More (Shapes) is Different (Symmetries)

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Dynamical Symmetry

$$
\begin{array}{ccccccc}
G_{\mathrm{dyn}} & \supset & G & \supset & \cdots & \supset & G_{\mathrm{sym}} \\
\downarrow & & \downarrow & & & & \downarrow \\
[N] & & \langle\Sigma\rangle & & & & \Lambda
\end{array}
$$

$$\hat{H} = \sum_{G} a_G \, \hat{C}_G$$

- *Solvability of the* ***complete*** *spectrum* $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$
- *Quantum numbers for* ***all*** *eigenstates* $|[N]\langle\Sigma\rangle\Lambda\rangle$

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- *Quantum numbers for **all** eigenstates* $|[N]\langle\Sigma\rangle\Lambda\rangle$

- IBM: s (L=0) , d (L=2) bosons, N conserved *(Arima, Iachello 75)*

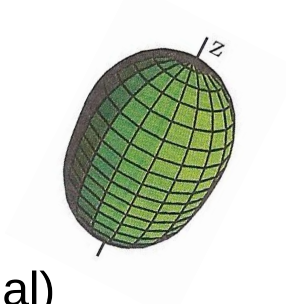
G_{dyn} = U(6), G_{sym} = SO(3)

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	$\vert [N]\ n_d\ \tau\ n_\Delta\ L \rangle$	Spherical vibrator
$U(6) \supset SU(3) \supset SO(3)$	$\vert [N]\ (\lambda, \mu)\ K\ L \rangle$	Prolate-deformed rotor
$U(6) \supset \overline{SU(3)} \supset SO(3)$	$\vert [N]\ (\bar{\lambda}, \bar{\mu})\ \bar{K}\ L \rangle$	Oblate-deformed rotor
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	$\vert [N]\ \sigma\ \tau\ n_\Delta\ L \rangle$	γ-unstable deformed rotor

Geometry

Coherent state
$$|\beta,\gamma;N\rangle = (N!)^{-1/2}(b_c^\dagger)^N\,|0\rangle$$
$$b_c^\dagger = (1+\beta^2)^{-1/2}\left[\beta\cos\gamma\, d_0^\dagger + \beta\sin\gamma\,\frac{1}{\sqrt{2}}\left(d_2^\dagger + d_{-2}^\dagger\right) + s^\dagger\right]$$

Energy surface
$$E_N(\beta,\gamma) = \langle\beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$$



Global min: equilibrium shape (β_0,γ_0)

$\beta_0 = 0$ spherical

$\beta_0 > 0$ deformed: $\gamma_0 = 0$ (prolate), $\gamma_0 = \pi/3$ (oblate), $0 < \gamma_0 < \pi/3$ (triaxial)

Intrinsic state ground band $|\beta_0,\gamma_0;N\rangle$, L-projected states $|\beta_0,\gamma_0;N,x,L\rangle$

$U(6) \supset \mathbf{G_1} \supset G_2 \supset \ldots SO(3)$ $\qquad |N, \lambda_1, \lambda_2,\ldots,L\rangle$

U(5)	$\beta_0 = 0$	$n_d = 0$
SU(3)	$(\beta_0 = \sqrt{2},\ \gamma_0 = 0)$	$(\lambda,\mu) = (2N,0)$
$\overline{SU(3)}$	$(\beta_0 = \sqrt{2},\ \gamma_0 = \pi/3)$	$(\bar{\lambda},\bar{\mu}) = (0,2N)$
SO(6)	$(\beta_0 = 1,\ \gamma_0$ arbitrary)	$\sigma = N$

- Dynamical symmetry corresponds to a particular shape (β_0,γ_0)
- $|\beta_0,\gamma_0;N\rangle$ lowest (highest) weight state in a particular irrep λ_1 of leading subalgebra $\mathbf{G_1}$

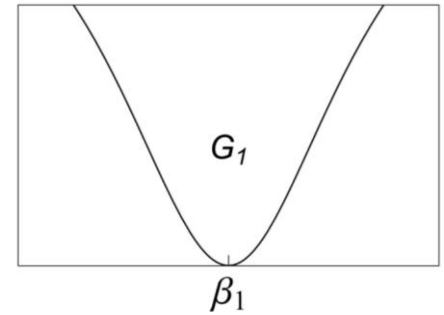
Dynamical Symmetry

$U(6) \supset G_1 \supset G_2 \supset \ldots SO(3)$ $\quad |N, \lambda_1, \lambda_2, \ldots, L\rangle$

- Complete solvability
- Good quantum numbers for all states
- DS: benchmark for a single shape

[G_1 = U(5), SU(3), $\overline{SU(3)}$, SO(6)]

Spherical, prolate- , oblate-, γ-unstable deformed



Dynamical Symmetry

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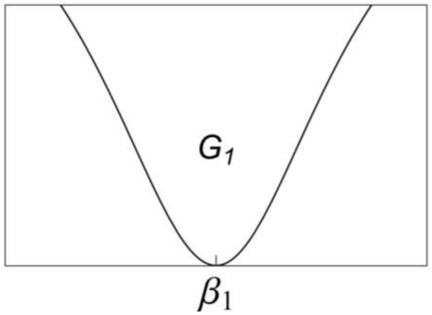
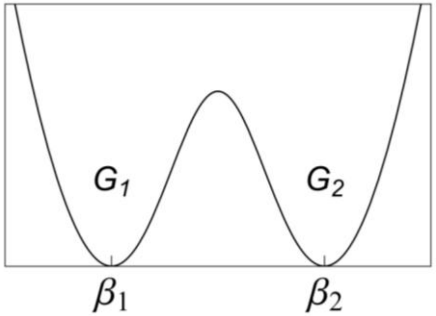
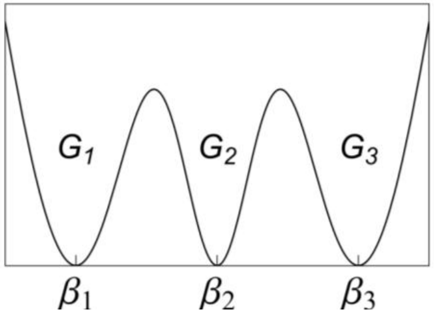


Partial Dynamical Symmetry

- Some states solvable and/or with good quantum numbers
- G_1, G_2 incompatible (non-commuting) symmetries
- PDS: benchmark for shape coexistence





Construction of Hamiltonians with a single PDS

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym}$$

$[N]$ $\langle\Sigma\rangle$ Λ

n-particle annihilation operator

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} \,|\, [N]\, \langle\Sigma_0\rangle\, \Lambda\rangle = 0$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} \,|\, [N]\, \langle\Sigma_0\rangle \rangle = 0$$

| **Lowest weight state** 〉

- Condition is satisfied if $\langle\sigma\rangle \otimes \langle\Sigma_0\rangle \not\ni [N-n]$

$$\hat{H} = \sum_{\alpha,\beta} u_{\alpha\beta} \hat{T}^{\dagger}_{\alpha} \hat{T}_{\beta}$$

DS is **broken** but
solvability of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

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DS is **broken** but **solvability** of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

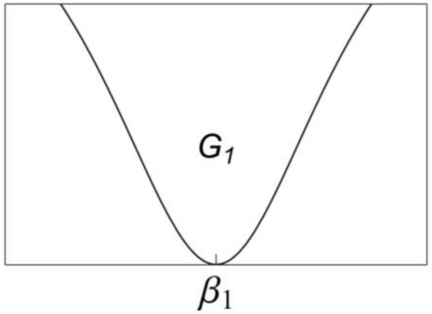
- PDS Hamiltonian $\hat{H}' = \hat{H} + \hat{H}_c$ Intrinsic collective resolution

Intrinsic part: $H\,|\,[N]\,\langle\Sigma_0\rangle\,\Lambda\rangle = 0$

Collective part: H_c composed of Casimir operators of conserved $G_i \subset G$ in the chain

Multiple PDS and Shape Coexistence

$$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$



Single PDS
Single shape

$$\hat{H}|\beta_1,\gamma_1;N,\lambda_1=\Lambda_0,\lambda_2,\ldots,L\rangle = 0$$

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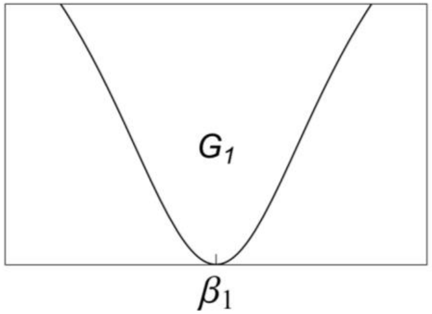

