

# A new approximate nuclear symmetry, Proxy-SU(3), and parameter-free predictions for deformed nuclei

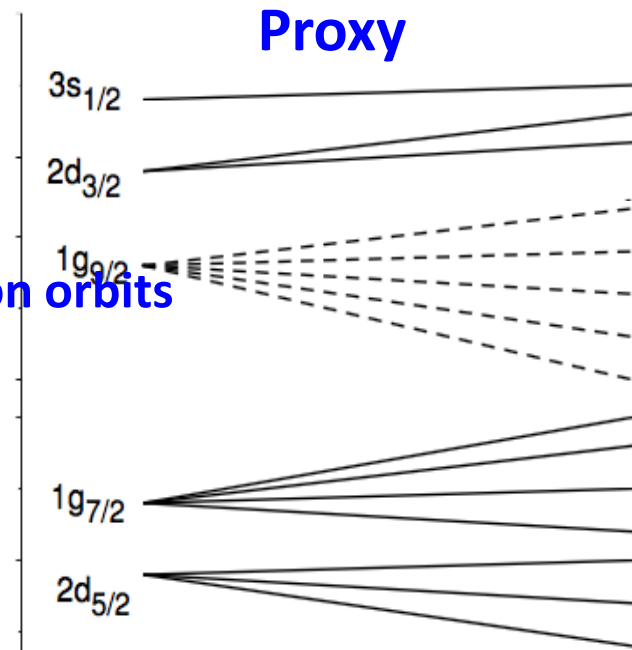
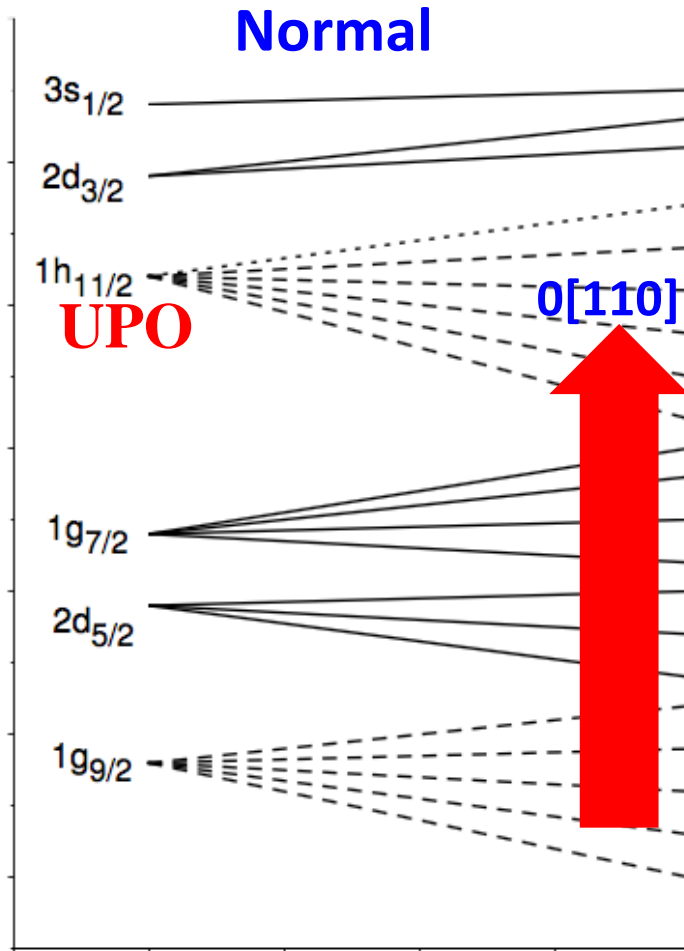
R. F. Casten

Yale and MSU-FRIB

Padova, May 22-25, 2018

**SU(3) (a la Elliott) breaks down in heavy nuclei.**

**Proxy-SU(3) – Approximate shell model to recover SU(3)**



**NOW have full set of sdg  $l, j$**

**$l = 0 \quad 2 \quad 4$**   
 **$J = 1/2, \quad 3/2, 5/2 \quad 7/2, 9/2$**

**Recovers H.O.  $\rightarrow$  SU(3). Lose one orbit: 30/32 particles**

**Big problem: Parity issue Spurious matrix elements.**

**N particles: group is  $U(N/2)$  with  $SU(3)$  sub-group**  
**G.s.— highest weight irreps labeled by**  
 **$SU(3)$  quantum numbers:  $(\lambda, \mu)$**

Relation of Proxy- $SU(3)$  irrep labels,  $(\lambda, \mu)$  to  $\beta$  and  $\gamma$

$$\beta^2 = \frac{4\pi}{5} \frac{1}{(Ar^2)^2} (\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$

$$\gamma = \arctan \left( \frac{\sqrt{3}(\mu + 1)}{2\lambda + \mu + 3} \right)$$

Note that, if  $\mu > \lambda$ , the nucleus is oblate ( $\gamma > 30$  deg.)

