

Coexistent Shapes in the Bohr Hamiltonian: Limitations and phenomenological challenges

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GENESE17
Geometries of Exotic
NuclEar StructurE



Padova, May 21, 2018

Bohr Hamiltonian

$$H = T + V(\beta, \gamma)$$

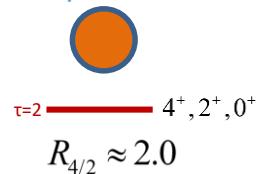
$$V(\beta, \gamma) = V(\beta) \quad \beta, \gamma \text{ decoupling}$$

$$H\Psi = E\Psi \Rightarrow \left[-\frac{\hbar^2}{2B} \left(\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} - \frac{\Lambda^2}{\beta^2} \right) + V(\beta) \right] \Psi = E\Psi, \quad \text{Moments of Inertia: } \mathfrak{I}(\beta) = B\beta^2$$

$$\Psi(\beta, \gamma, \Omega_3) = f(\beta)Y(\gamma, \Omega_3), \underbrace{\Lambda^2 Y_{\tau n_\Lambda LM}(\gamma, \Omega_3)}_{\text{SO}(5) \text{ Invariance: } \tau} = \tau(\tau+3)Y_{\tau n_\Lambda LM}(\gamma, \Omega_3), R(\beta) = \beta^2 f(\beta),$$

SO(5) Invariance: τ

Spherical



$$R_{4/2} \approx 2.0$$

$$\left(-\frac{\partial^2}{\partial \beta^2} + \frac{(\tau+1)(\tau+2)}{\beta^2} + v(\beta) \right) R(\beta) = \varepsilon R(\beta)$$

$$\frac{2B}{\hbar^2} V(\beta)$$

$$\frac{2B}{\hbar^2} E$$

γ -unstable deformed



$$R_{4/2} \approx 2.5$$

$$\tau=1 \quad 2^+$$

$$\tau=0 \quad 0^+$$

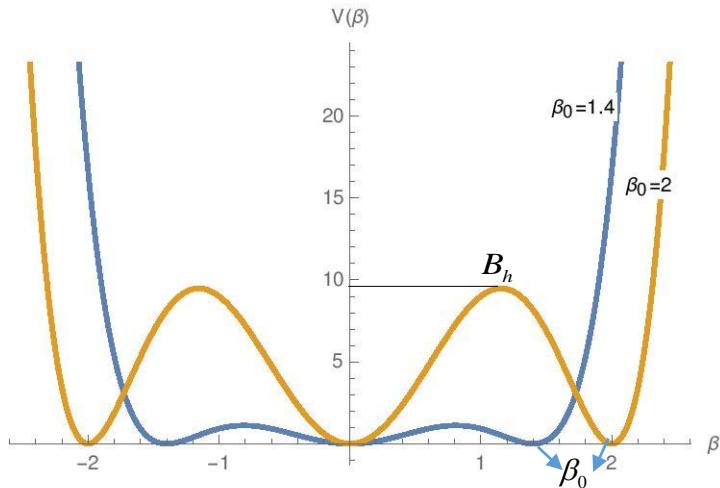
$$\langle \beta^2 \rangle > 0$$

Radial Equation

L. Wilets and M Jean,
Phys. Rev. 102, 3 (1956), 788.

Coexistent Shapes in the Bohr Hamiltonian: Potentials with double minima

SO(5) Invariant Potential Energy Surface



Two main characteristics:

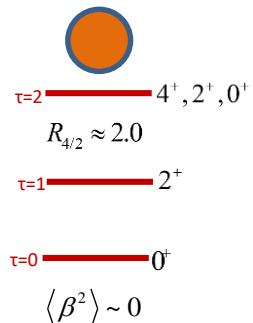
$$\text{1st: } B_h = \frac{4\beta_0^6 v_0}{27}$$

2nd: Non-constant β stiffness.

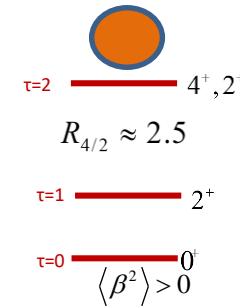
A Leviatan and N Gavrielov, Phys. Scr. 92, 11, 2017

$$v(\beta) = v_0 \beta^2 (\beta^2 - \beta_0^2)^2 = v_0 \beta_0^4 \beta^2 - 2v_0 \beta_0^2 \beta^4 + v_0 \beta^6$$

Spherical: $U(5)$



γ -unstable deformed: $O(6)$



$$v_{eff}^\tau(\beta) = \frac{(\tau+1)(\tau+2)}{\beta^2} + v_0 \beta_0^4 \beta^2 - 2v_0 \beta_0^2 \beta^4 + v_0 \beta^6$$

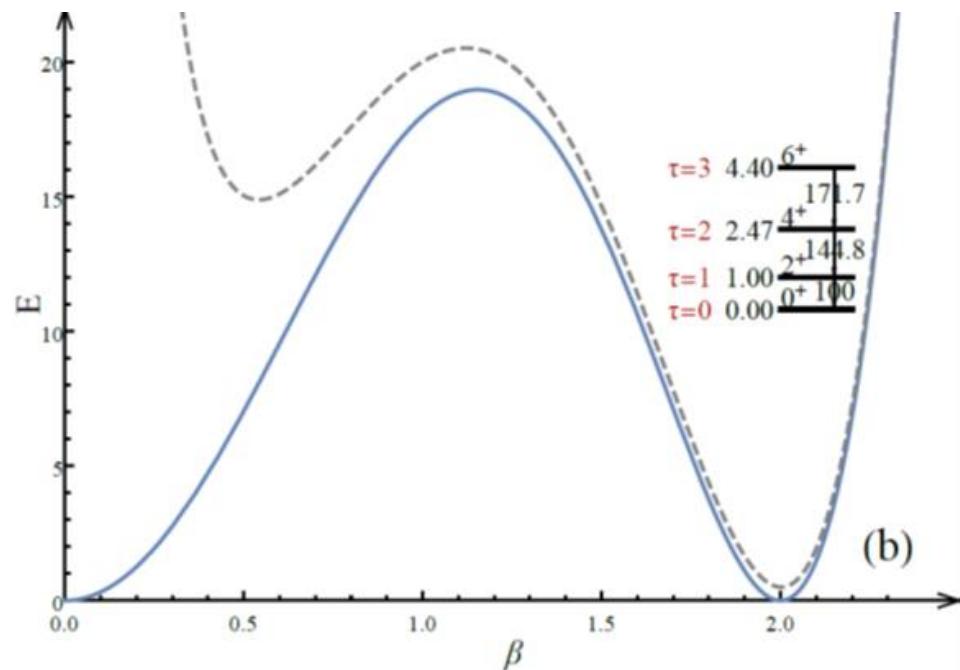
Limitations from the Bohr Hamiltonian

1. Where are the spherical states?

They need the appropriate basis. Spherical and Deformed shapes in the Bohr Hamiltonian are manifested through the use of two bases: The same principle is followed in Hartree-Fock Theories and recently in the IBM.