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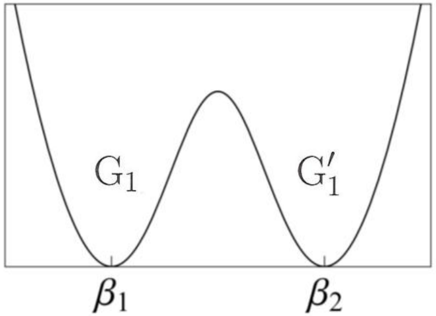
$$U(6) \supset G_1 \supset G_2 \supset \ldots \supset SO(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$

$$U(6) \supset G_1' \supset G_2' \supset \ldots \supset SO(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$$



$$G_1 \neq G_1'$$

Multiple PDS
Multiple shapes

$$\begin{cases} \hat{H}|\beta_1,\gamma_1; N, \lambda_1 = \Lambda_0, \lambda_2, \ldots, L\rangle = 0 \\ \hat{H}|\beta_2,\gamma_2; N, \sigma_1 = \Sigma_0, \sigma_2, \ldots, L\rangle = 0 \end{cases}$$

Critical-point Hamiltonian $\quad \hat{H}' = \hat{H} + \hat{H}_c$

G_1 -PDS & G'_1 -PDS

Intrinsic part: $\hat{H}$ determines $E(\beta,\gamma)$ band structure

Collective part: $\hat{H}_c = \sum_{G_i} a_{G_i} \hat{C}_{G_i}$ rotational splitting

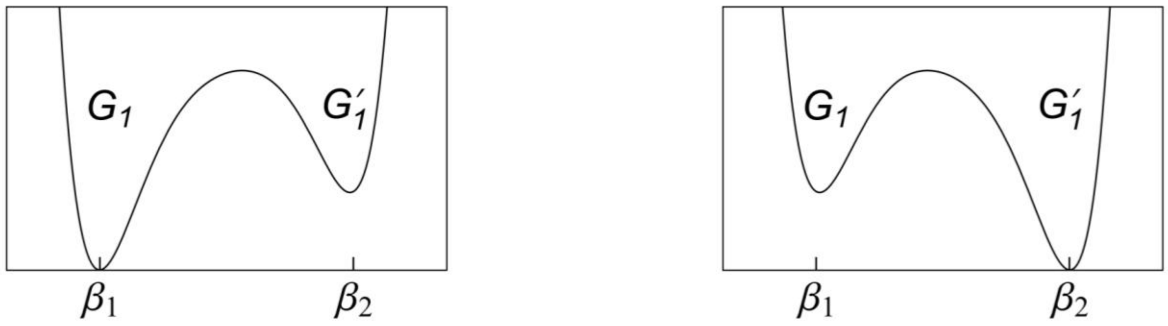
→ conserved G_i in both chains

Departure from the Critical Point

$$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$

$$\mathrm{U}(6) \supset \mathrm{G}'_1 \supset \mathrm{G}'_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$$

$$\mathrm{G}_1 \neq \mathrm{G}'_1$$



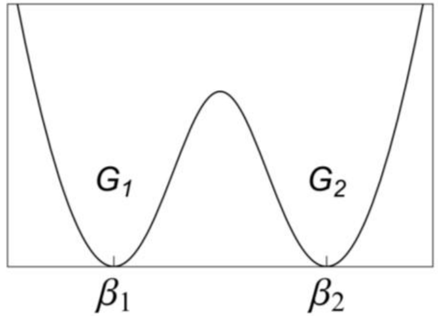
$$\hat{H}' = \hat{H}'_{\rm cp} + \alpha\, \hat{C}[\mathrm{G}_1] \qquad\qquad \hat{H}' = \hat{H}'_{\rm cp} + \alpha\, \hat{C}[\mathrm{G}'_1]$$

Symmetry Approach to Shape-Coexistence

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	Spherical vibrator	$\beta = 0$
$U(6) \supset SU(3) \supset SO(3)$	Prolate-deformed rotor	$\beta = \sqrt{2},\ \gamma = 0$
$U(6) \supset \overline{SU(3)} \supset SO(3)$	Oblate-deformed rotor	$\beta = \sqrt{2},\ \gamma = \pi/3$
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	γ-unstable deformed rotor	$\beta = 1$, γ arbitrary

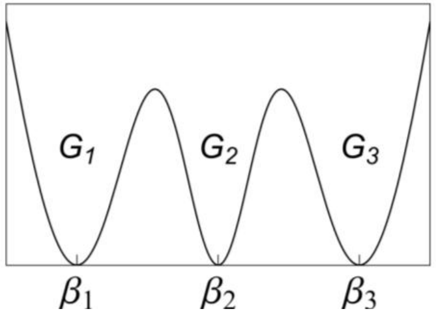
Multiple PDS and Multiple Shapes

$G_1 = U(5)$ $G_2 = SU(3)$ spherical – prolate
$G_1 = SU(3)$ $G_2 = \overline{SU(3)}$ prolate – oblate ♣
$G_1 = U(5)$ $G_2 = SO(6)$ spherical - γ-unstable ♣



Triple coexistence

$G_1 = U(5)$ $G_2 = SU(3)$ $G_3 = \overline{SU(3)}$ spherical-prolate-oblate ♣



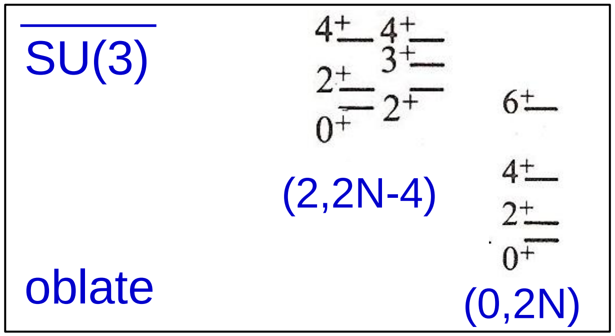
♣ Leviatan, Shapira, PRC **93**, 051302(R) (2016)

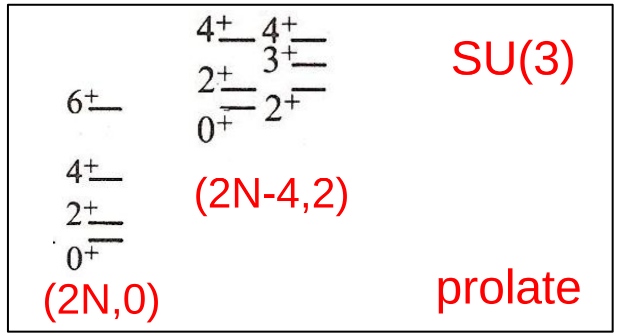
♣ Leviatan, Gavrielov, Phys. Scr. **92**, 114005 (2017)
arXiv:1803.03982 [nucl-th] (2018)

SU(3) and $\overline{SU(3)}$ Dynamical Symmetries

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\, (\lambda, \mu)\, K\, L\rangle$ Prolate-deformed rotor

$U(6) \supset \overline{SU(3)} \supset SO(3)$ $\quad |[N]\, (\bar{\lambda}, \bar{\mu})\, K\, L\rangle$ Oblate-deformed rotor





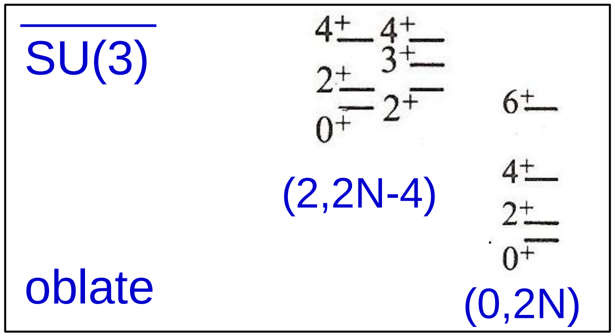
DS spectra are identical

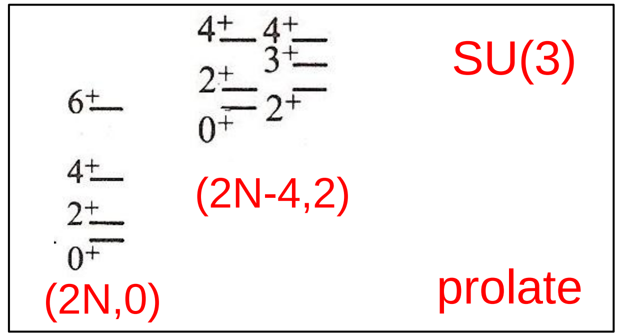
Quadrupole moments of corresponding states differ in sign

SU(3) and $\overline{SU(3)}$ Dynamical Symmetries

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\,(\lambda,\mu)\,K\,L\rangle$ **Prolate**-deformed rotor

$U(6) \supset \overline{SU(3)} \supset SO(3)$ $\quad |[N]\,(\bar{\lambda},\bar{\mu})\,K\,L\rangle$ **Oblate**-deformed rotor





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Quadrupole moments of corresponding states differ in sign

Prolate-Oblate Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\,(\lambda,\mu)=(2N,0),\,K=0,\,L\rangle = 0 \\ \hat{H}|N,\,(\bar{\lambda},\bar{\mu})=(0,2N),\,\bar{K}=0,\,L\rangle = 0 \end{cases}$$

