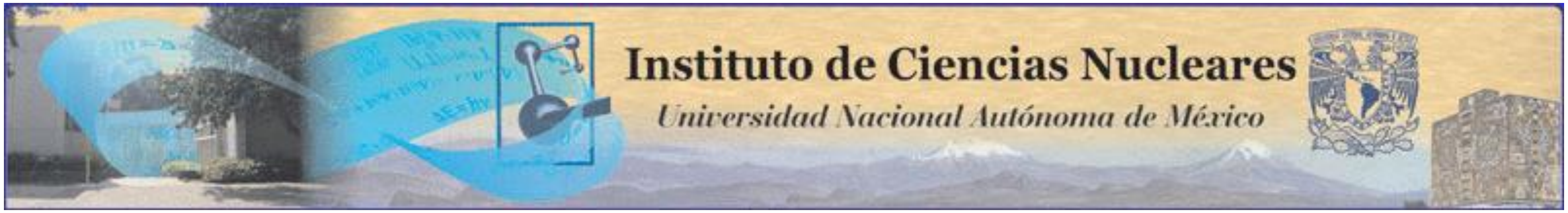
QPT in Cluster Nuclei

- Introduction
- ACM: Algebraic Cluster Model
- Appliciations: ^{8}Be, ^{12}C y ^{16}O
- Energies and em transitions
- Quantum phase transitions
- Summary and conclusions



Algebraic Cluster Model (ACM)

- IBM for few-body systems
- 2-body system: U(4) model
- 3-body system: U(7) model
- 4-body system: U(10) model
- Applications: hadrons, molecules, alpha-cluster nuclei

ACM for 3-Body Systems

6 relative degrees of freedom: Jacobi vectors

$$\vec{\rho} = (\vec{r}_1 - \vec{r}_2)/\sqrt{2}$$
$$\vec{\lambda} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}$$

Introduce 2 dipole bosons and a scalar boson

$$b_\rho^\dagger \, , \, b_\lambda^\dagger \, , \, s^\dagger$$

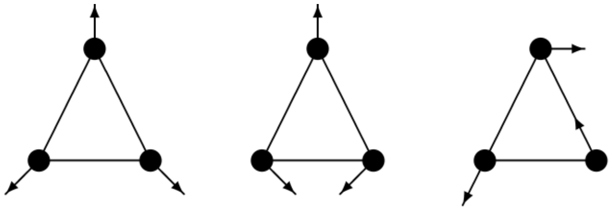
Identical particles: S_3 invariant Hamiltonian

- U(6) limit (harmonic oscillator)
- SO(7) limit (deformed oscillator)
- Oblate Top (equilateral triangle)

Oblate Top: Triangle

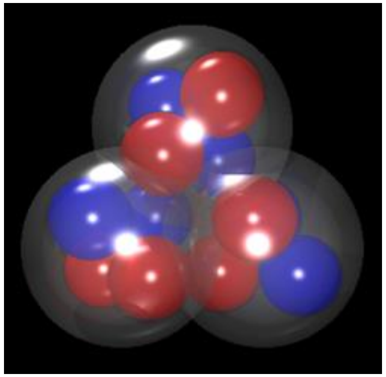
$$H_{\rm vib} = \xi_1\,(s^\dagger s^\dagger - b^\dagger_\rho\cdot b^\dagger_\rho - b^\dagger_\lambda\cdot b^\dagger_\lambda)\,(\text{h.c.}) + \xi_2\left[(b^\dagger_\rho\cdot b^\dagger_\rho - b^\dagger_\lambda\cdot b^\dagger_\lambda)\,(\text{h.c.}) + 4(b^\dagger_\rho\cdot b^\dagger_\lambda)(\text{h.c.})\right]$$

Equilibrium shape:
equilateral triangle



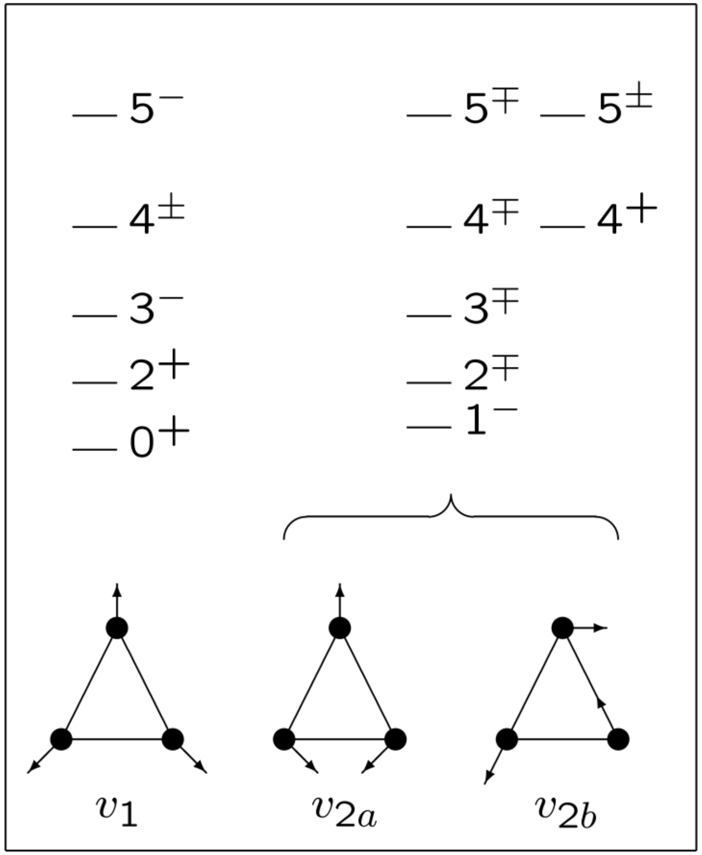
$$E_{\rm vib} \approx \omega_1(v_1+\frac{1}{2}) + \omega_2(v_2+1)$$

