

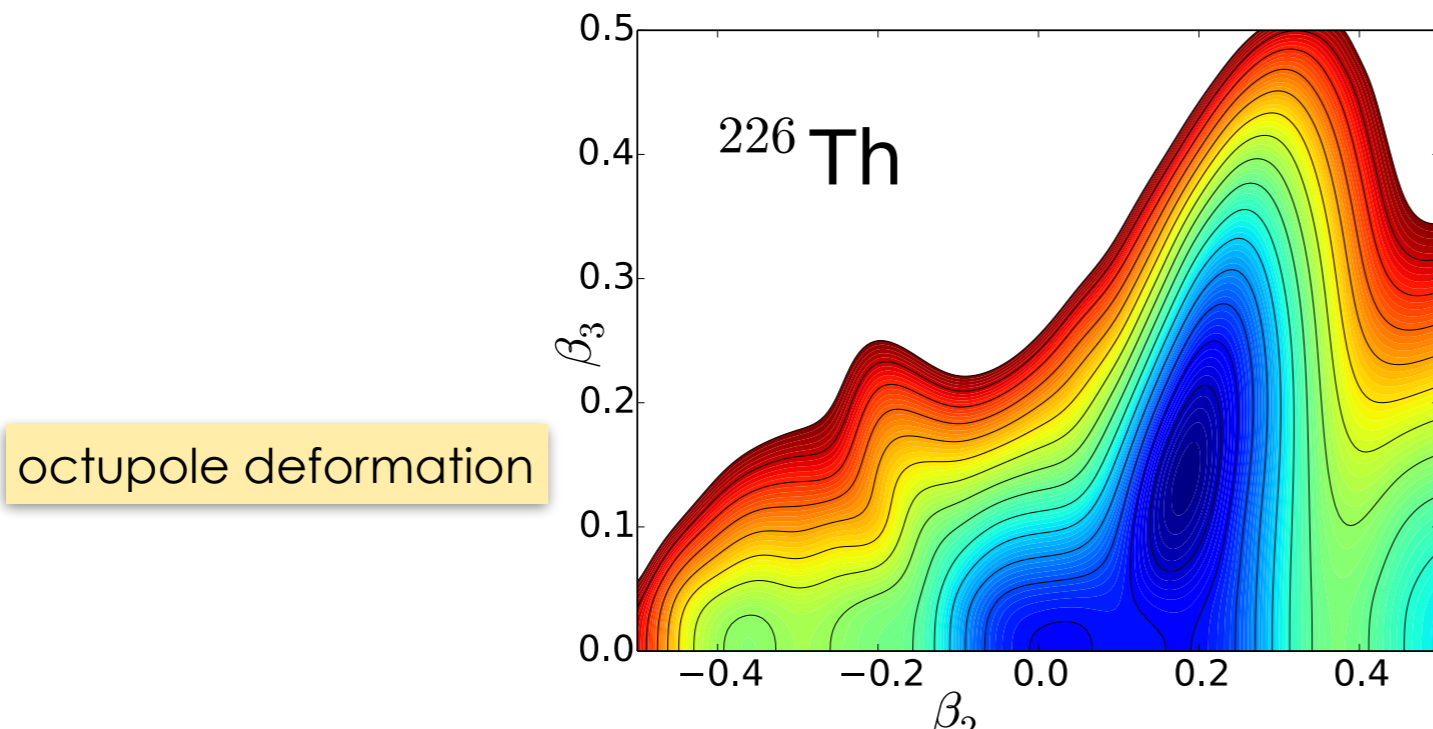
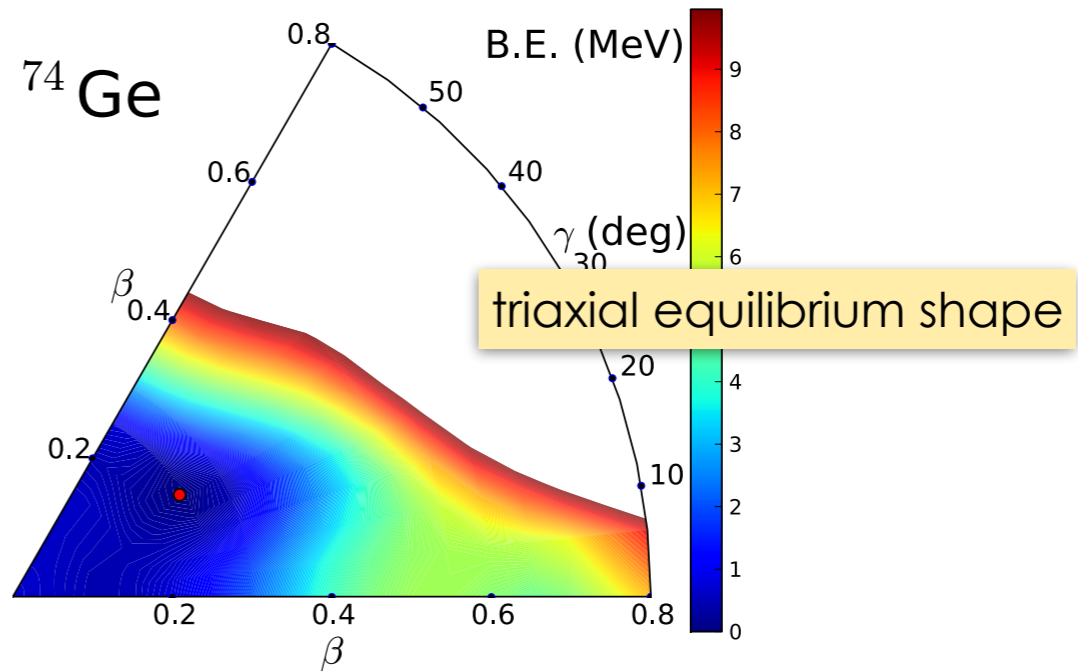
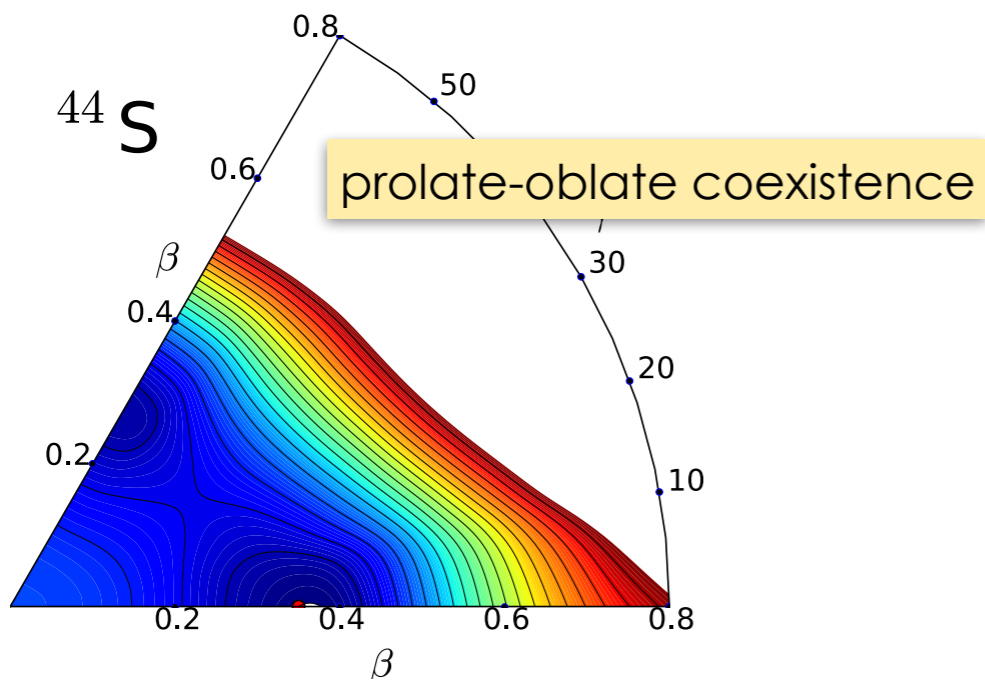
QPT vs shape coexistence in $N \approx 90$ rare-earth nuclei



Dario Vretenar
University of Zagreb



The self-consistent mean field method → produces semi-classical energy surfaces as functions of intrinsic deformation parameters.

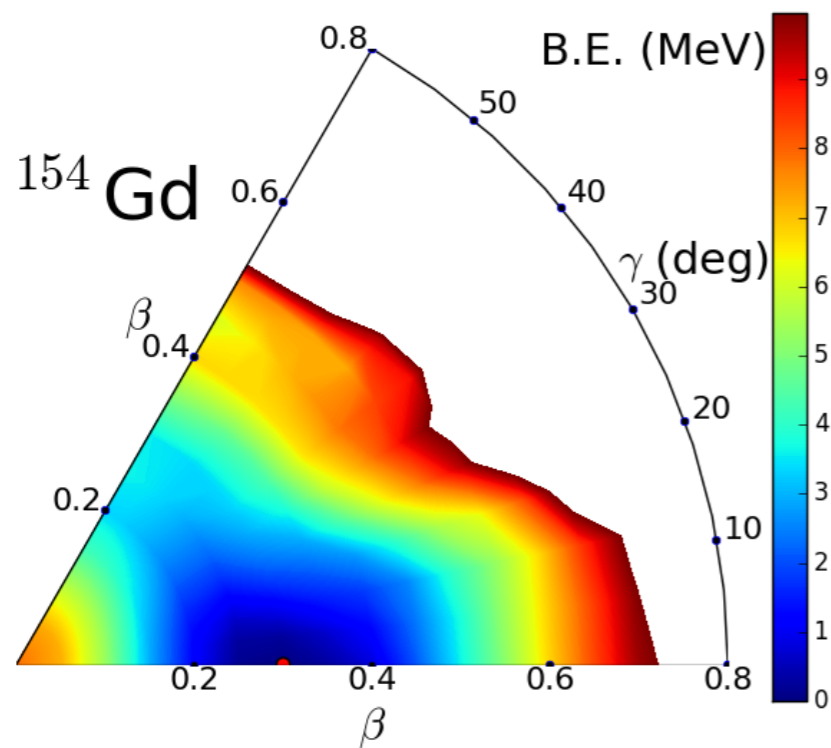


→ include **static correlations**: deformations & pairing

→ do not include **dynamic (collective) correlations** that arise from symmetry restoration and quantum fluctuations around mean-field minima

Collective Hamiltonian

Prog. Part. Nucl. Phys. **66**, 519 (2011).
 Phys. Rev. C **79**, 034303 (2009).



... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom:

$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta, \gamma) + \mathcal{T}_{\text{rot}}(\beta, \gamma, \Omega) + \mathcal{V}_{\text{coll}}(\beta, \gamma)$$

$$\mathcal{T}_{\text{vib}} = \frac{1}{2} B_{\beta\beta} \dot{\beta}^2 + \beta B_{\beta\gamma} \dot{\beta} \dot{\gamma} + \frac{1}{2} \beta^2 B_{\gamma\gamma} \dot{\gamma}^2$$

$$\mathcal{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{I}_k \omega_k^2$$

The dynamics is determined by: the self-consistent collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia \mathcal{I}_k , functions of the intrinsic deformations β and γ .