P E Georgoudis and A Leviatan, J.Phys.Conf.Ser. 966 (2018) no.1, 012043.

$$v_{eff}^0(\beta) = \frac{(0+1)(0+2)}{\beta^2} + v_0 \beta_0^4 \beta^2 - 2v_0 \beta_0^2 \beta^4 + v_0 \beta^6$$



D.J.Rowe, T.A. Welsh,
M.A.Caprio, Phys.Rev.C
79,054304 ,(2009)

The SU(1,1)xSO(5) Basis

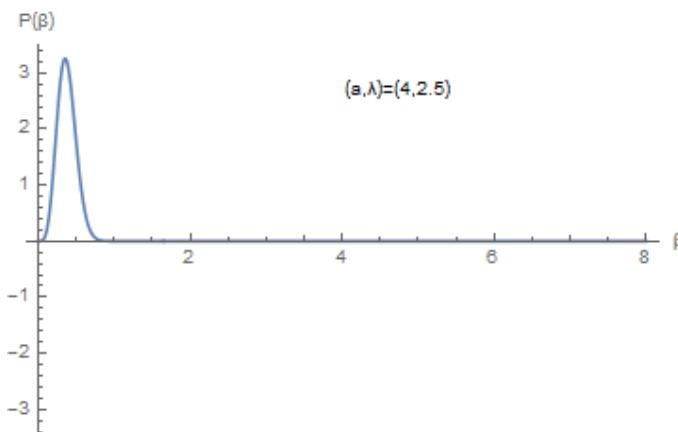
$$\Psi_{\lambda\nu;\tau n_\Delta LM}(\beta, \gamma, \Omega_3) = \frac{1}{\beta^2} R_\nu^\lambda(\beta) Y_{\tau n_\Delta LM}(\gamma, \Omega_3)$$

$$R_\nu^\lambda(a\beta) = (-1)^\nu \sqrt{\frac{2\nu!a}{\Gamma(\nu+\lambda)}} (a\beta)^{\lambda-1/2} e^{-a^2\beta^2/2} L_\nu^{(\lambda-1)}(a^2\beta^2)$$

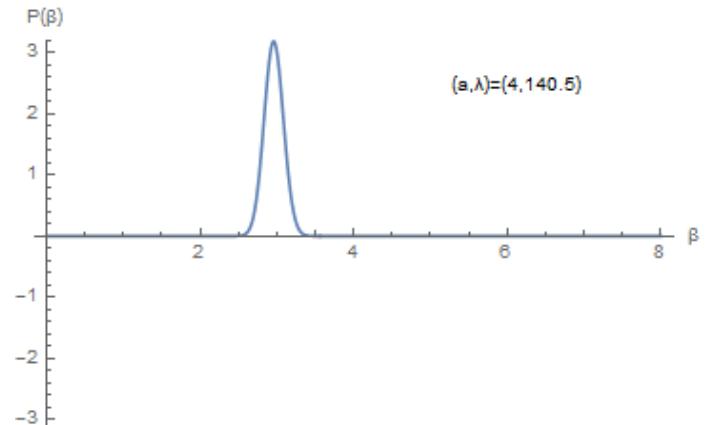
J.P. Elliott, J.A. Evans and P.Park
Phys. Lett. 169B,4 (1986), 309.