$$\omega_1 = 4N\xi_1$$
$$\omega_2 = 2N\xi_2$$



Bijker, Iachello & Leviatan, AP 236, 69 (1994)
Bijker & Iachello, AP 298, 334 (2002)

Energy Spectrum



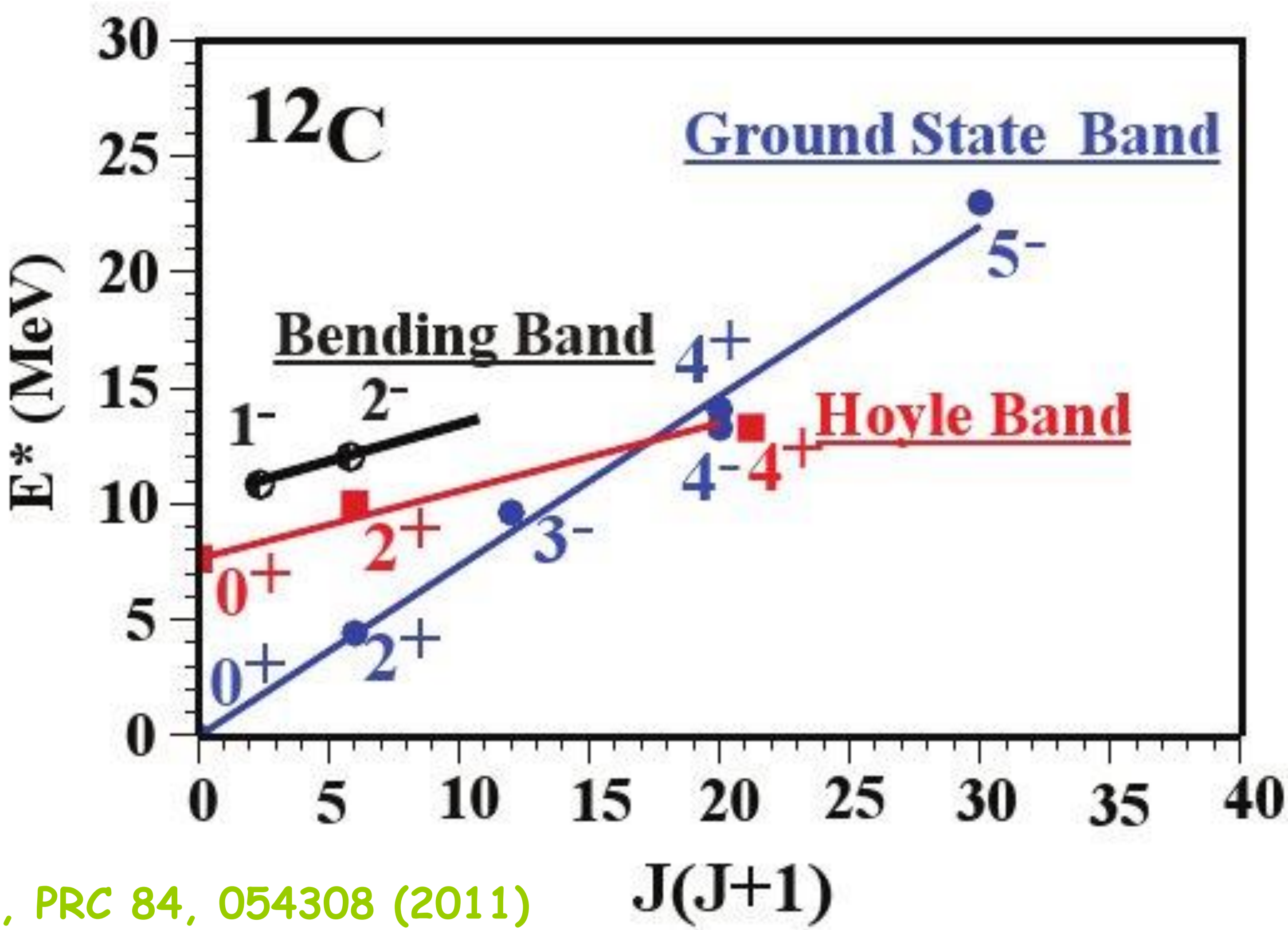
Ground state and Hoyle band (breathing vibration)