# Combined (proton + neutron)

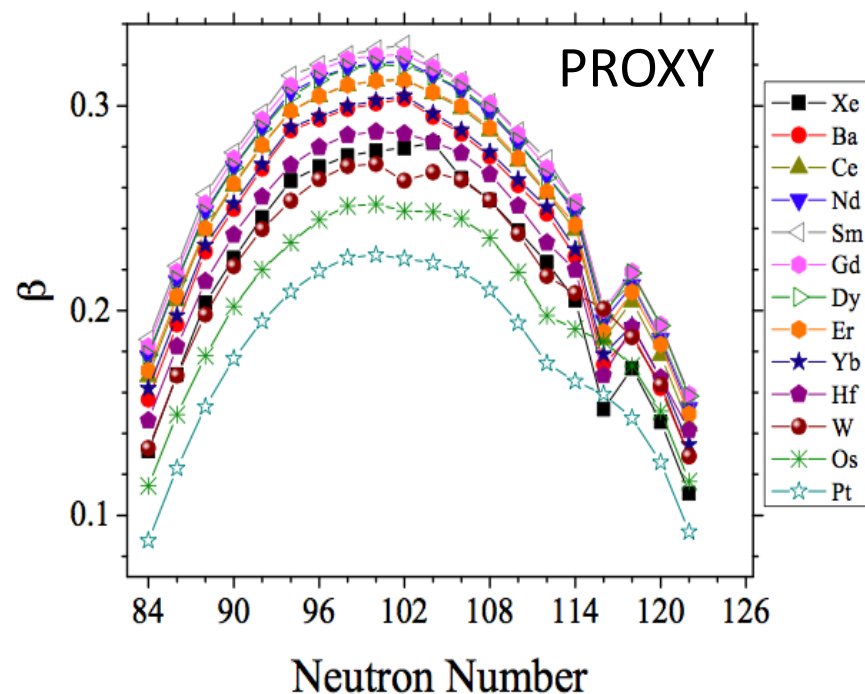
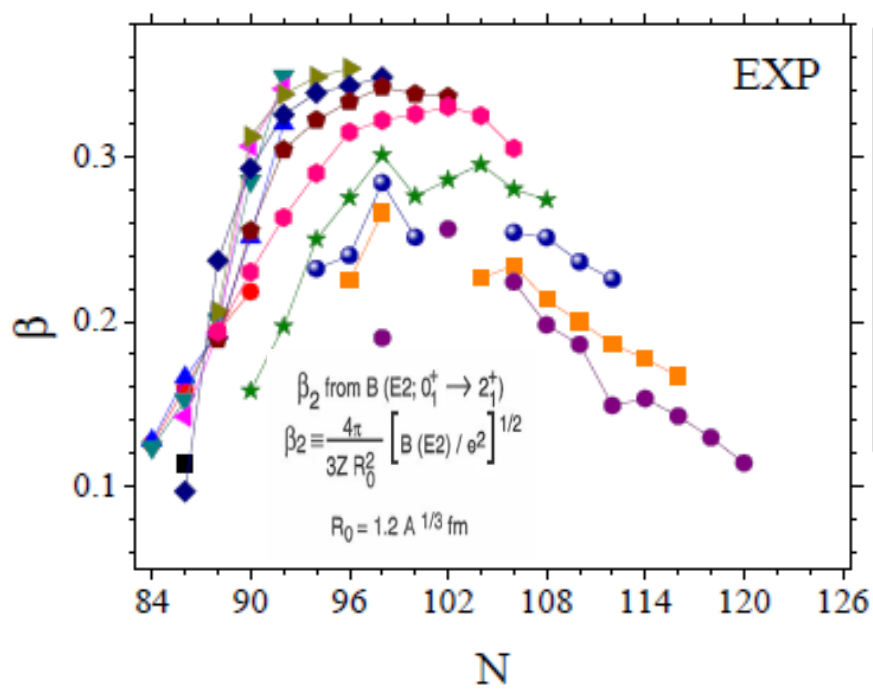
## Proxy-SU(3) irreps: prolate-oblate shapes

TABLE II: Most leading SU(3) irreps [34, 35] for nuclei with protons in the 50-82 shell and neutrons in the 82-126 shell. Boldface numbers indicate nuclei with  $R_{4/2} = E(4_1^+)/E(2_1^+) \geq 2.8$ , while \* denotes nuclei with  $2.8 > R_{4/2} \geq 2.5$ , and \*\* labels a few nuclei with  $R_{4/2}$  ratios any other nuclei with  $R_{4/2} < 2.5$ . For the rest of th still unknown [47]. Oblate irre d) the  $R_{4/2}$  ratios are

