

Nuclear Tetrahedral and Octahedral Symmetries: Research Lines after the First Identified Case^{*)}

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^{*)} *Non-alpha-cluster – heavy ‘mean-field’ nucleus – ^{152}Sm*

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Motto:

**Symmetries
determine the variety of islands of stability
of Atomic Nuclei**

Before starting – a few remarks:

**The year 2018 marks the 15th anniversary of
the TetraNuc Project and Collaboration**

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A part of the following presentation is based on a recent article
PHYSICAL REVIEW C 97, 021302(R) (2018)

Spectroscopic criteria for identification of nuclear tetrahedral and octahedral symmetries: Illustration on a rare earth nucleus

Focus on Tetrahedral and Octahedral Symmetries

or

How to Establish Their Presence in Subatomic Physics

Tetrahedral Symmetry: General Representation

Only special combinations of spherical harmonics may form a basis for surfaces with tetrahedral symmetry and only odd-order except 5

Three Lowest Order Solutions:

Rank \leftrightarrow Multipolarity λ

$$\lambda = 3 : \quad \alpha_{3,\pm 2} \equiv t_3$$

$$\lambda = 5 : \quad \text{no solution possible}$$

$$\lambda = 7 : \quad \alpha_{7,\pm 2} \equiv t_7; \quad \alpha_{7,\pm 6} \equiv -\sqrt{\frac{11}{13}} \cdot t_7$$

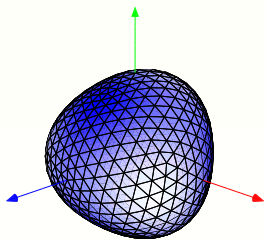
$$\lambda = 9 : \quad \alpha_{9,\pm 2} \equiv t_9; \quad \alpha_{9,\pm 6} \equiv +\sqrt{\frac{28}{198}} \cdot t_9$$

- Problem presented in detail in:

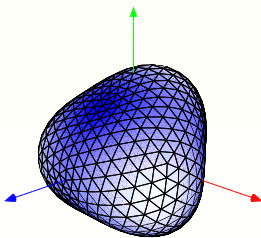
JD, J. Dobaczewski, N. Dubray, A. Gózdź, V. Pangon and N. Schunck,
Int. J. Mod. Phys. E16, 516 (2007) [516-532].

Nuclear Tetrahedral Shapes – 3D Examples

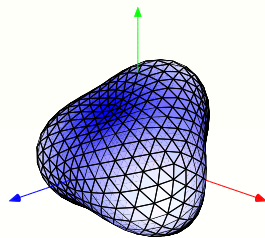
Illustrations below show the tetrahedral-symmetric surfaces at three increasing values of rank $\lambda = 3$ deformations α_{32} : 0.1, 0.2 and 0.3



$$\alpha_{32} \equiv t_3 = 0.1$$



$$\alpha_{32} \equiv t_3 = 0.2$$



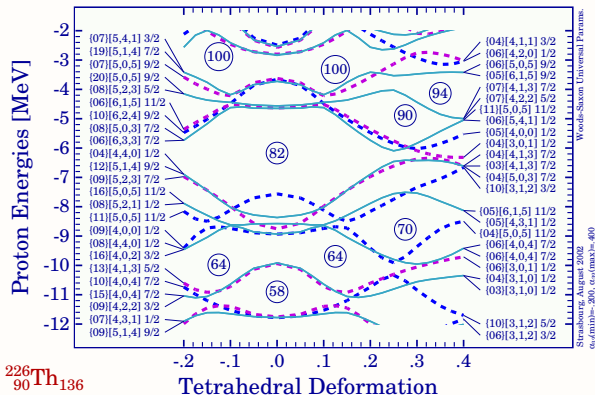
$$\alpha_{32} \equiv t_3 = 0.3$$

Observations:

- There are infinitely many tetrahedral-symmetric surfaces
- Nuclear 'pyramids' do not resemble pyramids very much!

Nuclear Tetrahedral Shapes - Proton Spectra

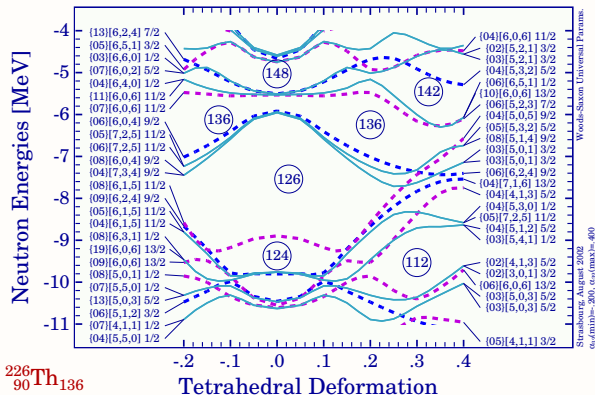
Double group T_d^D has two 2-dimensional - and one 4-dimensional irreducible representations: Three distinct families of nucleon levels



Full lines \leftrightarrow 4-dimensional irreducible representations - marked with double Nilsson labels. Observe huge gaps at N=64, 70, 90-94, 100.

