

Quartet structure of $N=Z$ nuclei in an IBM
formalism:
 ^{28}Si as a nucleus at the $U(5)-\overline{SU(3)}$
phase-transitional point

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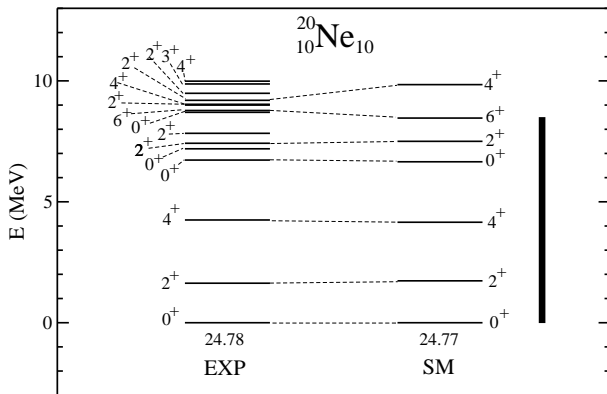
Outline

- ▶ Quartetting and proton-neutron pairing (\Rightarrow Sandulescu's talk)
- ▶ Quartet structure of even-even $N=Z$ nuclei:
 - ▶ a study of ^{24}Mg
- ▶ Bosonic approach to quartetting in even-even $N=Z$ nuclei:
 - ▶ a study of ^{28}Si
- ▶ Conclusions

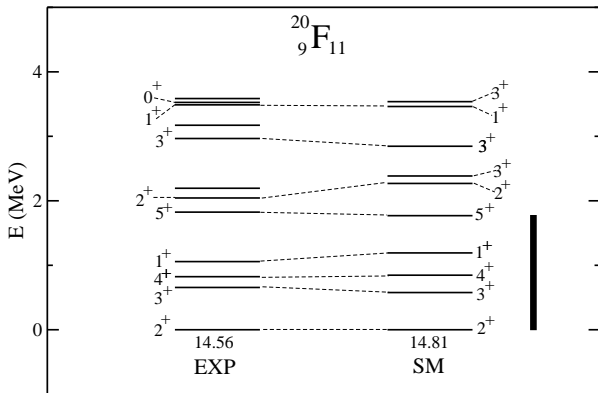
Quartets

$$Q_{\alpha, JM, TT_z}^+ = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2}^{(\alpha)} \times \left[[a_{i_1}^+ a_{j_1}^+]^{J_1 T_1} [a_{i_2}^+ a_{j_2}^+]^{J_2 T_2} \right]_{MT_z}^{JT}$$

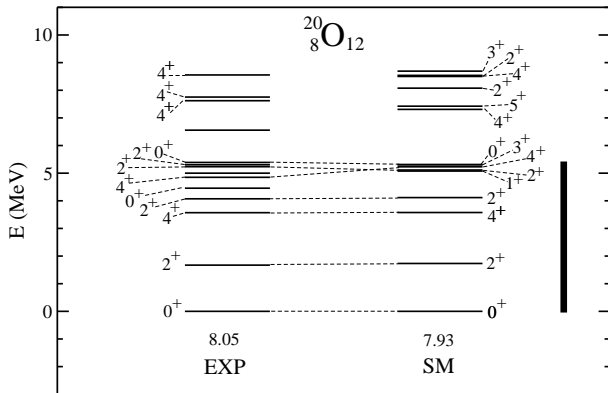
^{20}Ne : $T=0$ quartets



^{20}F : T=1 quartets



^{20}O : T=2 quartets



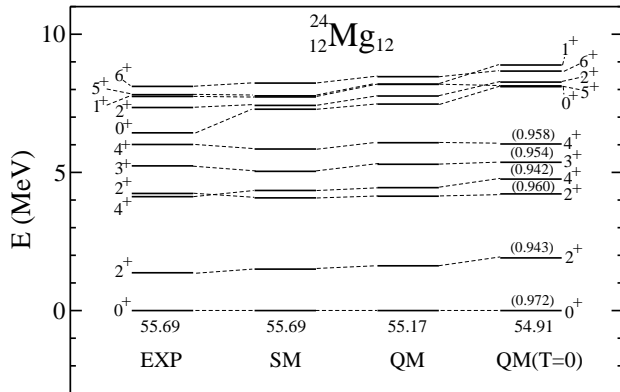
^{24}Mg in a formalism of quartets

$^{24}_{12}\text{Mg}_{12} = 4 \text{ protons} + 4 \text{ neutrons outside the } ^{16}\text{O} \text{ core} = 2 \text{ quartets}$