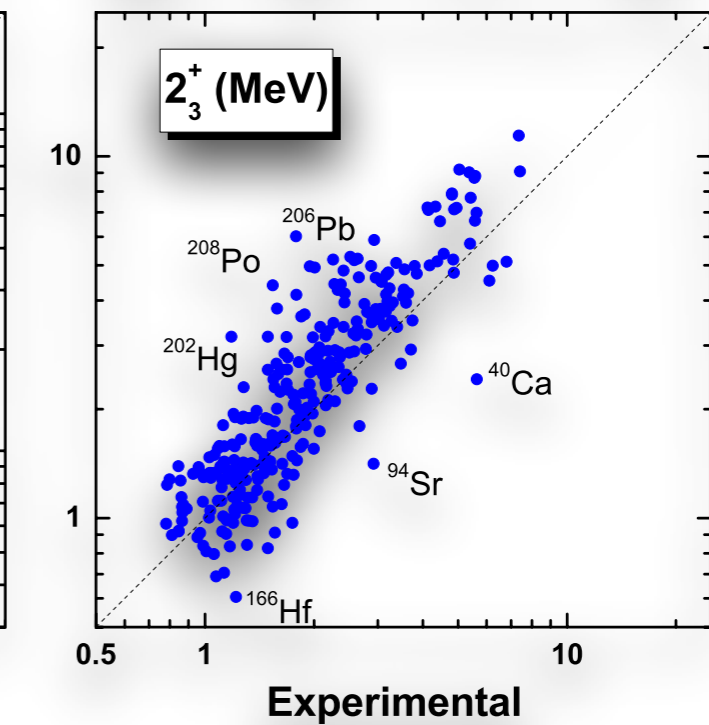
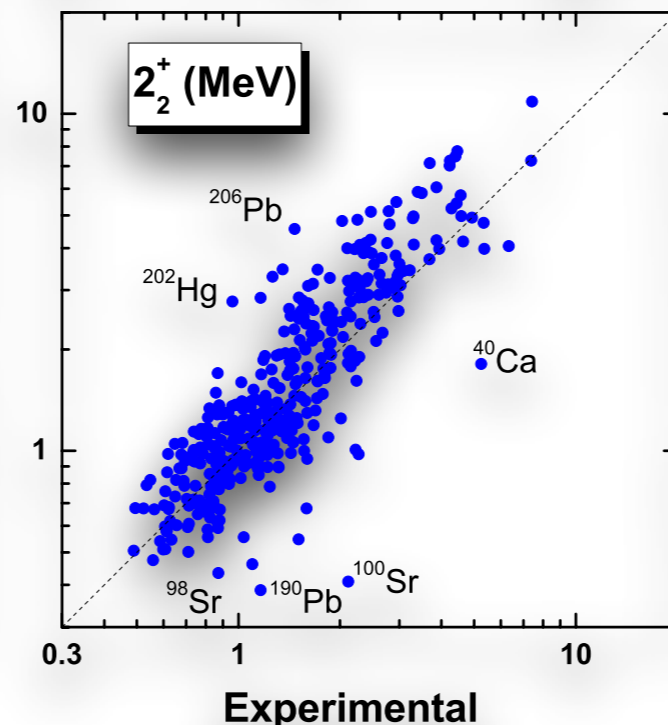
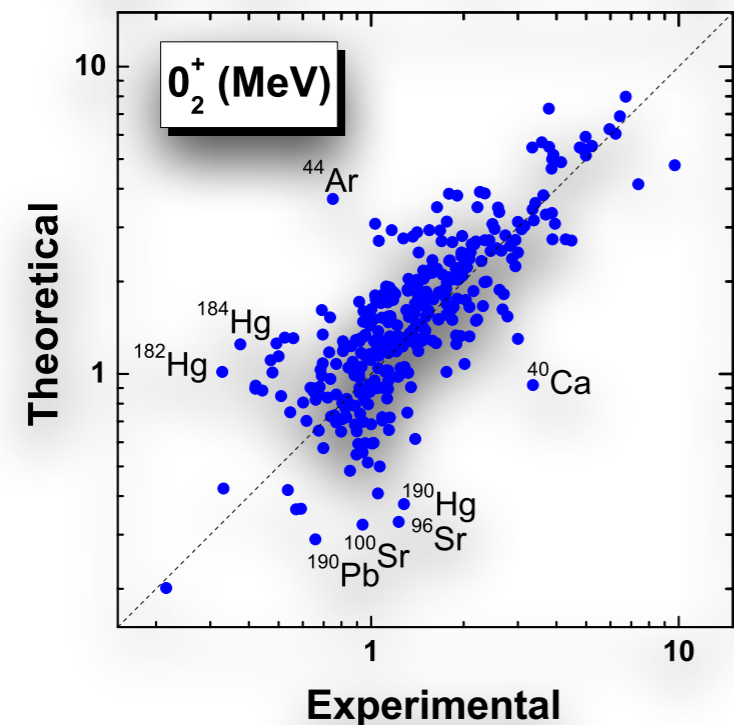
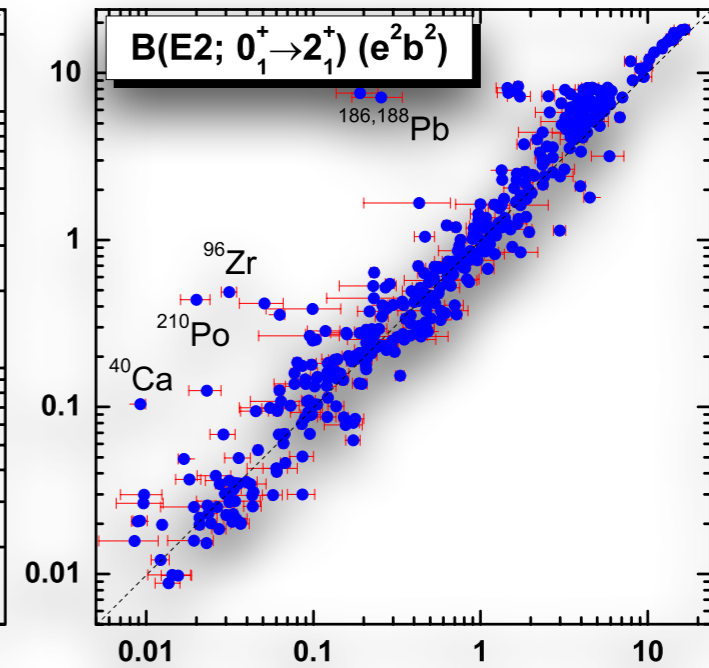
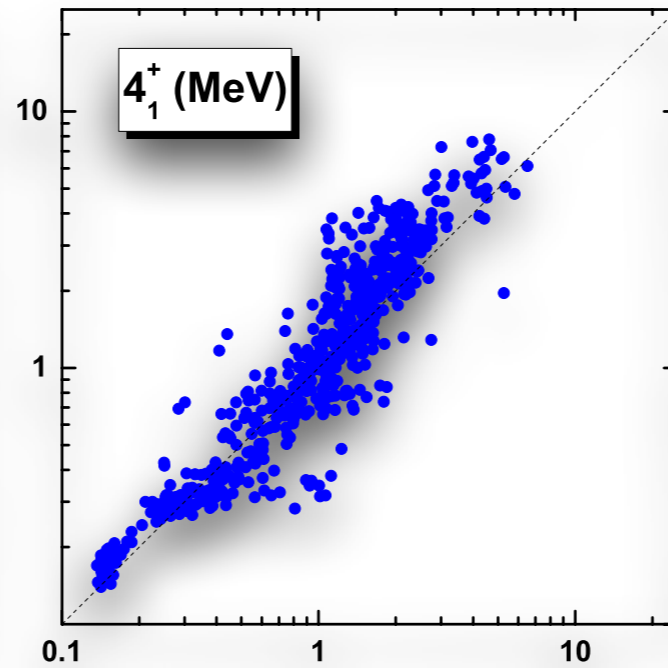
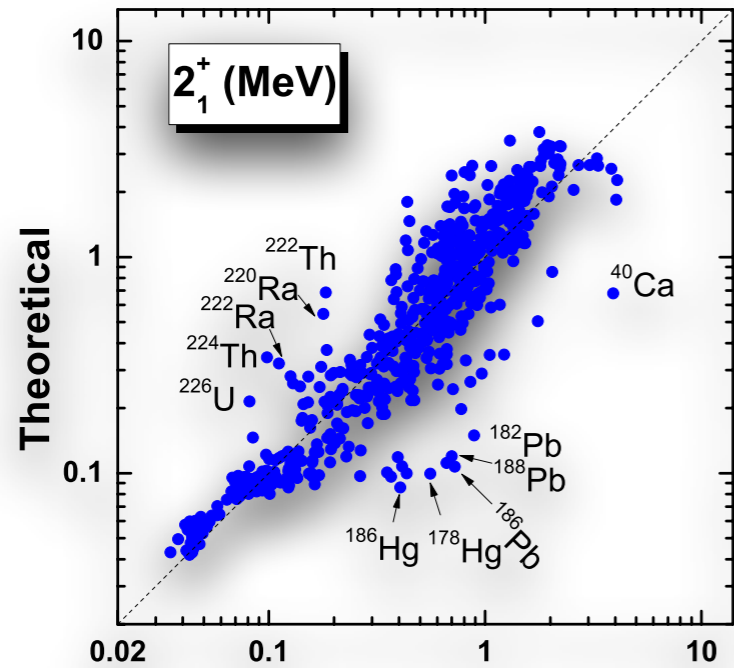
✓ an intuitive interpretation of mean-field results in terms of ***intrinsic shapes*** and ***single-particle states***

✓ the ***full model space*** of occupied states can be used; no distinction between core and valence nucleons, ***no need for effective charges!***

Global analysis of quadrupole shape invariants

Phys. Rev. C **95**, 054321 (2017).

621 even-even nuclei:



...the lowest-order quadrupole invariants:

$$q_2(0_i^+) = \langle 0_i^+ | Q^2 | 0_i^+ \rangle = \sum_j \langle 0_i^+ || Q || 2_j^+ \rangle \langle 2_j^+ || Q || 0_i^+ \rangle$$

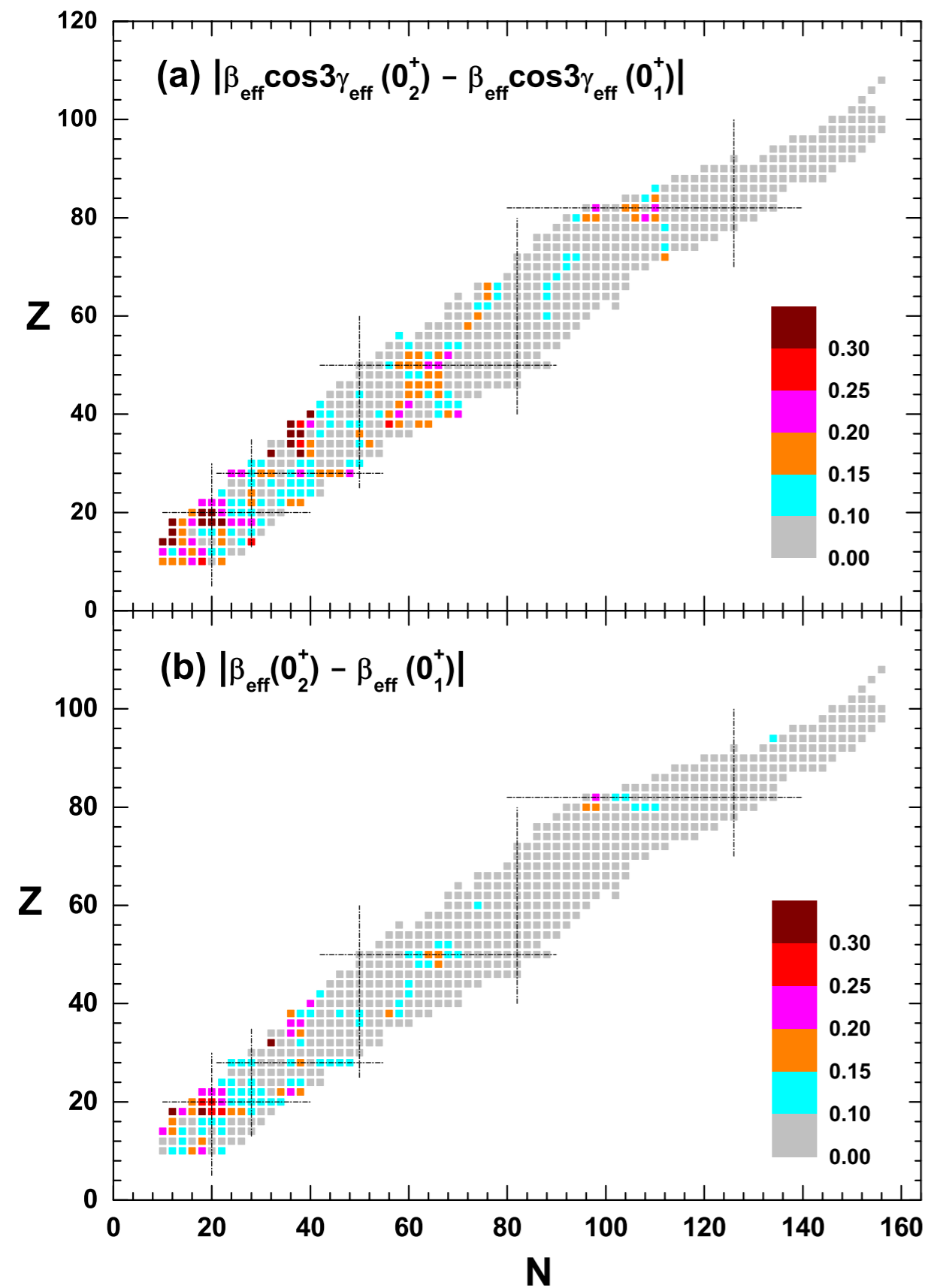
$$q_3(0_i^+) = \sqrt{\frac{35}{2}} \langle 0_i^+ | Q^3 | 0_i^+ \rangle = \sqrt{\frac{7}{10}} \sum_{jk} \langle 0_i^+ || Q || 2_j^+ \rangle \langle 2_j^+ || Q || 2_k^+ \rangle \langle 2_k^+ || Q || 0_i^+ \rangle$$