For a spherical
 $\tau=0$ state, $\lambda=2.5$

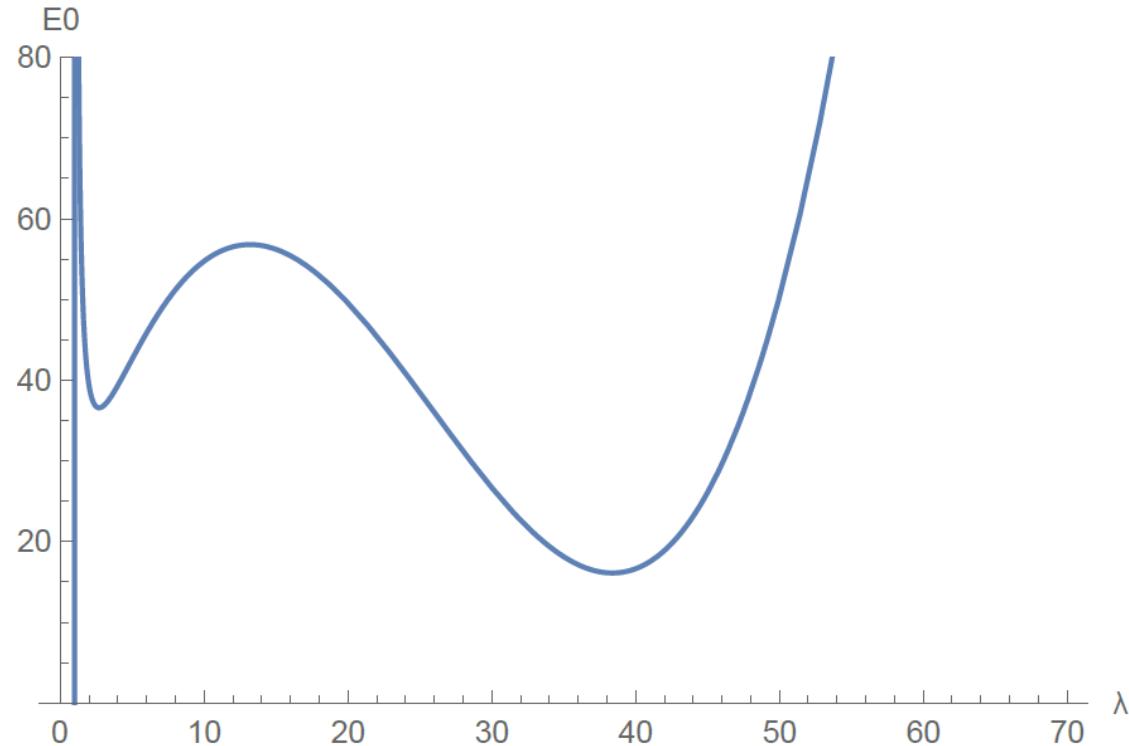
a : Width
 λ : Position



For a deformed
 $\tau=0$ state $\lambda >> 2.5$

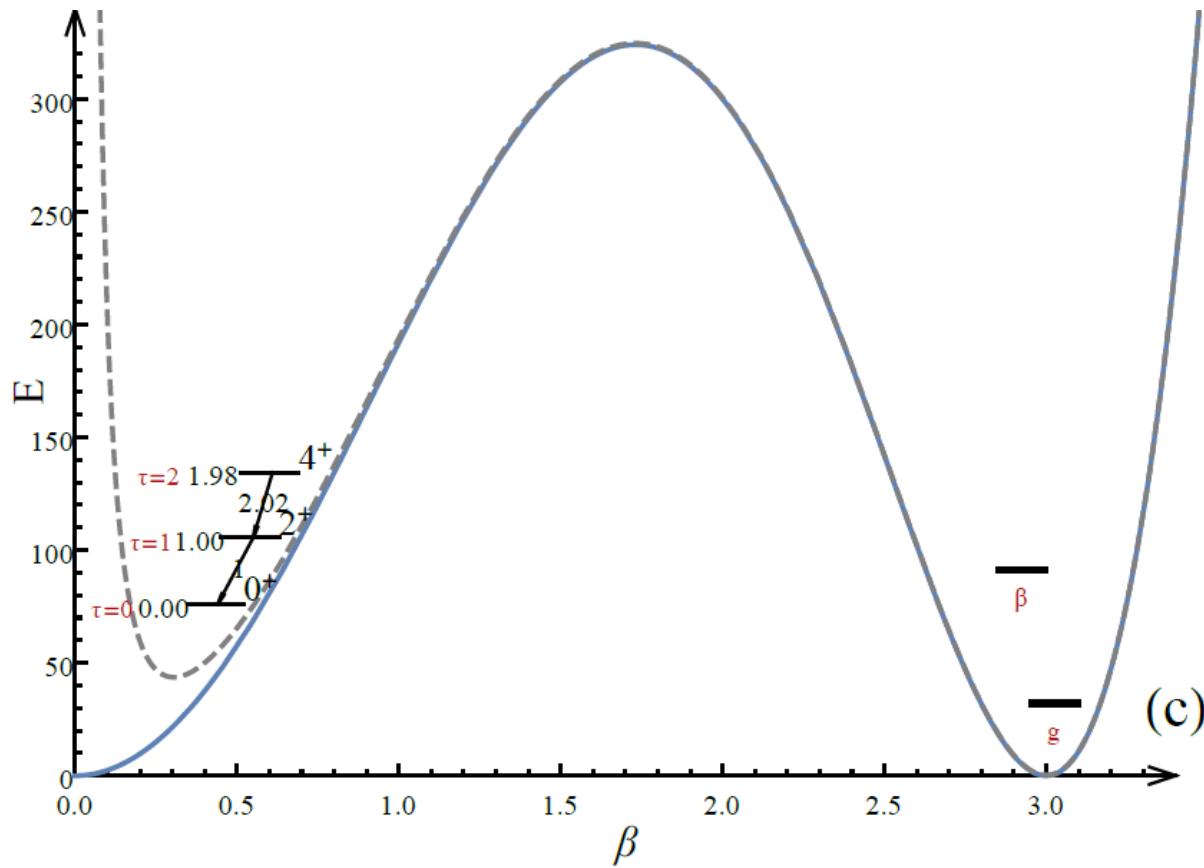


**Choose the best basis wavefunction by minimizing the expectation
Value of the Bohr Hamiltonian with the sextic potential in a $\tau=0$ state: E_0**



*This work performed under the supervision of Ami Leviatan
at the Racah Institute of Physics and supported partially by
a Golda Meir Fellowship during 2016-2017.*

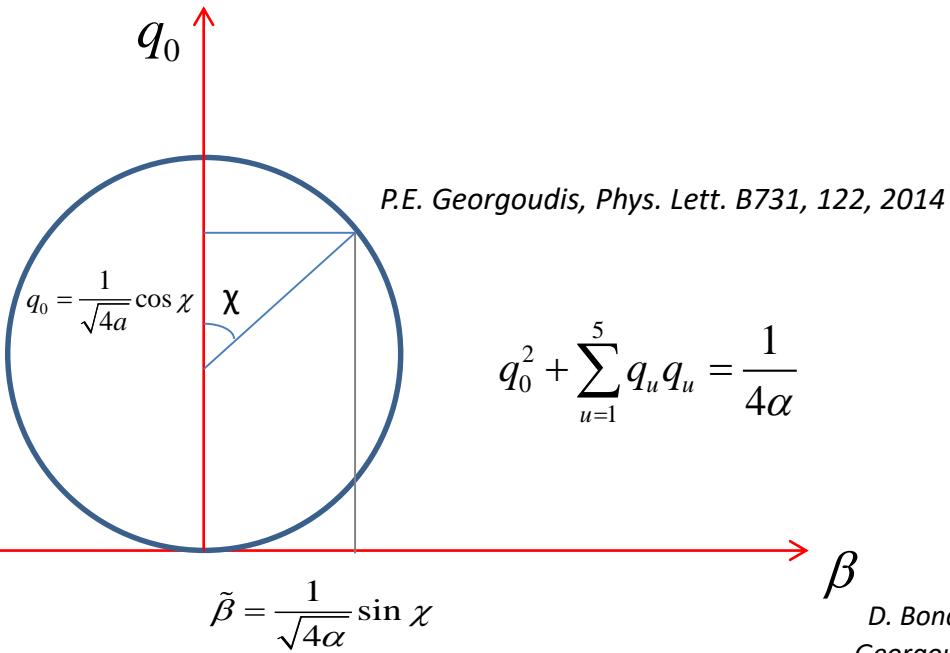
So, we do have spherical and deformed states but not coexistence.



