Excitation energy dependence of the moment of inertia of the well deformed nuclei

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Problem

^{156}Gd

$E^*(2^+_{ms})$=3089 keV

$E^*(1^+_{ms})$=3070 keV

$E^*(2^+_1) - E^*(0^+_1)$=89 keV

$\frac{\Im_{gs}}{\hbar^2} = 33.7$ MeV^{-1}
$\frac{\Im_{rig}}{\hbar^2} = 62.9$ MeV^{-1}
$\frac{\Im_{ms}}{\hbar^2} = 105$ MeV^{-1}

– Inglis formula: $\Im_{gs} \approx \Im_{rigid\,body}$

– $\Im_{exp} = (\frac{1}{2} - \frac{2}{3})\Im_{rigid\,body}$

– Residual interaction, pairing , S.T.Belyaes and A.B.Migdal

$$\Im^{(\rho_1,\rho_2)} =$$

$$2\sum_{s_1,s_2}\frac{|\langle s_1|j_x|s_2\rangle|^2\left(u_{s_1}(\rho_1\rho_2)v_{s_2}(\rho_1\rho_2)-u_{s_2}(\rho_1\rho_2)v_{s_1}(\rho_1\rho_2)\right)^2}{\varepsilon_{s_1}(\rho_1\rho_2)+\varepsilon_{s_2}(\rho_1\rho_2)}$$

$$+\sum_{s_1}\frac{|\langle \rho_1|j_x|s_1\rangle|^2\left(u_{s_1}(\rho_1\rho_2)u_{\rho_1}(\rho_1\rho_2)+v_{s_1}(\rho_1\rho_2)v_{\rho_1}(\rho_1\rho_2)\right)^2}{\varepsilon_{s_1}(\rho_1\rho_2)-\varepsilon_{\rho_1}(\rho_1\rho_2)}$$

$$+\sum_{s_2}\frac{|\langle \rho_2|j_x|s_2\rangle|^2\left(u_{s_2}(\rho_1\rho_2)u_{\rho_2}(\rho_1\rho_2)+v_{s_2}(\rho_1\rho_2)v_{\rho_2}(\rho_1\rho_2)\right)^2}{\varepsilon_{s_2}(\rho_1\rho_2)-\varepsilon_{\rho_2}(\rho_1\rho_2)}$$

This expression is derived assuming applicability of the perturbation treatment of the Coriolis term in the Cranking Hamiltonian.

In the second and the third sums there are only one-two terms separately for neutrons and protons, which practically exhausts the contribution to the sums. The contribution of the other terms is less than 3% of the total value of the moment of inertia. At the same time just these terms can not be treated in perturbation theory because of the large matrix elements of j_x and small energy differences in denominators. This happens when the single quasiparticle states ρ and s in the matrix element $\langle s|j_x|\rho\rangle$ satisfy the following selection rules for the asymptotic quantum numbers:

$$|n_s - n_\rho| = 1, \quad |\Lambda_s - \Lambda_\rho| = 1$$

The examples are: $\rho = n[642]$ and $s = n[651]$// $\rho = n[514]$ and $s = n[523]$// $\rho = p[541]$ and $s = p[532]$.

The number of terms with large matrix elements of $|\langle\rho|j_x|s\rangle|$ and small energy denominators $|\varepsilon_\rho - \varepsilon_s|$ is quite restricted. Single quasiparticle states coupled by the strong matrix elements of j_x have the following excitation energies:

For neutrons:

$\varepsilon(n[606])$=7.12 MeV, $\varepsilon(n[615])$=5.56 MeV, $\varepsilon(n[624])$=3.93 MeV, $\varepsilon(n[633])$=2.36 MeV, $\varepsilon(n[642])$=1.26 MeV, $\varepsilon(n[651])$=1.24 MeV, $\varepsilon(n[660])$=1.57 MeV.

For protons:

$\varepsilon(p[505])$=5.12 MeV, $\varepsilon(p[514])$=3.18 MeV, $\varepsilon(p[523])$=1.52 MeV, $\varepsilon(p[532])$=1.39 MeV, $\varepsilon(p[541])$=2.46 MeV, $\varepsilon(p[550])$=3.18 MeV.

