Bohr model description of the critical point for the first order shape phase transition

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9th International workshop on Quantum Phase Transitions in Nuclei and Many – Body Systems, Padova (Italy), 22 – 25 May 2018

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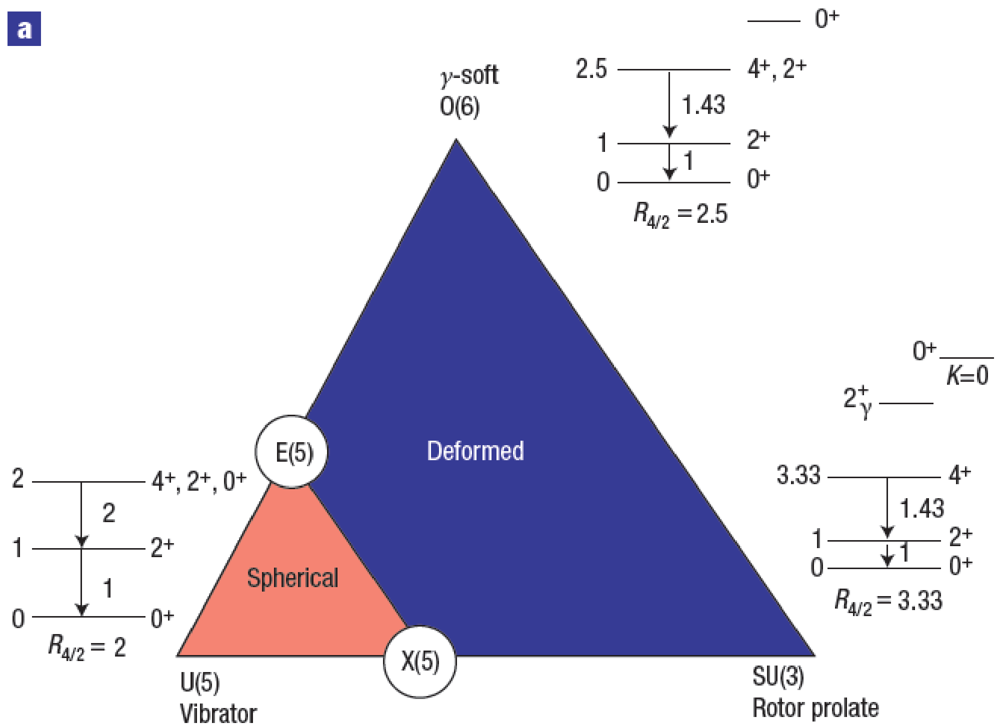
Motivation for the present study



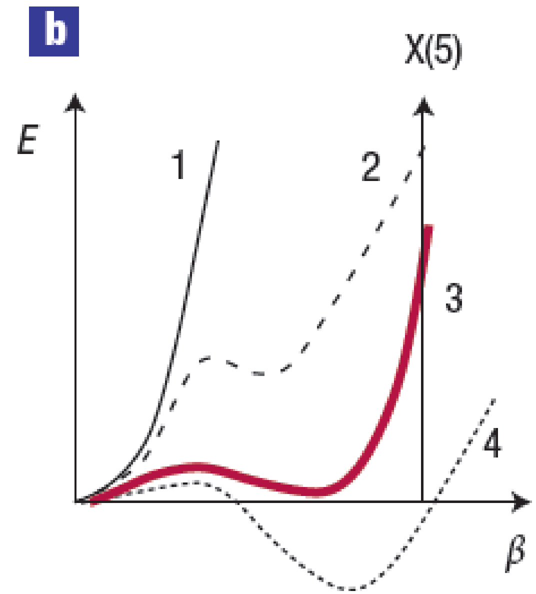
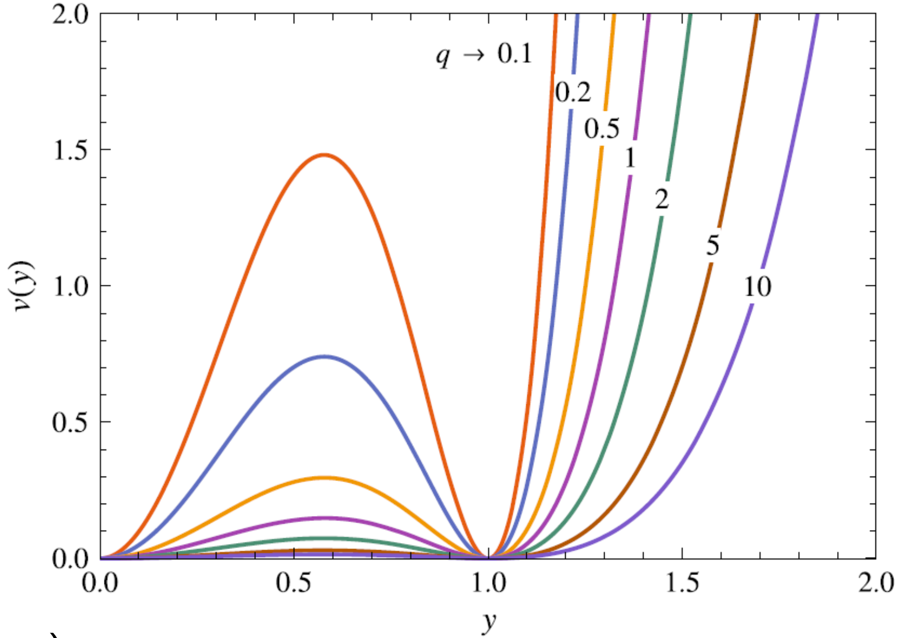