2. What happens with the rotational states within the deformed minimum?
- We need to alter the β dependence of the moments of inertia.

Local Scale Factor and Shape Coexistence

$$q_u \rightarrow \frac{q_u}{1 + \alpha q \cdot q}$$



$$g_{ij} \rightarrow \tilde{g}_{ij} = \frac{1}{(1 + \alpha \beta^2)^2} g_{ij}$$

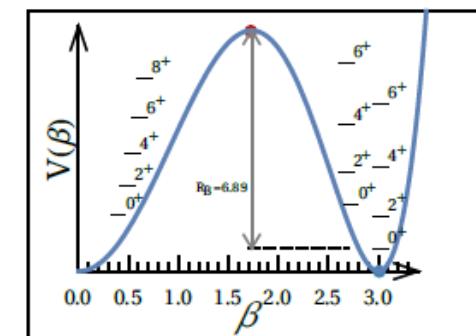
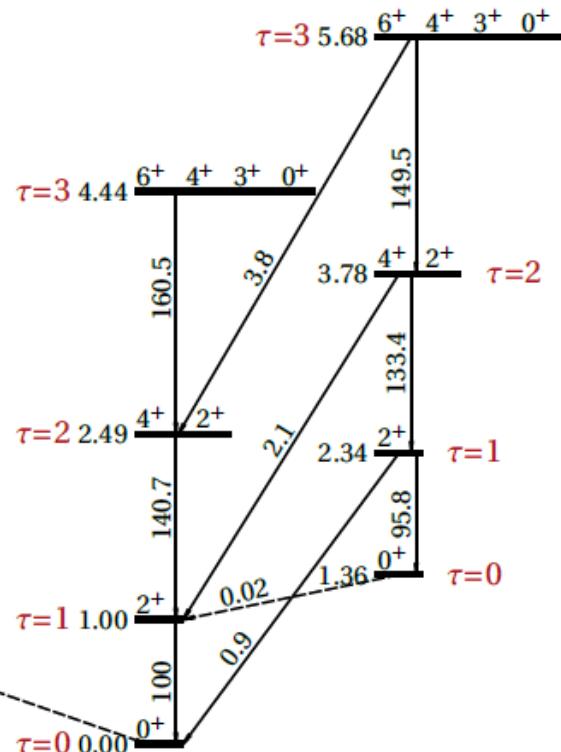
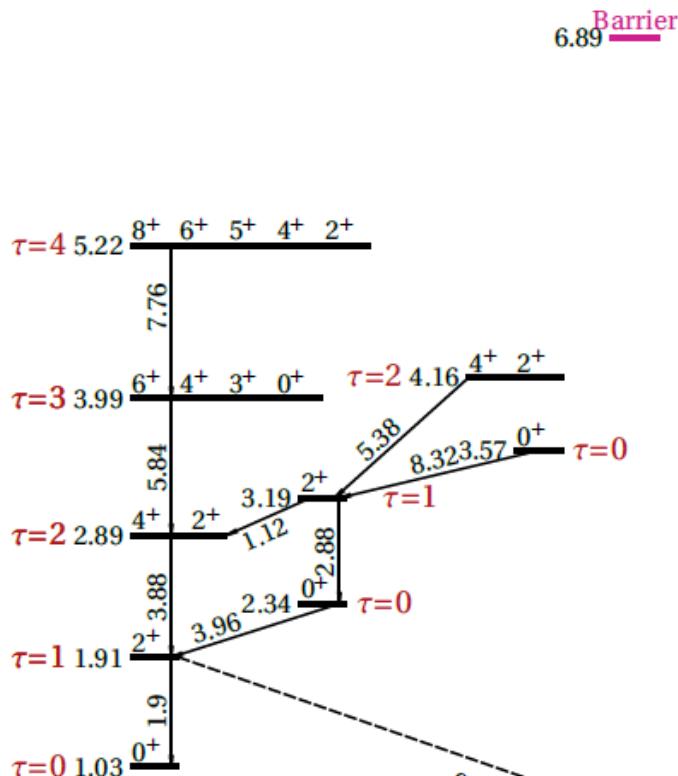


$$\Im(\beta) = \frac{\beta^2}{(1 + \alpha \beta^2)^2}$$

$$v_{eff}^\tau(\beta) = \frac{(\tau+1)(\tau+2)}{\beta^2} + \alpha \beta^2 \tau (\tau+3) + \alpha^2 \tau (\tau+3) + v_0 \beta_0^4 \beta^2 - 2v_0 \beta_0^2 \beta^4 + v_0 \beta^6$$

D. Bonatsos, P. E.
Georgoudis, D. Lenis, N.
Minkov, and C. Quesne,
Phys. Rev. C **83**, 044321
(2011).

Final Limitations from the degenerate minima:



The $B(E2)$ s and $B(E0)$ s between the spherical and the deformed states are 0. The ground State is the deformed.