⇒ effective deformation parameters:

$$q_2(0_i^+) = \left(\frac{3ZeR^2}{4\pi} \right)^2 \langle \beta^2 \rangle \equiv \left(\frac{3ZeR^2}{4\pi} \right)^2 \beta_{\text{eff}}^2$$

$$\frac{q_3(0_i^+)}{q_2^{3/2}(0_i^+)} = \frac{\langle \beta^3 \cos 3\gamma \rangle}{\langle \beta^2 \rangle^{3/2}} \equiv \cos 3\gamma_{\text{eff}}$$

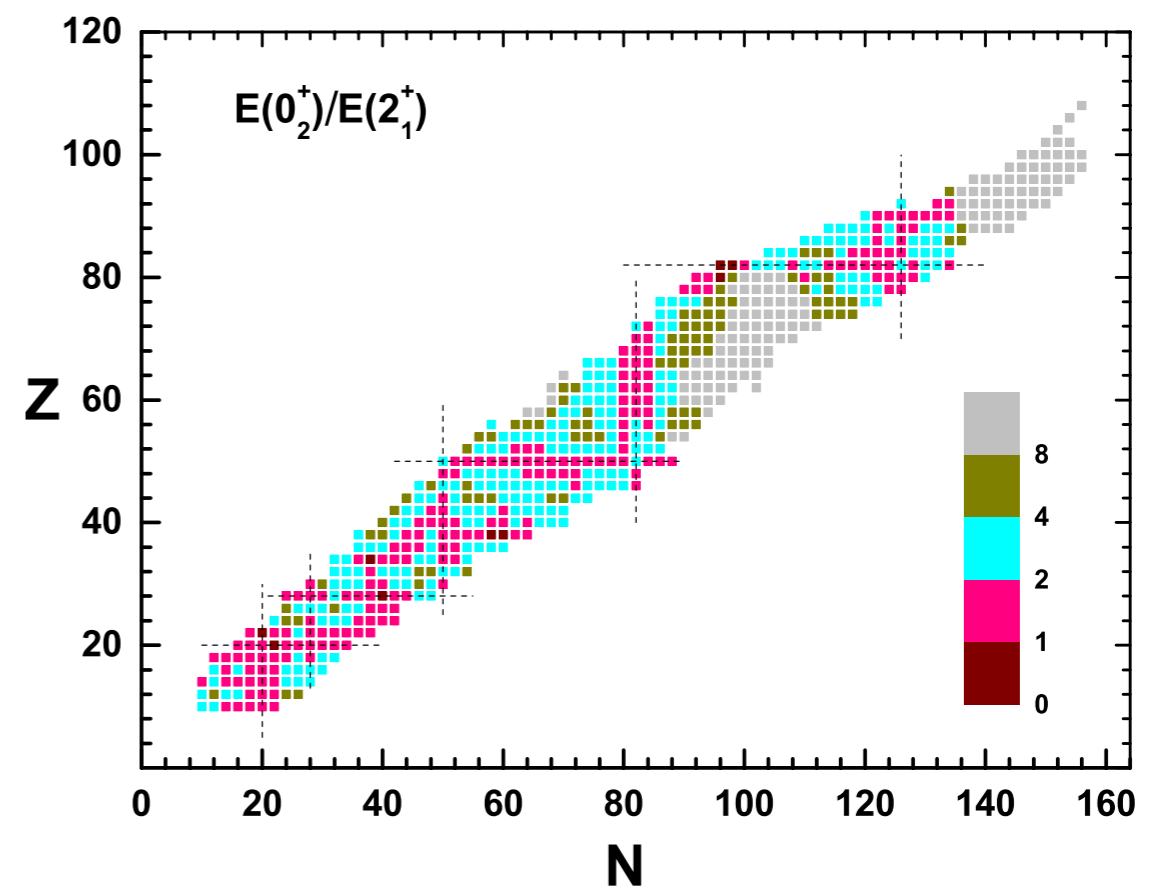
⇒ signatures of shape coexistence



⇒ signatures of shape coexistence:

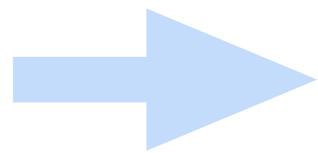
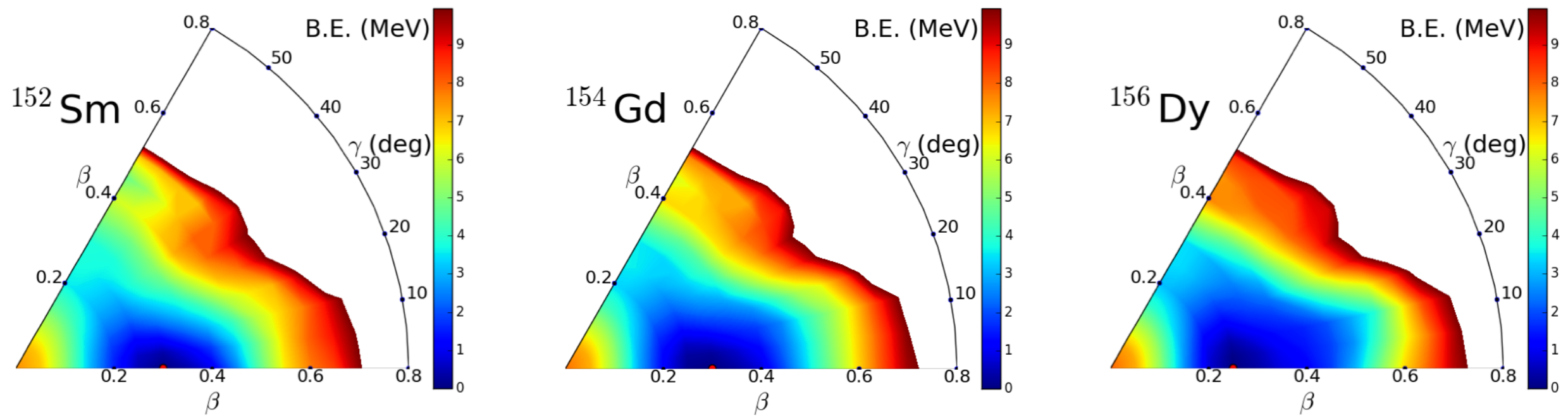
Large values of $|\beta_{\text{eff}} \cos 3\gamma_{\text{eff}}(0_2^+) - \beta_{\text{eff}} \cos 3\gamma_{\text{eff}}(0_1^+)|$
and $|\beta_{\text{eff}}(0_2^+) - \beta_{\text{eff}}(0_1^+)|$

First excited 0^+ state low in energy compared to the first 2^+ .

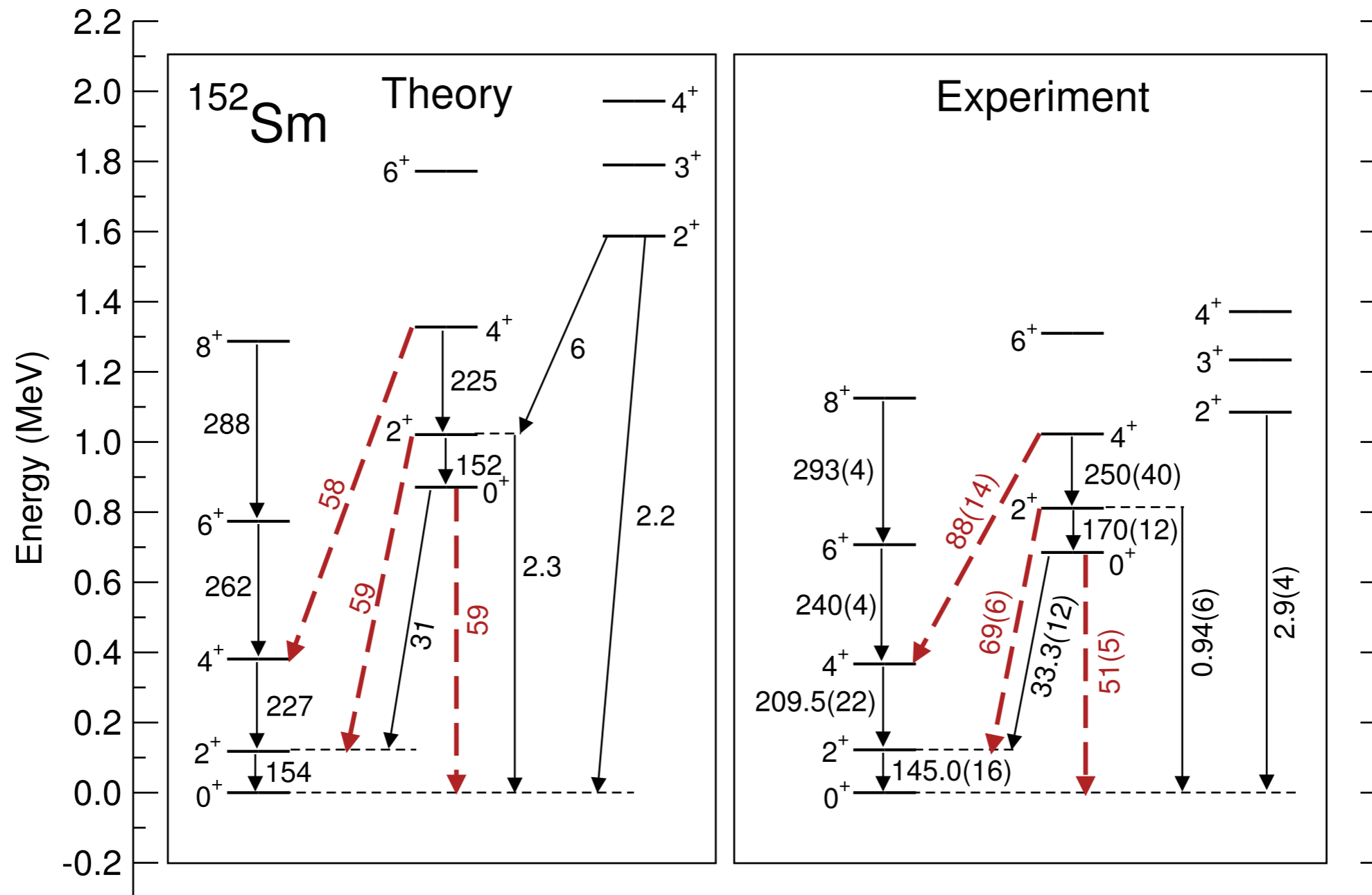


Lowest 0^+ excitations in $N \approx 90$ rare-earth nuclei

J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005



... eigenspectra of the 5D quadrupole collective Hamiltonian:

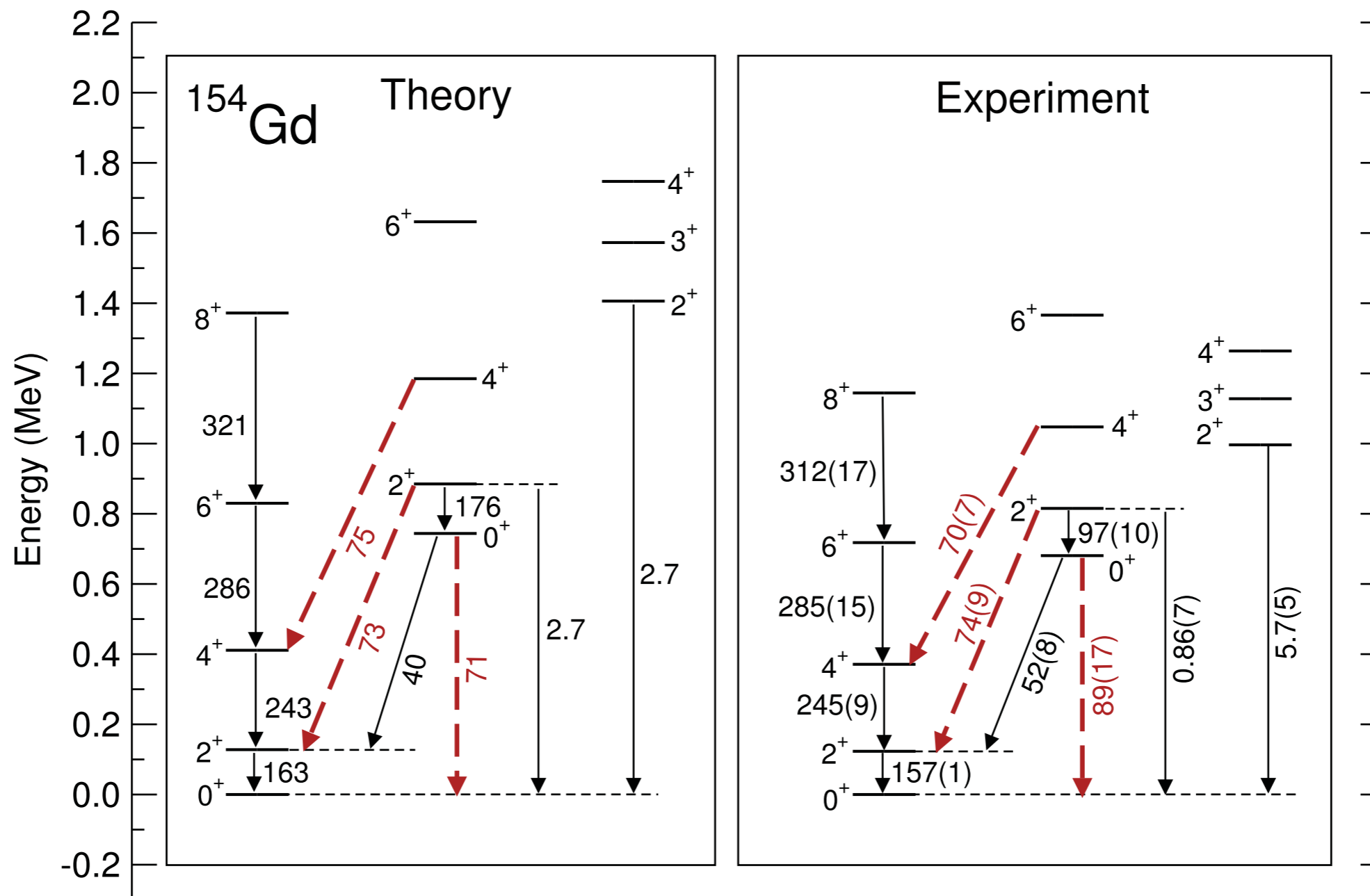


Criteria for the excited 0^+ state to be labelled as a β -vibration:

$$B(E2; 0^+_{\beta} \rightarrow 2^+_1) \approx 12 - 33 \text{ W.u.}$$

$$B(E2; 2^+_{\beta} \rightarrow 0^+_1) \approx 2.5 - 6 \text{ W.u.}$$

$$\rho^2(E0; 0^+_2 \rightarrow 0^+_1) \approx (85 - 230) \times 10^{-3}$$

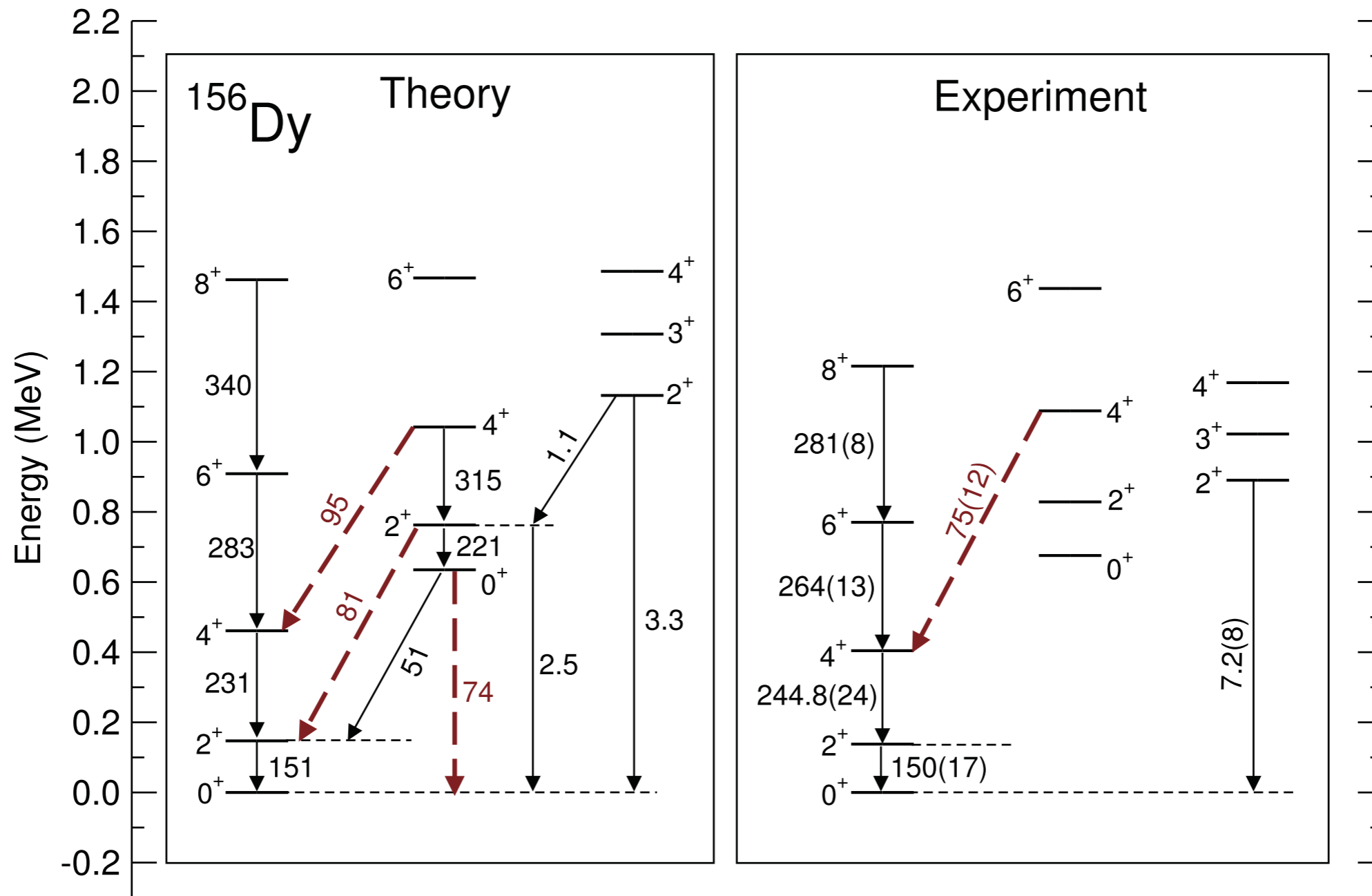


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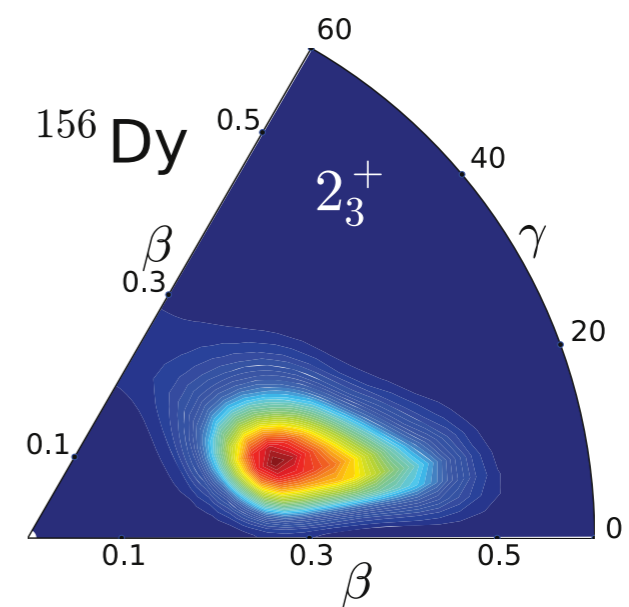
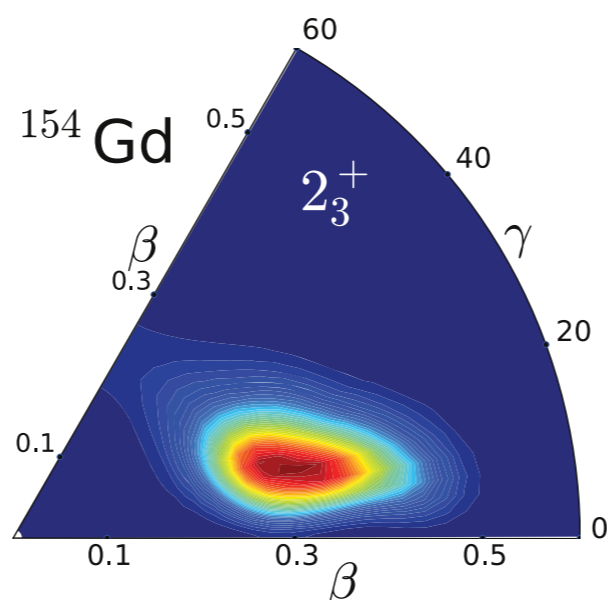
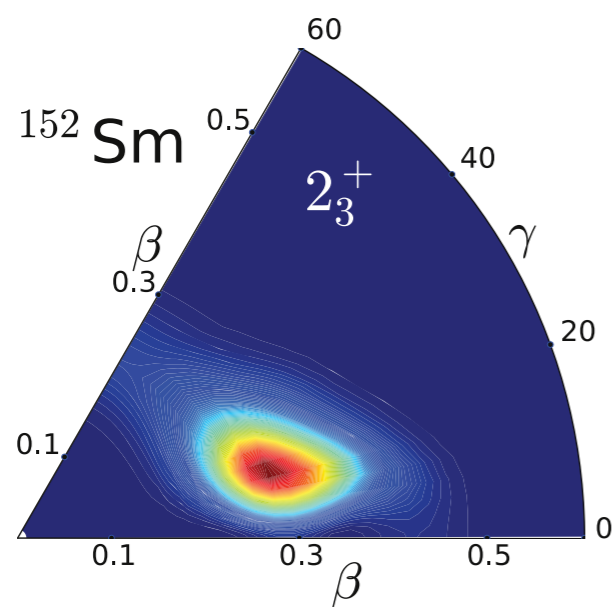
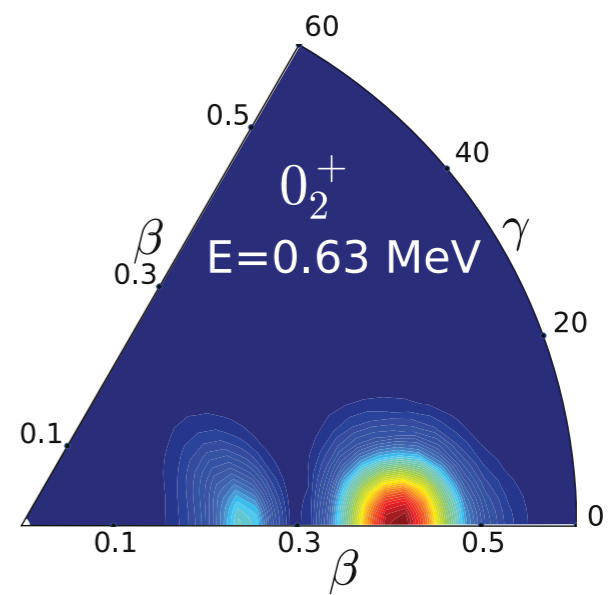
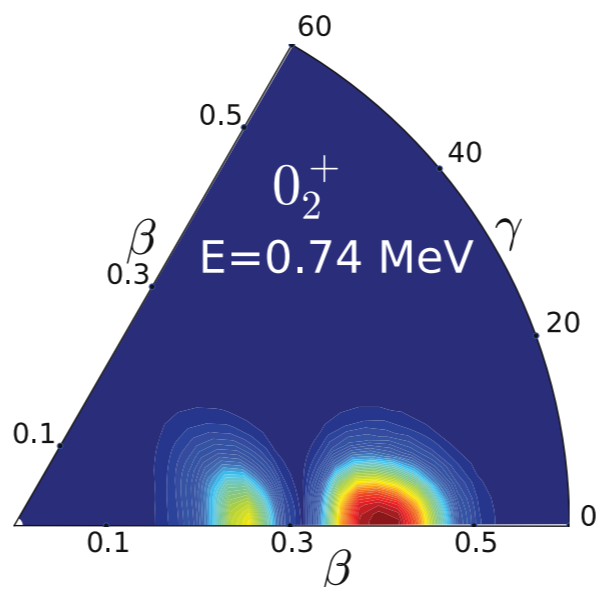
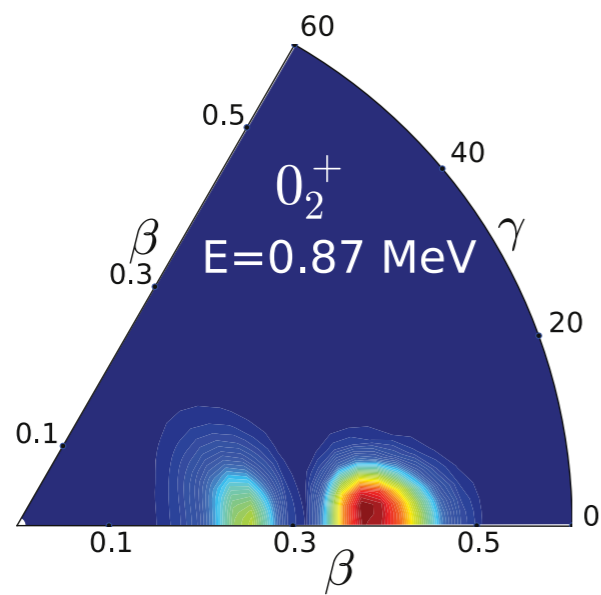
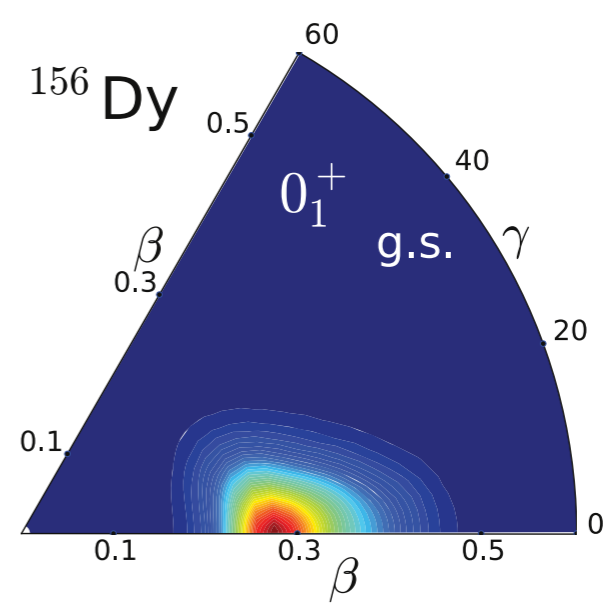
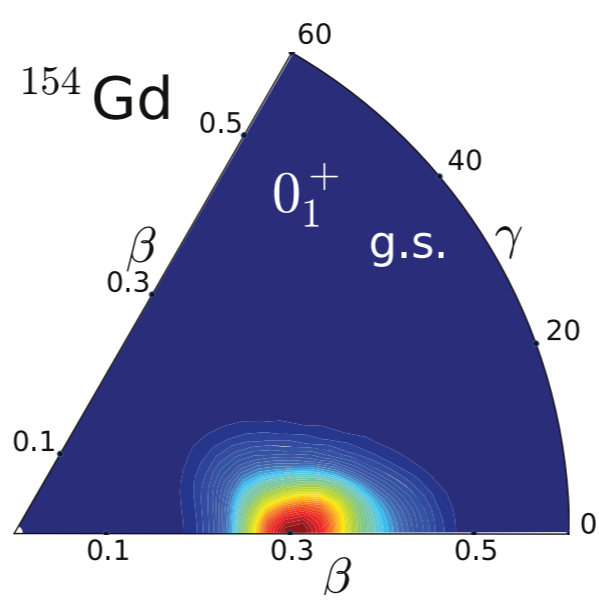
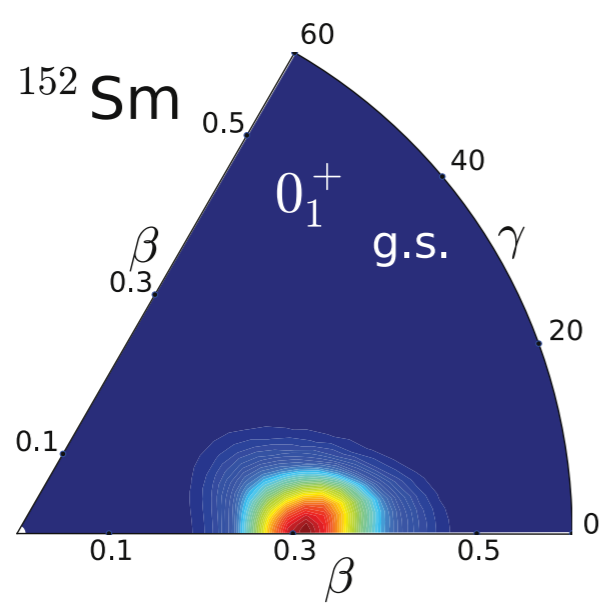
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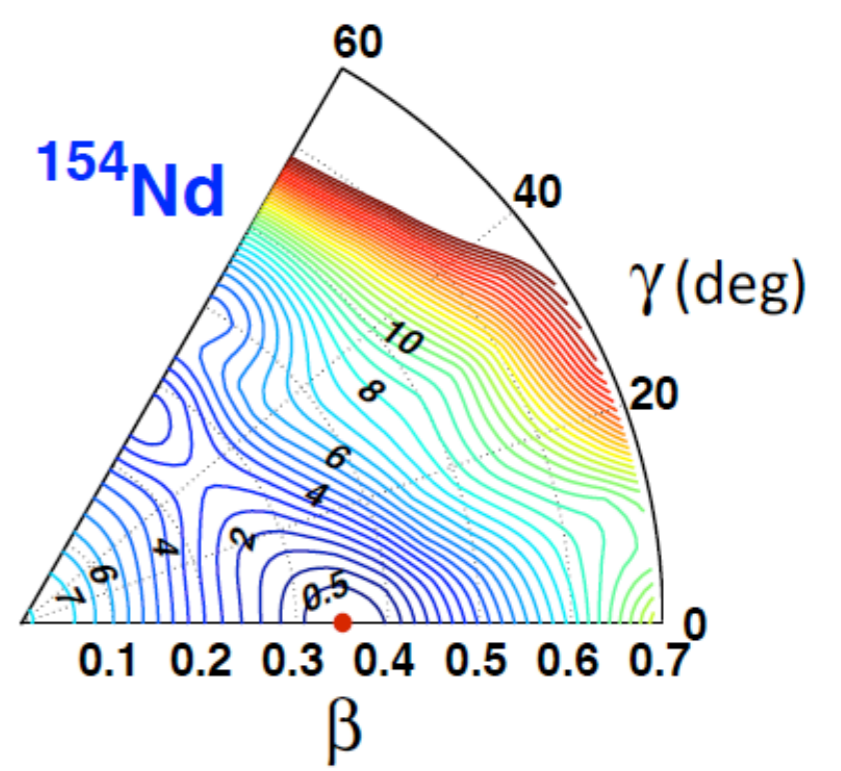
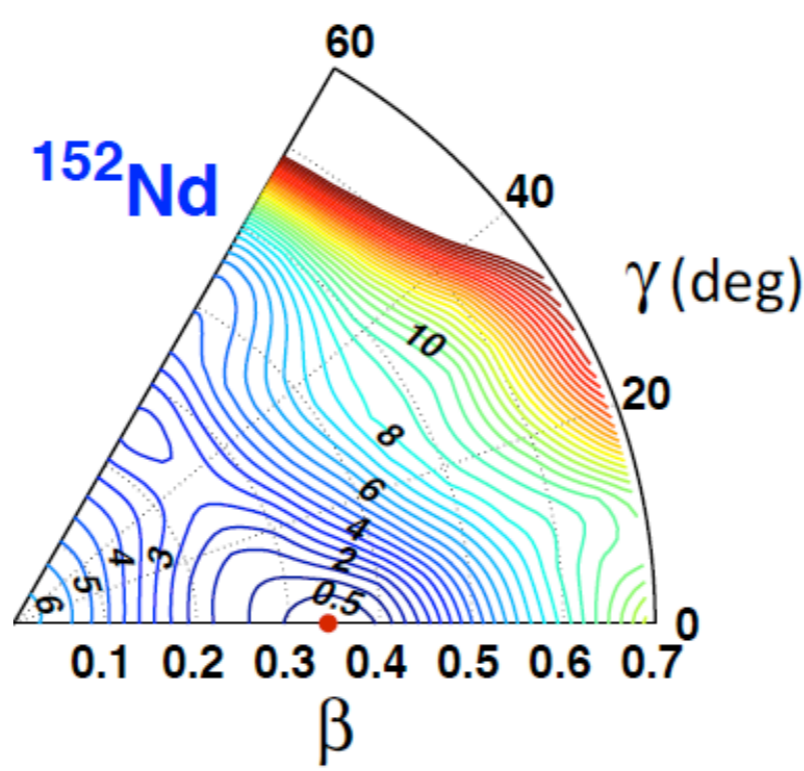
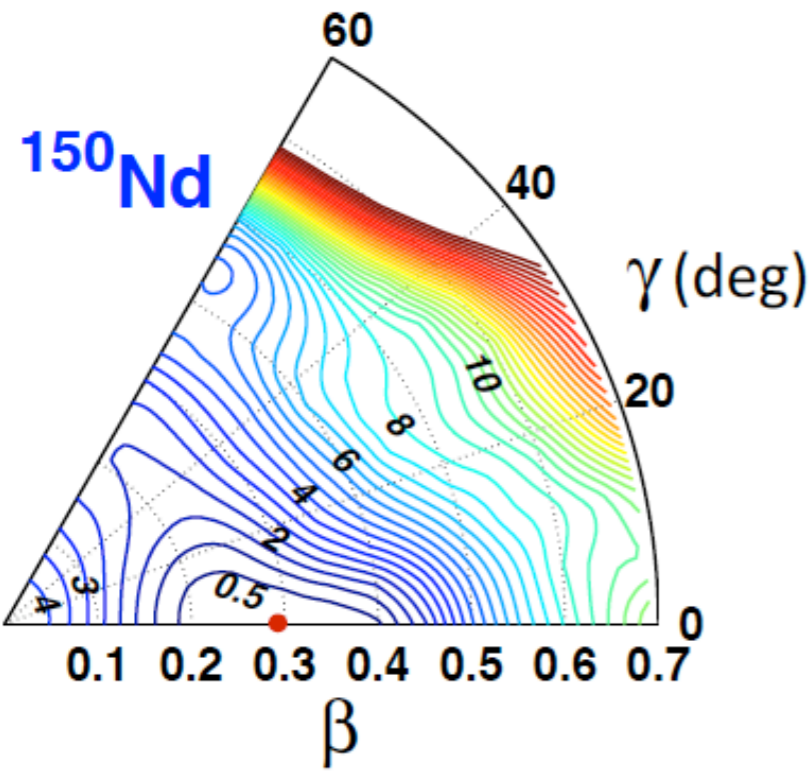
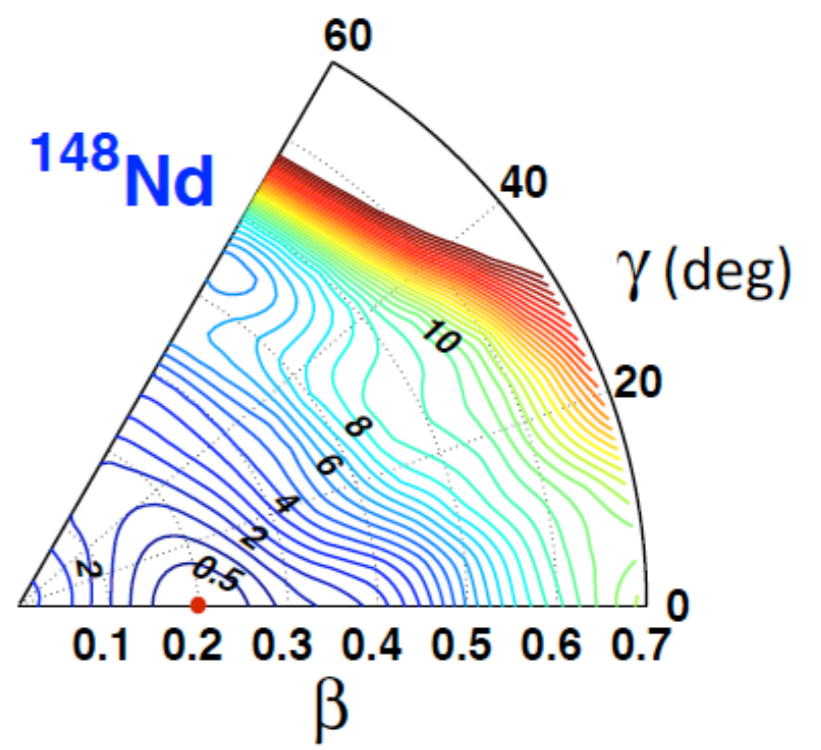
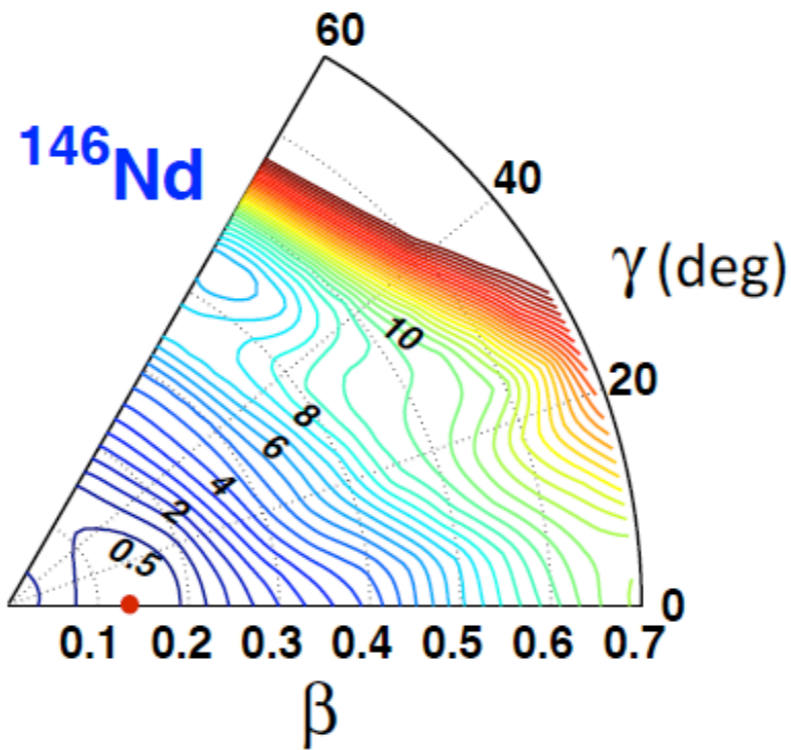
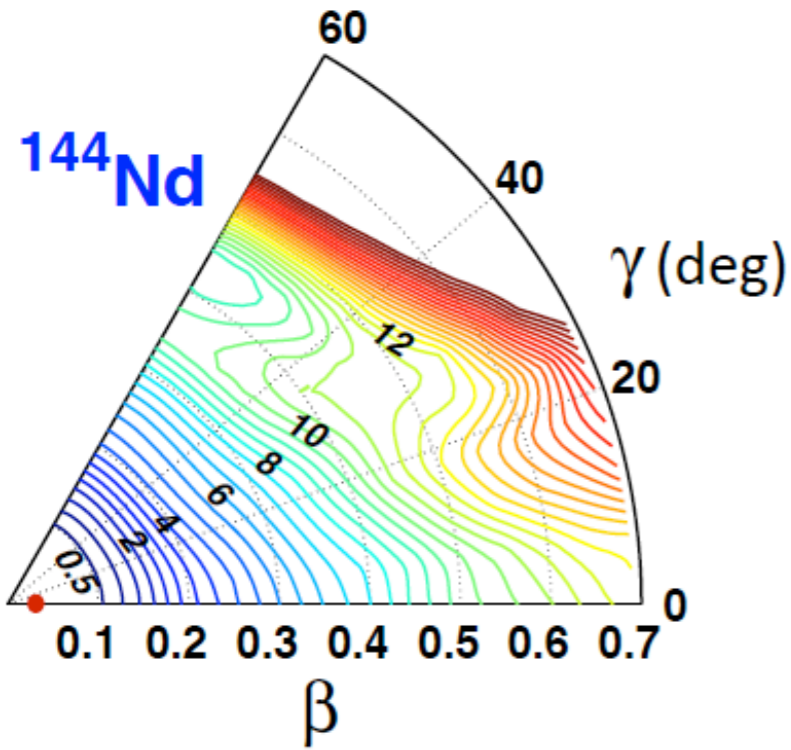
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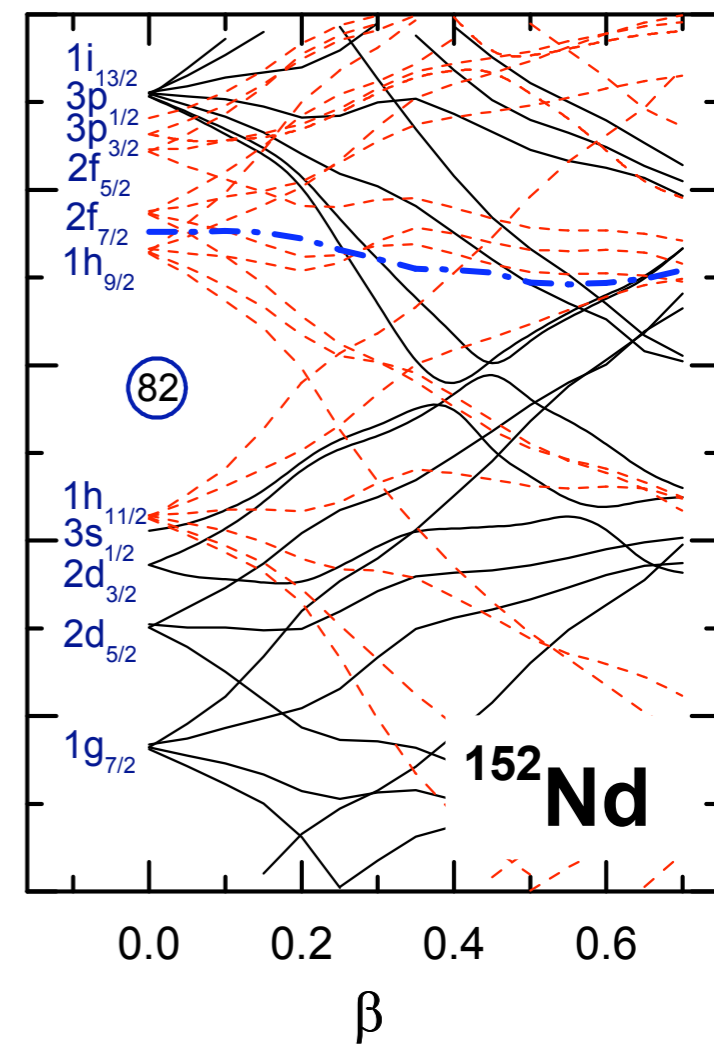
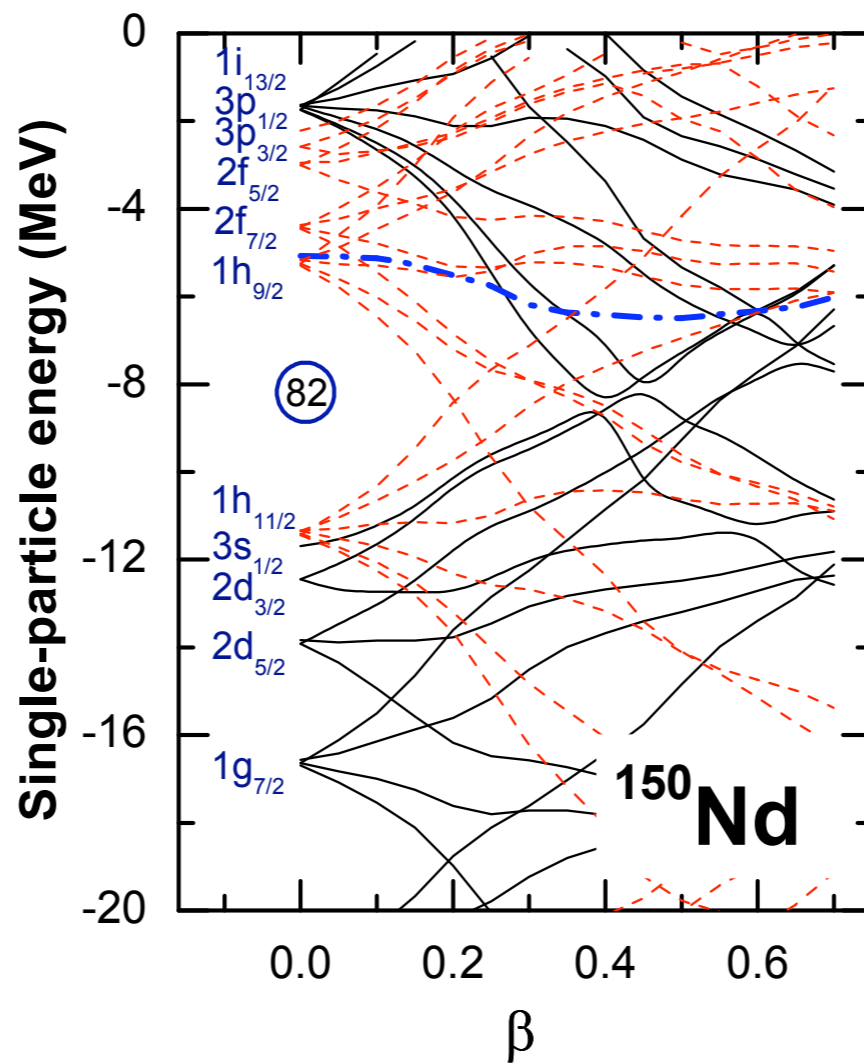
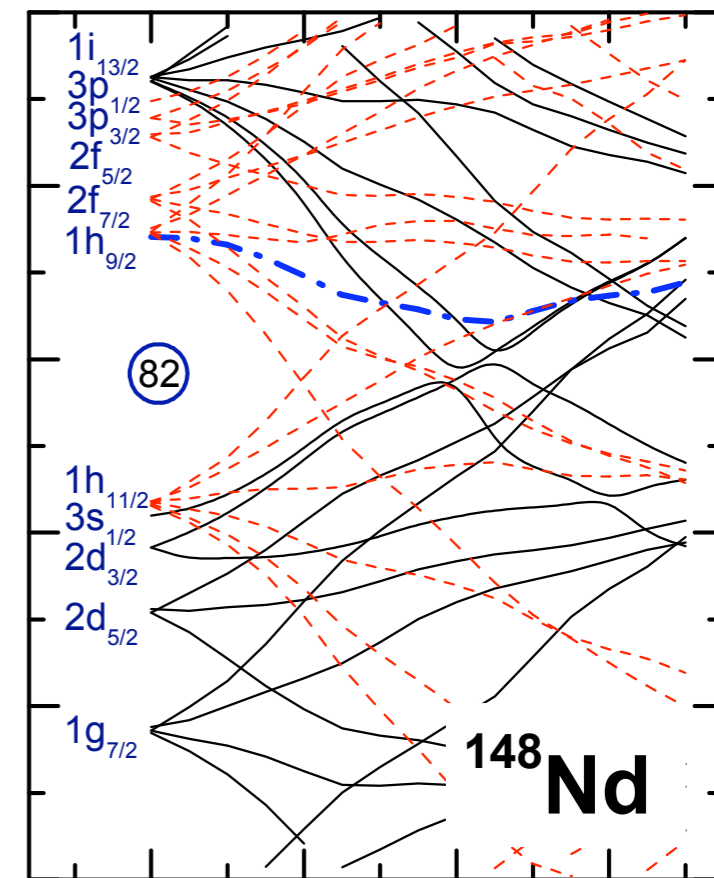
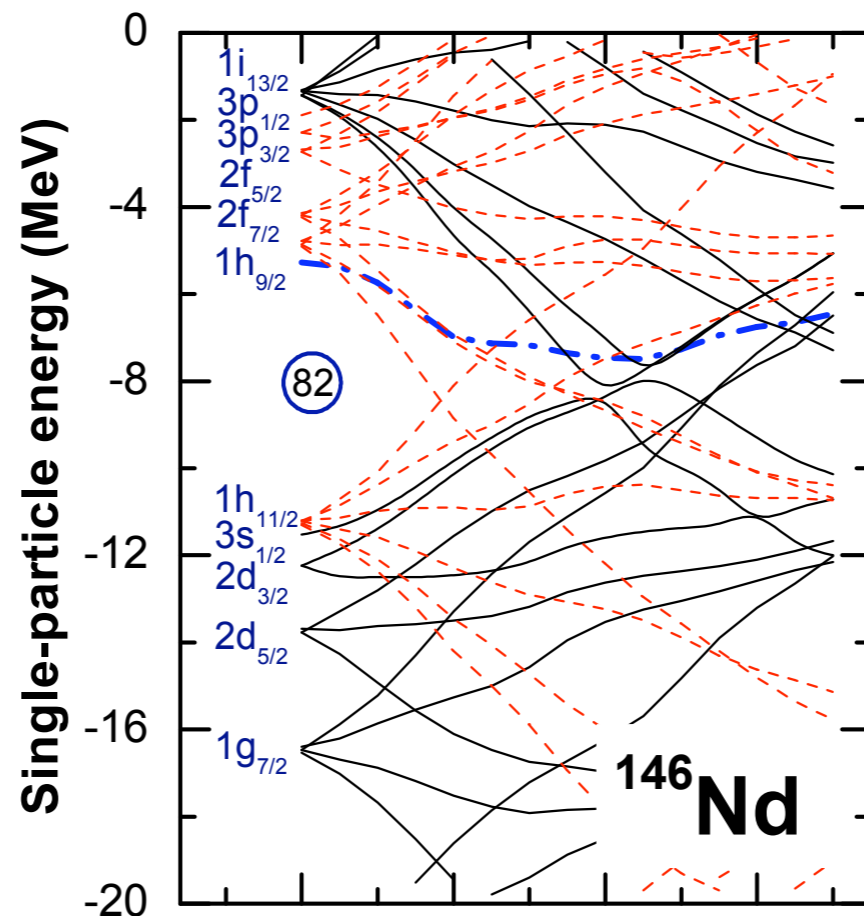
β -vibration or shape coexistence?