Fig. 3b, for a first-order transition, a coexisting, deformed minimum appears (curve 2) as an excited configuration. With increasing valence nucleon number, its energy decreases, eventually becoming the equilibrium shape (curve 4). The point of degeneracy (curve 3) of the coexisting shapes is the critical point.

Figures taken from Ref. [R. F. Casten, Nature Physics 2 (2006) 811-820.]

X(5): [F. Iachello, Phys. Rev. Lett. 87 (2001) 052502]

Bohr Hamiltonian with a sextic potential having simultaneous spherical and deformed minima of the same depth



Radial like differential equation for the β variable:
[F. Iachello, Phys. Rev. Lett. 87 (2001) 052502]

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{L(L+1)}{3\beta^2}+v(\beta)\right]\Psi(\beta)=\epsilon^{\beta}\Psi(\beta).$$

A general sextic potential:
[R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26 - 31]

$$v(\beta)=a\beta^2+b\beta^4+c\beta^6,$$

The eigenvalue is scaled as fallows:

$$\epsilon^{\beta}(a,b,c)=a^{1/2}\epsilon^{\beta}(1,ba^{-3/2},ca^{-2}).$$

$$v(\beta)=\beta^2+\mu\beta^4+\nu\beta^6.$$

Potential with both minima degenerated (at the same energy):

