Alternative approaches to phase transition

Section speakers:

1, Y. Zhang, Alternative approaches to phase transition

2, R. Budaca, Tilted-axis wobbing in odd-mass nuclei

3, M. Böyükata, Quantum phase transitions in odd-A nuclei: The effect of the odd particle along the critical line

Two-parameter (control) consistent-Q IBM Hamiltonian

$$\hat{H}(\eta,\chi) = \varepsilon\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$

$$\hat{Q}^\chi = (d^\dagger s + s^\dagger \tilde{d})^{(2)} + \chi(d^\dagger \tilde{d})^{(2)}$$

$$\hat{n}_d = d^\dagger \cdot \tilde{d}$$

$$|N;g\rangle = \frac{1}{\sqrt{N!}}(B_g^\dagger)^N|0\rangle \qquad B_g^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_{-2}^\dagger + d_{+2}^\dagger)\right]$$

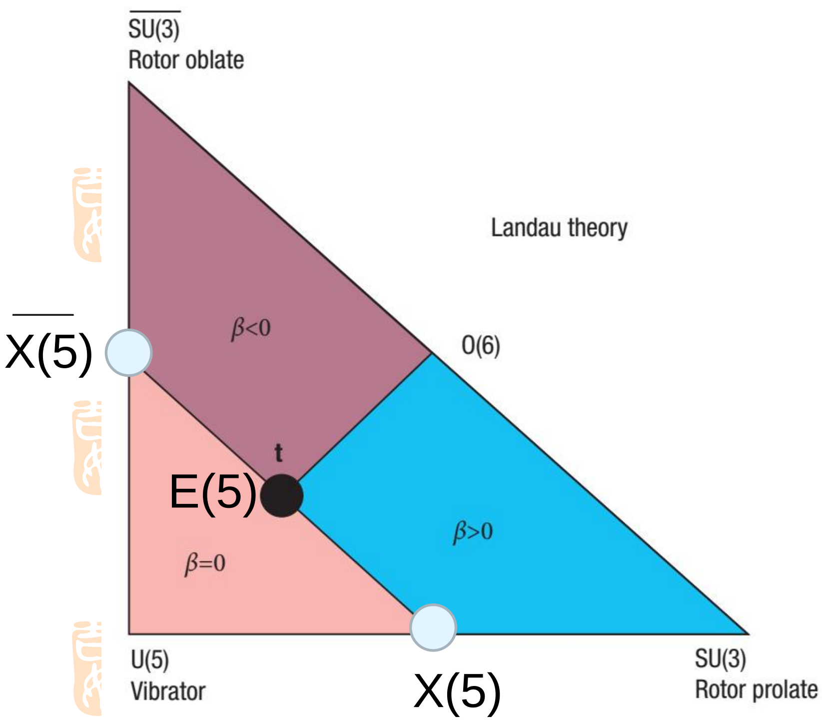
$$V(\beta,\gamma) \equiv \langle N;g|\hat{H}(\eta,\chi)|N;g\rangle$$