In the case of mixing of only two single quasiparticle states the expression for $\Im(\rho_1, \rho_2)$ take the form :

$$\Im^{(\rho_1,\rho_2)} =$$

$$2\sum_{s_1,s_2} \frac{|\langle s_1|j_x|s_2\rangle|^2 \left(u_{s_1}(\rho_1\rho_2)v_{s_2}(\rho_1\rho_2) - u_{s_2}(\rho_1\rho_2)v_{s_1}(\rho_1\rho_2)\right)^2}{\varepsilon_{s_1}(\rho_1\rho_2) + \varepsilon_{s_2}(\rho_1\rho_2)}$$

$$+\frac{(\varepsilon_{s_1} - \varepsilon_{\rho_1})}{2\Omega_{rot}^2}\left(1 - \frac{1}{\sqrt{1 + 4\Omega_{rot}^2 \frac{|\langle s_1|j_x|\rho_1\rangle|^2 (u_{s_1}u_{\rho_1} + v_{s_1}v_{\rho_1})^2}{(\varepsilon_{s_1} - \varepsilon_{\rho_1})^2}}}\right)$$

$$+\frac{(\varepsilon_{s_2} - \varepsilon_{\rho_2})}{2\Omega_{rot}^2}\left(1 - \frac{1}{\sqrt{1 + 4\Omega_{rot}^2 \frac{|\langle s_2|j_x|\rho_2\rangle|^2 (u_{s_2}u_{\rho_2} + v_{s_2}v_{\rho_2})^2}{(\varepsilon_{s_2} - \varepsilon_{\rho_2})^2}}}\right)$$

Table : Calculated values of the excitation energies and the moments of inertia of the γ-vibrational state ($\frac{\Im(2^+_\gamma)}{\hbar^2}$) in several rare-earth nuclei. The experimental values of these quantities and of the ground state moment of inertia are shown for comparison.

Nucleus	$E(2^+_\gamma)$ in MeV		$\frac{\Im(2^+_\gamma)}{\hbar^2}$ in MeV		$\frac{\Im(gs)}{\hbar^2}$ in MeV
	exp	cal	exp	cal	exp
^{156}Gd	1.154	1.046	34.8	42.6	33.7
^{158}Gd	1.187	1.238	40.9	46.2	37.7
^{160}Dy	0.966	1.057	36.8	40.1	34.6
^{162}Dy	0.888	0.915	40.5	48.2	37.2
^{166}Er	0.786	0.860	41.8	50.2	37.2
^{178}Hf	1.175	1.196	33.3	35.1	32.2

Table : Calculated values of the excitation energies (E^*_{cal}), B(M1) in μ_n^2 and the moments of inertia of the 1^+ states of ^{156}Gd. The contribution of the blocking effect $\left(\Delta\frac{\Im(1_n^+)}{\hbar^2}\right)_{block}$ and the quasiparticle interaction $\left(\Delta\frac{\Im(1_n^+)}{\hbar^2}\right)_{qp\ int}$ are given separately in the last two columns. All quantities are given in MeV

E^*_{cal}	B(M1)	$\left(\frac{\Im(1_n^+)}{\hbar^2}\right)_{cal}$	$\left(\Delta\frac{\Im(1_n^+)}{\hbar^2}\right)_{block}$	$\left(\Delta\frac{\Im(1_n^+)}{\hbar^2}\right)_{qp\ int}$
1.900	0.0012	53.6	13.4	6.5
2.182	0.889	100.8	9.6	57.5
2.450	0.0041	60.3	4.4	22.2
2.795	0.263	64.7	6.1	24.9
2.909	1.19	52.4	6.0	12.7
3.109	0.090	48.9	6.2	9.0

- In the case of the γ-vibrational states both blocking effect and quasiparticle – rotating core coupling give a comparable contribution.
- In the case of the 1^+ state the effect of the quasiparticle – rotating core coupling can be significantly higher. This effect is especially large if the single quasiparticle states contributing into the sums have very close energies and the matrix element of the angular momentum operator connecting these states is large.