Higher-rank discrete symmetries: Octahedral shapes in *sdg*-IBM

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Motivation

Quadrupole and hexadecapole shapes

The *sdg*-IBM

Octahedral shapes in *sdg*-IBM

Conclusion

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Motivation

How to realize higher-rank discrete symmetries in the context of an algebraic model.

Derive the necessary conditions on the parameters of the Hamiltonian.

Example: Octahedral shape in *sdg*-IBM.

4-pole and 16-pole shapes

Nuclear surface with quadrupole and hexadecapole deformation:

$$R(\theta,\phi) = R_0 \Big(1 + a_{20} Y_{20}(\theta,\phi) + a_{22}\left[Y_{2-2}(\theta,\phi) + Y_{2+2}(\theta,\phi)\right]$$

$$+ a_{40} Y_{40}(\theta,\phi) + a_{42}\left[Y_{4-2}(\theta,\phi) + Y_{4+2}(\theta,\phi)\right]$$

$$+ a_{44}\left[Y_{4-4}(\theta,\phi) + Y_{4+4}(\theta,\phi)\right]\Big)$$

4-pole and 16-pole shapes

4-pole variables ($\beta_2 \geq 0, 0 \leq \gamma_2 \leq \pi/3$):

$$a_{20} = \beta_2 \cos\gamma_2$$

$$a_{22} = \beta_2 \sin\gamma_2$$

16-pole variables ($\beta_4 \geq 0, 0 \leq \gamma_4 \leq \pi/3, 0 \leq \delta_4 \leq \pi$):

$$a_{40} = \beta_4 \left(\sqrt{\tfrac{7}{12}} \cos\delta_4 + \sqrt{\tfrac{5}{12}} \sin\delta_4 \cos\gamma_4 \right)$$

$$a_{42} = -\sqrt{\tfrac{1}{2}} \beta_4 \sin\delta_4 \sin\gamma_4$$

$$a_{44} = \beta_4 \left(\sqrt{\tfrac{5}{24}} \cos\delta_4 - \sqrt{\tfrac{7}{24}} \sin\delta_4 \cos\gamma_4 \right)$$

S.G. Rohoziński and A. Sobiczewski, Acta Phys. Pol. B **12** (1981) 1001

Octahedral symmetry

A shape with octahedral symmetry is obtained for

$$a_{20} = a_{22} = 0, \quad a_{42} = 0, \quad a_{44} / a_{40} = \pm\sqrt{5/14}$$

Three cases in the alternative parameterisation:

$$\text{(a) octahedron} : \beta_2 = 0, \gamma_2 \text{ free}, \beta_4 > 0, \gamma_4 \text{ free}, \delta_4 = 0$$

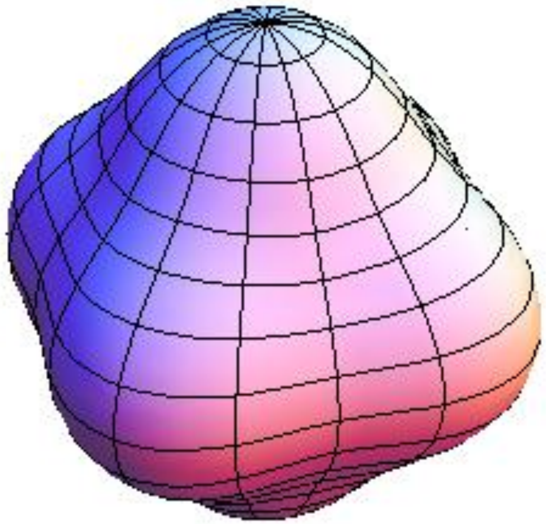
$$\text{(b) cube} : \beta_2 = 0, \gamma_2 \text{ free}, \beta_4 > 0, \gamma_4 \text{ free}, \delta_4 = \pi$$

$$\text{(c) octahedron} : \beta_2 = 0, \gamma_2 \text{ free}, \beta_4 > 0, \gamma_4 = 0, \delta_4 = \arccos\left(\tfrac{1}{6}\right)$$