$$v(\beta)=\beta^2-2q\beta^4+q^2\beta^6.$$

q is connected to the height of the barrier which separates the two minima

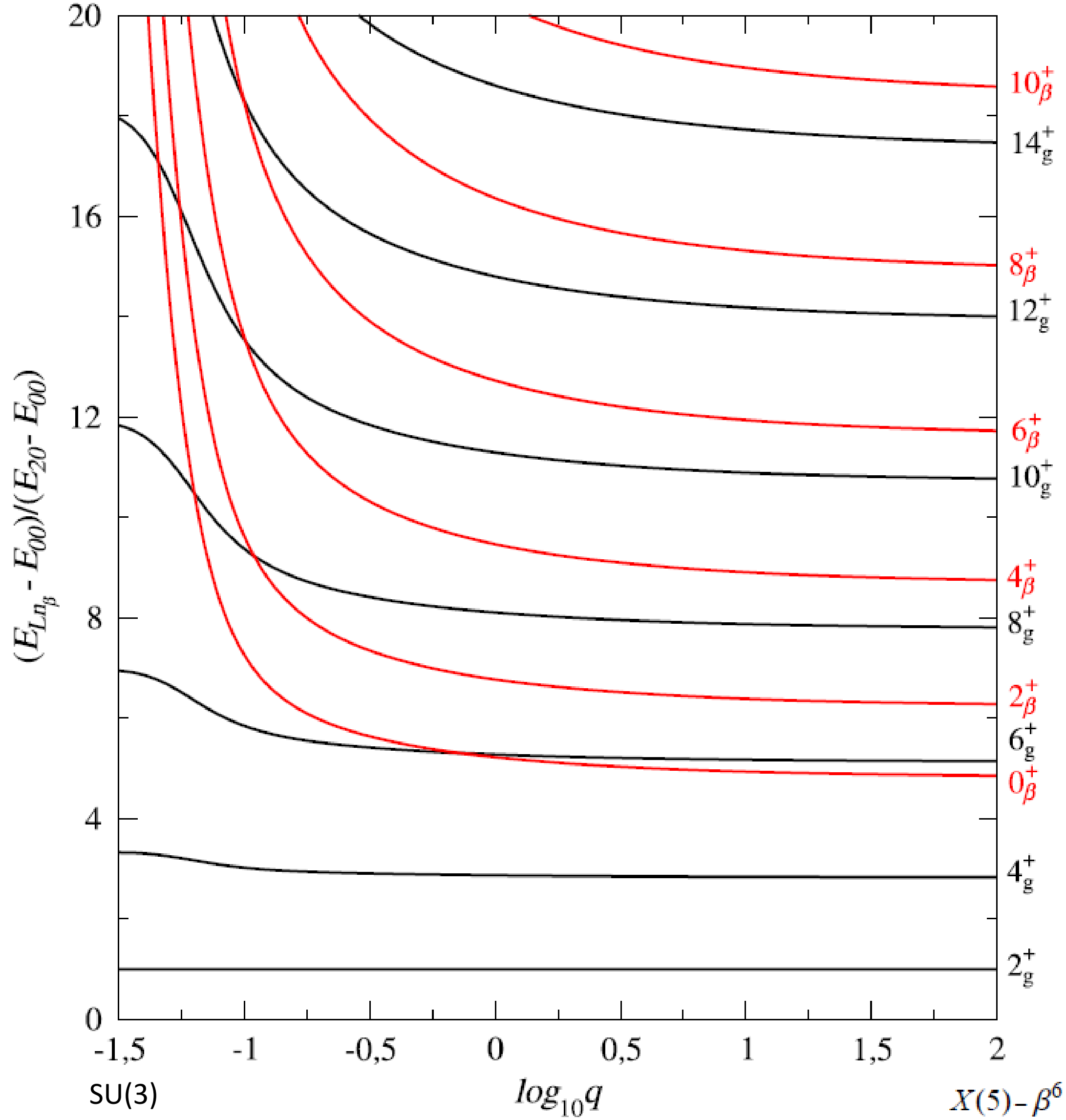
A final change of variable: $y=\sqrt{q}\beta.$

$$\left[-\frac{\partial^2}{\partial y^2}-\frac{4}{y}\frac{\partial}{\partial y}+\frac{L(L+1)}{3y^2}+v(y)\right]\Psi(y)=E\Psi(y),\quad v(y)=\frac{1}{q^2}\left(y^2-2y^4+y^6\right)\quad E=\epsilon^{\beta}/q.$$

Diagonalization by using Bessel functions of first kind (for ISW potential) as a basis:

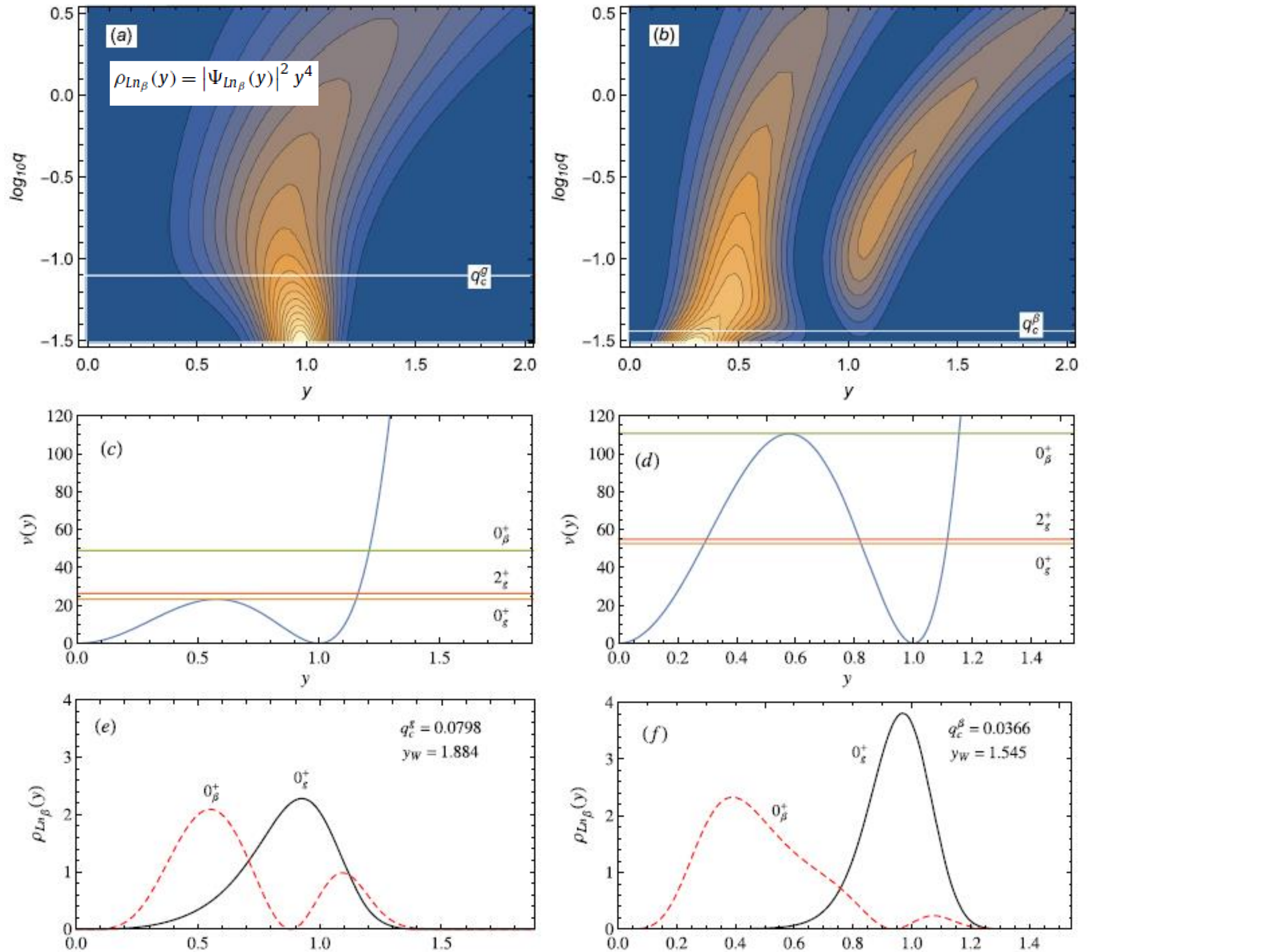
$$\Psi(y_W)=0\quad \tilde{v}(y)=\begin{cases}0, & y\le y_W,\\ \infty, & y>y_W,\end{cases}\quad \Psi_{Lk}(y)=\sum_{n}^{n_{max}}A_n^k\tilde{\Psi}_{\nu n}(y),\quad \nu=\sqrt{\frac{9}{4}+\frac{L(L+1)}{3}},\quad k=n_{\beta}+1,$$

H. Tașeli, A. Zafer, Int. J. Quant. Chem. 61 (1997) 759; Int. J. Quant. Chem. 63 (1997) 935; J. Comput. Appl. Math. 95 (1998) 83.