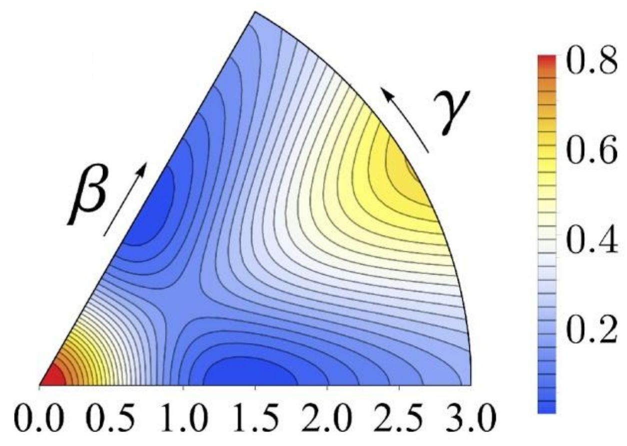
$$\hat{H} = h_0\,P_0^\dagger \hat{n}_s P_0 + h_2\,P_0^\dagger \hat{n}_d P_0 + \eta_3\,G_3^\dagger \cdot \tilde{G}_3 \qquad P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G_{3,\mu}^\dagger = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]_\mu^{(3)}$$

Energy Surface
$$\tilde{E}(\beta,\gamma) = (1+\beta^2)^{-3}\left\{(\beta^2-2)^2\left[h_0 + h_2\beta^2\right] + \eta_3\beta^6\sin^2(3\gamma)\right\}$$
$$= z_0 + (1+\beta^2)^{-3}[A\beta^6 + B\beta^6\Gamma^2 + D\beta^4 + F\beta^2] \qquad \Gamma = \cos 3\gamma$$

- Two degenerate P-O global minima

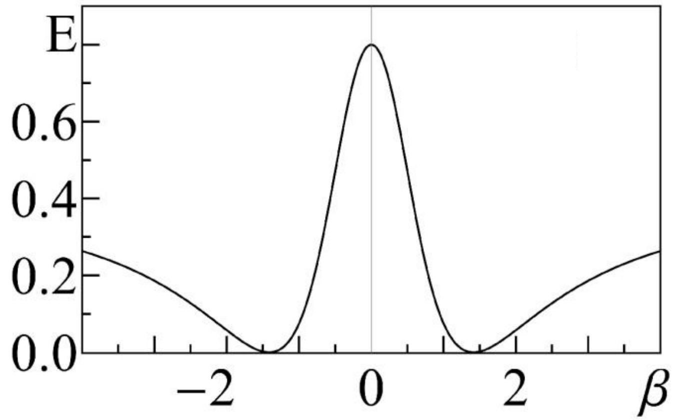
(β=√2,γ = 0) and (β=√2,γ = π/3) [or equivalently (β= -√2,γ 0)]

oblate-prolate



E(β,γ)

Saddle points support a barrier separating the various minima

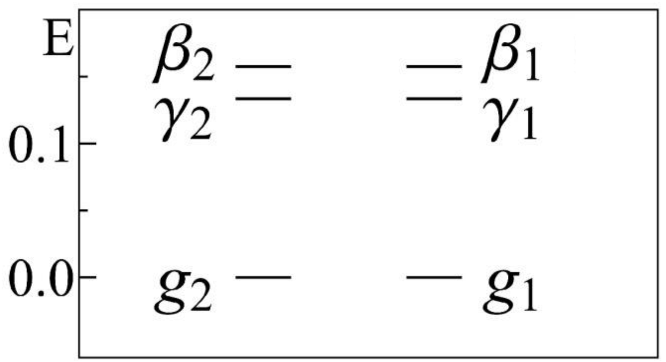


E(β,γ=0)

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{8}{3}(h_0 + 2h_2)N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

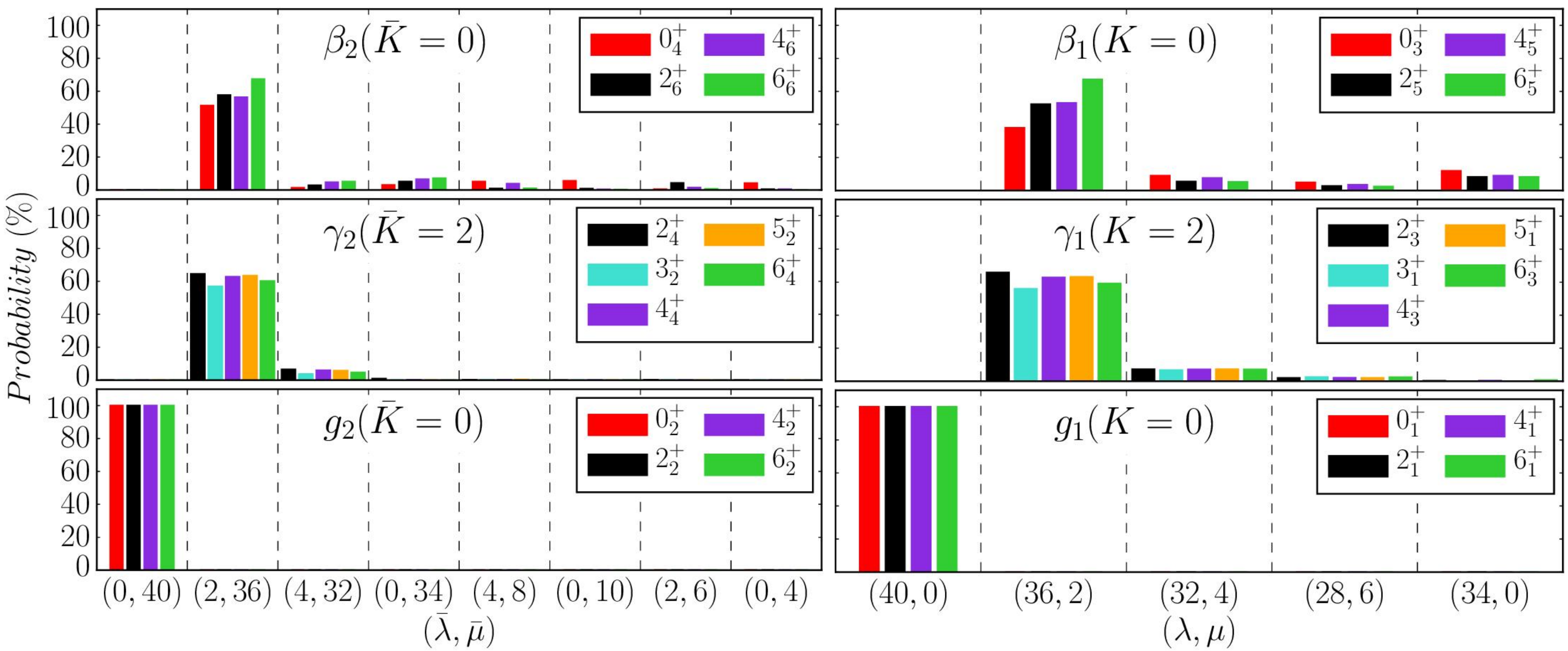


Complete Hamiltonian $\hat{H}' = \hat{H}(h_0, h_2, \eta_3) + \alpha\,\hat{\theta}_2 + \rho\,\hat{C}_2[\mathrm{SO}(3)]$

$$\alpha\,\hat{\theta}_2 = \alpha\,[-\hat{C}_2[SU(3)] + 2\hat{N}(2\hat{N}+3)]$$

$$\tilde{\alpha}(1+\beta^2)^{-2}[(\beta^2-2)^2+2\beta^2(2-2\sqrt{2}\beta\Gamma+\beta^2)] \qquad \tilde{\alpha} = \alpha/(N-2)$$