Nuclear Tetrahedral Shapes - Neutron Spectra

Double group T_d^D has two 2-dimensional - and one 4-dimensional irreducible representations: Three distinct families of nucleon levels



Full lines \leftrightarrow 4-dimensional irreducible representations - marked with double Nilsson labels. Observe huge gaps at N=112, 136.

First Goal: Obtain Tetrahedral Magic Numbers

- After inspecting many single-particle diagrams as functions of tetrahedral deformation we read-out all magic numbers (Z_t, N_t)

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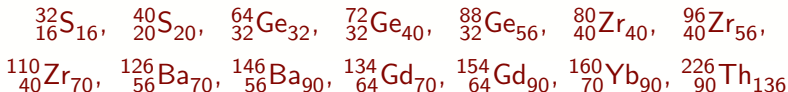
$$\{Z_t, N_t\} = \{16, 20, 32, 40, 56, 64, 70, 90, 136\}$$

First Goal: Obtain Tetrahedral Magic Numbers

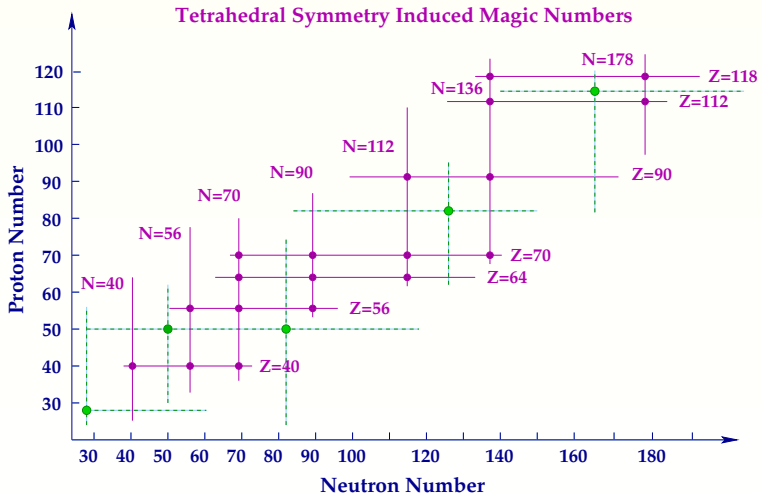
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- ... and more precisely around the following nuclei:

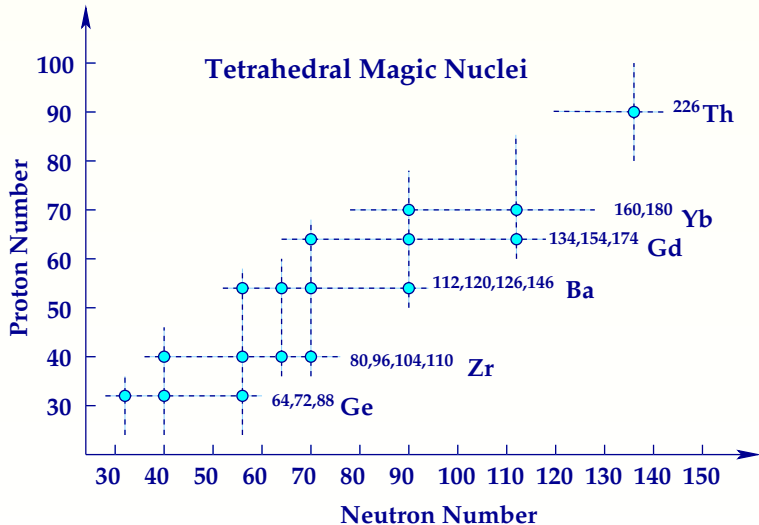


Tetrahedral Symmetry Can Be Present Many Nuclei



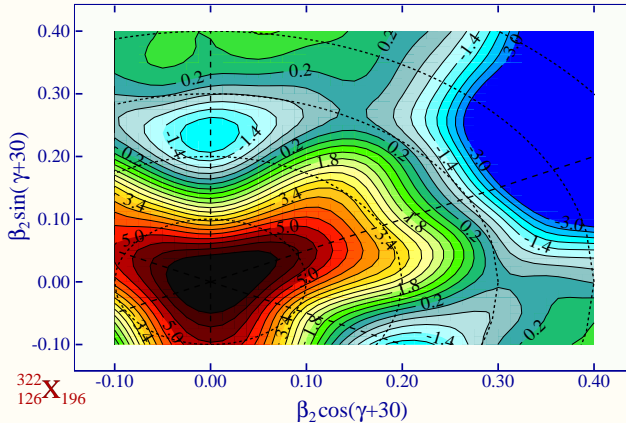
Observe that we have here only 5 spherical doubly-magic nuclei and 19 tetrahedral doubly-magic nuclei, nearly 4 times more