J. Dudek *et al.*, Acta Phys. Pol. B **34** (2003) 2491

Octahedral symmetry

(a) octahedron

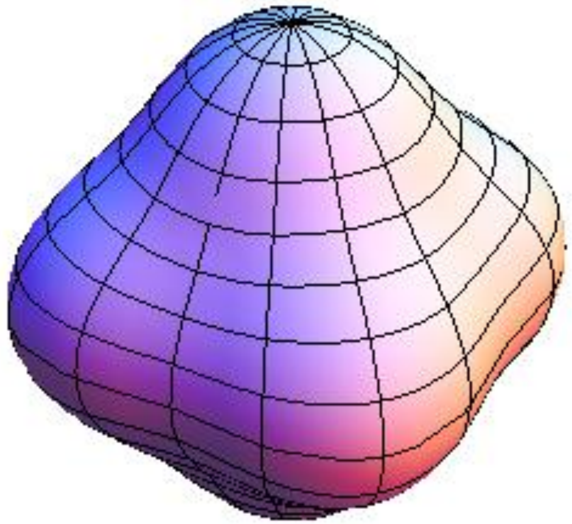


$$a_{40} = +0.2, \quad a_{42} = 0, \quad a_{44} / a_{40} = +\sqrt{5/14}$$

$$\beta_4 = 0.2, \quad \delta_4 = 0$$

Octahedral symmetry

(c) octahedron

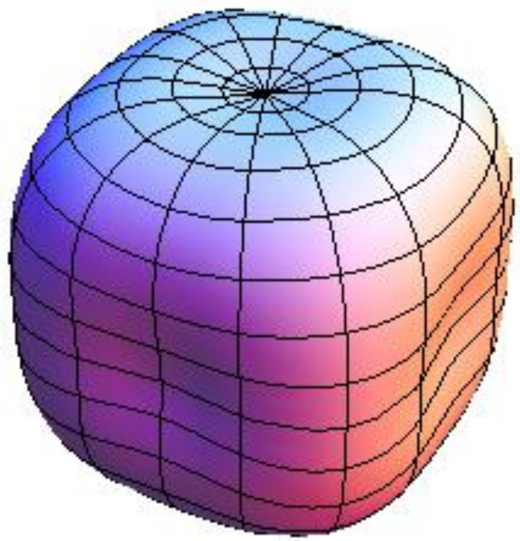


$$a_{40} = +0.2, \quad a_{42} = 0, \quad a_{44} / a_{40} = -\sqrt{5/14}$$

$$\beta_4 = 0.2, \quad \delta_4 = \arccos(1/6)$$

Octahedral symmetry

(b) cube

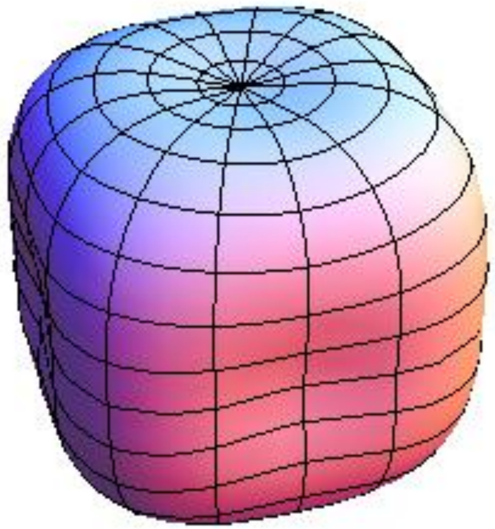


$$a_{40} = -0.2, \quad a_{42} = 0, \quad a_{44} / a_{40} = +\sqrt{5/14}$$

$$\beta_4 = 0.2, \quad \delta_4 = \pi$$

Octahedral symmetry

(d) cube



$$a_{40} = -0.2, \quad a_{42} = 0, \quad a_{44} / a_{40} = -\sqrt{5/14}$$

not possible

Hamiltonian of the *sdg*-IBM

Hamiltonian with up to 2-body interactions:

$$\hat{H}_{sdg} = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \varepsilon_g \hat{n}_g$$

$$+ \sum_{l_1 \le l_2 l_1' \le l_2' L} \frac{(-)^L \upsilon^L_{l_1 l_2 l_1' l_2'}}{\sqrt{\left(1+\delta_{l_1 l_2}\right)\left(1+\delta_{l_1' l_2'}\right)}} \left(b^+_{l_1} \times b^+_{l_2}\right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'}\right)^{(L)}$$

H_{sdg} contains 3 single-boson energies and 32 two-body boson-boson interactions

$$\varepsilon_l = \langle l | \hat{H}_{sdg} | l \rangle$$

$$\upsilon^L_{l_1 l_2 l_1' l_2'} = \langle l_1 l_2; L | \hat{H}_{sdg} | l_1' l_2'; L \rangle$$

Classical limit of the *sdg*-IBM

Two quadrupole (β_2,γ_2) and three hexadecapole $(\beta_4,\gamma_4,\delta_4)$ shape parameters.