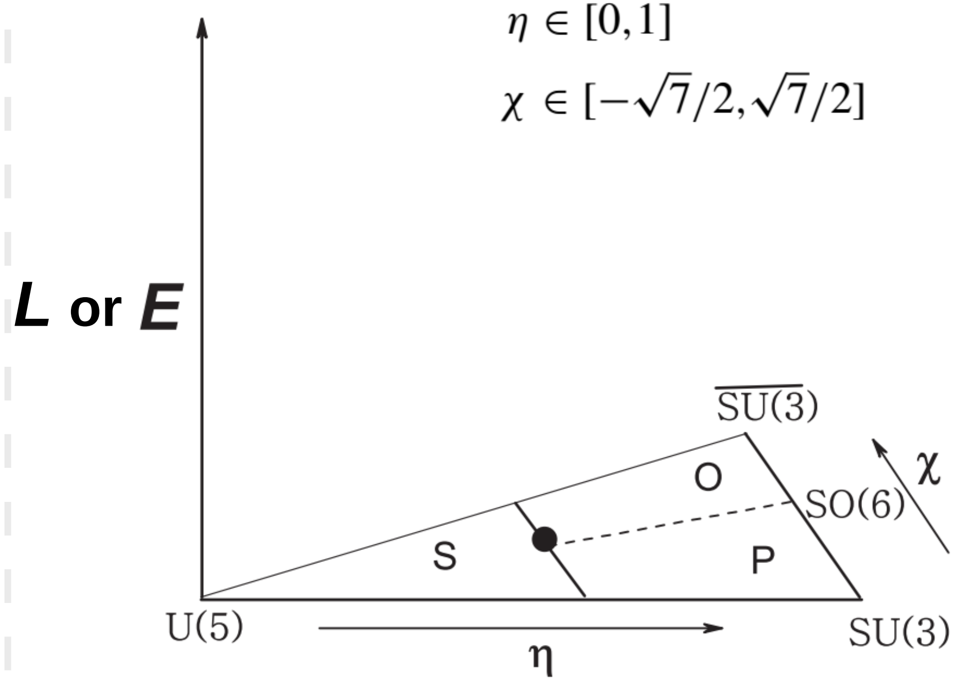
$$= \frac{\varepsilon_0 N\beta^2}{1+\beta^2}\left[(1-\eta)-(\chi^2+1)\frac{\eta}{4N}\right] - \frac{5\varepsilon_0\eta}{4(1+\beta^2)} - \frac{\varepsilon_0\eta(N-1)}{4(1+\beta^2)^2}\left[4\beta^2 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos3\gamma + \frac{2}{7}\chi^2\beta^4\right]$$

Casten, Nature Physics, 2 (2006) 813

$$\eta \in [0,1]$$

$$\chi \in [-\sqrt{7}/2, \sqrt{7}/2]$$





Outline

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- 1, Euclidean dynamical symmetry
- 2, Nucleon-pair transfer reactions as a test of shape phase transitions
- 3, Jacobi-type transitions in deformed nuclei
- 4, Conclusion

1.1 Dynamical symmetries in the s d interacting boson system

Iachello, Arima,
The Interacting Boson Model
(Cambridge England, 1987)

$$U(6) \supset U(5) \supset SO(5) \supset SO(3),$$
$$U(6) \supset O(6) \supset SO(5) \supset SO(3),$$
$$U(6) \supset SU(3) \supset SO(3).$$

$$(d^\dagger \times \tilde{d})^{(k)}_m = \sum_{u,u'} < 2u2u'|km > d^\dagger_u d_{u'}, \quad k = 0,\ 1,\ 2,\ 3,\ 4$$

$$d^\dagger_u s, \quad s^\dagger \tilde{d}_u, \quad s^\dagger s$$

$$Eu(5) = T_5 \oplus_s SO(5)$$

$$Eu(5) \supset T_5 \oplus_s SO(3) \supset SO(3),$$
$$Eu(5) \supset SO(5) \supset SO(3)$$

Y. Zhang, et al., PRC 90 (2014) 064318
Y. Zhang, et al., PLB 732 (2014) 55

T_5(g):

$$\hat{Q}^{(2)}_u = \frac{1}{\sqrt{2}}[\tilde{d}_u - d^\dagger_u],$$

SO(5)(g):

$$\hat{T}^{(\lambda)}_u = \sqrt{2}(d^\dagger \tilde{d})^{(\lambda)}_u, \quad \lambda = 1{,}3,$$

$$\hat{n}_d = \sum_m d^\dagger_m d_m$$

$$(P_d)^\dagger = P^\dagger_d = \sum_m (-)^m d^\dagger_m d^\dagger_{-m}$$

Casimir operators:

$$\hat{C}_2[Eu(5)] = \hat{n}_d + \tfrac{5}{2} - \tfrac{1}{2}(\hat{P}^\dagger_d + \hat{P}_d)$$
$$\hat{C}_2[SO(5)] = \hat{T}^3 \cdot \hat{T}^3 + \hat{T}^1 \cdot \hat{T}^1,$$
$$\hat{C}_2[SO(3)] = 5\hat{T}^1 \cdot \hat{T}^1,$$