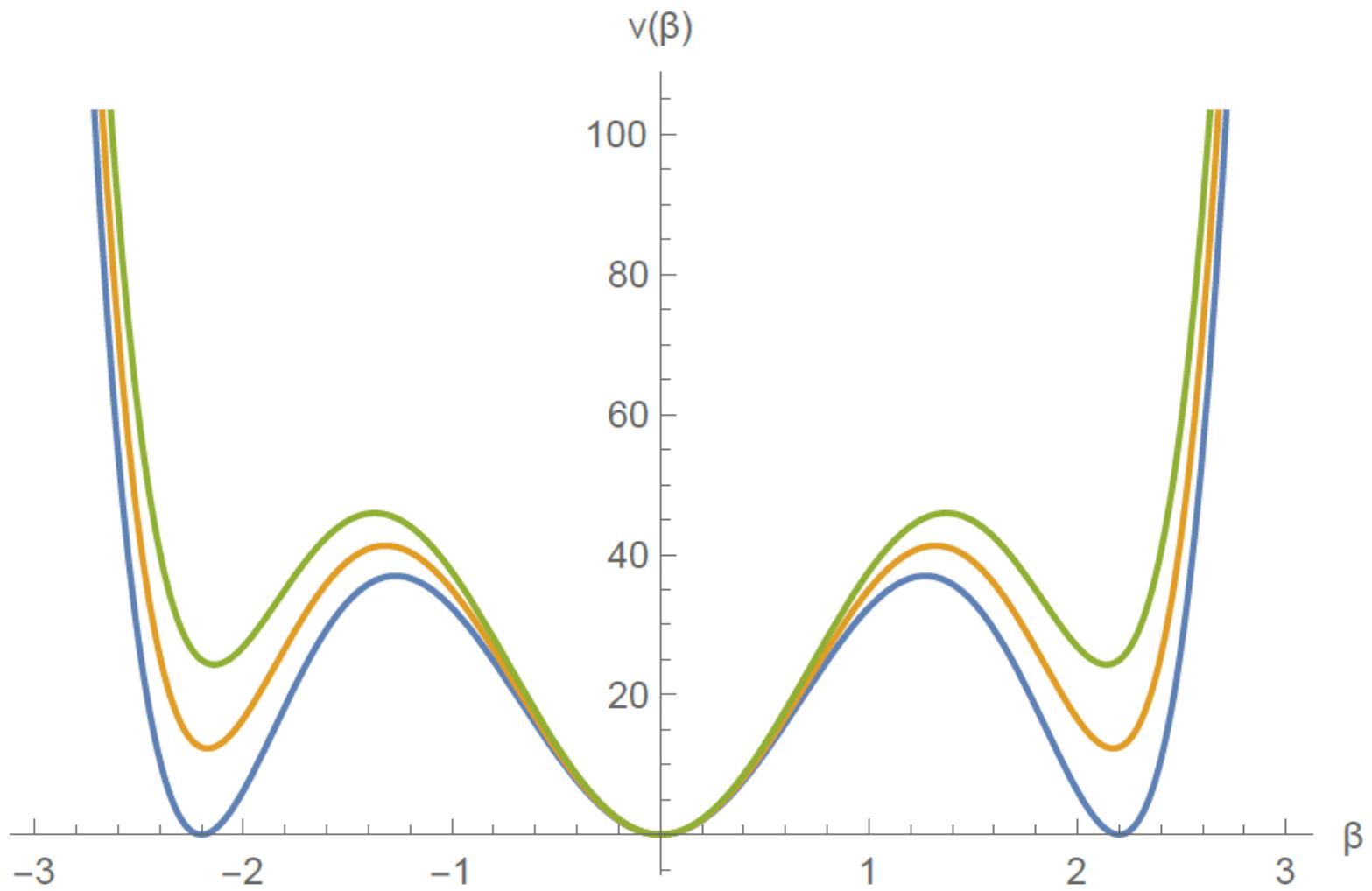
Realistic Applications: Predictions from microscopic Theories:

- *Relativistic Hartree Fock: (44,46)Ar* G A Lalazissis, D Vretenar, P Ring, M Stoitsov and L M Robledo, *Phys Rev C* 60, 014310 (1999)
- *IBM to Cogny: 72Ge* K Nomura, Rodriguez Guzman and L M Robledo *Phys Rev C* 95, 064310 (2017)

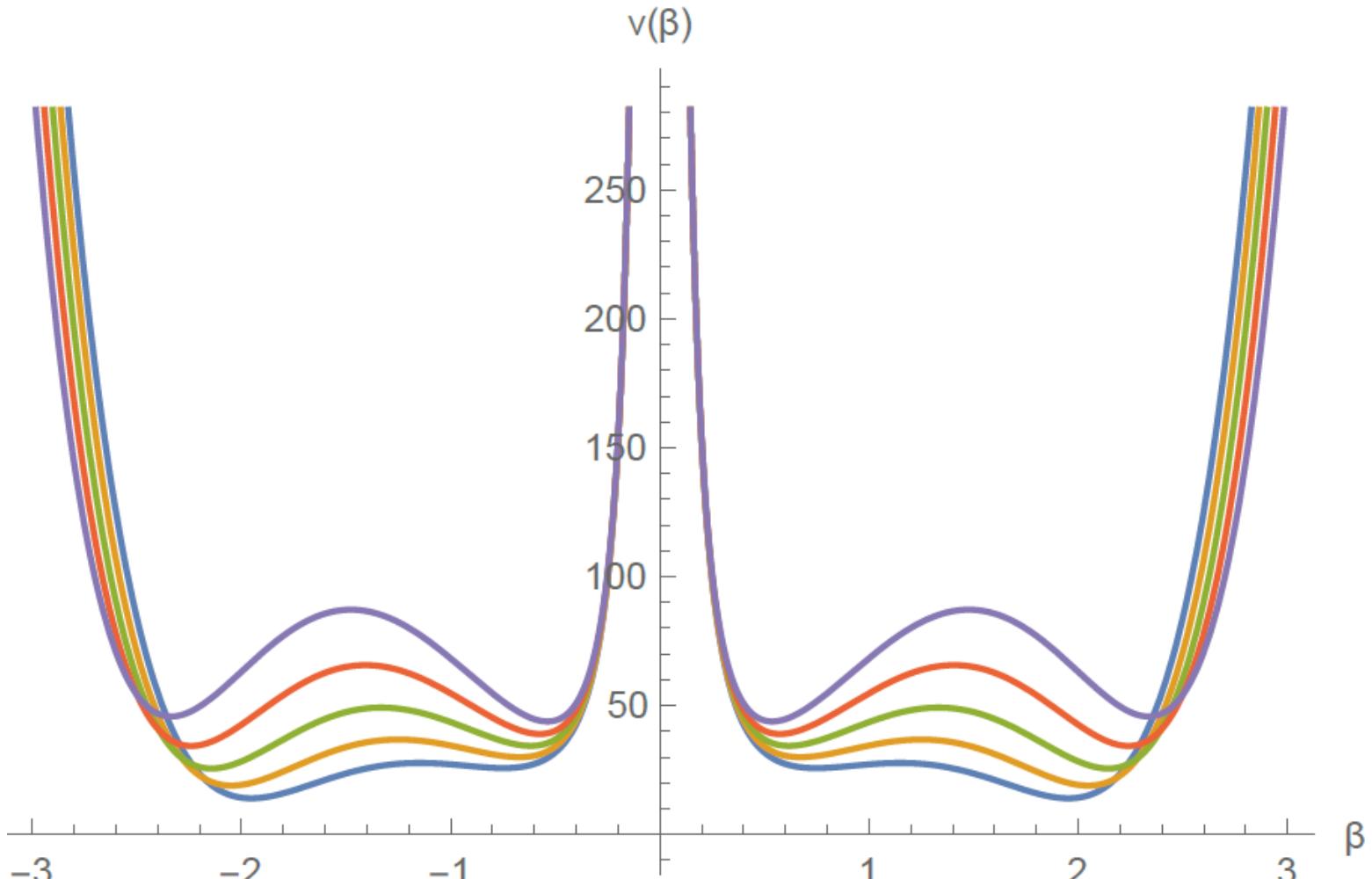
Realistic case: 110-112Cd: plethora of experimental data.