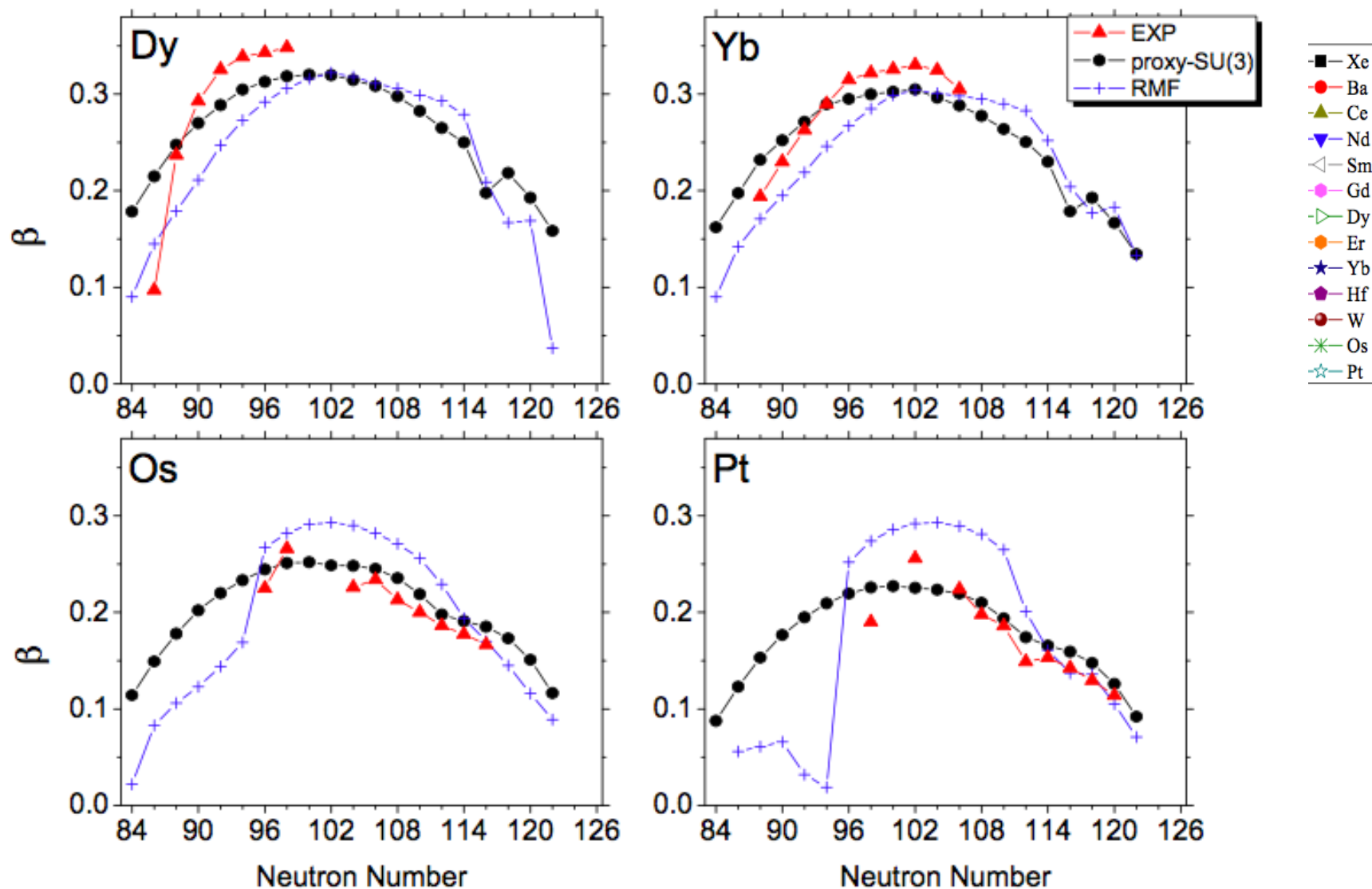
## Predictions using $(\lambda, \mu)$

			Ba	Ce	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt
		Z	56	58	60	62	64	66	68	70	72	74	76	78
		Z <sub>val</sub>	6	8	10	12	14	16	18	20	22	24	26	28
N	N <sub>val</sub>	irrep	(18,0)	(18,4)	(20,4)	(24,0)	(20,6)	(18,8)	(18,6)	(20,0)	(12,8)	(6,12)	(2,12)	(0,8)
88	6	(24,0)	(42,0)*	(42,4)*	(44,4)*									
90	8	(26,4)	(44,4)	(44,8)	(46,8)	(50,4)	(46,10)	(44,12)	(44,10)*	(46,4)*	(38,12)*			
92	10	(30,0)	(48,4)	(48,8)	(50,8)	(54,4)	(50,10)	(48,12)	(48,10)	(50,4)	(42,12)*			
94	12	(36,0)	(52,4)	(52,8)	(56,4)	(60,0)	(56,6)	(54,8)	(54,6)	(56,0)	(48,8)	(42,12)	(38,12)*	
100	14	(40,0)	(56,4)	(56,8)	(60,8)	(64,0)	(60,12)	(58,14)	(58,12)	(60,4)	(52,12)	(46,12)	(42,12)	(38,12)*
102	16	(44,0)	(60,4)	(60,8)	(64,8)	(68,0)	(64,12)	(62,14)	(62,12)	(64,4)	(56,12)	(50,12)	(46,12)	(42,12)
104	18	(48,0)	(64,4)	(64,8)	(68,8)	(72,0)	(68,12)	(66,14)	(66,12)	(68,4)	(60,12)	(54,12)	(50,12)	(46,12)
106	20	(52,0)	(68,4)	(68,8)	(72,8)	(76,0)	(72,12)	(70,14)	(70,12)	(72,4)	(64,12)	(58,12)	(54,12)	(50,12)
108	22	(56,0)	(72,4)	(72,8)	(76,8)	(80,0)	(76,12)	(74,14)	(74,12)	(76,4)	(68,12)	(62,12)	(58,12)	(54,12)
110	24	(60,0)	(76,4)	(76,8)	(80,8)	(84,0)	(80,12)	(78,14)	(78,12)	(80,4)	(70,12)	(64,12)	(60,12)	(56,12)
112	26	(64,0)	(80,4)	(80,8)	(84,8)	(88,0)	(84,12)	(82,14)	(82,12)	(84,4)	(74,12)	(68,12)	(64,12)	(60,12)
114	28	(68,0)	(84,4)	(84,8)	(88,8)	(92,0)	(88,12)	(86,14)	(86,12)	(88,4)	(78,12)	(72,12)	(68,12)	(64,12)
116	30	(72,0)	(88,4)	(88,8)	(92,8)	(96,0)	(92,12)	(90,14)	(90,12)	(92,4)	(82,12)	(76,12)	(72,12)	(68,12)
118	32	(76,0)	(92,4)	(92,8)	(96,8)	(100,0)	(96,12)	(94,14)	(94,12)	(96,4)	(86,12)	(80,12)	(76,12)	(72,12)
120	34	(80,0)	(96,4)	(96,8)	(100,8)	(104,0)	(100,12)	(98,14)	(98,12)	(100,4)	(90,12)	(84,12)	(80,12)	(76,12)
122	36	(84,0)	(100,4)	(100,8)	(104,8)	(108,0)	(104,12)	(102,14)	(102,12)	(104,4)	(94,12)	(88,12)	(84,12)	(80,12)
124	38	(88,0)	(104,4)	(104,8)	(108,8)	(112,0)	(108,12)	(106,14)	(106,12)	(108,4)	(98,12)	(92,12)	(88,12)	(84,12)
126	40	(92,0)	(108,4)	(108,8)	(112,8)	(116,0)	(112,12)	(110,14)	(110,12)	(112,4)	(102,12)	(96,12)	(92,12)	(88,12)
128	42	(96,0)	(112,4)	(112,8)	(116,8)	(120,0)	(116,12)	(114,14)	(114,12)	(116,4)	(106,12)	(100,12)	(96,12)	(92,12)
130	44	(100,0)	(116,4)	(116,8)	(120,8)	(124,0)	(120,12)	(118,14)	(118,12)	(120,4)	(110,12)	(104,12)	(100,12)	(96,12)
132	46	(104,0)	(120,4)	(120,8)	(124,8)	(128,0)	(124,12)	(122,14)	(122,12)	(124,4)	(114,12)	(108,12)	(104,12)	(100,12)
134	48	(108,0)	(124,4)	(124,8)	(128,8)	(132,0)	(128,12)	(126,14)	(126,12)	(128,4)	(118,12)	(112,12)	(108,12)	(104,12)
136	50	(112,0)	(128,4)	(128,8)	(132,8)	(136,0)	(132,12)	(130,14)	(130,12)	(132,4)	(122,12)	(116,12)	(112,12)	(108,12)
138	52	(116,0)	(132,4)	(132,8)	(136,8)	(140,0)	(136,12)	(134,14)	(134,12)	(136,4)	(126,12)	(120,12)	(116,12)	(112,12)
140	54	(120,0)	(136,4)	(136,8)	(140,8)	(144,0)	(140,12)	(138,14)	(138,12)	(140,4)	(130,12)	(124,12)	(120,12)	(116,12)
142	56	(124,0)	(140,4)	(140,8)	(144,8)	(148,0)	(144,12)	(142,14)	(142,12)	(144,4)	(134,12)	(128,12)	(124,12)	(120,12)
144	58	(128,0)	(144,4)	(144,8)	(148,8)	(152,0)	(148,12)	(146,14)	(146,12)	(148,4)	(138,12)	(132,12)	(128,12)	(124,12)
146	60	(132,0)	(148,4)	(148,8)	(152,8)	(156,0)	(152,12)	(150,14)	(150,12)	(152,4)	(142,12)	(136,12)	(132,12)	(128,12)
148	62	(136,0)	(152,4)	(152,8)	(156,8)	(160,0)	(156,12)	(154,14)	(154,12)	(156,4)	(146,12)	(140,12)	(136,12)	(132,12)
150	64	(140,0)	(156,4)	(156,8)	(160,8)	(164,0)	(160,12)	(158,14)	(158,12)	(160,4)	(150,12)	(144,12)	(140,12)	(136,12)
152	66	(144,0)	(160,4)	(160,8)	(164,8)	(168,0)	(164,12)	(162,14)	(162,12)	(164,4)	(154,12)	(148,12)	(144,12)	(140,12)
154	68	(148,0)	(164,4)	(164,8)	(168,8)	(172,0)	(168,12)	(166,14)	(166,12)	(168,4)	(158,12)	(152,12)	(148,12)	(144,12)
156	70	(152,0)	(168,4)	(168,8)	(172,8)	(176,0)	(172,12)	(170,14)	(170,12)	(172,4)	(162,12)	(156,12)	(152,12)	(148,12)
158	72	(156,0)	(172,4)	(172,8)	(176,8)	(180,0)	(176,12)	(174,14)	(174,12)	(176,4)	(166,12)	(160,12)	(156,12)	(152,12)