Tetrahedral Symmetry Can Be Present Many Nuclei



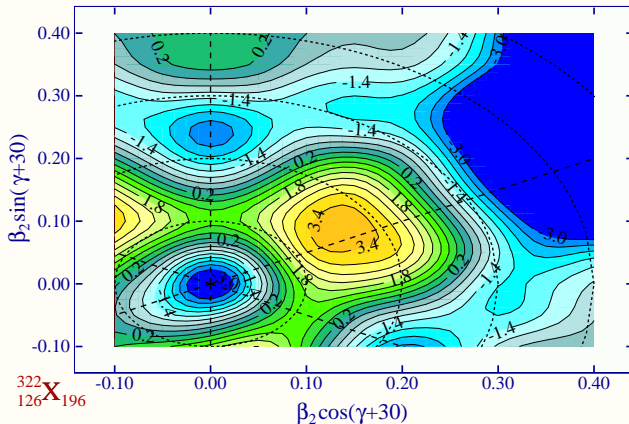
It may be instructive to think about this diagram when discussing, among others, the r-process

Symmetry Concepts Impact Our Ideas about Stability



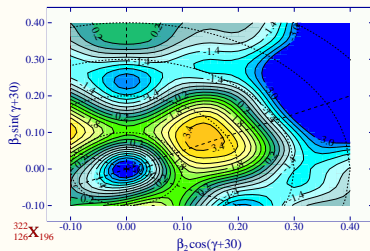
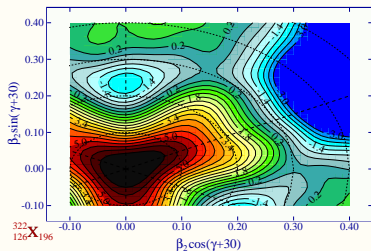
- Consider a total energy for a super-heavy nucleus in the form of the standard (β, γ) -representation

Symmetry Concepts Impact Our Ideas about Stability



- Consider the similar standard (β, γ) -representation but now let us introduce an *extra minimisation over the tetrahedral deformation*

These Concepts Change Our Ideas about Stability



- *The mechanism discussed may provide new challenges for the exotic nuclei projects: Observe a qualitative change of the landscape*
- *Totally different fission barriers - thus experimental search criteria*
- *The ground-state expected to be otherwise quadrupole deformed may obtain e.g. zero-quadrupole and non-zero-tetrahedral geometry*

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- Therefore we decided to focus first on the nuclei which can be populated with a big number of nuclear reactions since we may expect that - in such nuclei - the states sought exist in the literature
- We have verified that the nucleus ^{152}Sm can be produced by about 25 nuclear reactions, whereas surrounding nuclei can be produced typically with about a dozen but usually much fewer reactions only

**It will be instructive at this point to recall
some elementary theorems
from the group representation theory**

Elementary Group-Theory Properties

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- Let $\{D_i, i = 1, 2, \dots, M\}$ be the irreducible representations of G
- The representation $D^{(I\pi)}$ of the rotor states with the definite spin-parity $I\pi$, can be decomposed in terms of D_i with multiplicities $a_i^{(I\pi)}$:

$$D^{(I\pi)} = \sum_{i=1}^M a_i^{(I\pi)} D_i$$

- Multiplicities [M. Hamermesh, *Group Theory*, 1962] are given by:

$$a_i^{(I\pi)} = \frac{1}{N_G} \sum_{R \in G} \chi_{I\pi}(R) \chi_i(R) = \frac{1}{N_G} \sum_{\alpha=1}^M g_{\alpha} \chi_{I\pi}(R_{\alpha}) \chi_i(R_{\alpha});$$

N_G =order of the group G ; $\{\chi_{I\pi}(R), \chi_i(R)\}$ =characters of $\{D^{(I\pi)}, D_i\}$
 R =group element; g_{α} =the number of elements in the class α , whose representative element is R_{α} .

Elementary T_d -Group-Theory Properties

- Tetrahedral group has 5 irreducible representations and 5 classes
- The representative elements $\{R\}$ are: E , $C_2 (= S_4^2)$, C_3 , σ_d , S_4
- The characters of irreducible representation of T_d are listed below

T_d	E	$C_3(8)$	$C_2(3)$	$\sigma_d(2)$	$S_4(6)$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
$F_1(T_1)$	3	0	-1	-1	1
$F_2(T_2)$	3	0	-1	1	-1

- The characters $\chi_{I\pi}(R_\alpha)$ for the rotor representations are as follows:

$$\chi_{I\pi}(E) = 2I+1, \quad \chi_{I\pi}(C_n) = \sum_{K=-I}^I e^{\frac{2\pi K}{n}i}, \quad \chi_{I\pi}(\sigma_d) = \pi \times \chi_{I\pi}(C_2), \quad \chi_{I\pi}(S_4) = \pi \times \chi_{I\pi}(C_4)$$

- From these relations we obtain 'employing the pocket calculator':

$$a_i^{(I\pi)} = \frac{1}{N_G} \sum_{\alpha=1}^M g_\alpha \chi_{I\pi}(R_\alpha) \chi_i(R_\alpha) \leftrightarrow a_{A_1}^{(I\pm)} = a_{A_2}^{(I\mp)}, \quad a_E^{(I+)} = a_E^{(I-)}, \quad a_{F_1}^{(I\pm)} = a_{F_2}^{(I\mp)}$$

T_d -Group-Theory Properties: User's Instructions

- The number of states $a_i^{(I\pi)}$ within five irreducible representations. If $a_i^{(I\pi)} = 0 \rightarrow$ states not allowed; $a_i^{(I\pi)} = 2 \rightarrow$ doubly degenerate