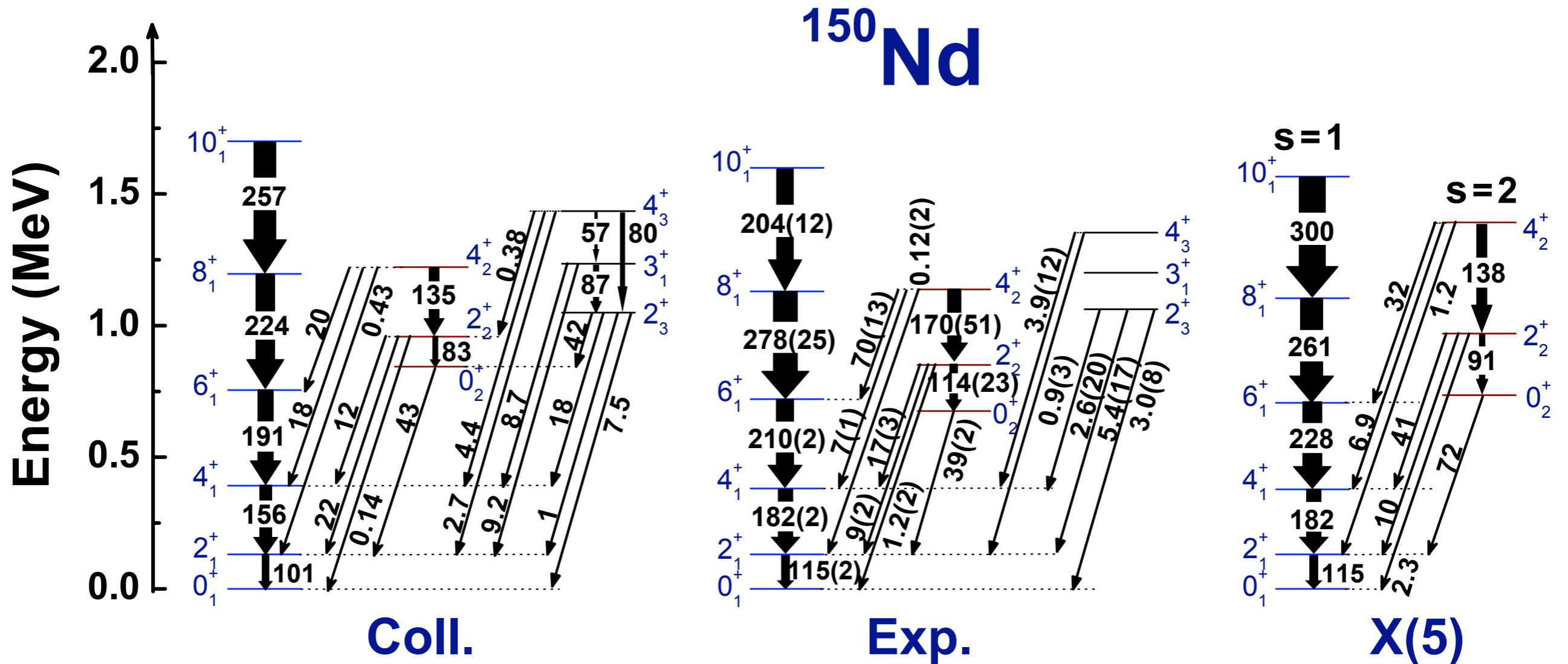
Transitions between spherical and axially deformed shapes in the chain of Nd-Sm-Gd isotopes.