J Jolie and H Lehmann, Phys Lett B, 342, 1, 1995

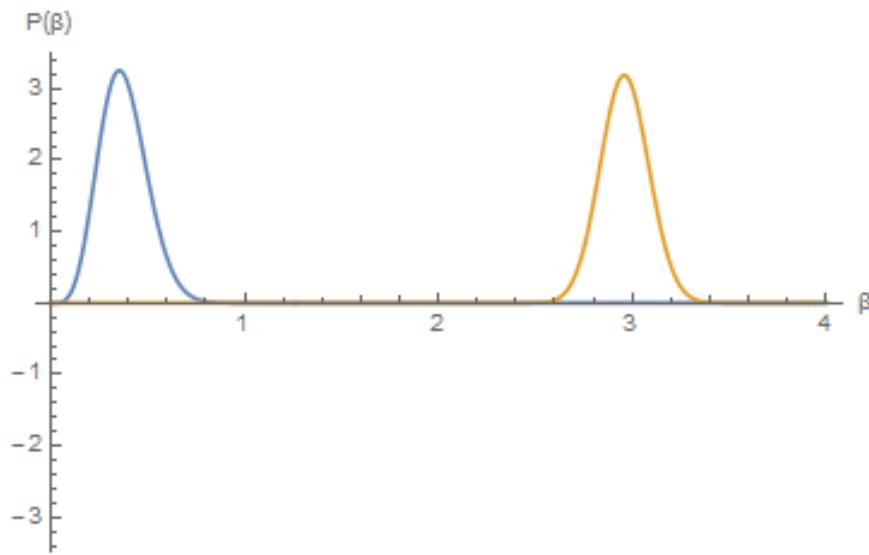
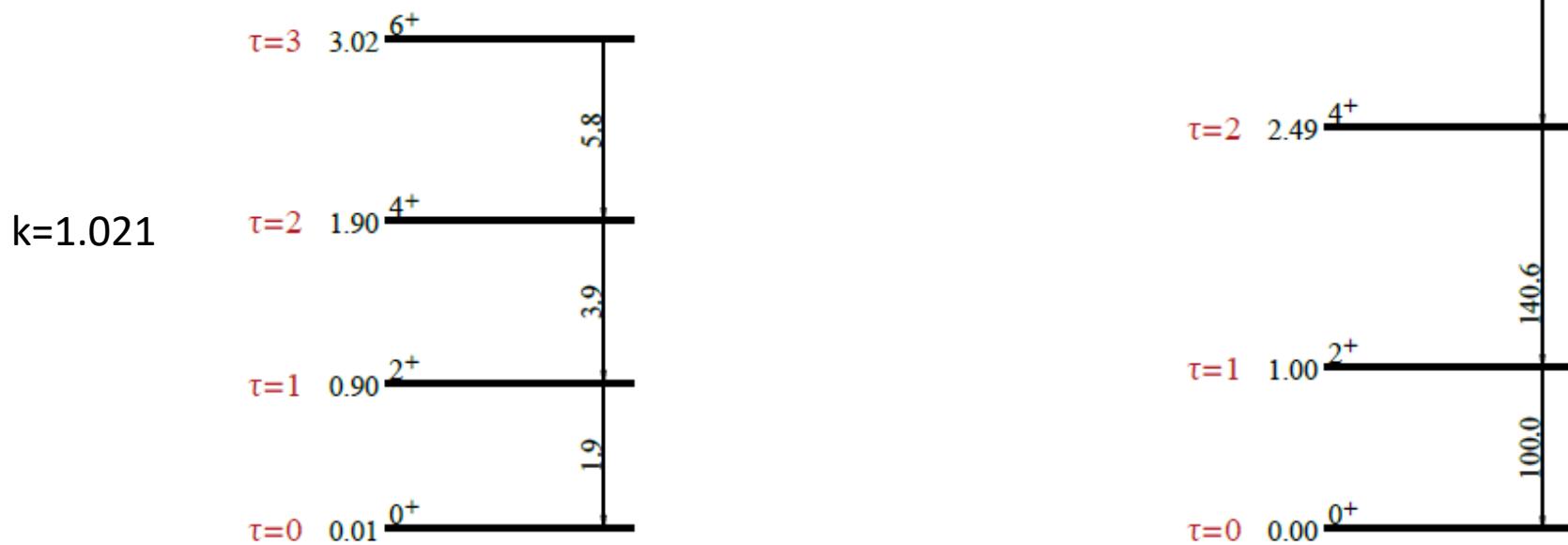
Need to obtain solutions with the spherical ground state:



$$v(\beta) = v_0 \left(\beta_0^4 k \beta^2 - 2\beta_0^2 \beta^4 + \beta^6 \right)$$