Eu(5) DS:

$$\hat{H}_{Eu(5)} = a\hat{C}_2[Eu(5)] + b\hat{C}_2[SO(5)] + c\hat{C}_2[SO(3)]$$

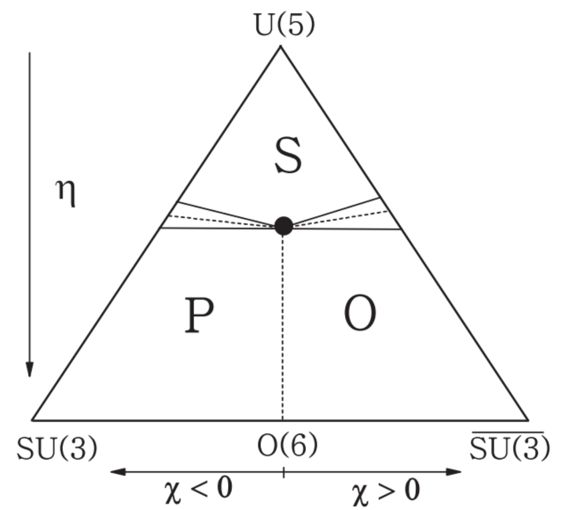
1.2, Eu(5) Dynamical symmetry and the interacting boson model

[*g*, *H*]=0

$$\hat{H}(\eta,\chi) = \varepsilon\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$

$$\hat{A}^{(2)}_q = (s^\dagger \tilde{d} + d^\dagger s)^{(2)}_q \text{ and } \hat{B}^{(k)}_q = (d^\dagger \tilde{d})^{(k)}_q$$

$$[\hat{T}^{(3)}_q, \hat{H}(\eta,\chi)]$$
$$= \frac{3\sqrt{5}\varepsilon\eta\chi}{28N}\{\sqrt{10}[(\hat{B}^{(2)}\hat{A}^{(2)})^{(3)}_q - (\hat{A}^{(2)}\hat{B}^{(2)})^{(3)}_q]$$
$$-2[(\hat{B}^{(4)}\hat{A}^{(2)})^{(3)}_q - (\hat{A}^{(2)}\hat{B}^{(4)})^{(3)}_q] - 2\chi[(\hat{B}^{(4)}\hat{B}^{(2)})^{(3)}_q$$
$$-(\hat{B}^{(2)}\hat{B}^{(4)})^{(3)}_q]\}, \quad (36)$$



$n_d/N \ll 1$ limit

Zhang, Pan, Liu, Luo, Draayer, PRC 90 (2014) 064318

$$[\hat{Q}^{(2)}_q, \hat{H}(\eta,\chi)] = \frac{\sqrt{2}\varepsilon}{2}(1-2\eta)\hat{C}^{(2)}_q$$
$$-\frac{\sqrt{2}\varepsilon\eta\chi}{8N}[(\hat{A}^{(2)}\hat{C}^{(2)})^{(2)}_q + (\hat{C}^{(2)}\hat{A}^{(2)})^{(2)}_q$$
$$+2\hat{B}^{(2)}_q + \chi(\hat{C}^{(2)}\hat{B}^{(2)})^{(2)}_q + \chi(\hat{B}^{(2)}\hat{C}^{(2)})^{(2)}_q],$$

the triple point $(\eta = 0.5, \chi = 0)$

$$\hat{H}_{\rm tri} \simeq \frac{\varepsilon}{4}\left[\hat{n}_d - \frac{5}{2} - \frac{1}{2}(P^\dagger_d + P_d)\right]$$
$$= \frac{\varepsilon}{4}[\hat{C}_2[\mathrm{Eu}(5)] - 5],$$

1.3 Critical point symmetries and Eu(5) Dynamcal symmetry

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_\kappa \frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + V(\beta,\gamma).$$

Iachello, PRL 85 (2000) 3580

E(5) CPS:

$$V(\beta,\gamma) = V(\beta) = \begin{cases} 0, & \beta \leqslant \beta_W, \\ \infty, & \beta > \beta_W. \end{cases}$$

$$\alpha_\mu = \beta\left[\mathscr{D}^{(2)}_{\mu 0}(\vartheta)\cos\gamma + \frac{1}{\sqrt{2}}[\mathscr{D}^{(2)}_{\mu 2}(\vartheta) + \mathscr{D}^{(2)}_{\mu -2}(\vartheta)]\sin\gamma\right].$$

In the well,

Caprio and Iachello, NPA 781 (2007) 26

$$H = \frac{\hbar^2}{2B}\tilde{\pi}\cdot\tilde{\pi} = a\hat{C}_2[\mathrm{Eu}(5)]$$

$$\Psi(\beta,\gamma,\theta_i) = f(\beta)\Phi(\gamma,\theta_i)$$

$$f_{\xi,\tau}(\beta) = c_{\xi,\tau}\beta^{-3/2}J_{\tau+3/2}(k_{\xi,\tau}\beta)$$