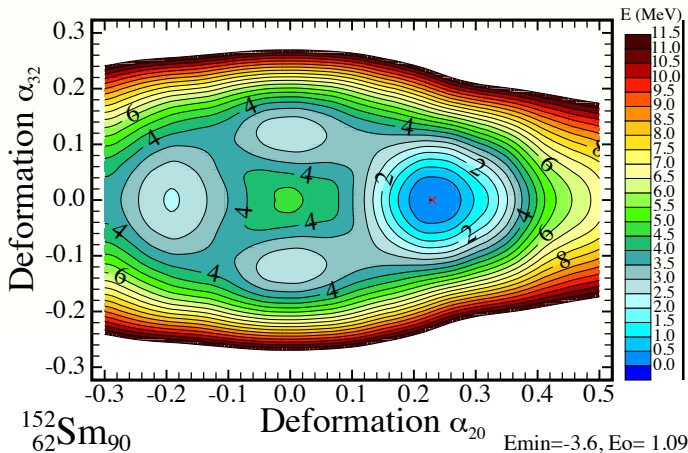
I^+	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+	10^+
A_1	1	0	0	0	1	0	1	0	1	1	1
A_2	0	0	0	1	0	0	1	1	0	1	1
E	0	0	1	0	1	1	1	1	2	1	2
$F_1(T_1)$	0	1	0	1	1	2	1	2	2	3	2
$F_2(T_2)$	0	0	1	1	1	1	2	2	2	2	3

I^-	0^-	1^-	2^-	3^-	4^-	5^-	6^-	7^-	8^-	9^-	10^-
A_1	0	0	0	1	0	0	1	1	0	1	1
A_2	1	0	0	0	1	0	1	0	1	1	1
E	0	0	1	0	1	1	1	1	2	1	2
$F_1(T_1)$	0	0	1	1	1	1	2	2	2	2	3
$F_2(T_2)$	0	1	0	1	1	2	1	2	2	3	2

- In this way we find the spin-parity sequence for A_1 -representation

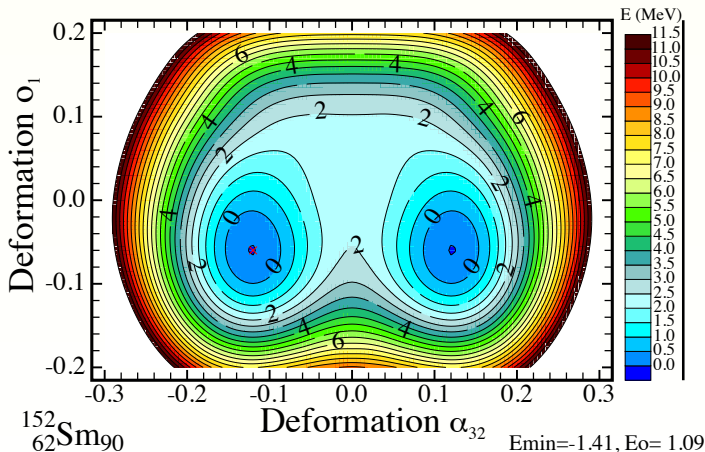
$$A_1 : 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2 \times 12^+, 12^-, \dots$$

Tetrahedral Symmetry Effect



Observe the presence of well defined tetrahedral minima at $\alpha_{32} \approx \pm 0.12$

Combined Octahedral and Tetrahedral Symmetry Effect



Allowing for octahedral deformation lowers the tetrahedral minimum by 2 MeV

Group-Theory Criteria Need Extension: T_d and O_h

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T_d – g.s. band : $0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, \dots$

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- Coexistence with the octahedral symmetry component implies that the positive-parity & negative-parity sequences form two bands
- We will revisit the group-theory criteria and compare T_d and O_h

Quantum Rotors: Tetrahedral vs. Octahedral

- The **tetrahedral** symmetry group has 5 irreducible representations
- The ground-state $l^\pi = 0^+$ belongs to A_1 representation given by:

$$A_1 : \quad 0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2 \times 12^+, 12^-}_{\text{triplet}}, \dots$$

Forming a common parabola

- There are no states with spins $l = 1, 2$ and 5 . We have parity doublets: $l = 6, 9, 10 \dots$, at energies: $E_{6^-} = E_{6^+}$, $E_{9^-} = E_{9^+}$, etc.

Consequently we should expect two independent parabolic structures

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- There are no states with spins $I = 1, 2$ and 5 . We have parity doublets: $I = 6, 9, 10 \dots$, at energies: $E_{6-} = E_{6+}$, $E_{9-} = E_{9+}$, etc.
- One shows that the analogue structure in the **octahedral** symmetry

$$A_{1g} : \quad 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \dots, \quad I^\pi = I^+$$

Forming a common parabola

$$A_{2u} : \quad 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \dots, \quad I^\pi = I^-$$

Forming another (common) parabola

Consequently we should expect two independent parabolic structures

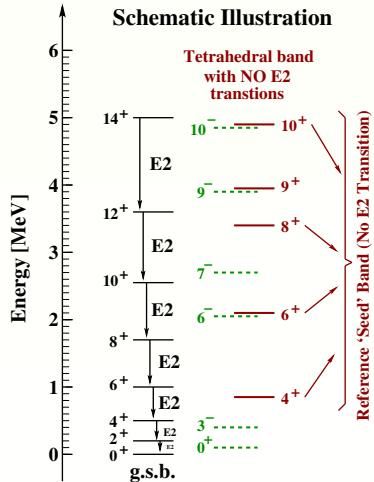
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What To Start With?