$\overline{\mathrm{SU(3)}}$ decomposition SU(3) decomposition

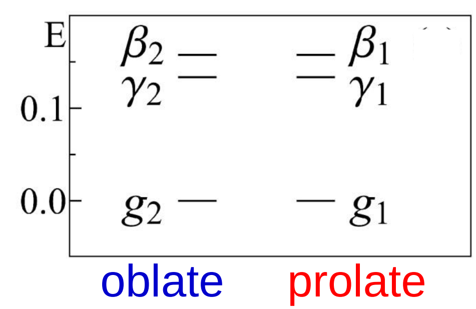


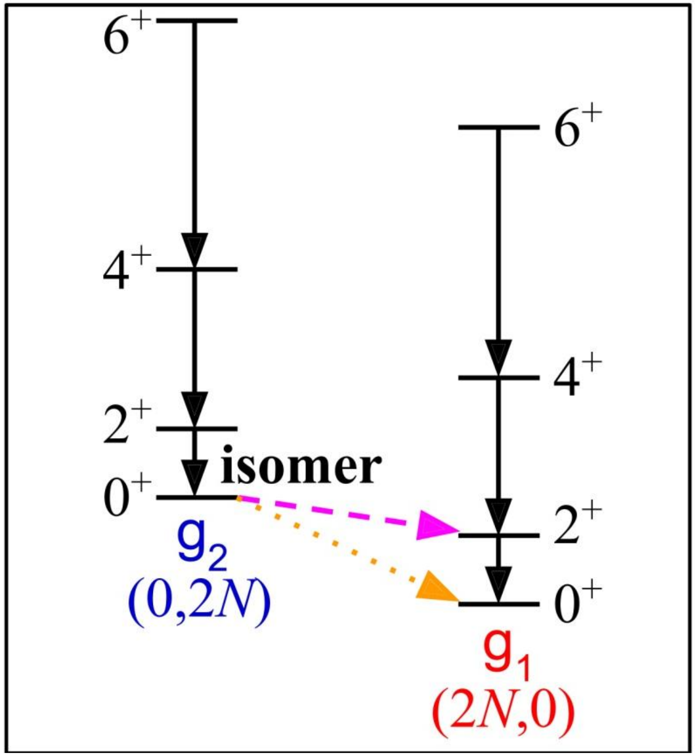
Ground g_1 band: pure SU(3)-DS states (2N,0)

Ground g_2 band: pure $\overline{\mathrm{SU(3)}}$-DS states (0,2N)

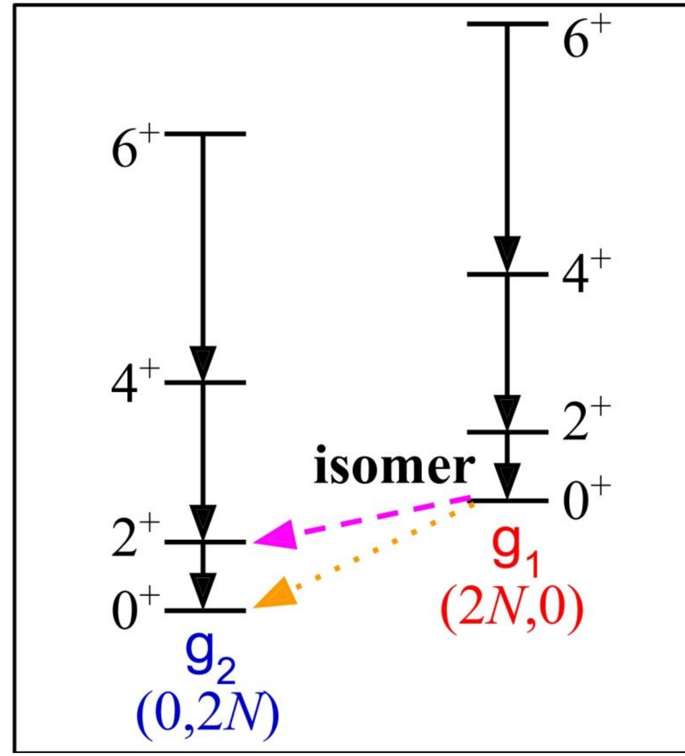
Excited β and γ bands: considerable mixing

⇒ SU(3)-PDS coexisting with $\overline{\mathrm{SU(3)}}$-PDS





P-O coexistence



$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ (1,1) ⊕ (2,2) tensor

$T(E0) \propto \hat{n}_d$ (0,0) ⊕ (2,2) tensor

E2 selection rule: $g_1 \not\leftrightarrow g_2$

E0 selection rule: $g_1 \not\leftrightarrow g_2$

$$Q_L = \mp e_B \sqrt{\frac{16\pi}{40}} \frac{L}{2L+3} \frac{4(2N-L)(2N+L+1)}{3(2N-1)}$$

ANALYTIC expressions !

$$B(E2; g_i,\ L+2 \to g_i,\ L) =$$

$$e_B^2 \frac{3(L+1)(L+2)}{2(2L+3)(2L+5)} \frac{(4N-1)^2(2N-L)(2N+L+3)}{18(2N-1)^2}$$

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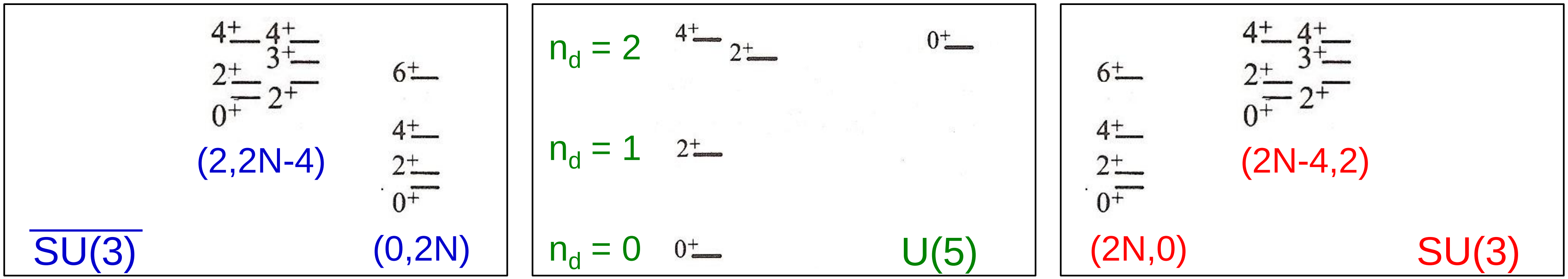
$U(6) \supset \overline{SU(3)} \supset SO(3)$	$\vert [N]\ (\bar{\lambda}, \bar{\mu})\ K\ L \rangle$	Oblate-deformed rotor



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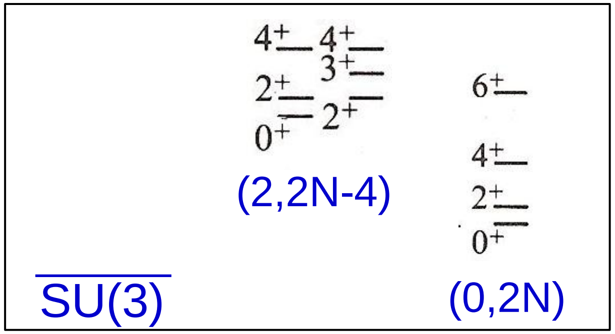
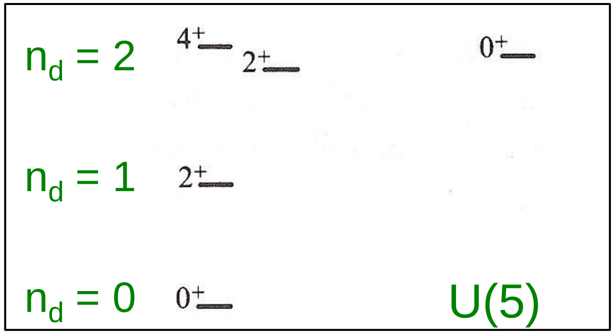
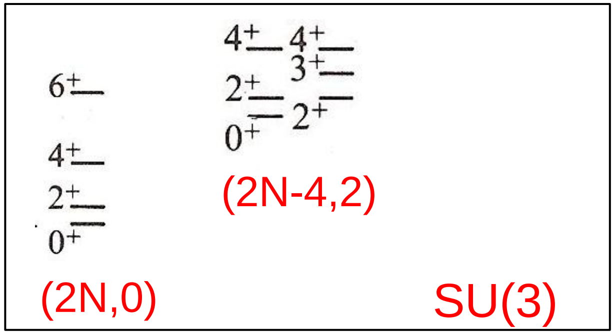






Spherical-Prolate-Oblate Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\, (\lambda,\mu) = (2N,0),\, K=0,\, L\rangle = 0 \\ \hat{H}|N,\, (\bar{\lambda},\bar{\mu}) = (0,2N),\, \bar{K}=0,\, L\rangle = 0 \\ \hat{H}|N,\, n_d = 0,\, \tau = 0,\, L = 0\rangle = 0 \end{cases}$$

$$\hat{H} = h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3 \qquad P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G_{3,\mu}^\dagger = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(3)}_\mu$$

Energy Surface

$$\tilde{E}(\beta,\gamma) = \beta^2[\, h_2(\beta^2-2)^2 + \eta_3\beta^4\sin^2(3\gamma)\,](1+\beta^2)^{-3}$$

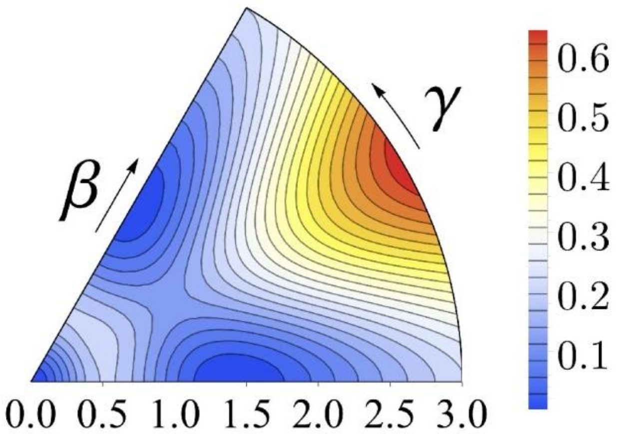
- Three degenerate S-P-O global minima: $\beta=0$, ($\beta = \pm\sqrt{2}, \gamma = 0$)