Iachello, PRL 87 (2001) 052502

X(5) CPS:

$$V(\beta,\gamma) = V_{\mathrm{well}}(\beta) + c(\gamma - \gamma_0)^2$$

$$\Psi(\beta,\gamma,\theta_i) = c_{s,L}\beta^{-3/2}J_\nu(k_{s,L}\beta)\,\eta_{n_\gamma,K}(\gamma)\mathcal{D}^L_{MK}(\theta_i)$$

1.4 Diagonalizing the Eu(5) Hamiltonian

E(5) and X(5) $\psi''(z) + \frac{\psi'(z)}{z} + \left(1 - \frac{\nu^2}{z^2}\right)\psi(z) = 0$

$$\nu = \left(1+\frac{2}{\sqrt{7}}\chi\right)\frac{L}{2} - \frac{2\chi}{\sqrt{7}}\left[\frac{-3+\sqrt{9+4L(L+1)/3}}{2}\right] + \frac{3}{2} \quad \chi \in [-\frac{\sqrt{7}}{2}, 0] \quad L = 2\tau$$

$$\hat{H}_{\mathrm{Eu}(5)} = C_2[\mathrm{Eu}(5)] = \hat{n}_d + \frac{5}{2} - \frac{1}{2}(P_d^\dagger + P_d) \qquad T_u = e(d^\dagger + \tilde{d})_u^{(2)}$$

- Diagonalize $\hat{H}_{\mathrm{Eu}(5)}$ under the projected U(5) basis $U(6) \supset U(5) \supset O(5) \supset O(3)$

$$\hat{P}^{\chi}_{\tau',\ \tau}|Nn_d\tau\Delta L >= |Nn_d\tau'\Delta L >, \quad \tau' = \nu - 3/2$$

- Diagonalize $\hat{H}'_{\mathrm{Eu}(5)}$ under the U(5) basis $|Nn_d\tau\Delta L >$

$$\hat{H}'_{\mathrm{Eu}(5)} = (\hat{P}^{\chi}_{\tau',\tau})^\dagger \hat{H}_{\mathrm{Eu}(5)} \hat{P}^{\chi}_{\tau',\tau} =$$

$$A + \frac{2\chi}{\sqrt{7}}\sqrt{B} - \frac{\chi}{\sqrt{7}}\sqrt{\frac{16}{3}B - \frac{40}{3}\sqrt{B} + 17} - \frac{5}{2}$$