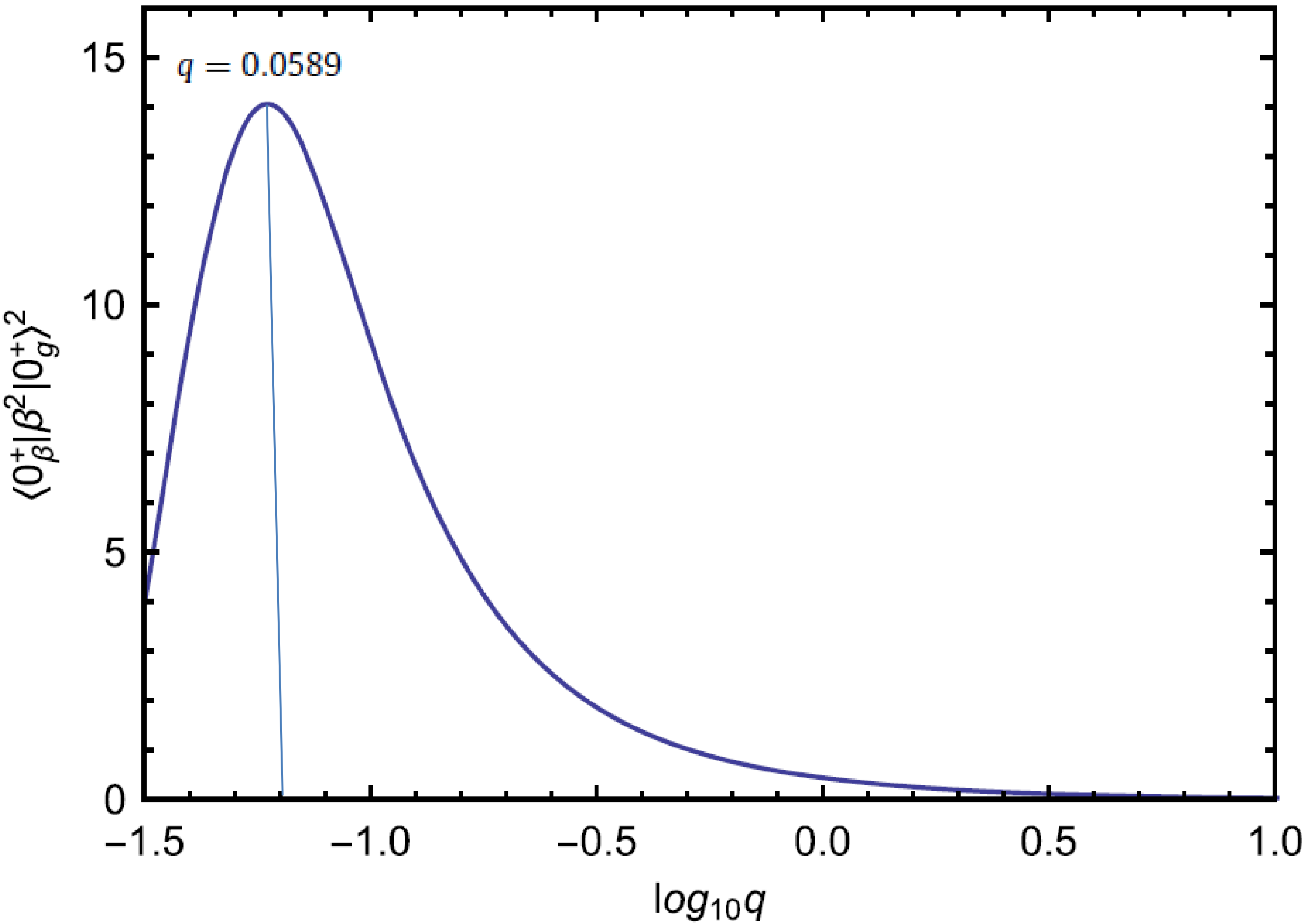
The low – lying energy spectrum of the ground band and first excited β band.

[R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26 - 31]



Contour plots of the density of probability distribution as a function of y and q for ground state (a) and for the first excited β state (b).
The potential and the absolute values for the 0_g^+, 2_g^+ and 0_β^+ energy levels are given in the same arbitrary units for the values q_c^g (c) and q_c^β (d).

[R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26 - 31]



Monopole transition matrix element.

[R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26 - 31]

“In general, nuclei characterized by coexisting shapes having different deformations will exibit strong ρ2(E0)”, J. L. Wood, E. F. Zganjar, C. De Coster, K. Heyde, Nucl. Phys. A 651 (1999) 323 – 368.