160	74	(160,0)	(176,4)	(176,8)	(180,8)	(184,0)	(180,12)	(178,14)	(178,12)	(180,4)	(170,12)	(164,12)	(160,12)	(156,12)
162	76	(164,0)	(180,4)	(180,8)	(184,8)	(188,0)	(184,12)	(182,14)	(182,12)	(184,4)	(174,12)	(168,12)	(164,12)	(160,12)
164	78	(168,0)	(184,4)	(184,8)	(188,8)	(192,0)	(188,12)	(186,14)	(186,12)	(188,4)	(178,12)	(172,12)	(168,12)	(164,12)
166	80	(172,0)	(188,4)	(188,8)	(192,8)	(196,0)	(192,12)	(190,14)	(190,12)	(192,4)	(182,12)	(176,12)	(172,12)	(168,12)
168	82	(176,0)	(192,4)	(192,8)	(196,8)	(200,0)	(196,12)	(194,14)	(194,12)	(196,4)	(186,12)	(180,12)	(176,12)	(172,12)
170	84	(180,0)	(196,4)	(196,8)	(200,8)	(204,0)	(200,12)	(198,14)	(198,12)	(200,4)	(190,12)	(184,12)	(180,12)	(176,12)
172	86	(184,0)	(200,4)	(200,8)	(204,8)	(208,0)	(204,12)	(202,14)	(202,12)	(204,4)	(194,12)	(188,12)	(184,12)	(180,12)
174	88	(188,0)	(204,4)	(204,8)	(208,8)	(212,0)	(208,12)	(206,14)	(206,12)	(208,4)	(198,12)	(192,12)	(188,12)	(184,12)
176	90	(192,0)	(208,4)	(208,8)	(212,8)	(216,0)	(212,12)	(210,14)	(210,12)	(212,4)	(202,12)	(196,12)	(192,12)	(188,12)
178	92	(196,0)	(212,4)	(212,8)	(216,8)	(220,0)	(216,12)	(214,14)	(214,12)	(216,4)	(206,12)	(200,12)	(196,12)	(192,12)
180	94	(200,0)	(216,4)	(216,8)	(220,8)	(224,0)	(220,12)	(218,14)	(218,12)	(220,4)	(210,12)	(204,12)	(200,12)	(196,12)
182	96	(204,0)	(220,4)	(220,8)	(224,8)	(228,0)	(224,12)	(222,14)	(222,12)	(224,4)	(214,12)	(208,12)	(204,12)	(200,12)
184	98	(208,0)	(224,4)	(224,8)	(228,8)	(232,0)	(228,12)	(226,14)	(226,12)	(228,4)	(218,12)	(212,12)	(208,12)	(204,12)
186	100	(212,0)	(228,4)	(228,8)	(232,8)	(236,0)	(232,12)	(230,14)	(230,12)	(232,4)	(222,12)	(216,12)	(212,12)	(208,12)
188	102	(216,0)	(232,4)	(232,8)	(236,8)	(240,0)	(236,12)	(234,14)	(234,12)	(236,4)	(226,12)	(220,12)	(216,12)	(212,12)
190	104	(220,0)	(236,4)	(236,8)	(240,8)	(244,0)	(240,12)	(238,14)	(238,12)	(240,4)	(230,12)	(224,12)	(220,12)	(216,12)
192	106	(224,0)	(240,4)	(240,8)	(244,8)	(248,0)	(244,12)	(242,14)	(242,12)	(244,4)	(234,12)	(228,12)	(224,12)	(220,12)
194	108	(228,0)	(244,4)	(244,8)	(248,8)	(252,0)	(248,12)	(246,14)	(246,12)	(248,4)	(238,12)	(232,12)	(228,12)	(224,12)
196	110	(232,0)	(248,4)	(248,8)	(252,8)	(256,0)	(252,12)	(250,14)	(250,12)	(252,4)	(242,12)	(236,12)	(232,12)	(228,12)
198	112	(236,0)	(252,4)	(252,8)	(256,8)	(260,0)	(256,12)	(254,14)	(254,12)	(256,4)	(246,12)	(240,12)	(236,12)	(232,12)
200	114	(240,0)	(256,4)	(256,8)	(260,8)	(264,0)	(260,12)	(258,14)	(258,12)	(260,4)	(250,12)	(244,12)	(240,12)	(236,12)
202	116	(244,0)	(260,4)	(260,8)	(264,8)	(268,0)	(264,12)	(262,14)	(262,12)	(264,4)	(254,12)	(248,12)	(244,12)	(240,12)
204	118	(248,0)	(264,4)	(264,8)	(268,8)	(272,0)	(268,12)	(266,14)	(266,12)	(268,4)	(258,12)	(252,12)	(248,12)	(244,12)
206	120	(252,0)	(268,4)	(268,8)	(272,8)	(276,0)	(272,12)	(270,14)	(270,12)	(272,4)	(262,12)	(256,12)	(252,12)	(248,12)
208	122	(256,0)	(272,4)	(272,8)	(276,8)	(280,0)	(276,12)	(274,14)	(274,12)	(276,4)	(266,12)	(260,12)	(256,12)	(252,12)
210	124	(260,0)	(276,4)	(276,8)	(280,8)	(284,0)	(280,12)	(278,14)	(278,12)	(280,4)	(270,12)	(264,12)	(260,12)	(256,12)
212	126	(264,0)	(280,4)	(280,8)	(284,8)	(288,0)	(284,12)	(282,14)	(282,12)	(284,4)	(274,12)	(268,12)	(264,12)	(260,12)
214	128	(268,0)	(284,4)	(284,8)	(288,8)	(292,0)	(288,12)	(286,14)	(286,12)	(288,4)	(278,12)	(272,12)	(268,12)	(264,12)
216	130	(272,0)	(288,4)	(288,8)	(292,8)	(296,0)	(292,12)	(290,14)	(290,12)	(292,4)	(282,12)	(276,12)	(272,12)	(268,12)
218	132	(276,0)	(292,4)	(292,8)	(296,8)	(300,0)	(296,12)	(294,14)	(294,12)	(296,4)	(286,12)	(280,12)	(276,12)	(272,12)
220	134	(280,0)	(296,4)	(296,8)	(300,8)	(304,0)	(300,12)	(298,14)	(298,12)	(300,4)	(290,12)	(284,12)	(280,12)	(276,12)
222	136	(284,0)	(300,4)	(300,8)	(304,8)	(308,0)	(304,12)	(302,14)	(302,12)	(304,4)	(294,12)	(288,12)	(284,12)	(280,12)
224	138	(288,0)	(304,4)	(304,8)	(308,8)	(312,0)	(308,12)	(306,14)	(306,12)	(308,4)	(298,12)	(292,12)	(288,12)	(284,12)
226	140	(292,0)	(308,4)	(308,8)	(312,8)	(316,0)	(312,12)	(310,14)	(					