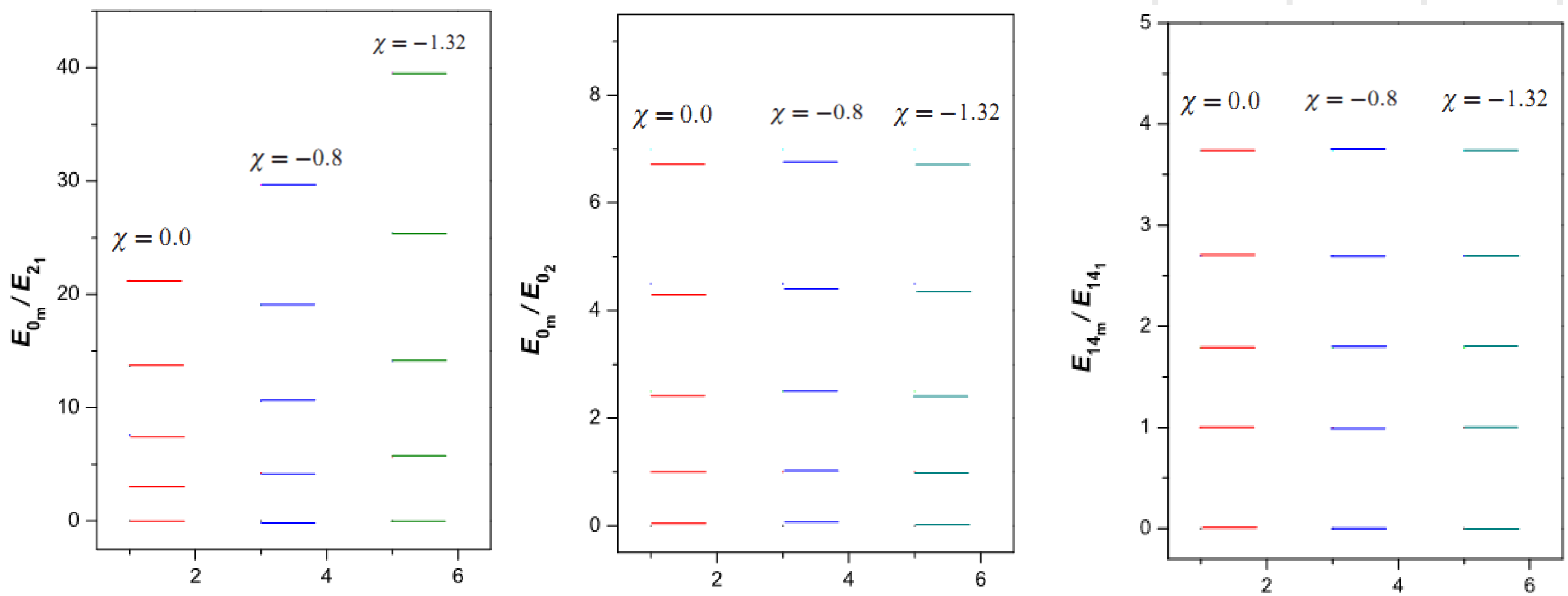
$$- \frac{A + (1+\frac{4\chi}{\sqrt{7}})\sqrt{B} - \frac{2\chi}{\sqrt{7}}\sqrt{\frac{16}{3}B - \frac{40}{3}\sqrt{B} + 17} + \frac{7}{2}}{2(A + \sqrt{B} + \frac{7}{2})} C^\dagger$$

$$+ C\,\frac{A + (1+\frac{4\chi}{\sqrt{7}})\sqrt{B} - \frac{2\chi}{\sqrt{7}}\sqrt{\frac{16}{3}B - \frac{40}{3}\sqrt{B} + 17} + \frac{7}{2}}{2(A + \sqrt{B} + \frac{7}{2})}$$

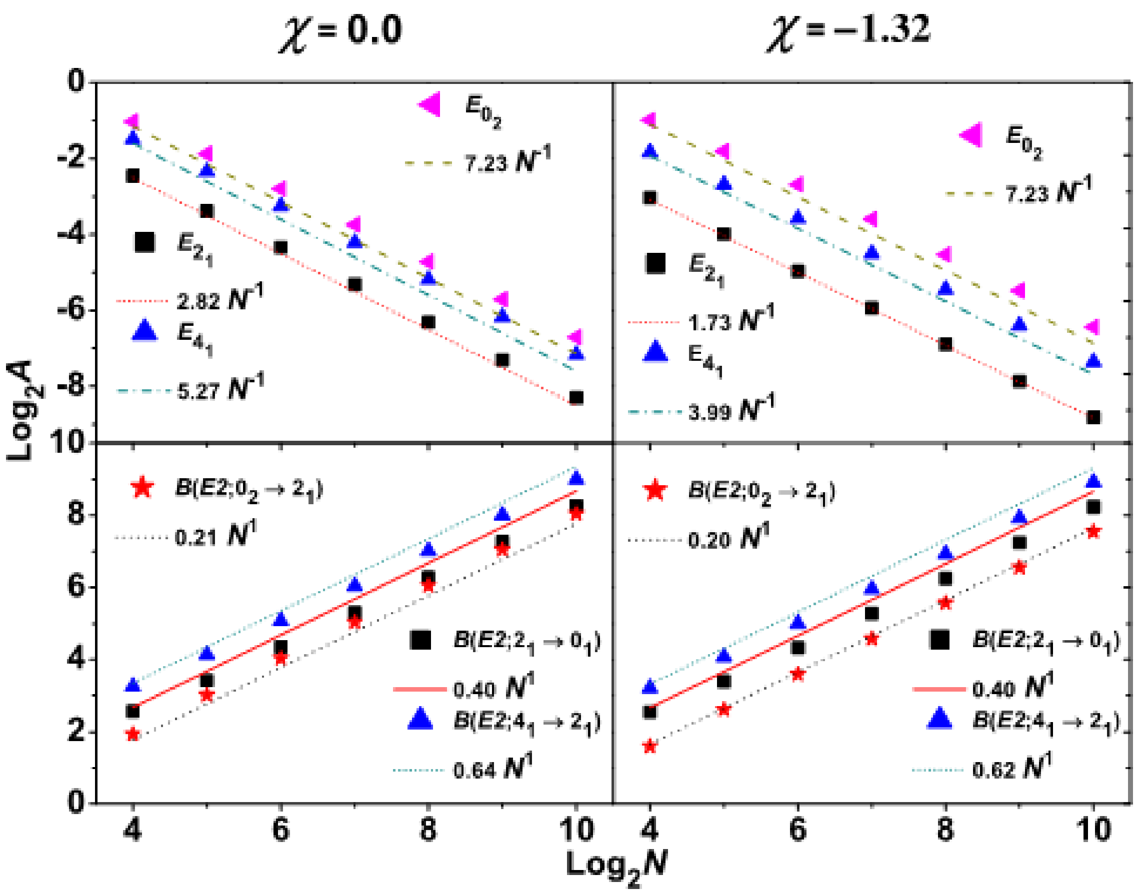
$$A = \hat{n}_d,\ B = \hat{n}_d(\hat{n}_d + 3) - 2P_d^\dagger P_d + \tfrac{9}{4},\ C = P_d$$