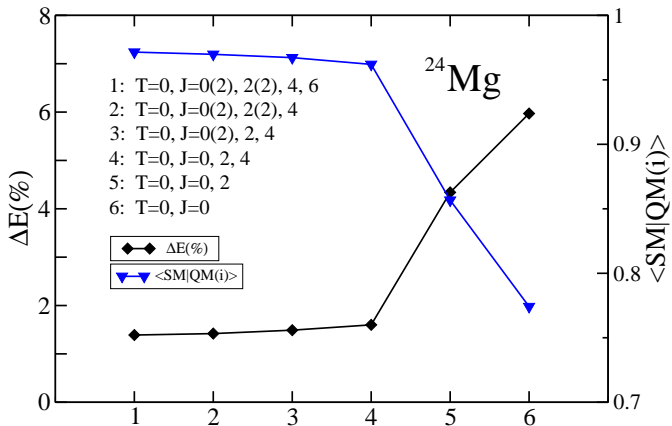
We want to represent its states as linear superpositions of

$$[Q_{\alpha J_1 T_1}^+ Q_{\beta J_2 T_2}^+]_{M, T_z=0}^{J, T=0} |0\rangle$$

^{24}Mg : the spectrum



^{24}Mg : the ground state



Bosonic approach to quartetting in even-even $N=Z$ nuclei

- ▶ We assume two basic building-blocks:

$$Q_{J=0, T=0}^+ \equiv S^+ \quad Q_{J=2, T=0}^+ \equiv D^+$$

- ▶ We replace them with two elementary bosons:

$$S^+ \quad \Longrightarrow \quad s^+$$

$$D^+ \quad \Longrightarrow \quad d^+$$

- ▶ We construct the most general one- plus two-body Hamiltonian

$$H_B = \epsilon_s \hat{n}_s + \epsilon_d \hat{n}_d + \sum_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2 \Lambda} V_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2}^{(\Lambda)} [[b_{\lambda_1}^+ b_{\lambda_2}^+]^{(\Lambda)} [\tilde{b}_{\lambda'_1} \tilde{b}_{\lambda'_2}]^{(\Lambda)}]^{(0)}$$

- ▶ It is: $H_B \equiv H_B^{(IBM)}$ but $N_B \equiv \frac{N_B^{(IBM)}}{2}$
- ▶ Previous use of this formalism:
J. Dukelsky et al, Phys. Lett. 115B, 359 (1982)

Parameters of H_B from a mapping procedure

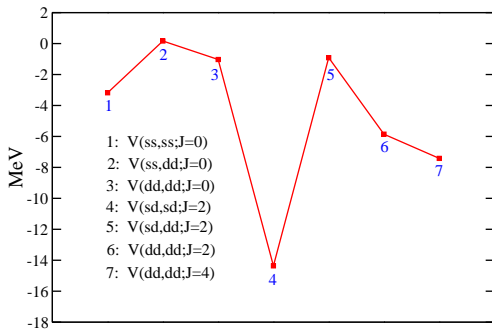
- ▶ H_B (one- plus two-body, hermitian) is generated from $H_F(USDB)$

- ▶ Single-boson energies ϵ_s and ϵ_d :