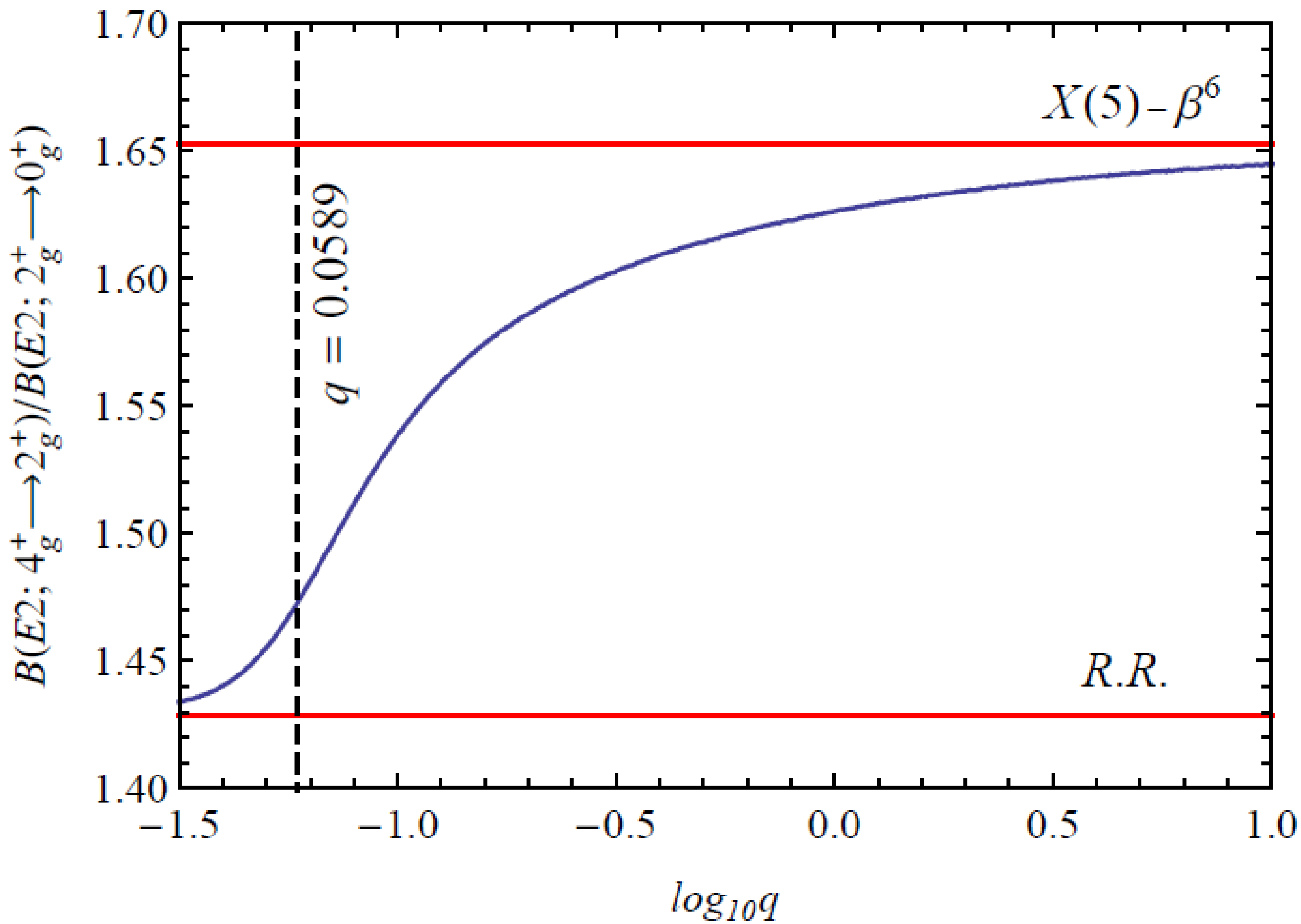


Figure 2. (color online) The ratio $B(E2; 4_g^+ \to 2_g^+)/B(E2; 2_g^+ \to 0_g^+)$ given as a function of $\log_{10} q$. The values corresponding to the rigid rotor estimation and that corresponding to the $X(5)$-β^6 prediction are presented as red lines for reference. The vertical dashed line marks the critical value of the transition between the rigid rotor picture and the critical point instance described by the $X(5)$-β^6 model.

[R. Budaca, P. Buganu, A. I. Budaca, Bulg. J. Phys. 44 (2017) 319 – 325.]

Conclusions

- The influence hand by the barrier, separating the spherical and deformed minima, for the critical point of the phase transition from spherical vibrator to axial rotor is investigated in the frame of the Bohr-Mottelson model using a sextic potential for the β variable.

- The Hamiltonian is diagonalized in a basis of Bessel functions of the first kind obtained solving the β equation for an infinite square well potential. The eigenvalues depend only on a free parameter related with the height of the barrier.

- Analyzing the density distribution probabilities for the ground state and for the first excited β state, but also the monopole transition between these two states one can see how the barrier influence the deformation of the ground and excited states and moreover, the fact that these states presents shape coexistence signatures.