Complete Hamiltonian

$$\hat{H}' = h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3 + \alpha\, \hat{\theta}_2 + \rho\, \hat{C}_2[SO(3)]$$

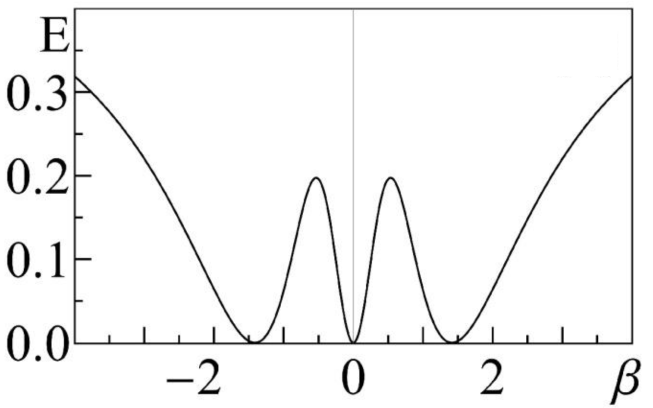
oblate-spherical-prolate

Triple coexistence



E(β,γ)

Saddle points support a barrier separating the various minima



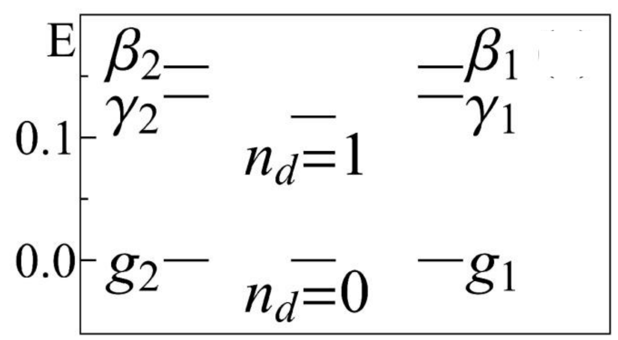
E(β,γ=0)

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{16}{3} h_2 N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

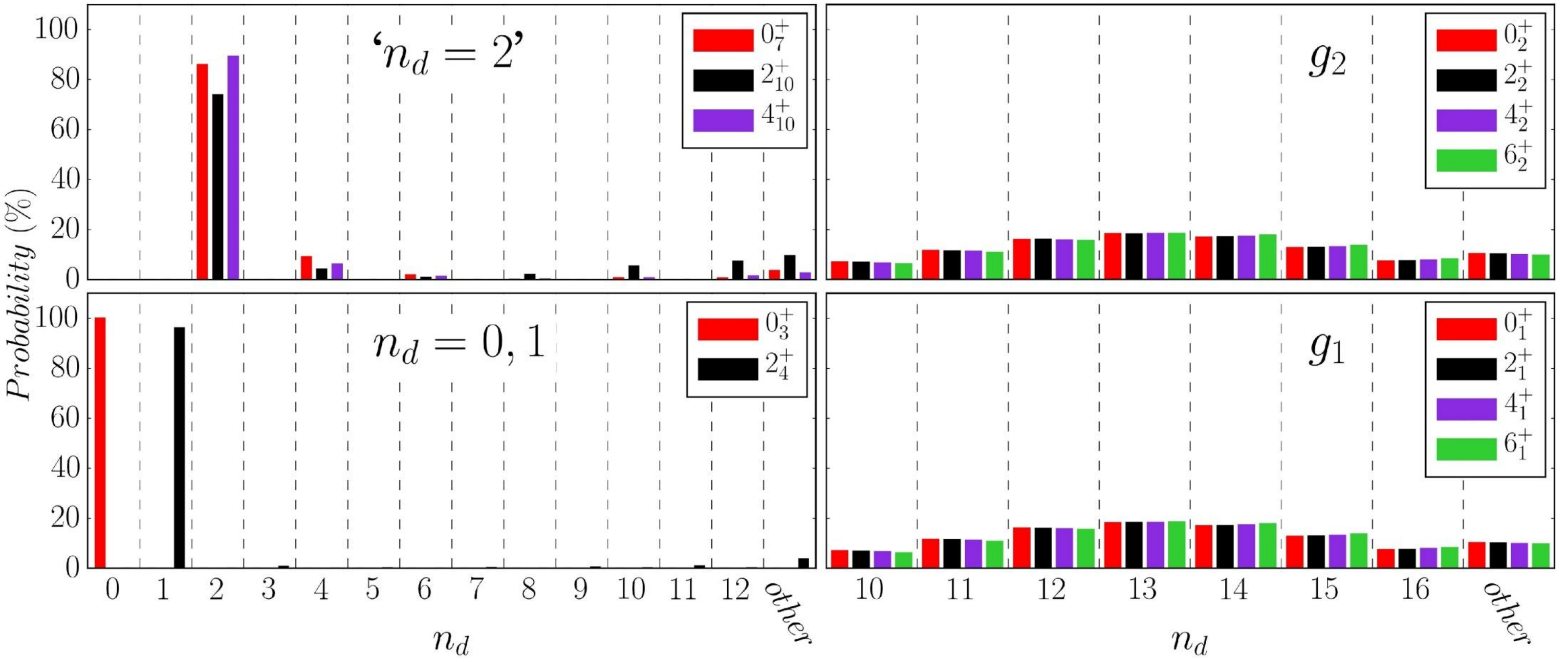
$$\epsilon = 4h_2 N^2$$



bandhead spectrum

Triple Spherical-Prolate-Oblate Coexistence

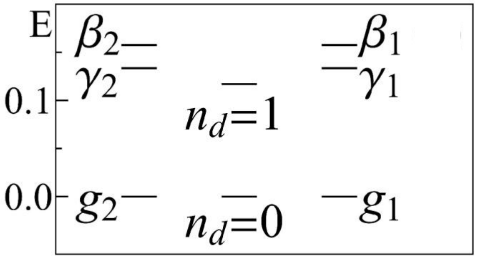
U(5) decompostion



P-O bands show similar behavior as in P-O coexistence

New aspect: occurrence of spherical type of states (n_d=L=0) and (n_d=1,L=2) pure U(5)-DS

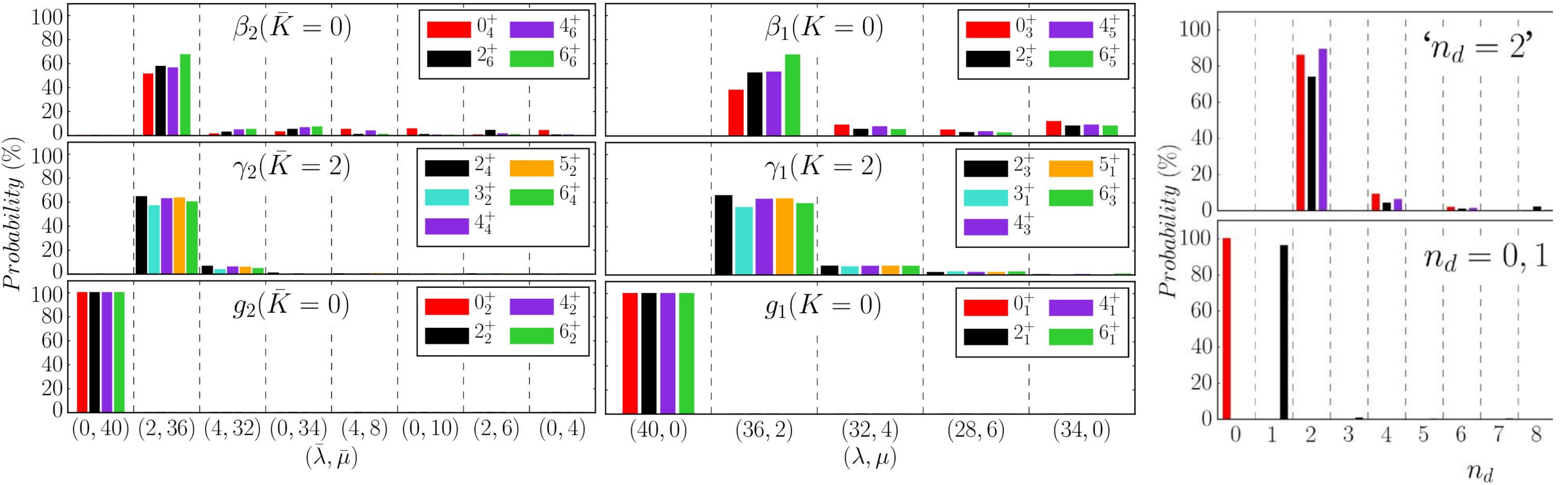
Higher spherical states: pronounced (~70%) n_d=2