$$L^P = 0^+, 2^+, 3^-, 4^\pm, 5^-, \ldots$$

Bending vibration

$$L^P = 1^-, 2^\mp, 3^\mp, \ldots$$

Fingerprint of triangular shape with D_{3h} symmetry

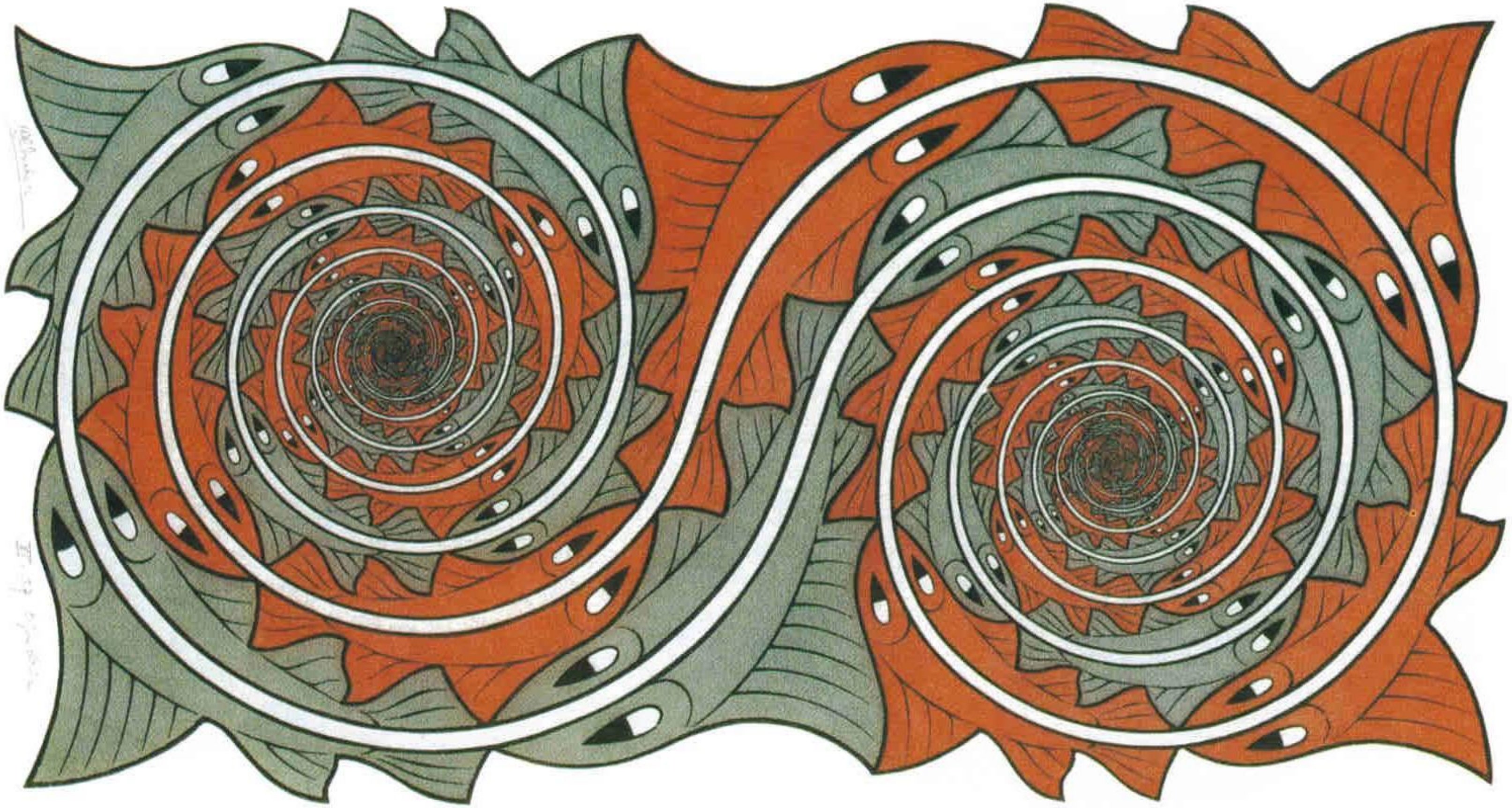


Itoh et al, PRC 84, 054308 (2011)
Freer et al, PRC 86, 034320 (2012)
Zimmerman et al, PRL 110, 152502 (2013)

Marín-Lámbarri, Bijker et al, PRL 113, 012502 (2014)

Experimental Studies

gs	3^-	Kokalova et al, PRC 87, 057307 (2013)
gs	4^-	Freer et al, PRC 76, 034320 (2007)
		Kirsebom et al, PRC 81, 064313 (2010)
gs	5^-	Marín-Lámbarri et al, PRL 113, 012502 (2014)
Hoyle	2^+	Itoh et al, PRC 84, 054308 (2011)
		Freer et al, PRC 86, 034320 (2012)
		Zimmerman et al, PRL 110, 152502 (2013)
Hoyle	4^+	Freer et al, PRC 83, 034314 (2011)
Hoyle	$3^-, 4^-$	Some evidence for negative parity strengths between 11 and 14 MeV
		Freer et al, PRC 76, 034320 (2007)