Neutron single-particle levels in Nd isotopes.

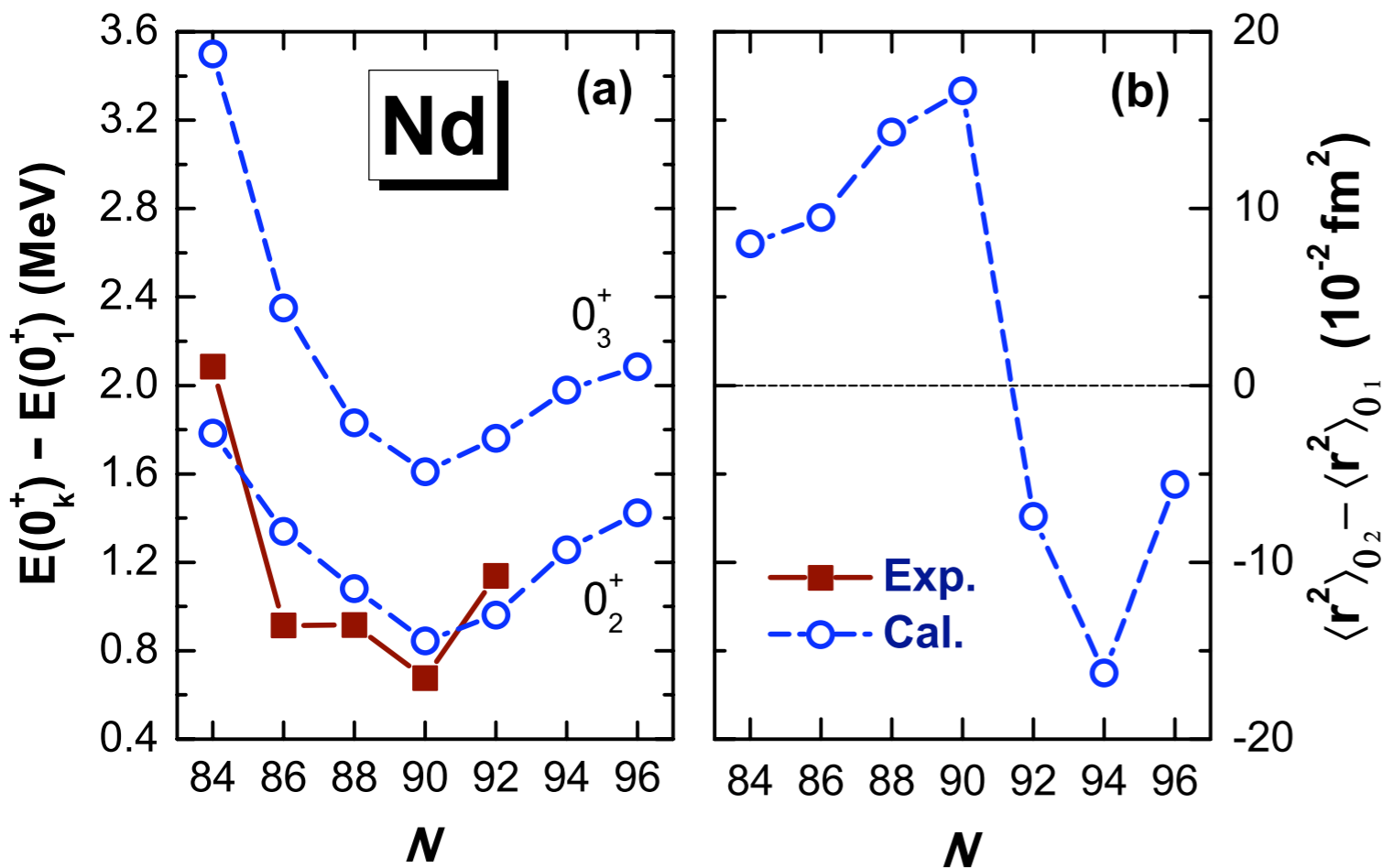
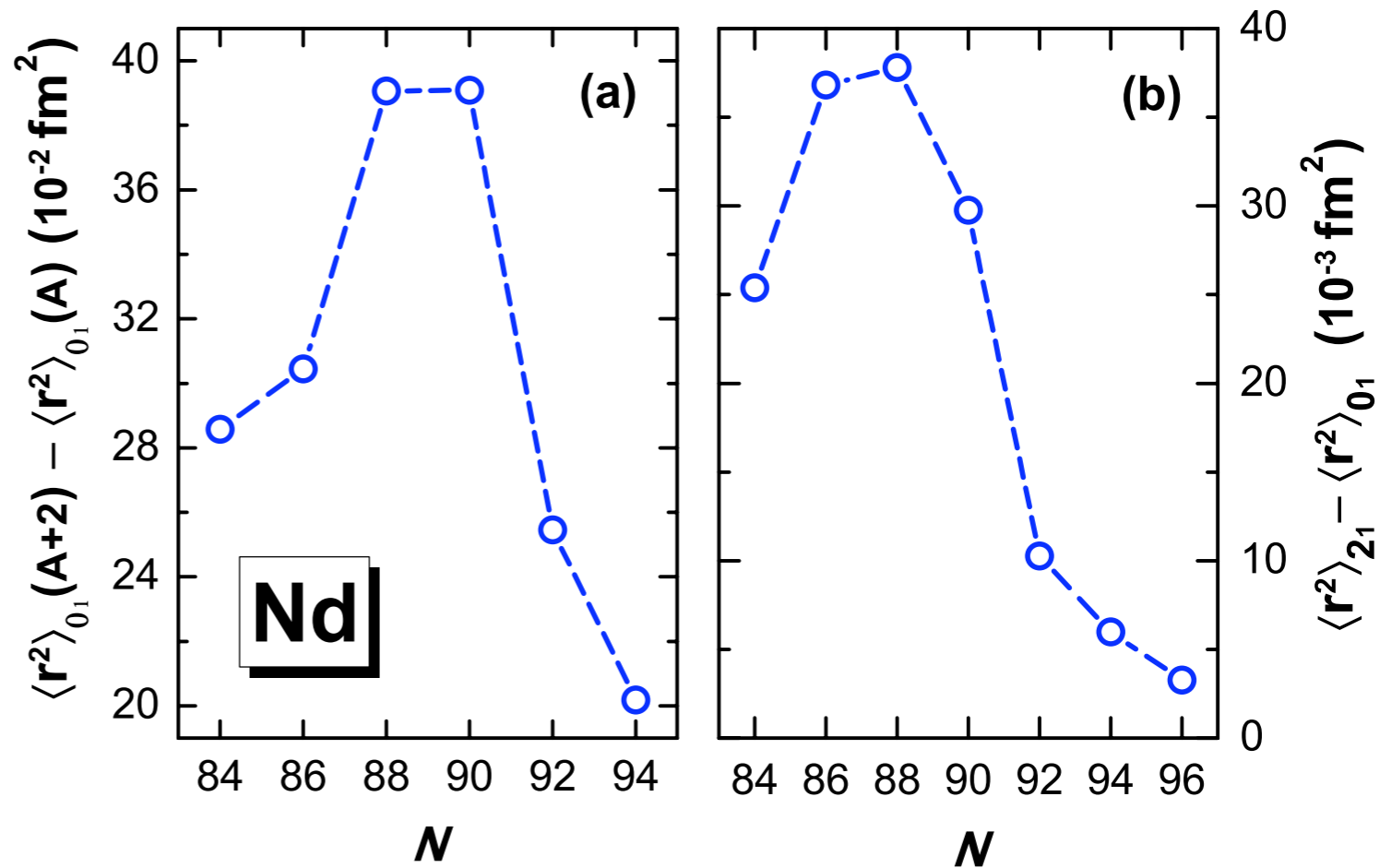