oblate spherical prolate

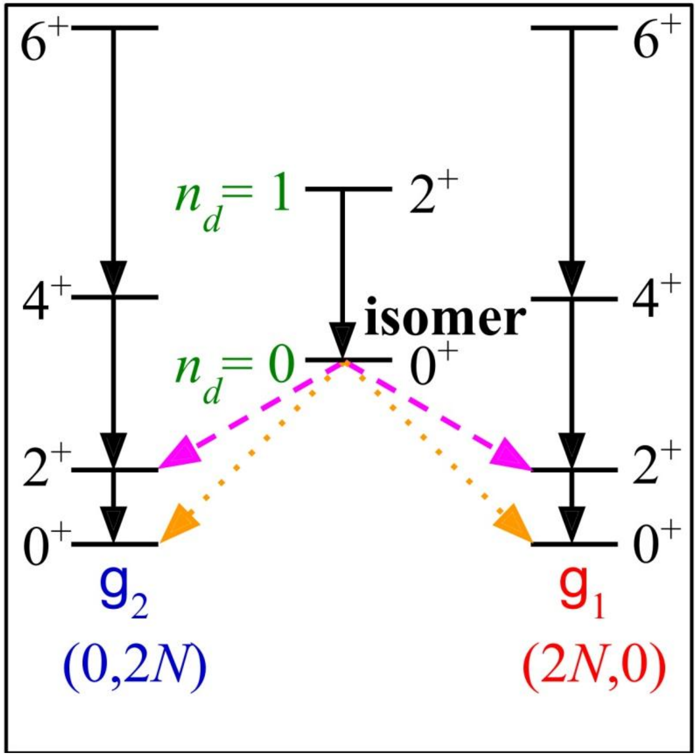
Coexisting Partial Dynamical Symmetries

$\overline{SU(3)}$ decomposition | SU(3) decompostion | U(5) decompostion

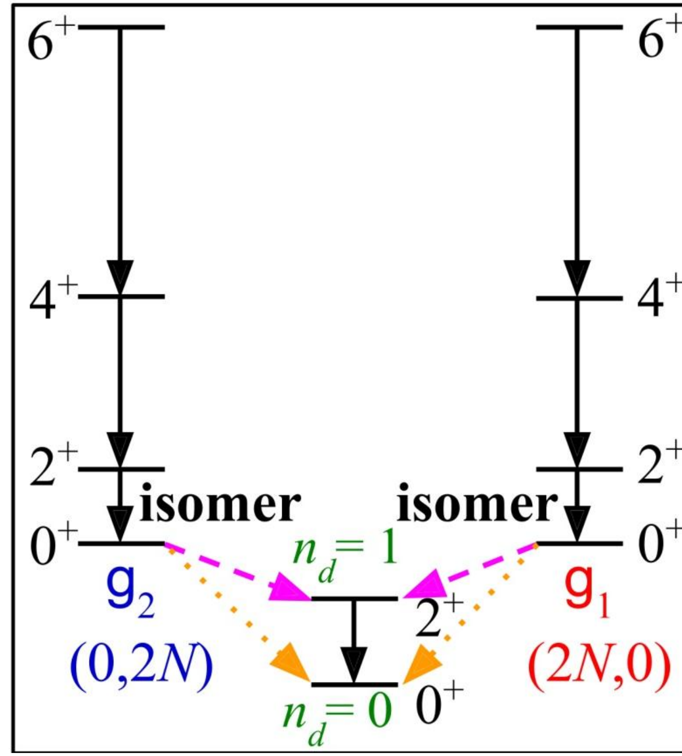


oblate | prolate | spherical

The purity of selected sets of states with respect to SU(3), $\overline{SU(3)}$ and U(5), in the presence of other mixed states, are the hallmarks of coexisting SU(3)-PDS, $\overline{SU(3)}$-PDS and U(5)-PDS



S-P-O coexistence



$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ $\Delta n_d = \pm 1$

Spherical → deformed E2 rates very weak

Deformed SU(3) & SU(3) DS states
($g_1 \to g_1$, $g_2 \to g_2$) Q_L & B(E2) KNOWN!

Spherical U(5)-DS states (n_d=1 → n_d=0)

Q(n_d=1,L=2) = 0

$B(E2; n_d = 1, L = 2 \to n_d = 0, L = 0) = e_B^2 N$

$T(E0) \propto \hat{n}_d$ diagnal in n_d

No E0 transitions involving these spherical states

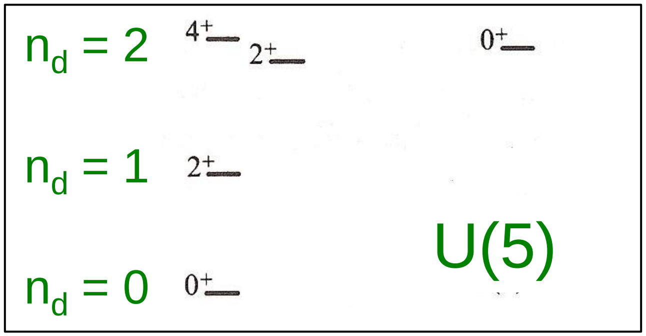
The spherical states exhaust the (n_d=0,1) irreps of U(5)

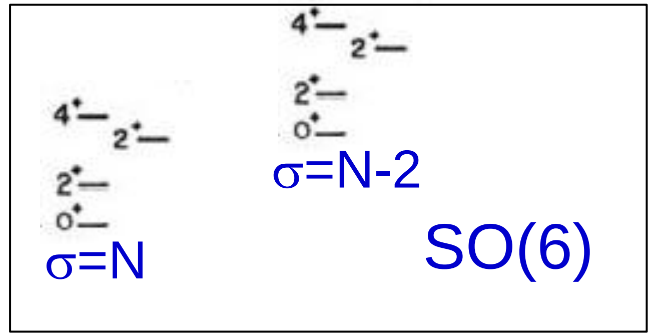
The n_d=2 component in the (L=0,2,4) states of the g_1 and g_2 bands is extremely small

U(5) and SO(6) Dynamical Symmetries

$U(6) \supset$ U(5) $\supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator

$U(6) \supset$ SO(6) $\supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ γ-unstable rotor





common segment

$SO(5) \supset SO(3)$

U(5) and SO(6) Dynamical Symmetries

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator

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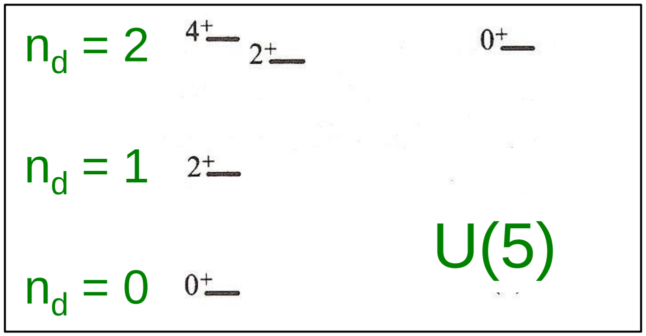
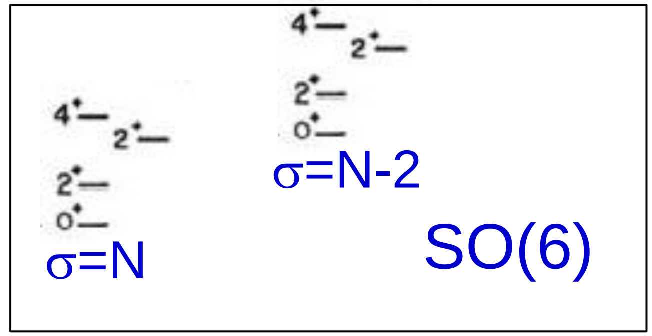




common segment

$SO(5) \supset SO(3)$

Spherical and γ-unstable deformed Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N, \sigma = N, \tau, L\rangle = 0 \\ \hat{H}|N, n_d = \tau = L = 0\rangle = 0 \end{cases}$$

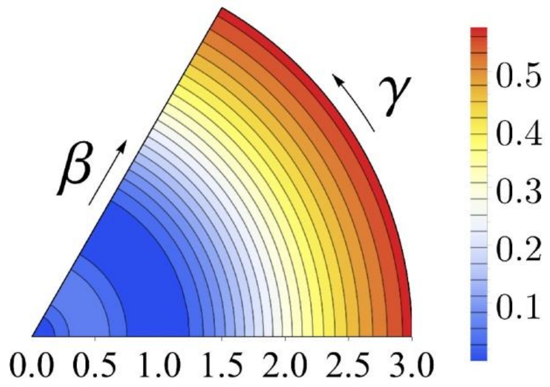
$$\hat{H} = r_2\, R_0^\dagger \hat{n}_d R_0 \qquad R_0^\dagger = d^\dagger \cdot d^\dagger - (s^\dagger)^2$$