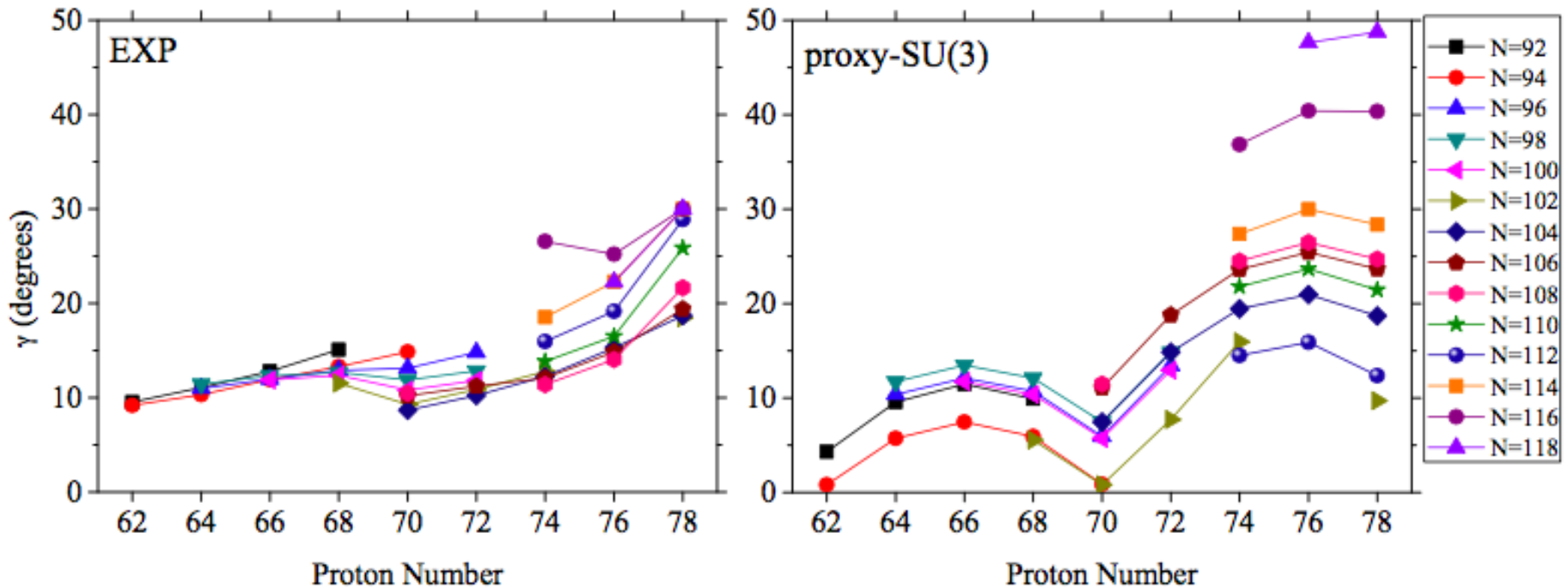
# Proxy-SU(3) predictions of the $\beta$ deformation variable