Electric Transitions

$$\rho(\vec{r}) = \frac{Ze}{k}\left(\frac{\alpha}{\pi}\right)^{3/2}\sum_{i=1}^{k} e^{-\alpha(\vec{r}-\vec{r}_i)^2}$$

$$\mathcal{F}_L(q) \rightarrow c_L\, j_L(q\beta)\, e^{-q^2/4\alpha}$$

$$\langle r^2\rangle^{1/2} = \sqrt{\frac{3}{2\alpha}+\beta^2}$$

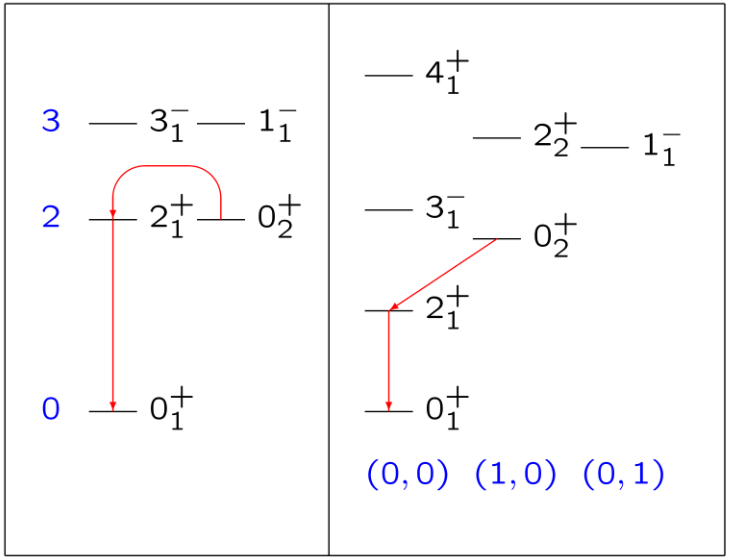
$$B(EL;0^+ \rightarrow L^P) \rightarrow \frac{(Ze)^2\, c_L^2\, \beta^{2L}}{4\pi}$$

$$c_L^2 = \begin{cases} \frac{2L+1}{2}\,[1+P_L(-1)] & 2\alpha\text{-cluster} \\ \frac{2L+1}{3}\left[1+2P_L(-\frac{1}{2})\right] & 3\alpha\text{-cluster} \\ \frac{2L+1}{4}\left[1+3P_L(-\frac{1}{3})\right] & 4\alpha\text{-cluster} \end{cases}$$

		Th.	Exp.	
^{12}C	$B(E2;2_1^+ \rightarrow 0_1^+)$	8.4	7.6 ± 0.4	e^2fm^4
	$B(E3;3_1^- \rightarrow 0_1^+)$	44	103 ± 17	e^2fm^6
	$B(E4;4_1^+ \rightarrow 0_1^+)$	73		e^2fm^8
	$B(E2;0_2^+ \rightarrow 2_1^+)$	1.3	13.1 ± 1.8	e^2fm^2

B(E2) values

Rotational and vibrational energies of the same order



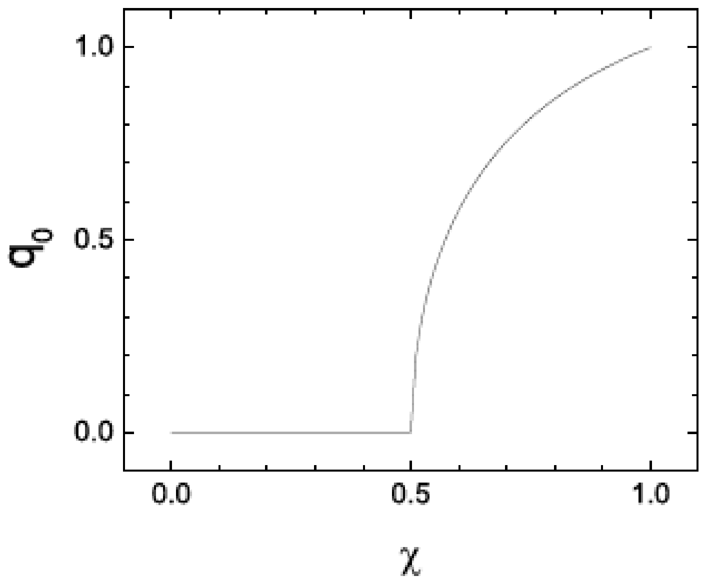
	Spherical	Deformed	^{12}C
$\frac{E_4}{E_2}$	2	$\frac{10}{3}$	3.17
$\frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	$\frac{20}{3}$	0	1.72 ± 0.25