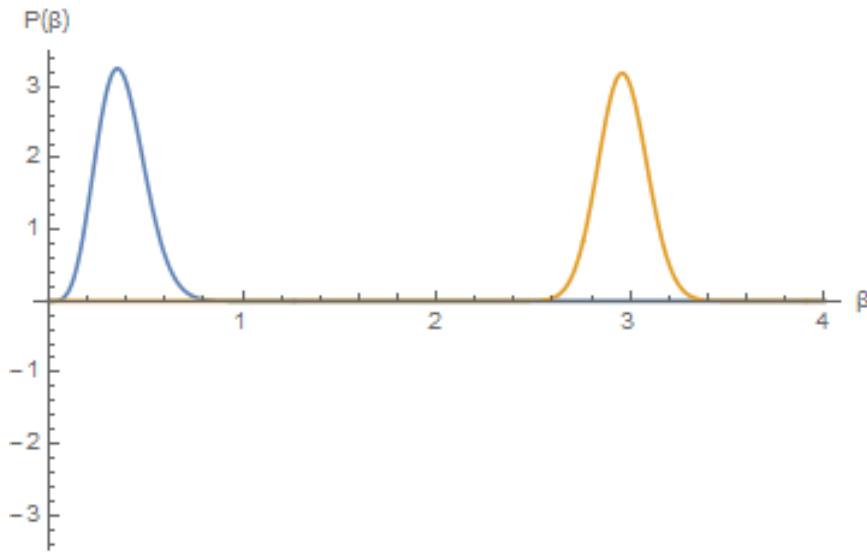
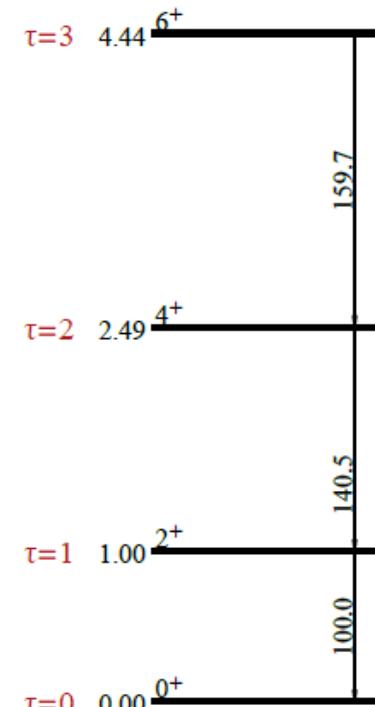
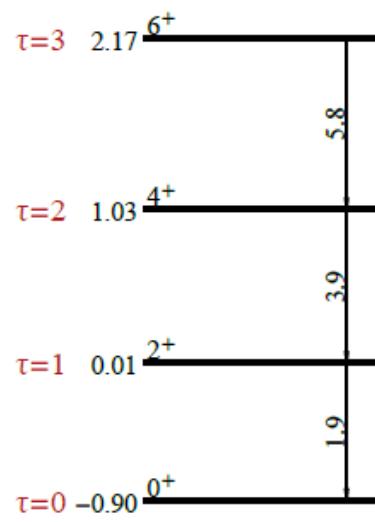


$$v_{eff}^0(\beta) = \frac{2}{\beta^2} + v_0 \left(\beta_0^4 k \beta^2 - 2 \beta_0^2 \beta^4 + \beta^6 \right)$$



A solution with degenerate spherical and deformed $\tau=0$ states and no transitions between them.

$k=1.04$



*A solution with a spherical ground
 $\tau=0$ state and no transitions between
Them.*

Phenomenological Challenges

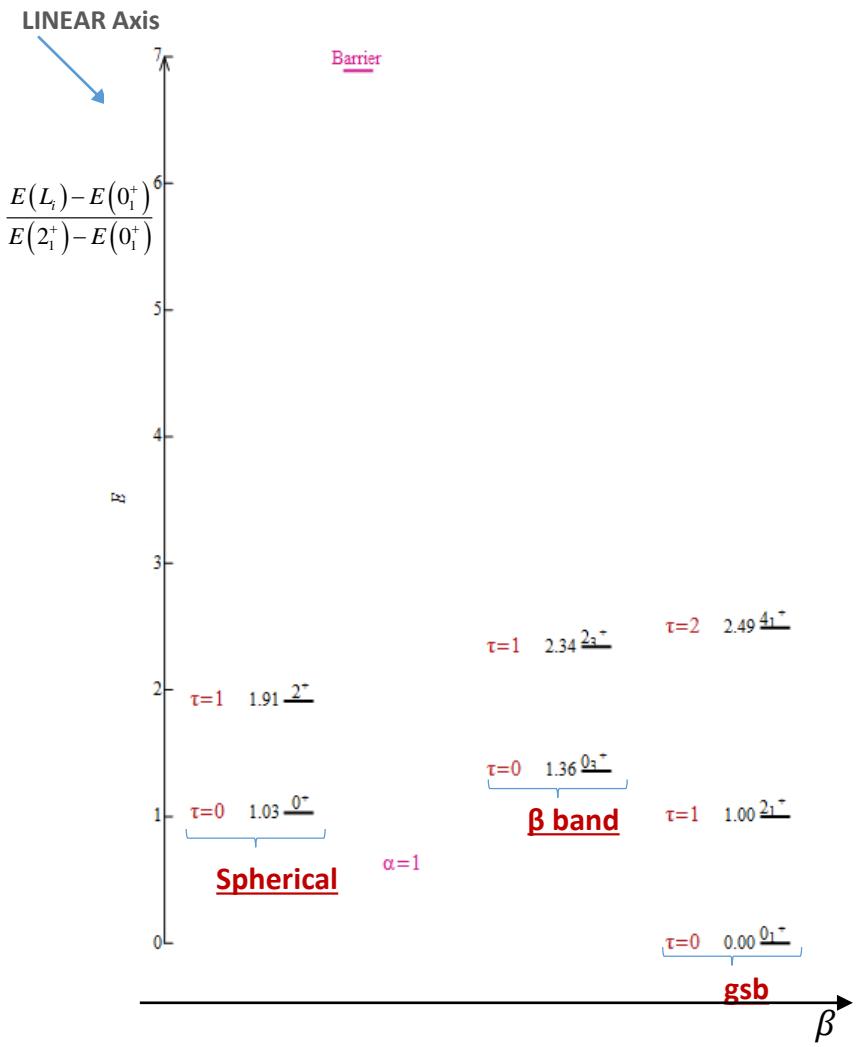
Realistic case: $^{110-112}\text{Cd}$: plethora of experimental data.

J Jolie and H Lehmann, Phys Lett B, 342, 1, 1995

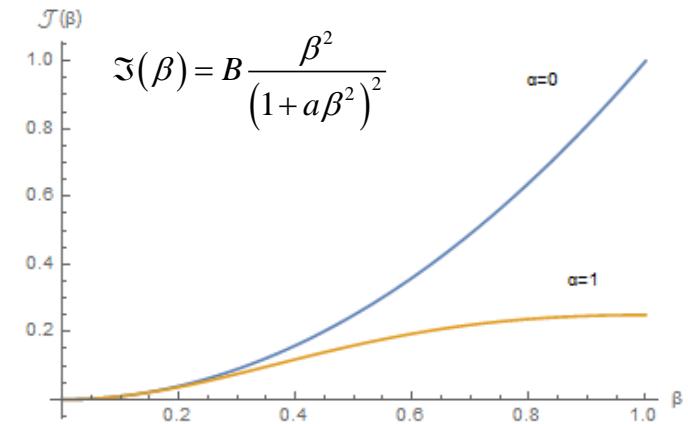
Challenge: The Transitions $B(E2)$ s and $B(E0)$ s between spherical and deformed States.