# Proxy-SU(3) predictions of the $\beta$ deformation variable



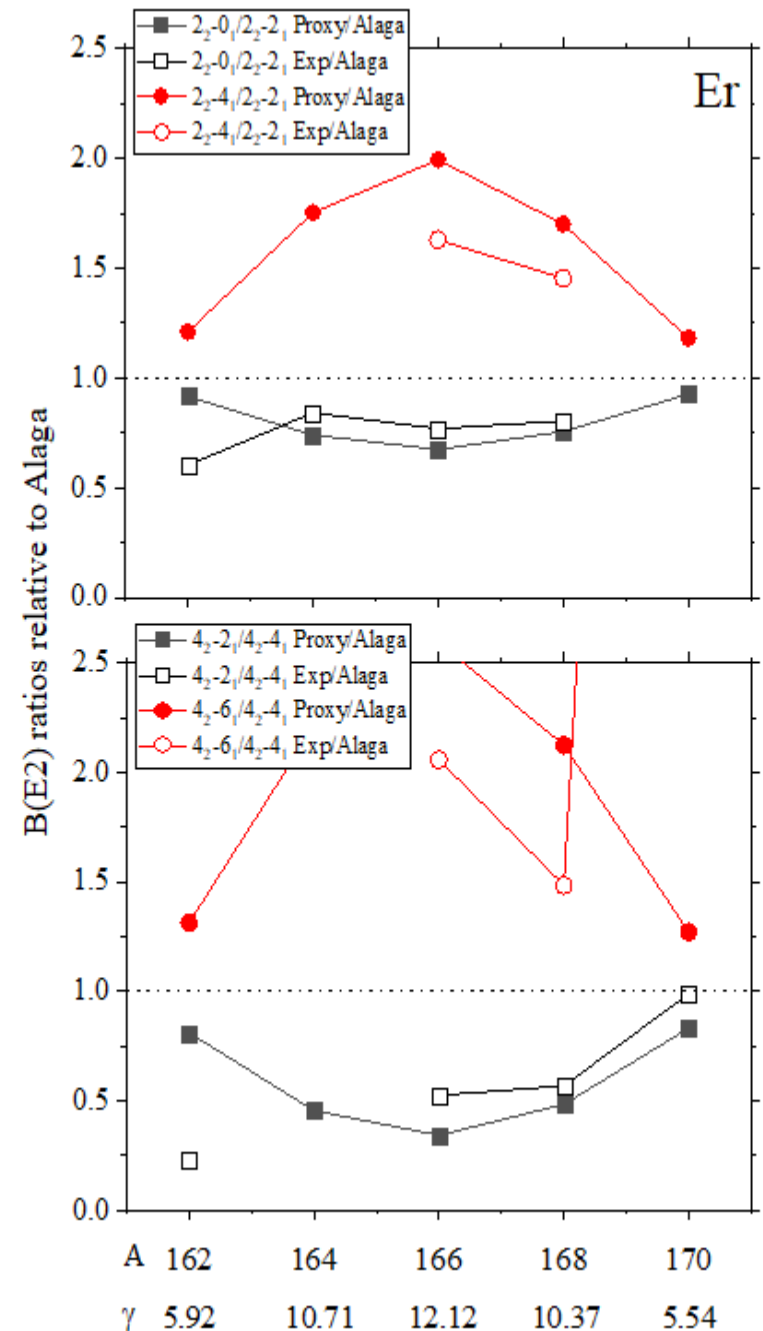
# Empirical values and Proxy-SU(3) predictions of $\gamma$ deformation values