Coherent state:

$$\left|N;\beta_2,\gamma_2,\beta_4,\gamma_4,\delta_4\right\rangle \propto$$

$$\Big\{s^+ + \beta_2\left[\cos\gamma_2\, d_0^+ + \sqrt{\tfrac{1}{2}}\sin\gamma_2\left(d_{-2}^+ + d_{+2}^+\right)\right]$$

$$+\beta_4\Big[\left(\sqrt{\tfrac{7}{12}}\cos\delta_4 + \sqrt{\tfrac{5}{12}}\sin\delta_4\cos\gamma_4\right)g_0^+$$

$$-\sqrt{\tfrac{1}{2}}\sin\delta_4\sin\gamma_4\left(g_{-2}^+ + g_{+2}^+\right)$$

$$+\left(\sqrt{\tfrac{5}{24}}\cos\delta_4 - \sqrt{\tfrac{7}{24}}\sin\delta_4\cos\gamma_4\right)\left(g_{-4}^+ + g_{+4}^+\right)\Big]\Big\}^N\left|\mathrm{o}\right\rangle$$

Classical limit of the *sdg*-IBM

Classical limit of H_{sdg}

$$\left\langle \hat{H}_{sdg} \right\rangle \equiv E\left(\beta_2,\gamma_2,\beta_4,\gamma_4,\delta_4;\boldsymbol{c}\right)$$

$$= N\frac{\varepsilon_s+\varepsilon_d\beta_2^2+\varepsilon_g\beta_4^2}{1+\beta_2^2+\beta_4^2}$$

$$+\frac{N\left(N-1\right)}{\left(1+\beta_2^2+\beta_4^2\right)^2}\sum_{kl}\beta_2^k\beta_4^l\left[c_{kl}+\sum_{ij}c_{kl}^{ij}\cos\left(i\gamma_2+j\gamma_4\right)\phi_{kl}^{ij}\left(\delta_4\right)\right]$$

in terms of trigonometric functions $\Phi(\delta_4)$ and with 22 coefficients c depending on interactions υ.

Catastrophe analysis

Energy surface has extremum at *p** if

$$\left.\frac{\partial E}{\partial \beta_2}\right|_{p^*} = \left.\frac{\partial E}{\partial \gamma_2}\right|_{p^*} = \left.\frac{\partial E}{\partial \beta_4}\right|_{p^*} = \left.\frac{\partial E}{\partial \gamma_4}\right|_{p^*} = \left.\frac{\partial E}{\partial \delta_4}\right|_{p^*} = 0$$

Shape of extremum is octahedral [(a), upper sign] or cubic [(b), lower sign] if

$$\beta_4\Big[-4c'_{00} + 2c'_{02} \pm 21 c^{00}_{03}\beta_4 - \left(2c'_{02} - 4c'_{04} - 76 c^{00}_{04}\right)\beta_4^2 \mp 7 c^{00}_{03}\beta_4^3\Big] = 0$$

A real solution $\beta_4 > 0$ always exists except for

$$c^{00}_{03} = \frac{2}{\sqrt{429}}\upsilon^4_{sg\cdot gg} = 0$$

Catastrophe analysis

Shape of extremum is octahedral (c) if

$$\beta_4^2\left[-3\left(\sqrt{35}c_{12}^{1-1}+35c_{12}^{12}\right)+\left(\sqrt{35}c_{13}^{1-1}-5c_{13}^{12}\right)\beta_4\right]=0$$

$$\beta_4\left[324\left(-2c'_{00}+c'_{02}\right)+63\left(4c_{03}^{00}+5\sqrt{35}c_{03}^{03}\right)\right.$$

$$+4\left(-81c'_{02}+162c'_{04}+1853c_{04}^{00}+35\sqrt{35}c_{04}^{03}\right)\beta_4^2$$

$$\left.-21\left(4c_{03}^{00}+5\sqrt{35}c_{03}^{03}\right)\beta_4^3\right]=0$$

$$\beta_4^3\left[9\left(-2\sqrt{35}c_{03}^{00}+7c_{03}^{03}\right)+224\left(\sqrt{35}c_{04}^{00}-c_{04}^{03}\right)\beta_4\right]=0$$

First and third conditions are identically satisfied; second reduces to previous one (upper sign).