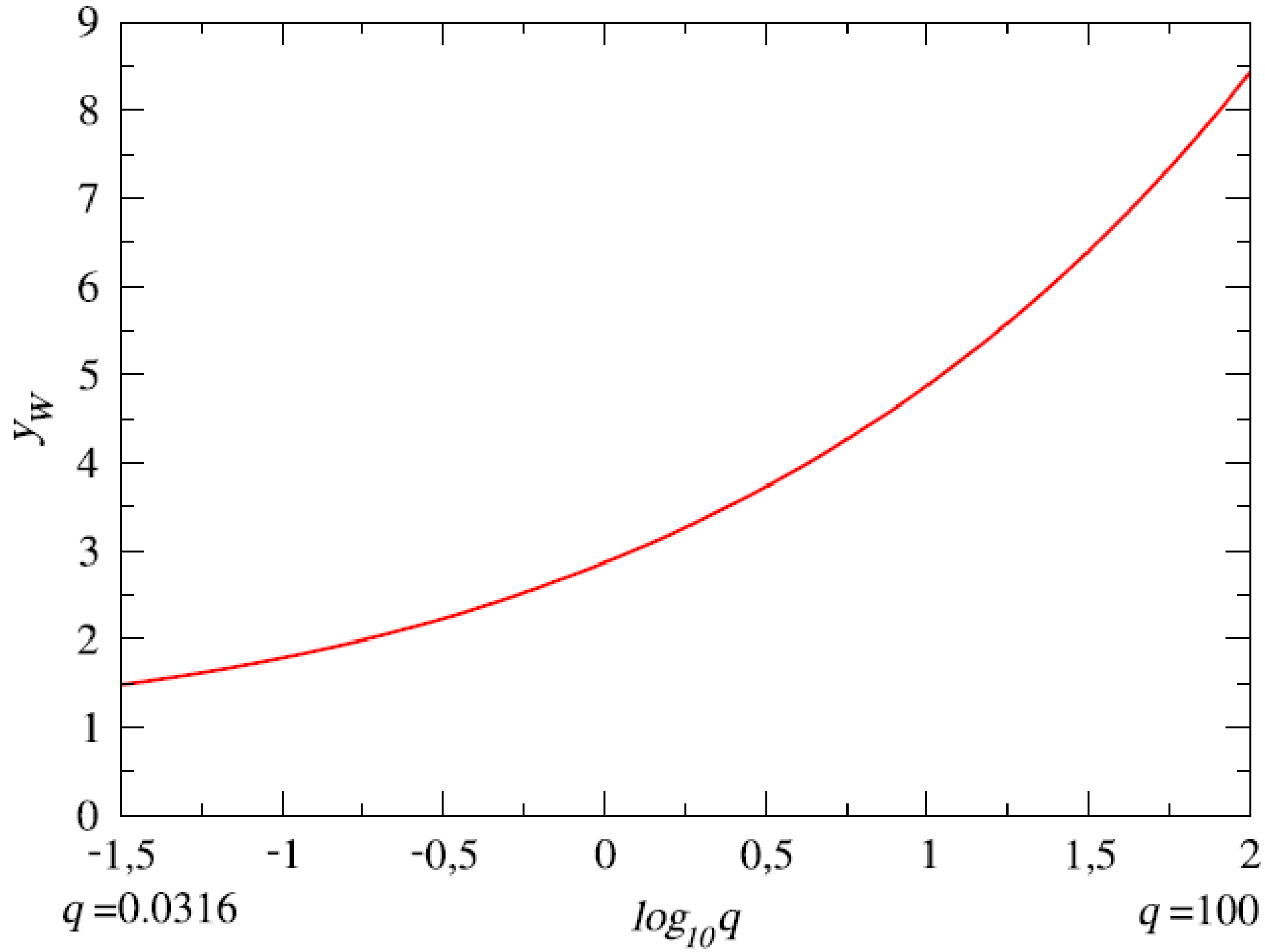


Fig. 2. The boundary value y_W as function of the potential parameter q determined for a basis with $n_{max} = 20$ and with a convergence precision $\varepsilon < 10^{-7}$.