**Similar trends: 10-15 deg. rising to ~30 degrees**  
**Oscillations in the predictions: Presumably due to neglect of pairing which would average out predictions over several neutron numbers**

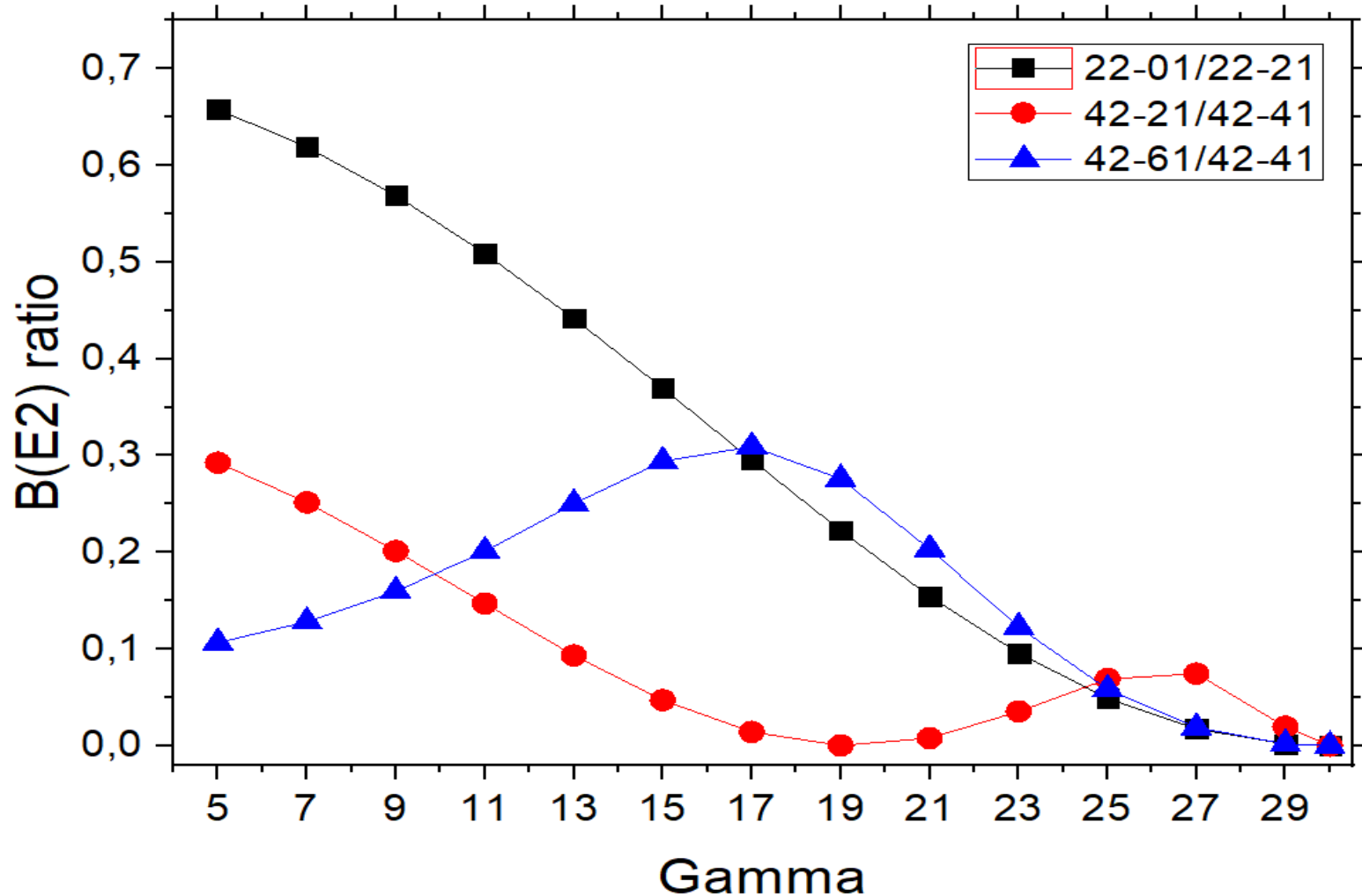
Proxy  $\rightarrow \gamma \rightarrow$  Davydov  $\rightarrow$   
 $B(E2: \gamma \rightarrow \text{ground})$  values

- Finite  $\gamma$  is equivalent to bandmixing
- Deviations from Alaga rules increase with  $\gamma$
- Spin **INcreasing** transitions INcrease rel to Alaga
- Spin **DEcreasing** transitions DEcrease rel to Alaga
- Er:  $\gamma$  values vary “parabolically”. So do predicted Proxy  $B(E2)$ s