QPT in 2-Body ACM

$$H = (1-\chi)H_{\text{sph}} + \chi H_{\text{def}}$$

$$V_{\text{cl}}(q) = \frac{1-\chi}{2}q^2 + \frac{\chi}{4}\left(1-q^2\right)^2$$

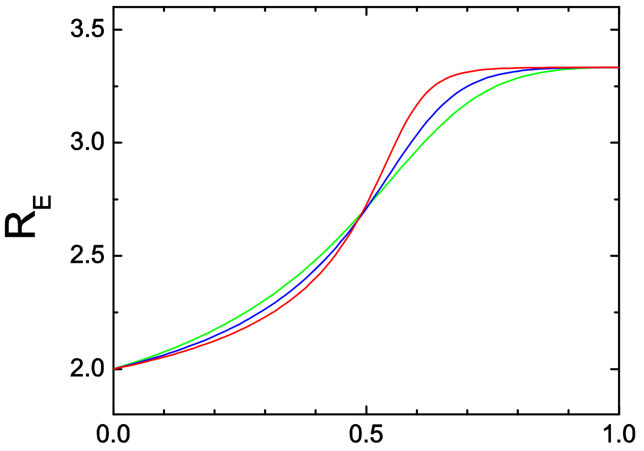
	Spherical $\chi < \chi_c = \frac{1}{2}$	Deformed $\chi > \chi_c = \frac{1}{2}$
q_0^2	0	$\frac{2\chi-1}{\chi}$
E_0	$\frac{\chi}{4}$	$\frac{(1-\chi)(3\chi-1)}{4\chi}$
$\frac{dE_0}{d\chi}$	$\frac{1}{4}$	$\frac{1-3\chi^2}{4\chi^2}$
$\frac{d^2E_0}{d\chi^2}$	0	$-\frac{1}{2\chi^3}$



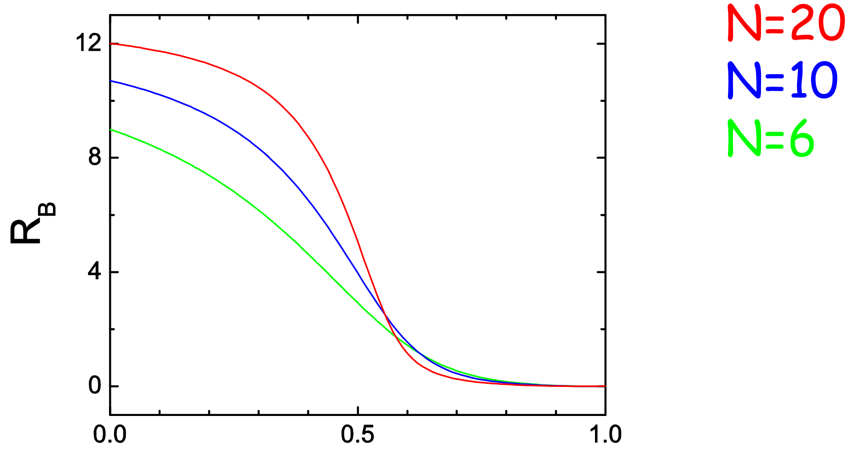
2nd order QPT

U(3) - SO(4)

Spherical - Deformed



	χ	$R_E = \frac{E_4}{E_2}$	$R_B = \frac{B(E2;0_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}$	N
Sph	0	2	40/3	∞
	0.59	3.00	1.74	10
Def	1	10/3	0	∞
^{12}C		3.17	1.72 ± 0.25	



Omar Alejandro Díaz Caballero
B.Sc. thesis, FC-UNAM, 2016

Phys. Scr. 91, 073005 (2016)

QPT in 3-Body ACM

$$
\begin{aligned}
H &= (1-\chi) H_{\text{sph}} + \chi H_{\text{def}} \\
H_{\text{sph}} &= \sum_m \left(b^\dagger_{\rho,m} b_{\rho,m} + b^\dagger_{\lambda,m} b_{\lambda,m} \right) \\
H_{\text{def}} &= \frac{1}{N-1} \Big[\frac{1-\zeta}{4} \left(s^\dagger s^\dagger - b^\dagger_\rho \cdot b^\dagger_\rho - b^\dagger_\lambda \cdot b^\dagger_\lambda \right) (\text{h.c.}) \\
&\quad + \zeta \left\{ \left(b^\dagger_\rho \cdot b^\dagger_\rho - b^\dagger_\lambda \cdot b^\dagger_\lambda \right) (\text{h.c.}) + 4 (b^\dagger_\rho \cdot b^\dagger_\lambda)(\text{h.c.}) \right\} \Big]
\end{aligned}
$$

	$\zeta = 0$	$U(6) - SO(7)$ QPT
	$\zeta = 1$	Anharmonic oscillator
$\chi = 0$		Harmonic oscillator
$\chi = 1$	$\zeta = 0$	Deformed oscillator
$\chi = 1$	$0 < \zeta < 1$	Oblate top

Equilibrium Shapes

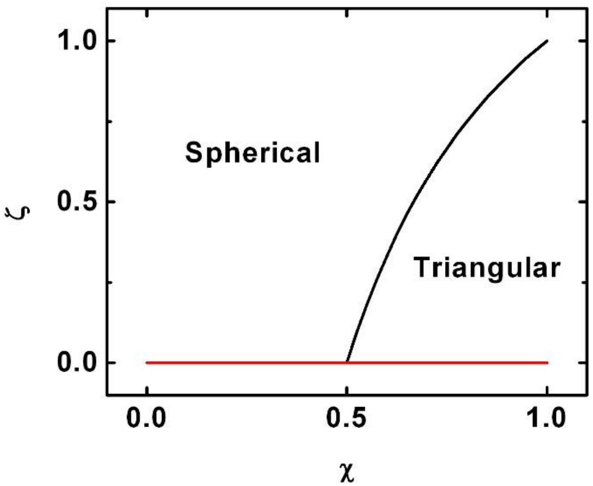
$$V_{\rm cl}(q,\phi,\theta) = \frac{1-\chi}{2} q^2 + \frac{\chi}{4}\left[(1-\zeta)\left(1-q^2\right)^2 + \zeta q^4 \left(\cos^2 2\phi + \sin^2 2\phi \cos^2 2\theta\right)\right]$$