Experimental evidence for a first-order shape phase transition at $N \approx 90$

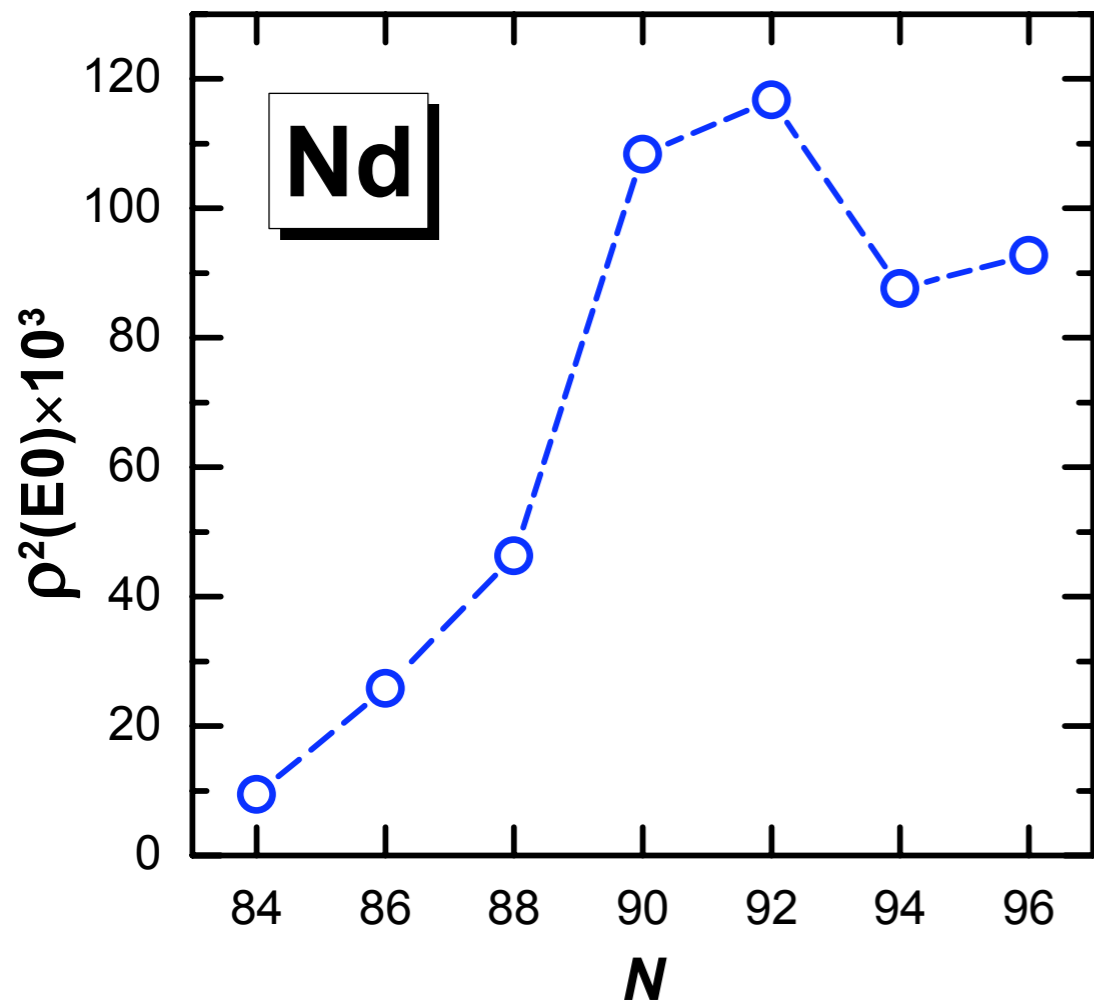


Nikšić, Vretenar, Lalazissis, Ring, Phys. Rev. Lett. **99**, 092502 (2007)
 Li, Nikšić, Vretenar, Meng, Lalazissis, Ring, Phys. Rev. C **79**, 054301 (2009)

... using collective wave functions obtained by diagonalization of the five-dimensional Hamiltonian ...



... microscopic calculation of order parameters for a first-order nuclear QPT between spherical and axially deformed shapes.

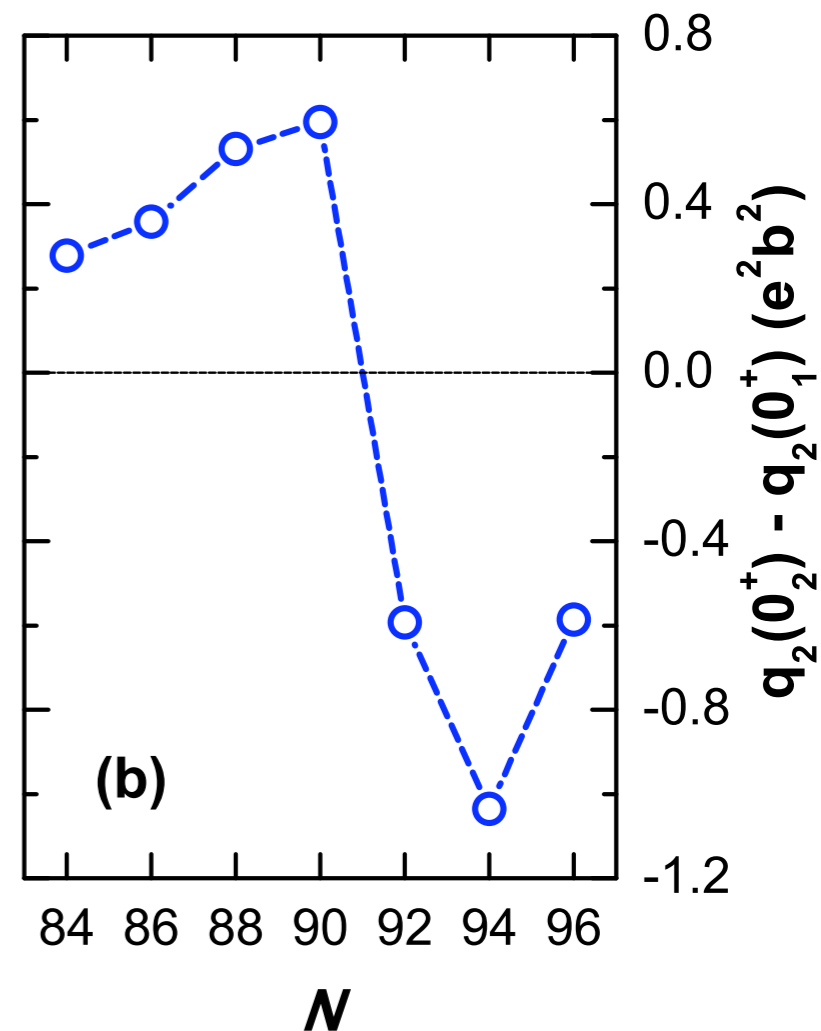
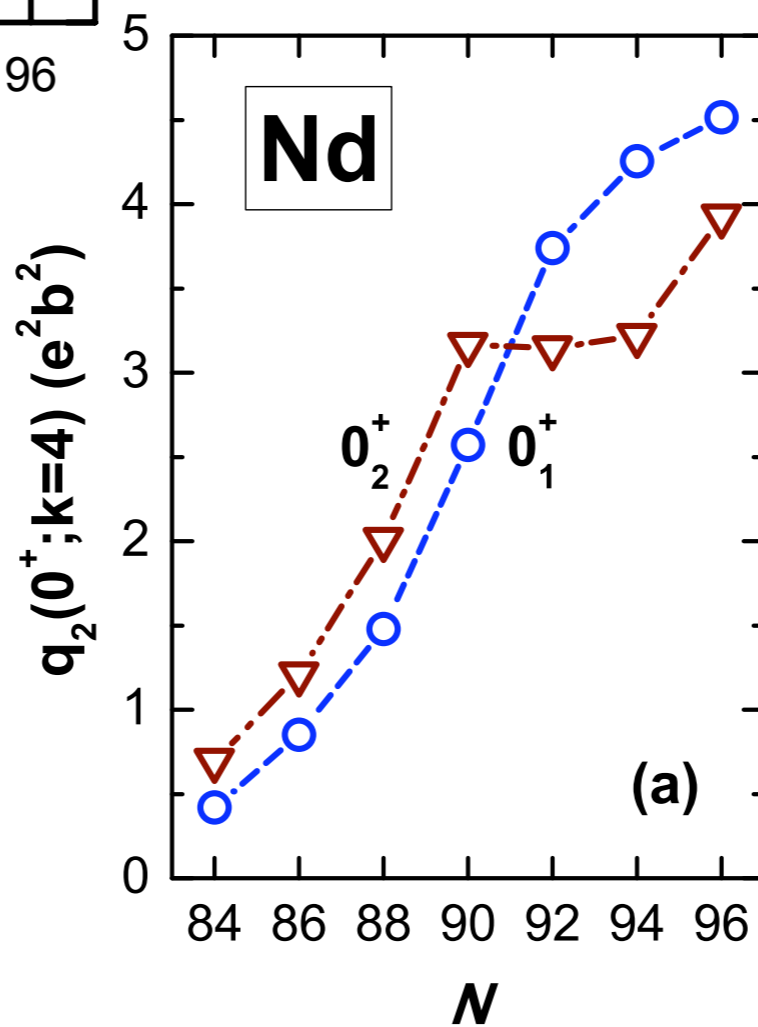


$$\hat{T}(E0) = \sum_k e_k r_k^2$$

$$\rho^2(E0; 0_2^+ \rightarrow 0_1^+) = \left| \frac{\langle 0_2^+ | \hat{T}(E0) | 0_1^+ \rangle}{eR^2} \right|^2$$

q shape invariants:

$$q_2(0_n^+; k) = \sum_{j=1}^k B(E2; 0_n^+ \rightarrow 2_j^+)$$



✘ identification of **order parameters**? Accuracy of the EDF-based collective models used to calculate excitation spectra and transition rates?

✘ How much are the discontinuities at a phase transitional point smoothed out in finite nuclei?

✘ discrete integer values for the control parameter - **nucleon number** → how precisely can a QPT point be assigned to a particular nucleus?