Energy Surface
$$\tilde{E}(\beta) = r_2 \beta^2 (\beta^2 - 1)^2 (1 + \beta^2)^{-3}$$
$$= (1 + \beta^2)^{-3} [\, A\beta^6 + D\beta^4 + F\beta^2\,]$$

- Two degenerate spherical and γ-unstable deformed global minima: β=0 and β=1

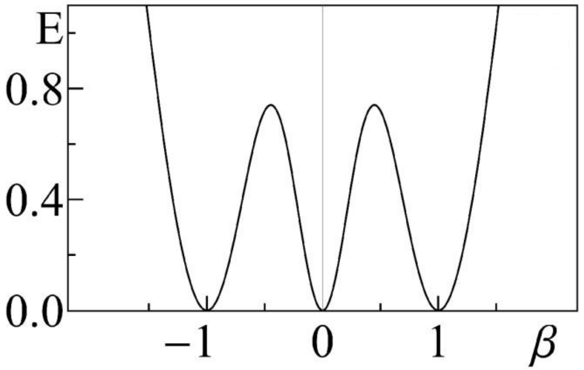
Spherical & γ-unstable deformed

Energy surface independent of γ
SO(5) symmetry



E(β,γ)

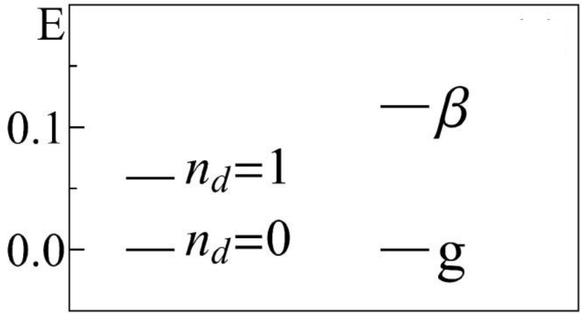
a barrier separates the spherical and γ-unstable deformed minima



E(β,γ=0)

Normal modes:

$$\epsilon_\beta = 2r_2\,N^2$$
$$\epsilon = r_2\,N^2$$

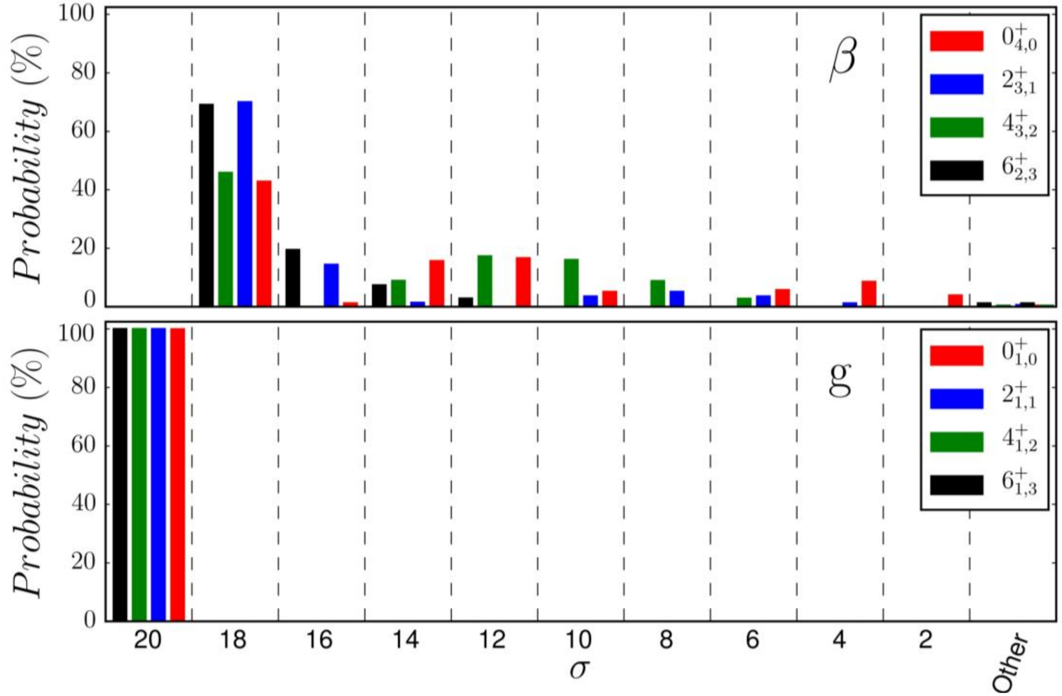


bandhead spectrum

Complete Hamiltonian

$$\hat{H}' = r_2\,R_0^\dagger \hat{n}_d R_0 + \rho_5\,\hat{C}_2[\mathrm{SO}(5)] + \rho_3\,\hat{\mathrm{C}}_2[\mathrm{SO}(3)]$$