	Spherical $\chi < \chi_c = \frac{1}{2-\zeta}$	Triangular $\chi > \chi_c = \frac{1}{2-\zeta}$
q_0^2	0	$\frac{\chi(2-\zeta)-1}{\chi(1-\zeta)}$
ϕ_0	$-$	$\frac{\pi}{4}$
θ_0	$-$	$\frac{\pi}{4}$
E_0	$\frac{\chi(1-\zeta)}{4}$	$\frac{(1-\chi)\{\chi[1+2(1-\zeta)]-1\}}{4\chi(1-\zeta)}$
$\frac{dE_0}{d\chi}$	$\frac{1-\zeta}{4}$	$\frac{1-\chi^2[1+2(1-\zeta)]}{4\chi^2(1-\zeta)}$
$\frac{d^2E_0}{d\chi^2}$	0	$-\frac{1}{2\chi^3(1-\zeta)}$

2nd order QPT

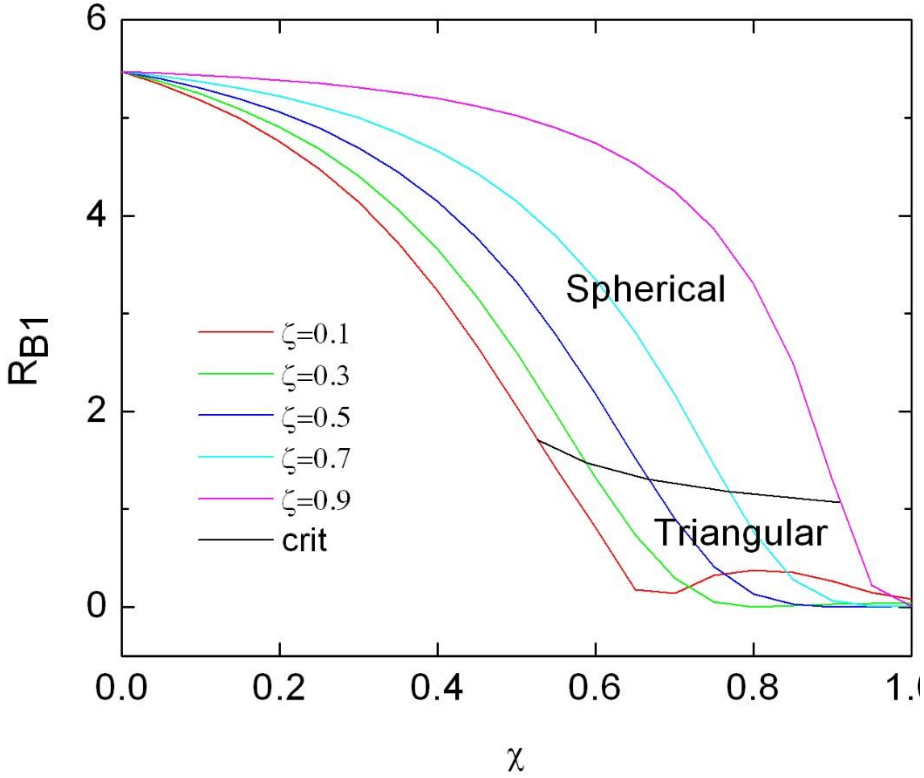
Critical line

$$\chi_c = \frac{1}{2-\zeta}$$

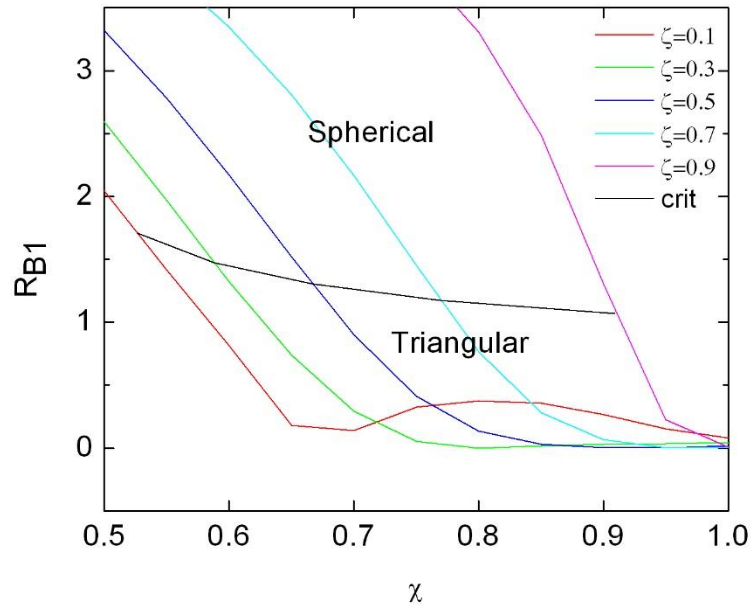


U(6) - SO(7)