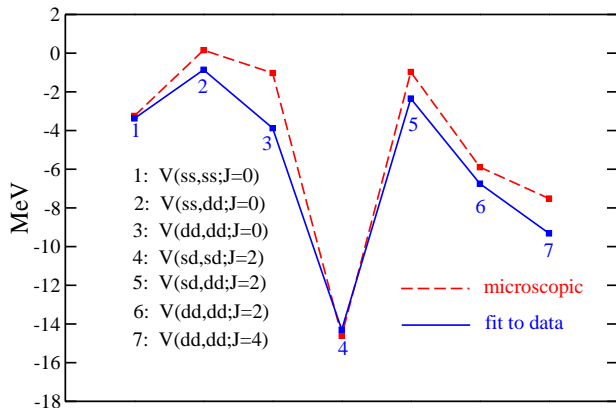
$$\epsilon_s = \langle Q_{J=0, T=0} | H_F | Q_{J=0, T=0} \rangle$$

$$\epsilon_d = \langle Q_{J=2, T=0} | H_F | Q_{J=2, T=0} \rangle$$

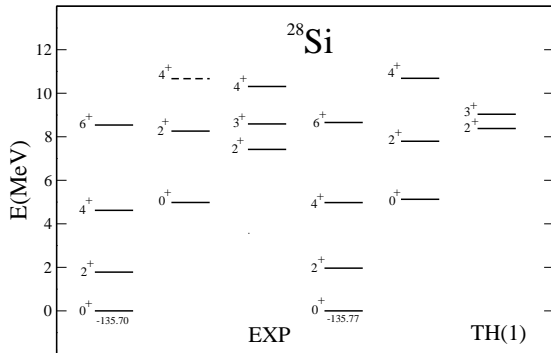
- ▶ Two-body matrix elements:



Parameters of H_B : microscopic vs phenomenological



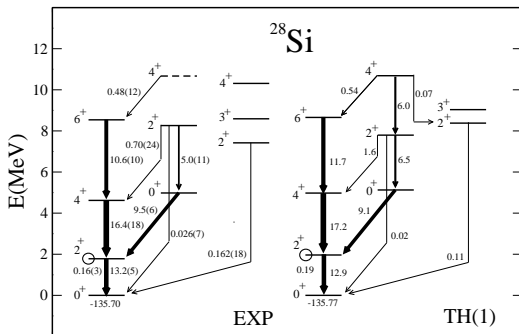
^{28}Si : spectrum



(EXP: R.K. Sheline et al., Phys. Lett. B 119, 263 (1982))

^{28}Si : spectrum and E2 transitions

$$T_{\mu}^{(E2)} = e_B([d_{\mu}^{\dagger}s + s^{\dagger}\tilde{d}_{\mu}]^2 + \chi[d^{\dagger}\tilde{d}]_{\mu}^2)$$

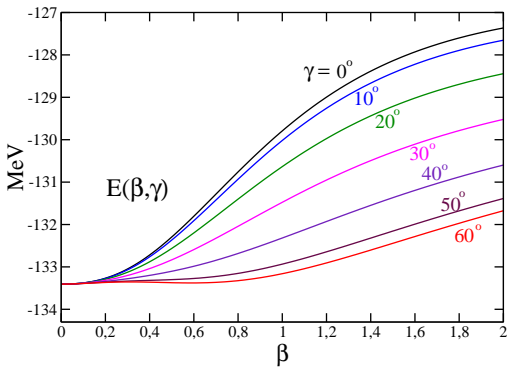


^{28}Si : geometric structure

$$|N; \beta, \gamma\rangle = \frac{1}{\sqrt{N!(1+\beta^2)^N}} (B^\dagger)^N |0\rangle$$

$$B^\dagger = s^\dagger + \beta[\cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\sin\gamma (d_{+2}^\dagger + d_{-2}^\dagger)]$$

$$E(N, \beta, \gamma) = \langle N; \beta, \gamma | H_B | N; \beta, \gamma \rangle$$

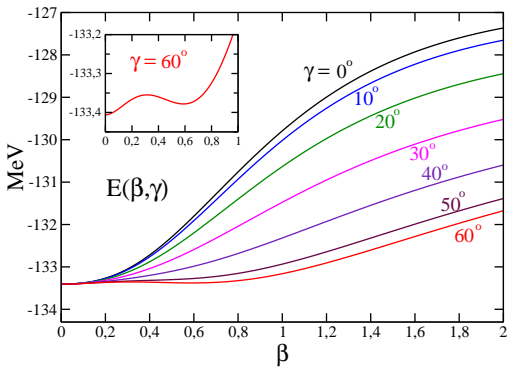


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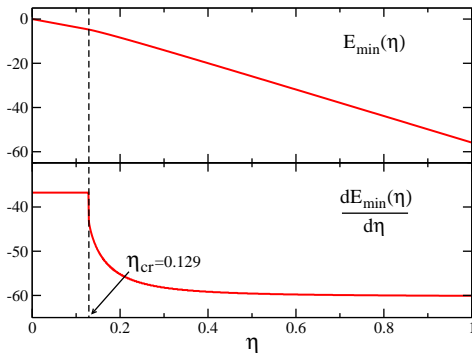