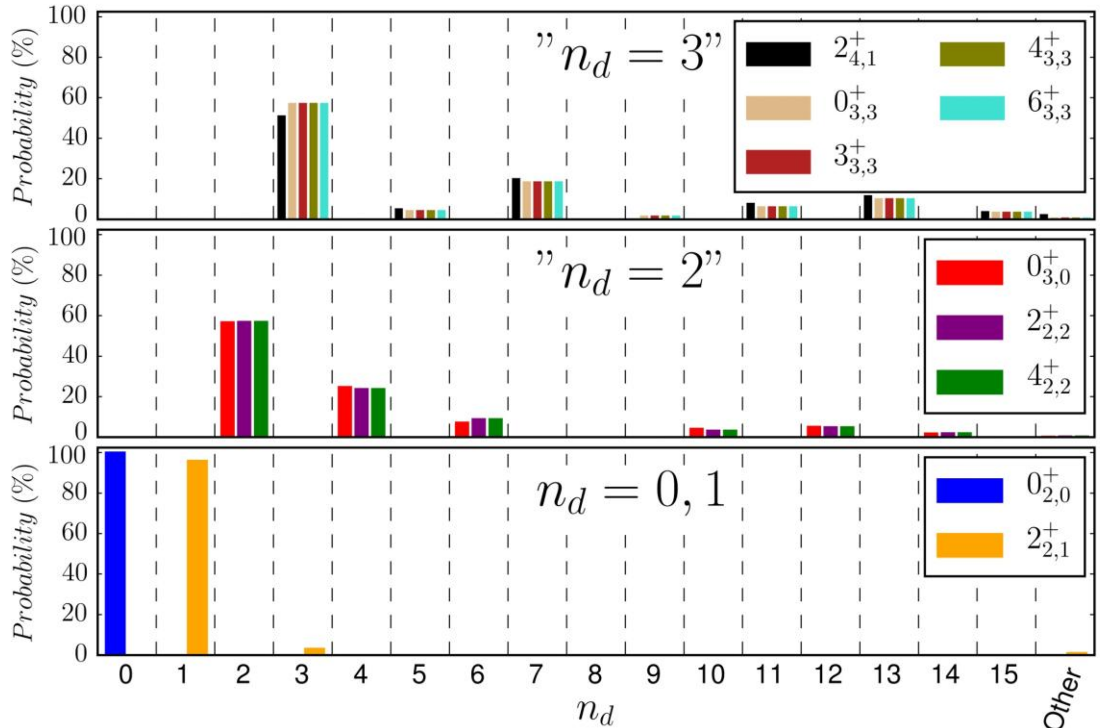
SO(6) decompostion



- g-band: pure SO(6)-DS ($\sigma=N$)
- Excited β bands: mixed

$\Rightarrow$ SO(6)-PDS

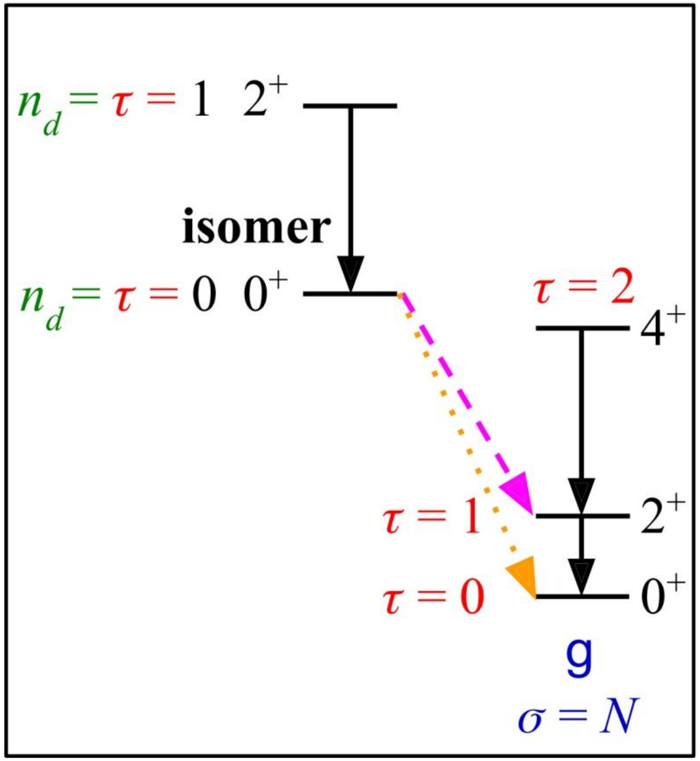
U(5) decompostion



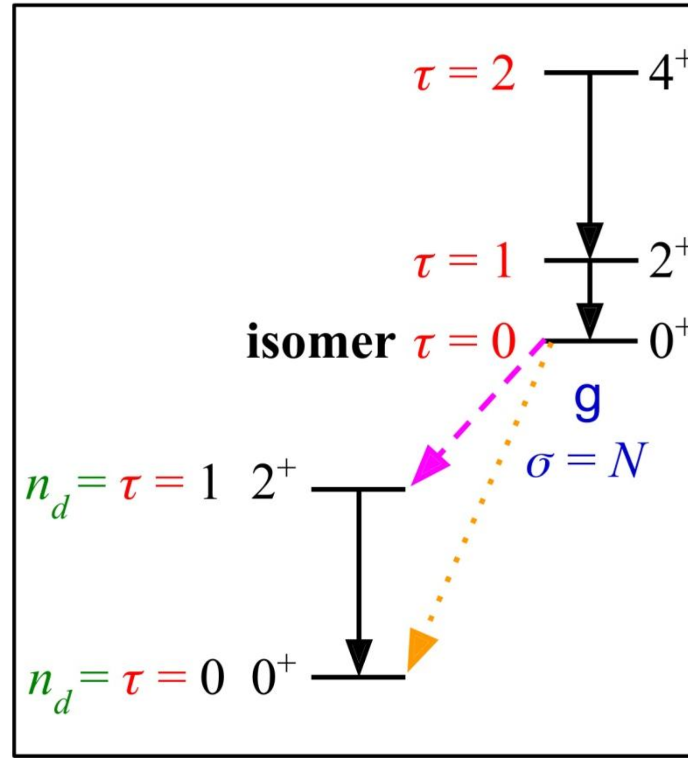
- Spherical states: pure U(5)-DS with ($n_d=\tau=L=0$) & ($n_d=\tau=1, L=2$)
- Higher spherical states: pronounced & coherent mixing

$\Rightarrow$ U(5)-PDS

Coexisting U(5)-PDS & SO(6)-PDS



Spherical and
γ-unstable deformed
coexistence



$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ $\Delta\sigma = 0$, Δn_d & $\Delta\tau = \pm 1$

$T(E0) \propto \hat{n}_d$ diagnal in n_d

deformed → spherical E2 rates very weak
g-band exhausts the σ=N irrep of SO(6)

No E0 transitions involving these spherical states

Deformed SO(6)-DS states (g → g)

$Q(\sigma{=}N,\tau) = 0$

$B(E2; g;\, \tau+1, L' = 2\tau+2 \rightarrow g;\, \tau, L = 2\tau)$

$= e_B^2 \frac{\tau+1}{2\tau+5}(N-\tau)(N+\tau+4)$

Spherical U(5)-DS states (n_d=1 → n_d=0)

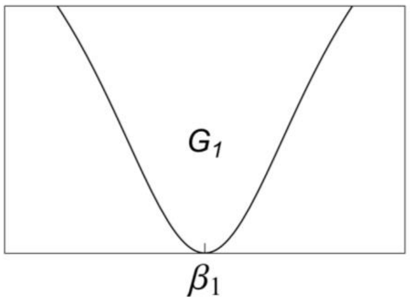
$Q(n_d{=}1,L{=}2) = 0$

$B(E2; n_d = 1, L = 2 \rightarrow n_d = 0, L = 0) = e_B^2 N$

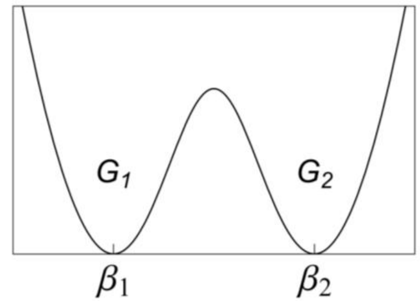
KNOWN !

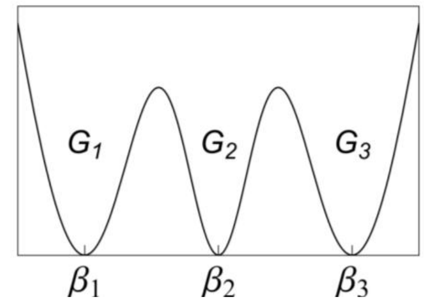
Concluding Remarks

Single DS or PDS



Multiple PDS





- A symmetry-based approach to shape coexistence
  Ingredients: spectrum generating algebra with several DS chains
  geometry: coherent states
  intrinsic-collective resolution of the Hamiltonian

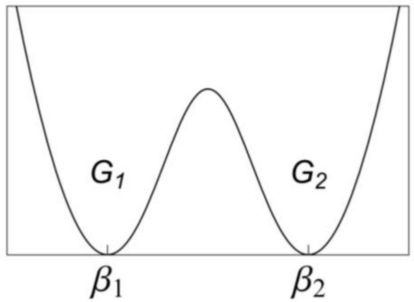
- A single number-conserving rotational invariant H which conserves the dynamical symmetry for selected bands
  Multiple Partial Dynamical Symmetries relevant for shape-coexistence

U(5) and SU(3) PDS	spherical-prolate
SU(3) and $\overline{\text{SU(3)}}$ PDS	prolate-oblate
U(5), SU(3) and $\overline{\text{SU(3)}}$ PDS	spherical-prolate-oblate
U5) and SO(6) PDS	spherical - γ-unstable deformed

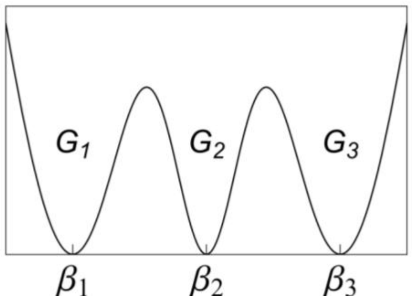
- Closed expressions for quadrupole moments and B(E2) values; selection rules for E2 & E0 transitions and isomeric states

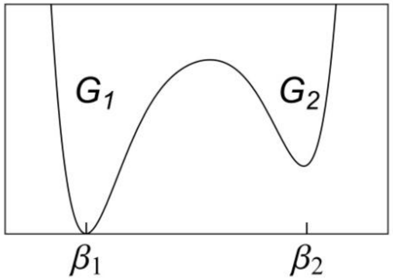
Concluding Remarks

G_1 -PDS
G_2 -PDS



G_1 -PDS
G_2 -PDS
G_3 -PDS





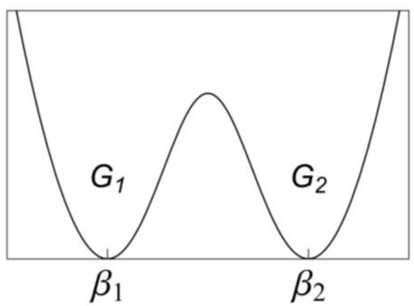
- Structure away from the critical point can be studied by adding the Casimir operator of a particular DS chain

- PDS: solvable bands are unmixed.
  Band mixing can be incorporated by including in H kinetic terms which do not affect $E(\beta,\gamma)$ but, if strong, may destroy the PDS

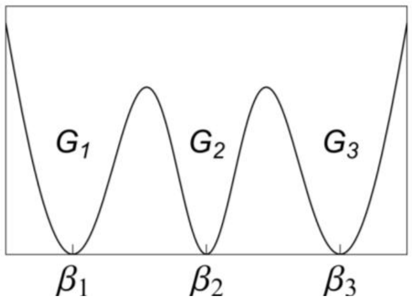
- PDS in the IBM with configuration mixing: Gavrielov
  Partial symmetry/solvability in the GCM: Levai, Georgoudis, Buganu

Concluding Remarks

G_1 -PDS
G_2 -PDS



G_1 -PDS
G_2 -PDS
G_3 -PDS



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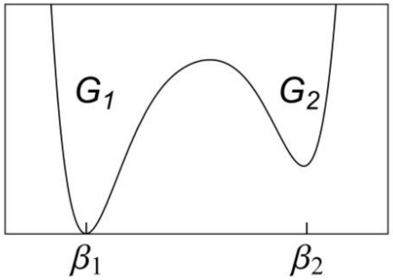

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More (Shapes) is Different (Symmetries)

Coexistence Multiple PDSs

Thank you