Ratio of B(E2) Values



$$^{12}C: \quad \omega_1 \sim \omega_2 \ \Rightarrow \ \zeta \sim \frac{1}{3}$$

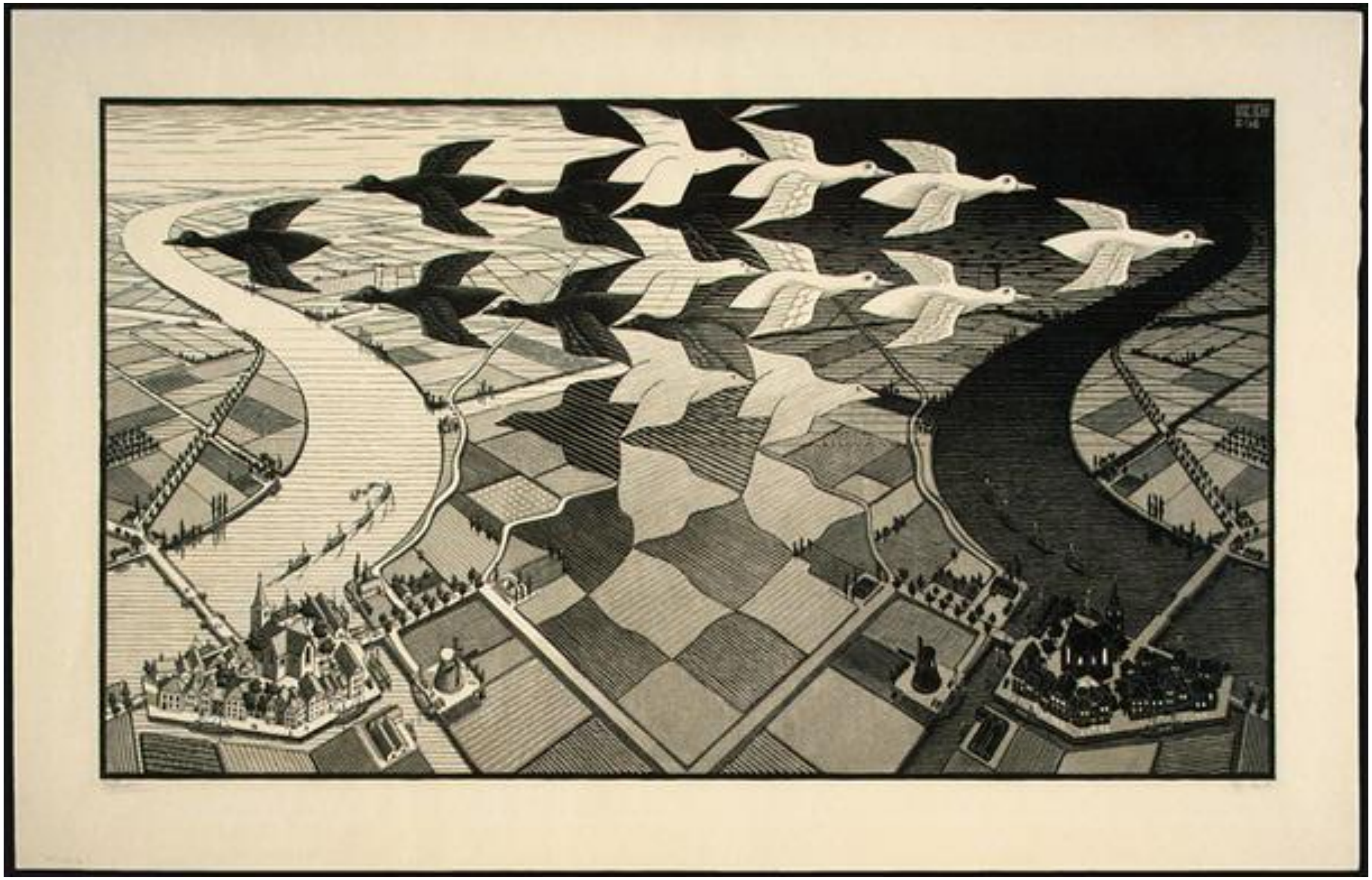


Algebraic Cluster Model

	2α	3α	4α
ACM	$U(4)$	$U(7)$	$U(10)$
Point group	$\mathcal{Z}_2$	$\mathcal{D}_{3h}$	$\mathcal{T}_d$
Geom. conf.	Linear	Triangle	Tetrahedron
Model	Rotor	Oblate top	Spherical top
Vibrations	1	3	6
Rotations	2	3	3
G.s. band	0^+	0^+	0^+
	2^+	2^+	
		3^-	3^-
	4^+	$4^{\pm}$	4^+
		5^-	
	6^+	$6^{\pm+}$	$6^{\pm}$
Large $E\lambda$ electric transitions!			

Summary and Conclusions

- Algebraic Cluster Model
- Discrete and continuous symmetries
- Special solutions: spherical and deformed oscillators, oblate top
- Rotational bands: fingerprints of geometric configurations of alpha particles
- QPT and decay of Hoyle state
- Higher order terms in quadrupole operator?