A schematic $U(5)\text{-}\overline{SU(3)}$ Hamiltonian

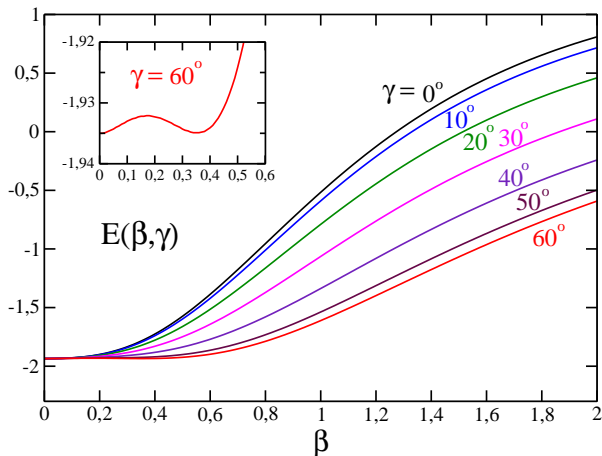
$$H_B^{(T)} = (1 - \eta)\hat{n}_d - \eta(Q^\dagger \cdot Q^\dagger)$$

$$Q^\dagger = [d^\dagger s + s^\dagger \tilde{d}]^{(2)} + \chi [d^\dagger \tilde{d}]^{(2)}, \quad \chi = +\frac{\sqrt{7}}{2}$$

First-order phase transition at $\eta = \eta_{cr}$

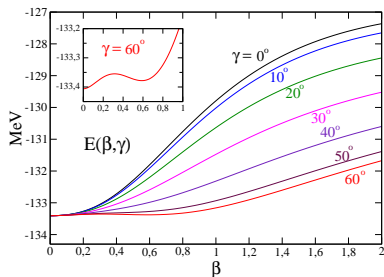


Potential energy surface at $\eta = \eta_{cr}$

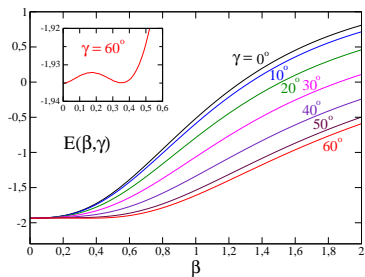


Comparison of potential energy surfaces

H_B



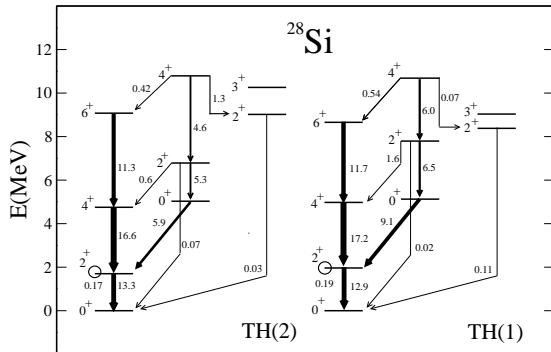
$H_B(\text{U}(5)\text{-}\overline{\text{SU}}(3))$ at $\eta = \eta_{cr}$