[R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26 - 31]

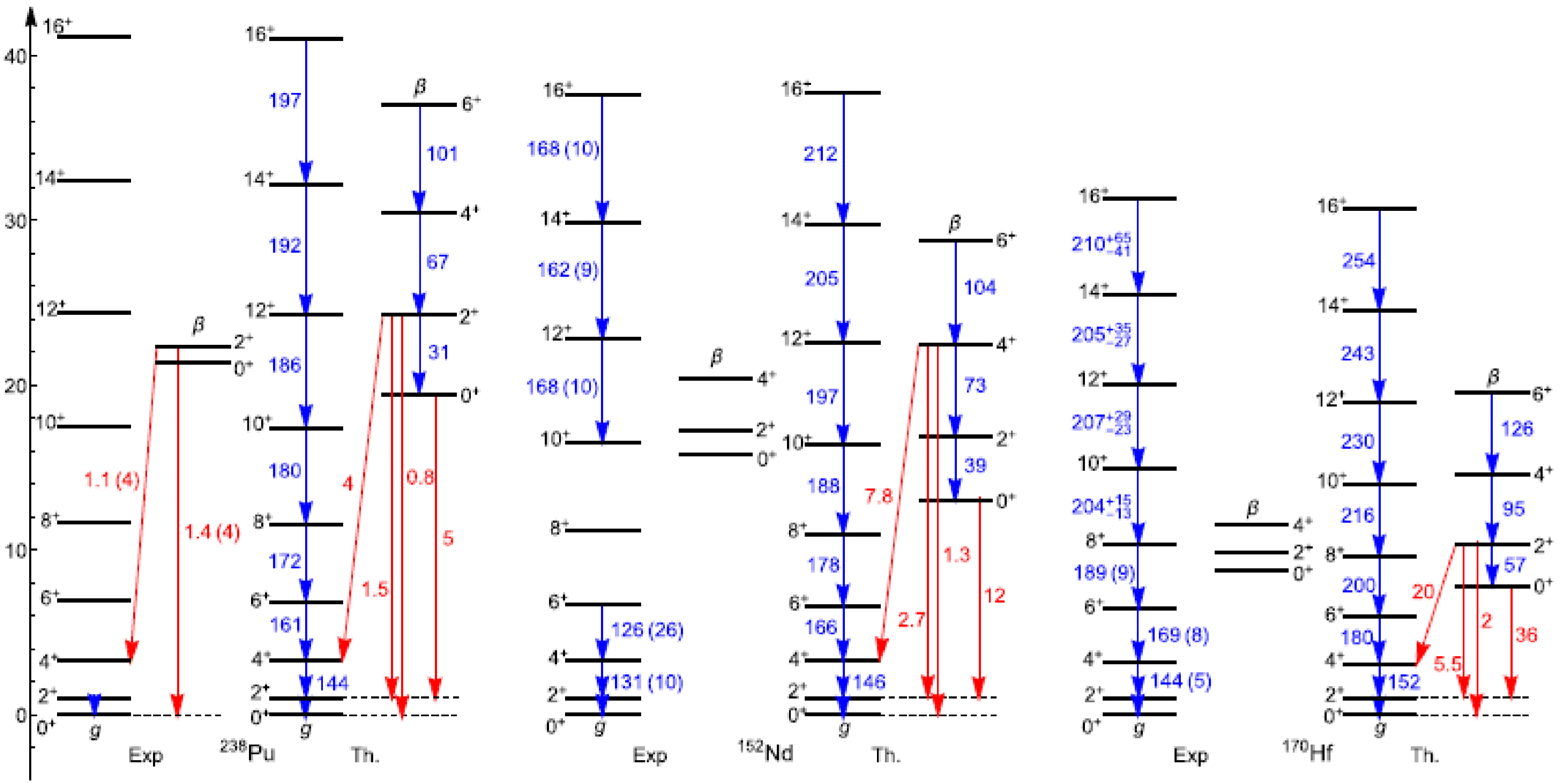


Fig. 5. Theoretical results for ground and β band energies normalized to the energy of the first excited state 2_g^+ and associated $E2$ transition probabilities similarly normalized to the $B(E2, 2_g^+ \to 0_g^+)$ value, are compared with the available experimental data for ^{238}Pu [45], ^{152}Nd [46] and ^{170}Hf [47,48] nuclei. Theoretical predictions of the fits against the free parameter q whose resulting values are 0.0422, 0.0535, 0.0833 correspond to boundary parameters $y_W = 1.607, 1.710, 1.928$ and rms deviations 0.93, 1.09 and 1.21, respectively.