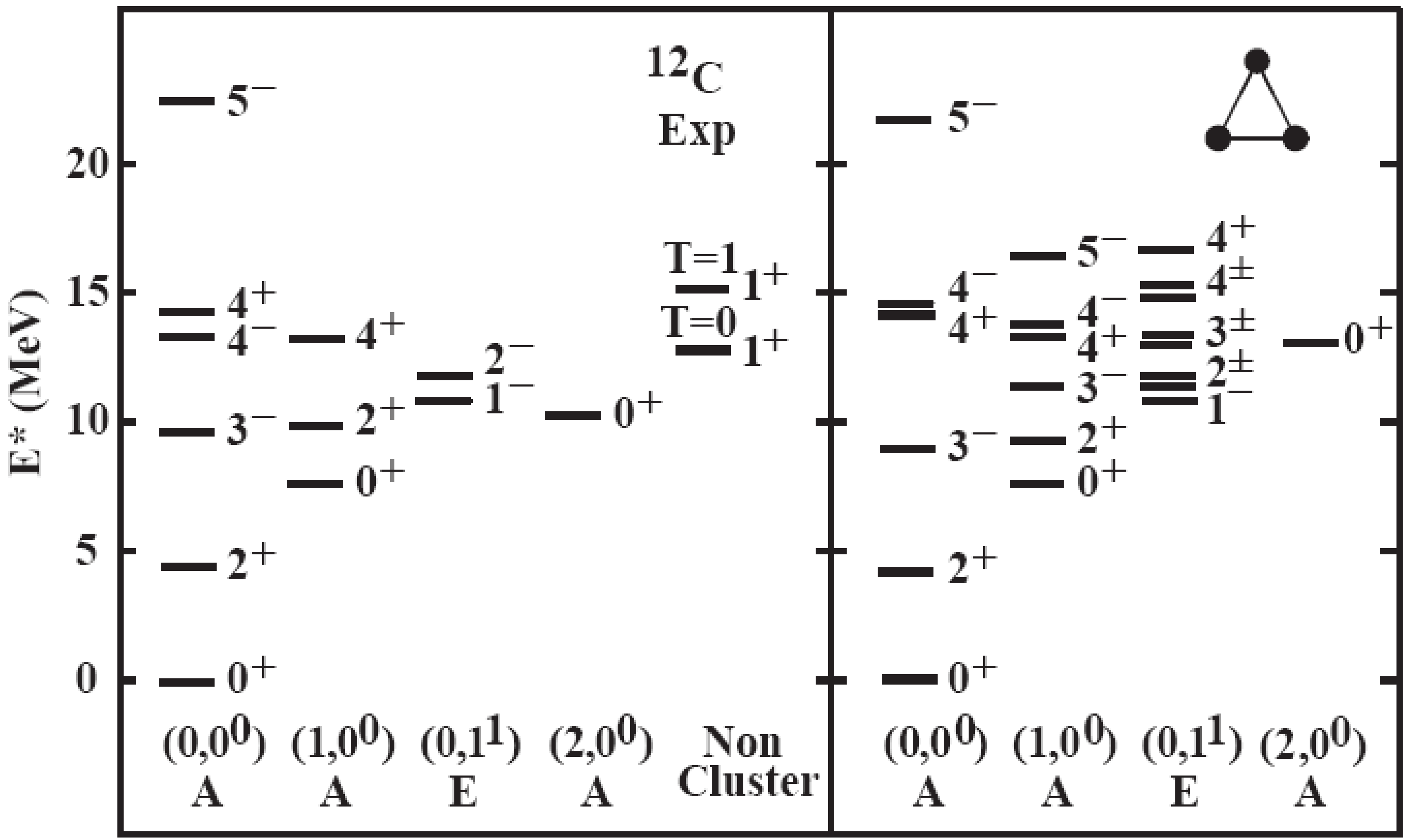


Recent Theoretical Work

- Alpha-cluster model (Wheeler 1937, Brink 1966, Robson, 1978)

- AMD (Kanada-Enyo, PTP, 2007)
- FMD model (Chernykh et al, PRL, 2007)
- BEC-like cluster model (Funaki et al, PRC, 2009)
- Ab initio no-core shell model (Roth et al, PRL, 2011)
- Lattice EFT (Epelbaum et al, PRL, 2011, 2012)
- No-core symplectic model (Dreyfuss et al, PLB, 2013)
- Algebraic Cluster Model (2000, 2002, 2014, 2017)
- and many others

- Recent reviews: Freer & Fynbo, PPNP 78, 1 (2014), Jenkins & Courtin, JPG 42, 034010 (2015)


12C
Exp
E* (MeV)
20
15
10
5
0
5−
4+
4−
3−
2+
0+
4+
2+
0+
2−
1−
0+
T=1
1+
T=0
1+
(0,0^0)
A
(1,0^0)
A
(0,1^1)
E
(2,0^0)
A
Non
Cluster
5−
4−
4+
3−
2+
0+
5−
4−
4+
3−
2+
0+
4+
4±
3±
2±
1−
0+
(0,0^0)
A
(1,0^0)
A
(0,1^1)
E
(2,0^0)
A

Parameters

	β (fm)	α (fm^{-2})	$\langle r^2 \rangle^{1/2}$ (fm)
^{8}Be	1.82^{a}	0.56^{b}	2.45
^{12}C	1.74^{c}	0.52^{d}	2.468 ± 0.012
^{16}O	2.07^{c}	0.60^{d}	2.710 ± 0.015

a	Moment of inertia
b	Form factor α particle
c	Elastic form factor
d	Charge radius

Energy Ratio