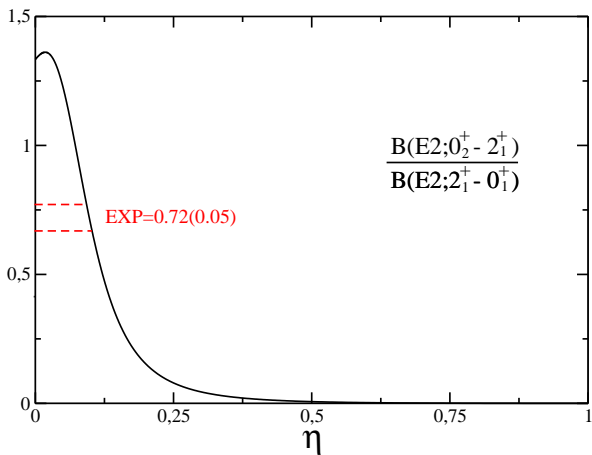
Comparison of theoretical spectra

TH(1): H_B

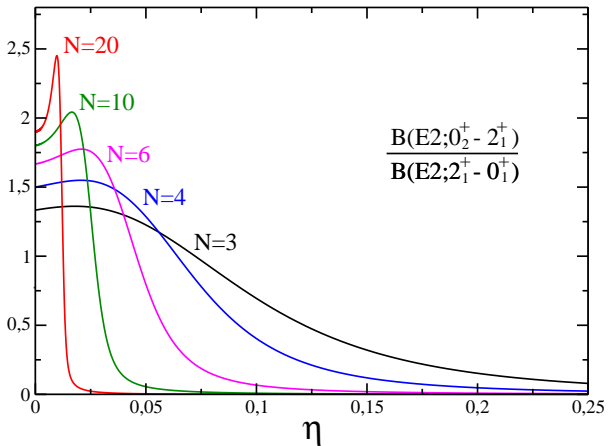
TH(2): $H_B(\text{U}(5)\text{-}\overline{\text{SU}}(3))$ at $\eta = \eta_{cr}$



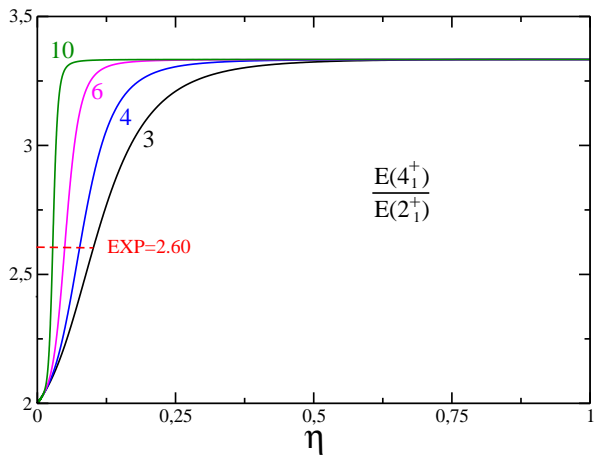
$$B(E2; 0_2^+ - 2_1^+)/B(E2; 2_1^+ - 0_1^+)$$



$$B(E2; 0_2^+ - 2_1^+)/B(E2; 2_1^+ - 0_1^+)$$



$$E(4_1^+)/E(2_1^+)$$

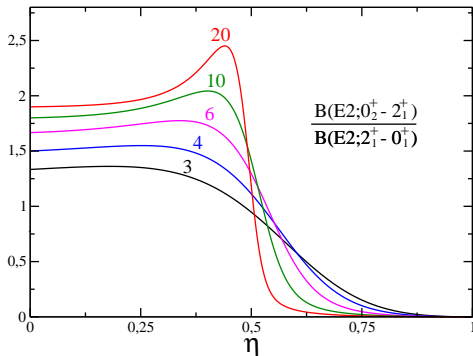


Conclusions

- ▶ We have presented a general approach to describe $N=Z$ nuclei in an IBM formalism where s and d bosons represent $T = 0, J = 0$ and $T = 0, J = 2$ quartets.
- ▶ We have provided a microscopic justification for this model.
- ▶ We have shown an application for ^{28}Si .
- ▶ An analysis of the potential energy surface has placed ^{28}Si at the $U(5)-\overline{SU(3)}$ phase-transitional point.
- ▶ Precursors of a phase transition in ^{28}Si have been evidenced.

$$B(E2; 0_2^+ - 2_1^+)/B(E2; 2_1^+ - 0_1^+)$$

$$H_B^{(T)} = (1 - \eta)\hat{n}_d - \frac{\eta}{4N}(Q^\dagger \cdot Q^\dagger)$$



$$E(4_1^+)/E(2_1^+)$$

$$H_B^{(T)} = (1 - \eta)\hat{n}_d - \frac{\eta}{4N}(Q^\dagger \cdot Q^\dagger)$$