Catastrophe analysis

Sufficient conditions that the *extremum* with octahedral shape is a *minimum*

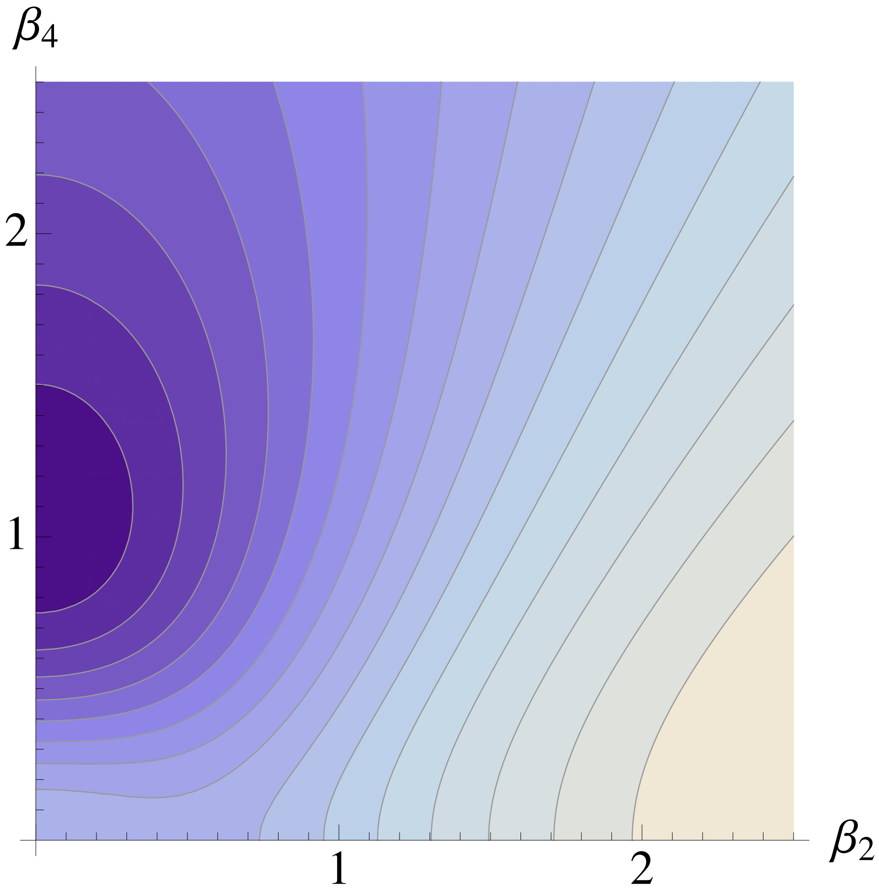
$$c_{02} + \frac{\varepsilon_g}{N-1} \leq 2c_{00} \leq c_{20} + \frac{\varepsilon_d}{N-1}$$

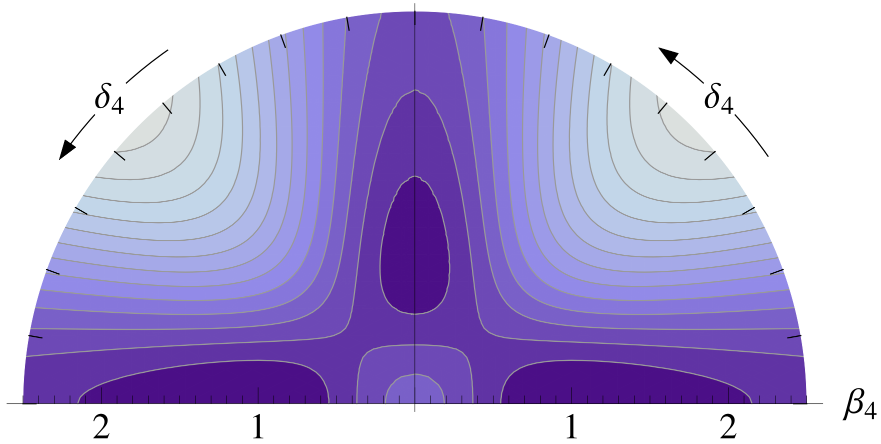
$$c_{02} \leq 2c_{04} + 38c_{04}^{00} + \frac{\varepsilon_g}{N-1} \leq c_{22} + \frac{\varepsilon_d}{N-1}$$

Reduce to conditions on interactions, e.g.

$$\tfrac{1}{3}\upsilon^0_{ss\cdot gg} + \upsilon^4_{sgsg} + \frac{\varepsilon_g}{N-1} \leq \upsilon^0_{ssss} \leq \sqrt{\tfrac{1}{5}}\upsilon^0_{ss\cdot dd} + \upsilon^2_{sdsd} + \frac{\varepsilon_d}{N-1}$$

Example





Conclusion

A procedure to treat higher-rank discrete symmetries in an algebraic framework.

Analysis completed for octahedral symmetry in the *sdg*-IBM:

➔ *s-g mixing strong enough compared to* $\varepsilon_g - \varepsilon_s$ *to develop a* $\beta_4 > 0$ *minimum;*

➔ *s-d mixing weak enough compared to* $\varepsilon_d - \varepsilon_s$ *not to develop a* $\beta_2 > 0$ *minimum.*

Analysis of tetrahedral case can be done similarly.