The Starting Point in Collecting Experimental Evidence

How to start finding specific levels satisfying very specific criteria?

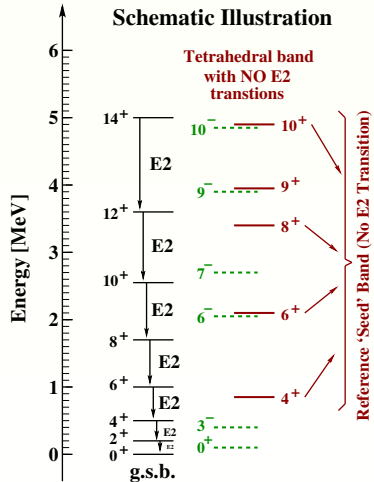


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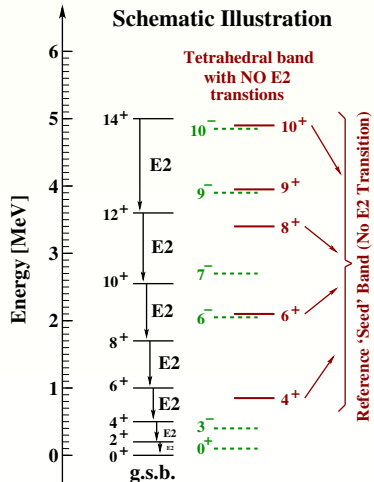
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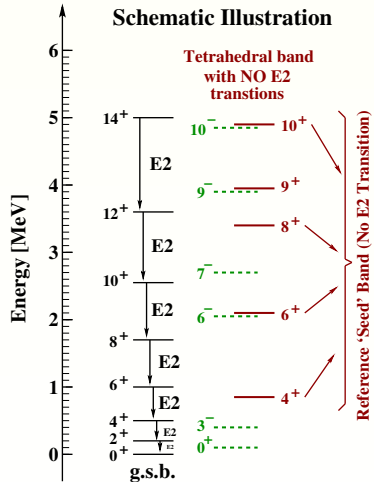
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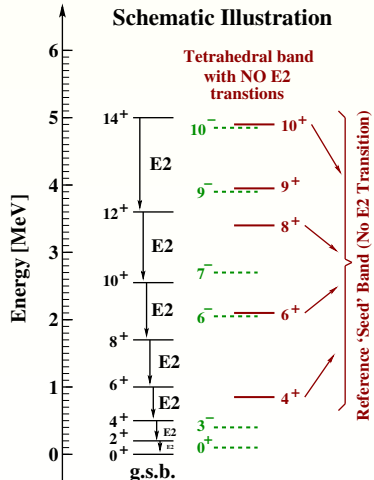
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- If successful, we will fit coefficients of the reference 'seed-band' parabola
- Once this parabola is known – we select other experimental candidate states close to reference seed-band

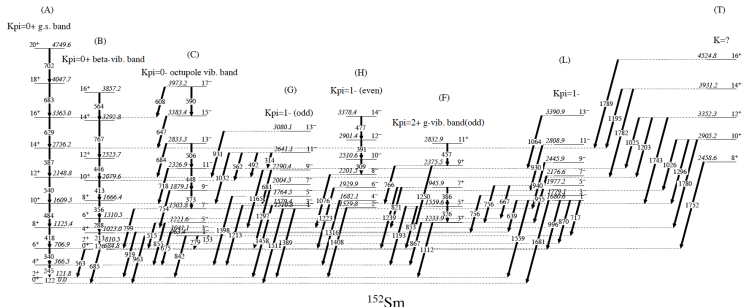


We begin by looking for experimental candidates for the 'reference seed band'

Start Looking for the Reference Band with no E2's

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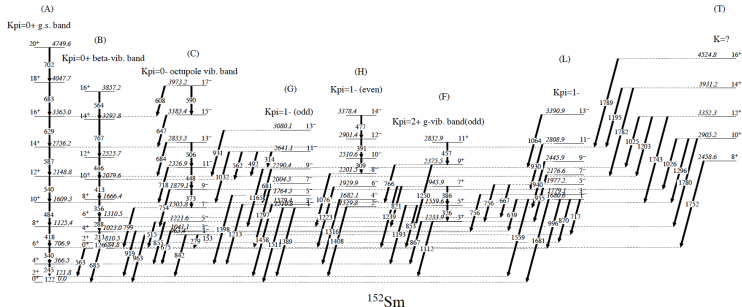