THE END

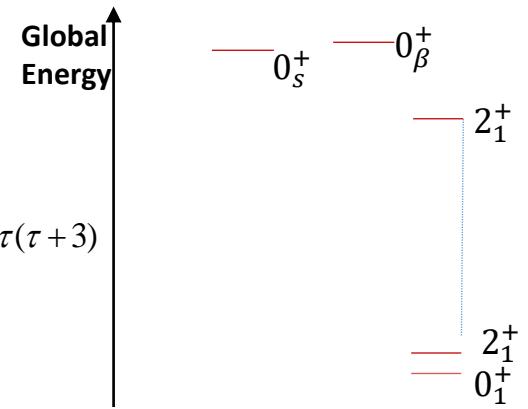
Effective moments of inertia



$$E(2_1^+) - E(0_1^+) = 43.77$$

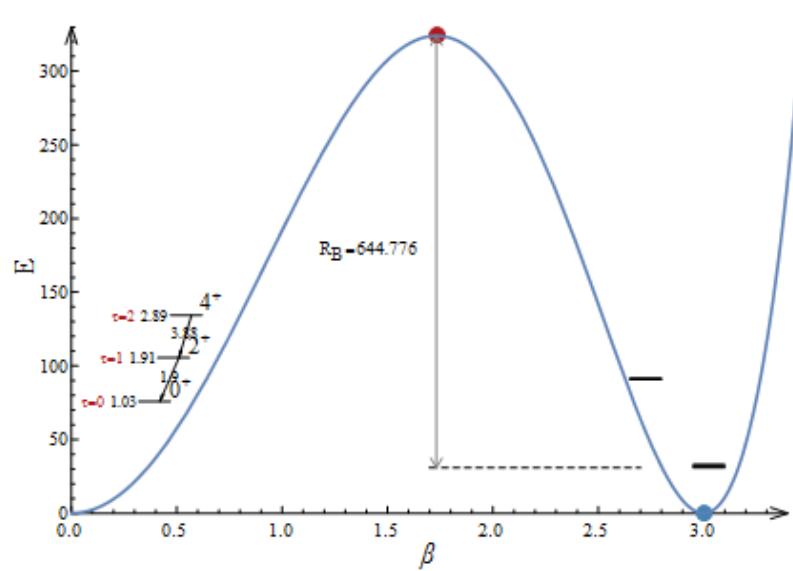


$$\frac{\Lambda}{\beta^2} + \beta^2 a^2 \Lambda + 2a\Lambda, \quad \Lambda = \tau(\tau+3)$$

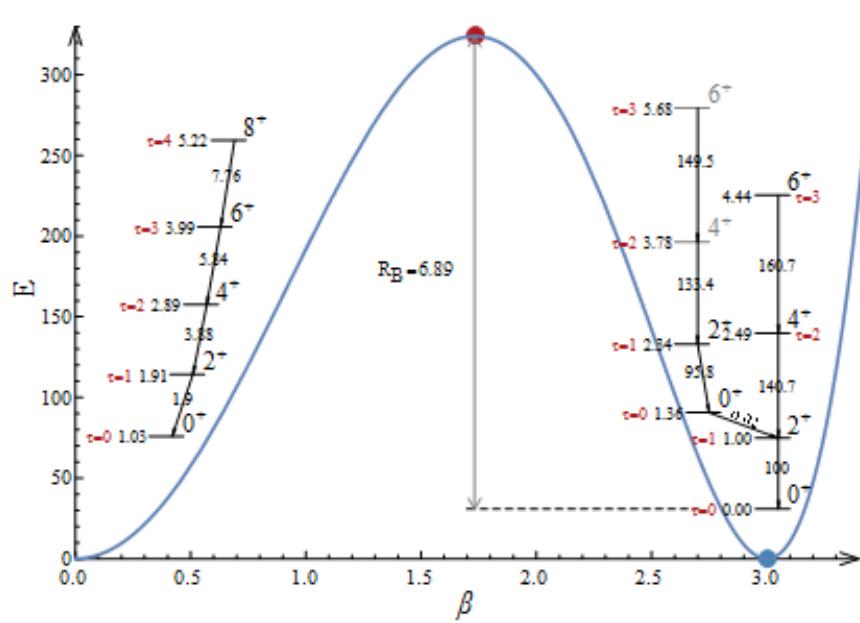


The effective moments of inertia affect the energy scale $E(2_1^+) - E(0_1^+)$.

$$\Im(\beta) = B\beta^2$$



$$\Im(\beta) = B \frac{\beta^2}{(1 + \beta^2)^2}$$



1st SGA-SU(1,1)xSO(5)

$$(a\beta)^2 = \hat{S}_+ + \hat{S}_- + 2\hat{S}_0$$

$$-\frac{1}{a^2} \frac{\partial^2}{\partial \beta^2} + \frac{(\tau+1)(\tau+2)}{(a\beta)^2} = 2\hat{S}_0 - \hat{S}_+ - \hat{S}_-$$

$$\left(-\frac{\partial^2}{\partial \beta^2} + \frac{(\tau+1)(\tau+2)}{\beta^2} + v_0 \beta_0^4 \beta^2 - 2\beta_0^2 \beta^4 + v_0 \beta^6 \right) \varphi(\beta) = \varepsilon \varphi(\beta), \quad \varepsilon = \frac{2B}{\hbar^2}$$

Spectrum Generating Algebra with cubic term

$$2\hat{S}_0 - \hat{S}_+ - \hat{S}_- + v_0 \beta_0^4 (\hat{S}_+ + \hat{S}_- + 2\hat{S}_0) - 2v_0 \beta_0^2 (\hat{S}_+ + \hat{S}_- + 2\hat{S}_0)^2 + v_0 (\hat{S}_+ + \hat{S}_- + 2\hat{S}_0)^3$$

$$\varphi_\nu^\lambda(\beta) = (-1)^\nu \sqrt{\frac{2\nu!a}{\Gamma(\nu+\lambda)}} (a\beta)^{\lambda-1/2} e^{-a^2\beta^2/2} L_\nu^{(\lambda-1)}(a^2\beta^2)$$

Diagonalize it in the basis of
Laguerre Polynomials,