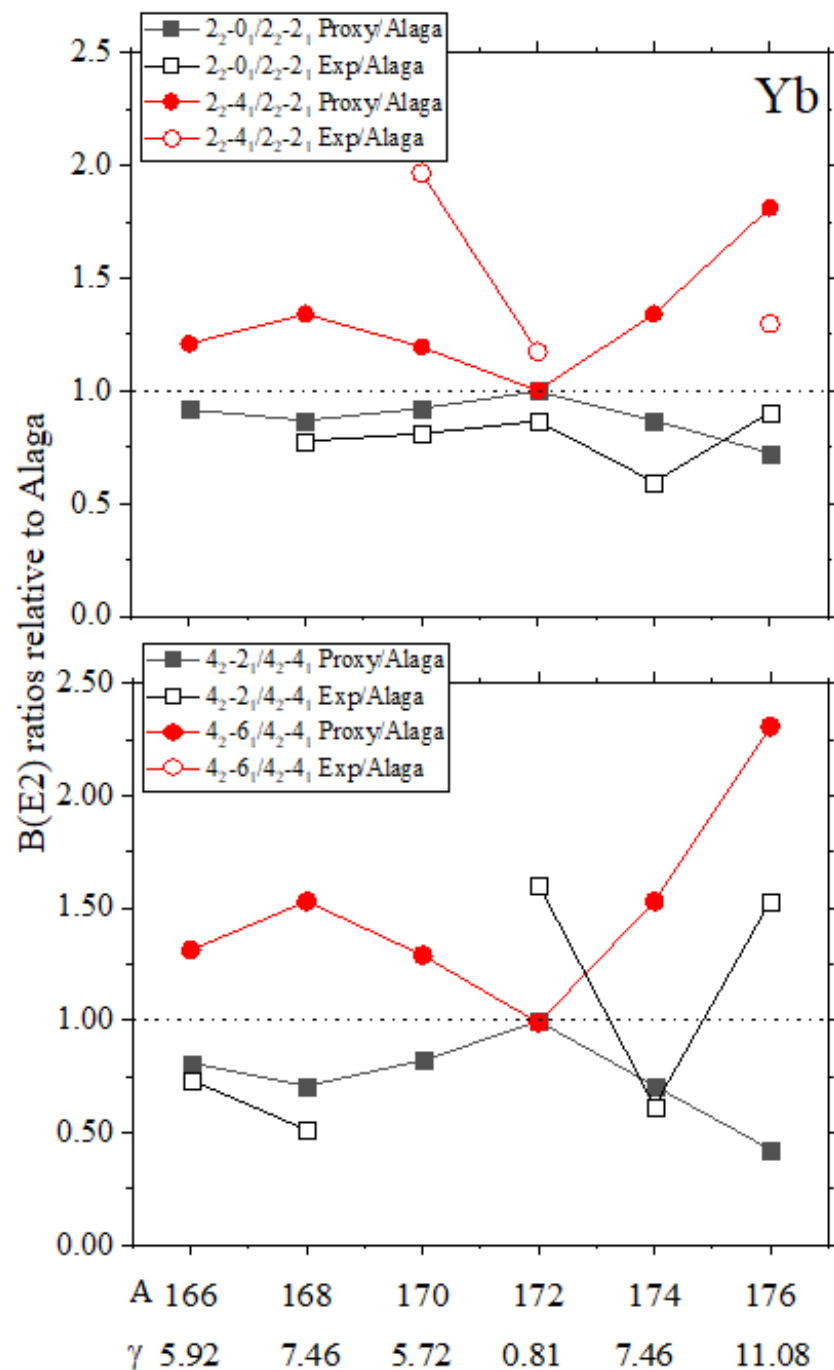
Use Proxy  $\gamma$  in Davydov model to predict  $B(E2: \gamma \rightarrow \text{ground})$  values: deviations from Alaga rules:  
Generic behavior of  $B(E2: \gamma \rightarrow \text{ground})$  values vs  $\gamma$



# Full set of Proxy-Davydov predictions for $^{168}\text{Er}$

$J_{\text{initial}}$	$J_{\text{final}}$	$^{168}\text{Er-EXP}$	Alaga	Proxy	$Z\gamma=0.035$	PDS	CQF
$2_{\gamma}$	$0^{+}$	56.2(11)	70	52.9	56.9	64.3	54
	$2^{+}$	100	100	100	100	100	100
	$4^{+}$	7.3(4)	5	8.5	7.6	6.3	8
$3_{\gamma}$	$2^{+}$	100	100	100	100	100	100
	$4^{+}$	62.6(14)	40	73	62.9	49.3	69
$4_{\gamma}$	$2^{+}$	19.3(4)	34	16.4	20.2	28.1	18
	$4^{+}$	100	100	100	100	100	100
	$6^{+}$	13.1(12)	8.6	18.7	16	12.5	16
$5_{\gamma}$	$4^{+}$	100	100	100	100	100	100
	$6^{+}$	123(14)	57.1	147.7	117	79.6	125
$6_{\gamma}$	$4^{+}$	11.2(10)	26.9	7.4	11	20.3	9
	$6^{+}$	100	100	100	100	100	100
	$8^{+}$	37.6(72)	10.6	27.9	23.6	18	20

- Yb:  $\gamma$  values vary erratically. So do predicted Proxy B(E2)s



# Summary

- Approximate shell model scheme – physics-motivated orbit substitution – recovers an  $SU(3)$  symmetry for heavy deformed nuclei
- The Proxy Scheme: simple but with inherent flaws to be assessed
- Analytic parameter-free predictions: ground state shapes, prolate-oblate transition. Overall agreement.
- $\gamma$ -band to ground band  $B(E2)$ s:  
Proxy  $\rightarrow \gamma \rightarrow$  Davydov  $\rightarrow B(E2)$ s: Relation to bandmixing
- Improvements (e.g., pairing) and extensions (e.g., higher intrinsic excitations)

# Collaborators

**Dennis Bonatsos**

**Nikolay Minkov**

**R. Burcu Cakirli**

**Klaus Blaum**

**Andriana Martinou**

**I.E. Assimakis**

**S. Sarantopoulou**

**RFC**

**PR. C 95, 064325(2017): Proxy-SU(3) symmetry in heavy def. nuclei**

**PR C 95, 064326(2017): Analytic predictions for nuclear shapes, prolate dominance, and the P-O shape transition in the proxy-SU(3) model**

**B(E2) values, in preparation**