Experimental spectrum of ^{152}Sm from the NNDC data base

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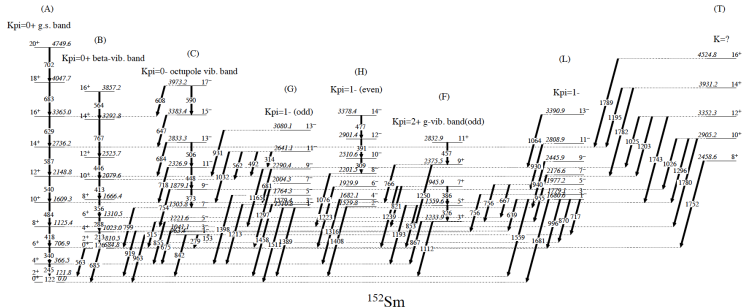
From NNDC data base: Notice the fantasist nomenclature of the bands
... invented long ago by an NNDC data base evaluator

“OUR BAND” is called ... Band (T) like ...

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Experimental spectrum of ¹⁵²Sm

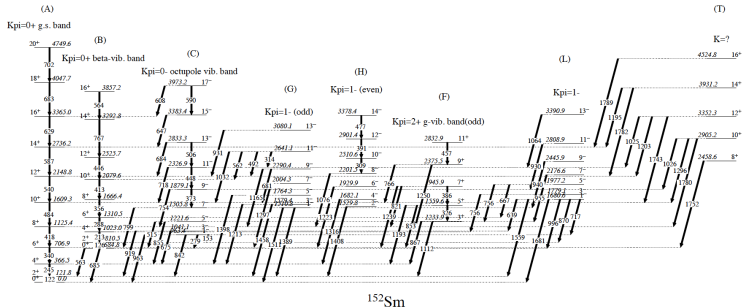
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“OUR BAND” is called ... Band (T) like ... Terrific

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Experimental spectrum of ^{152}Sm

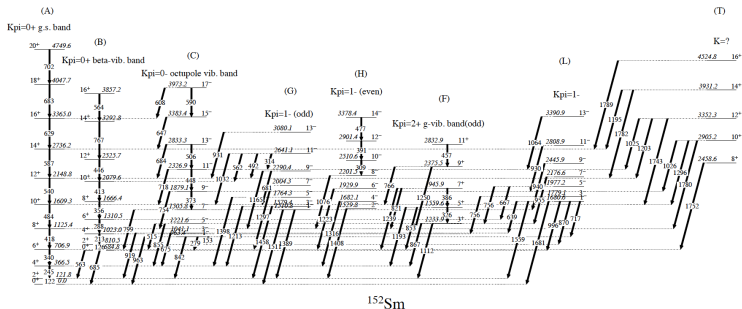
From NNDC data base: Notice the fantasist nomenclature of the bands
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“OUR BAND” is called ... Band (T) like ... Terrific or Terrible

I could not stop laughing seeing it for the first time

- We must try to find the sequence which is parabolic, no E2 transitions

4^+ , 6^+ , 8^+ , 10^+ ...



Experimental spectrum of ^{152}Sm

From NNDC data base: Notice the fantasist nomenclature of the bands
... invented long ago by an NNDC data base evaluator

“OUR BAND” is called ... Band (T) like ... Terrific or Terrible
... or Tetrahedral ... or ...

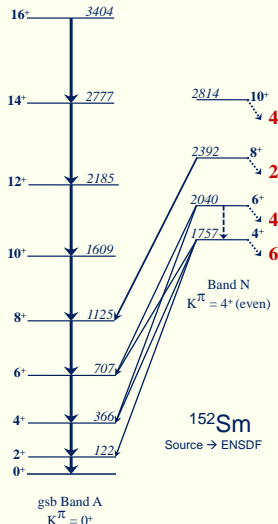
Possible Candidate as a Reference Band

- The sequence $4^+, 6^+, 8^+, 10^+ \dots$ of experimental energies turns out to be (very) parabolic and with no E2 transitions

- In this way we obtain the coefficients of the reference parabola

$$E_I = a * I^2 + b I + c$$

- Numbers marked in red count observed distinct depopulating transitions as a certain measure of **exoticity**



We Proceed Looking for the Other Candidate States

Criterion no. 1:

Accepted states must neither be populated nor depopulated by any strong E1 or E2 transitions, preferably populated by nuclear reaction

Criterion No. 2:

Their energies should be 'reasonably' close to the reference parabola

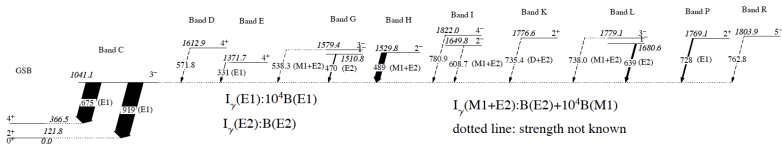
Observation:

Since they do not decay via a single strong transition it is instructive verifying that they decay into several states – with weak intensities

Next Steps in the Procedure: Part II

A typical diagram among a hundred in this analysis
Decay from the tetrahedral $I^\pi = 3^-$ candidate (among five others)

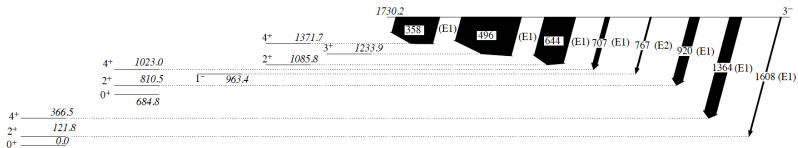
Feeding and Decay of 1041 keV 3^- level



Let us note that 3^- does not decay to the 0^+ ground-states (suggesting that it is not an octupole vibrational state built on the other) and that there are numerous states populating it suggesting that its structure is exotic from our point of view.

