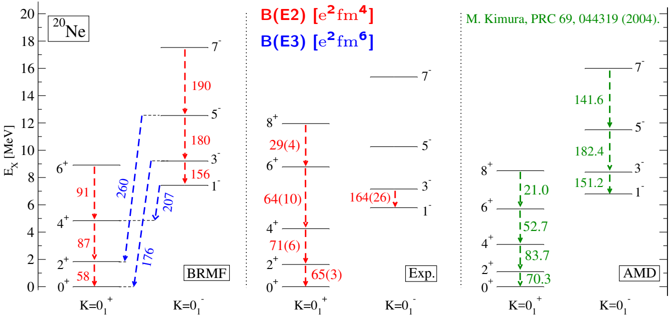

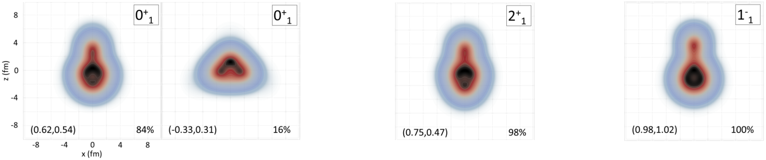

Conclusion

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- RMF description of ground-state properties
    - clustering due to the depth of confining potential
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- formation of clusters as transitional phenomenon
- RMF description of ground-state properties
  - clustering due to the depth of confining potential
  - relativistic functionals: deeper potentials and smaller α values
  - systematic prediction for nucleon localization over nuclide chart
- beyond RMF description
  - systematics of neon isotopic chain
  - collective properties and cluster structures in ^{20}Ne
  - applicable over the entire nuclide chart

Thank you for your attention!