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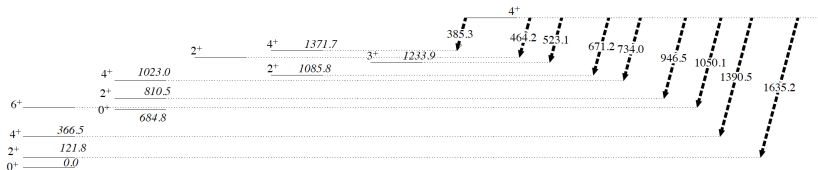
A typical diagram among a hundred in this analysis
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Let us observe that this state decays to many others suggesting its 'exotic' structure as in the previous case

Next Steps in the Procedure: Part II

A typical diagram among a hundred in this analysis
Decay from the tetrahedral $I^\pi = 4^+$ candidate level



Let us observe that this state decays to many others via very weak transitions suggesting no resemblance to quadrupole-deformed rotational states

Proceeding Towards a Summary

Proposed experimental energy levels candidates members of the tetrahedral band in ^{152}Sm after analysing numerous hypotheses. Columns 3 and 4 give the numbers of decay-out transitions and feeding transitions, respectively.

Spin	E[keV]	No. D-out	No. Feed	Reaction
3^-	1579.4	10	none	CE & α
4^+	1757.0	9	$1+(1)$	CE & α
6^-	1929.9	2	(1)	CE & α
6^+	2040.1	7	none	CE & α
7^-	2057.5	6	$2+(1)$	CE & α
8^+	2391.7	3	1	CE & α
9^-	2388.8	4	3	CE & α
9^+	2588	2	1	α
10^-	2590.7	4	1	α
(10^+)	2810	2	none	α
11^-	2808.9	2	none	CE

Full Collection of Experimental T_d -Band Candidates

- We expect the tetrahedral band composed of spins:

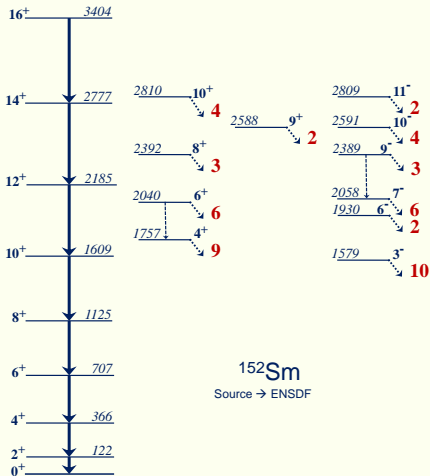
$$I^\pi = 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \dots$$

- ... and at the same time of the negative parity states:

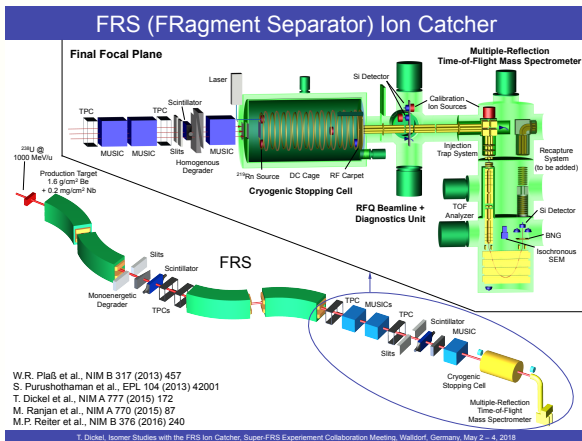
$$I^\pi = 3^-, 6^-, 7^-, 9^-, 10^-, 11^- \dots$$

- Both sequences are expected to form a common parabola

- Each of the tetrahedral states once populated is expected to give rise to an isomer



Plans: Joining Super-FRS Experiment collaboration, GSI



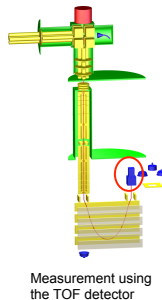
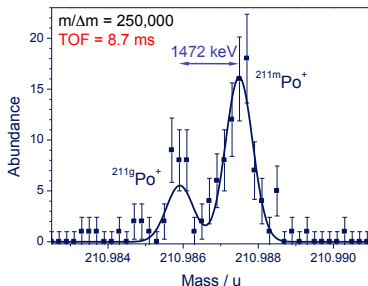
- Mass spectrometry can detect and identify the isomers without measuring their decay: This is the method of choice particularly for the long lived isomers

Courtesy: Dr T. Dickel, GSI Darmstadt and Giessen University

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Measurement and Separation of Isomers

- Identification of ^{211g}Po and ^{211m}Po by using PID detectors in the FRS, by alpha decay on Si detector and by mass spectrometry
- Measurement of excitation energy:
(1472 ± 120) keV Lit.: (1462 ± 5) keV



T. Dickel et al., Phys. Lett. B 744 (2015) 137

T. Dickel, Isomer Studies with the FRS Ion Catcher, Super-FRS Experiment Collaboration Meeting, Walldorf, Germany, May 2 – 4, 2018

Courtesy: Dr T. Dickel, GSI Darmstadt and Giessen University

Tetrahedral Symmetry Hypothesis: One Parabolic Branch

$$A_1 : \quad 0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2 \times 12^+, 12^-}_{\text{triplet}}, \dots$$

Forming a common parabola

- We performed the test of the tetrahedral A_1 -type hypothesis by fitting the parameters of the parabola to the energies in the Table. The obtained root-mean-square deviation:

$$T_d : A_1 \rightarrow r.m.s. \approx 80.5 \text{ keV} \leftrightarrow 11 \text{ levels } I^\pi = I^\pm$$

For comparison:

$$\text{G.s.b.} \rightarrow r.m.s. \approx 52.4 \text{ keV} \leftrightarrow 7 \text{ levels } I^\pi = I^+$$

Octahedral Symmetry Hypothesis: Two Parabolic Branches

$$A_{1g} : 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \dots, I^\pi = I^+$$

Forming a common parabola

$$A_{2u} : 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \dots, I^\pi = I^-$$

Forming another (common) parabola

- We performed the test of the octahedral A_{1g} - A_{2u} hypothesis by fitting the parameters of the parabolas to the energies in the Table. The obtained root-mean-square deviations:

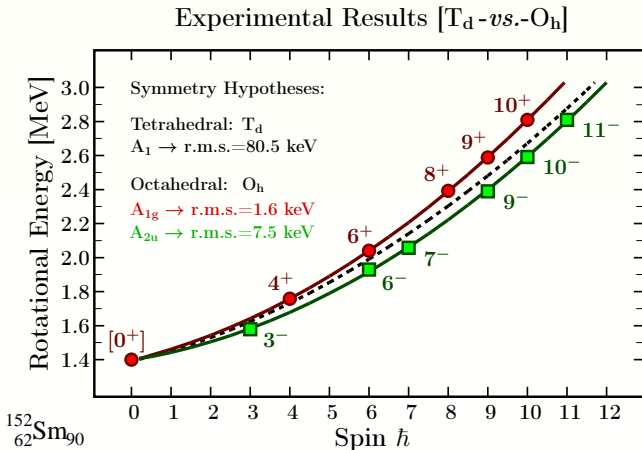
$$O_h : A_{1g} \rightarrow r.m.s. \approx 1.6 \text{ keV} \leftrightarrow 5 \text{ levels } I^\pi = I^+,$$

$$O_h : A_{2u} \rightarrow r.m.s. \approx 7.5 \text{ keV} \leftrightarrow 6 \text{ levels } I^\pi = I^-.$$

For comparison:

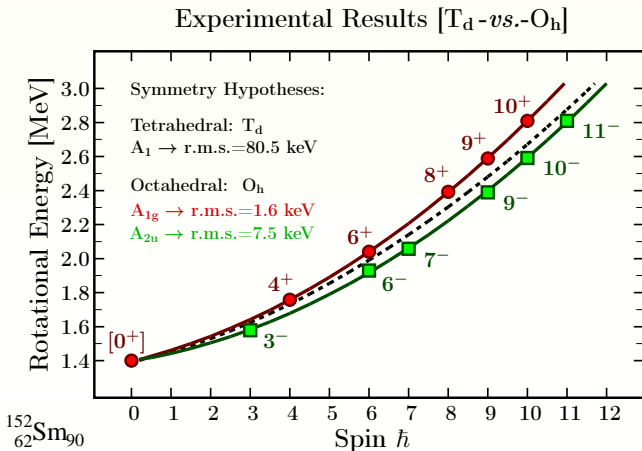
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Dominating Octahedral-Symmetry Hypothesis



Graphical representation of the experimental data from the summary Table. Curves represent the fit and are *not* meant 'to guide the eye'. Emphasise: the point $[I^\pi = 0^+]$ is a prediction by extrapolation - not an experimental datum.

A Comment About Extrapolation to $I^\pi \rightarrow 0^+$



Notice: The negative parity sequence lies entirely below the positive parity one. Extrapolating the parabolas to zero-spin we find $E_{I=0}^- = 1.3968 \text{ MeV}$ compared to $E_{I=0}^+ = 1.3961 \text{ MeV}$, the difference of 0.7 keV at the level 1.4 MeV excitation!

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$$T_d - \text{incidental} : P_{11 \text{ levels}}^{\sqrt{\sigma^2}=80 \text{ keV}} \approx 1.1 \cdot 10^{-14}$$

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- Emphasise: None of the geometrical nuclear symmetries can be considered exact because of the zero-point motion (Bohr model) and various polarisation mechanism, e.g. by nucleons outside shells
- Consequently relatively weak electromagnetic transitions are to be expected and this mechanism can/should be used to obtain a **more complete information about electromagnetic decay, spectra and possibly phase transitions.**