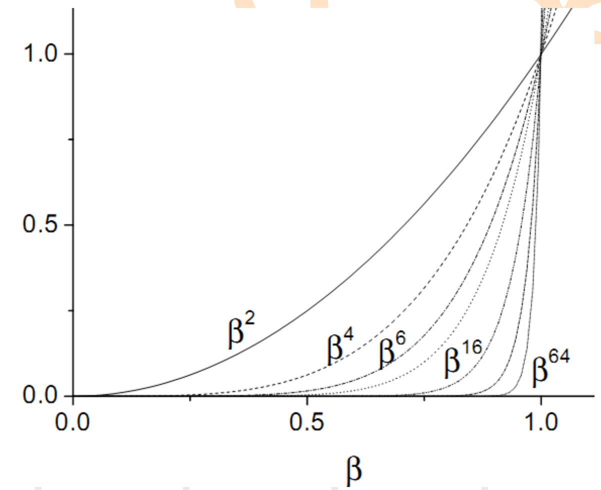
1.5 Spectral characters of the Eu(5) DS

	E(5)	Eu(5) at $N=1000$						X(5)	U(5)	SO(6)	SU(3)
		$\chi=0.0$	$\chi=-0.4$	$\chi=-0.8$	$\chi=-1.0$	$\chi=-1.1$	$\chi=-1.32$				
E_{4_1}/E_{2_1}	2.20	2.19	2.33	2.51	2.63	2.71	2.89	2.91	2.00	2.50	3.33
E_{6_1}/E_{0_2}	1.18	1.19	1.12	1.05	1.01	1.00	0.96	0.96	1.50	1.00	$\frac{21}{8N-4}$
E_{0_2}/E_{2_1}	3.03	3.02	3.53	4.22	4.67	4.93	5.61	5.67	2.00	4.50	$\frac{4(2N-1)}{3}$
$\frac{B(E2;\,4_1\to 2_1)}{B(E2;\,2_1\to 0_1)}$	1.68	1.67	1.65	1.63	1.62	1.61	1.60	1.58	$\frac{2(N-1)}{N}$	$\frac{10(N^2+4N-5)}{7(N^2+4N)}$	$\frac{10(4N^2+6N-10)}{7(4N^2+6N)}$
$\frac{B(E2;\,0_2\to 2_1)}{B(E1;\,2_1\to 0_1)}$	0.86	0.86	0.79	0.72	0.68	0.66	0.62	0.63	$\frac{2(N-1)}{N}$	0.00	0.00



1.6, Scaling characters of Eu(5) DS





$$\hat{H} = -\frac{\nabla^2}{2M} + k\beta^{2n} \quad (\text{if } k \propto M^t)$$

$$E \propto M^{(t-n)/(n+1)}|_{n\to\infty} = M^{-1}$$

Rowe, Turner, Rosensteel, PRL 93 (2004) 232502

If $M \propto N$, then $\hat{H}_{\text{Eu}(5)} \to \hat{H}_B = -\frac{\nabla^2}{2M} + V(\beta)$ with

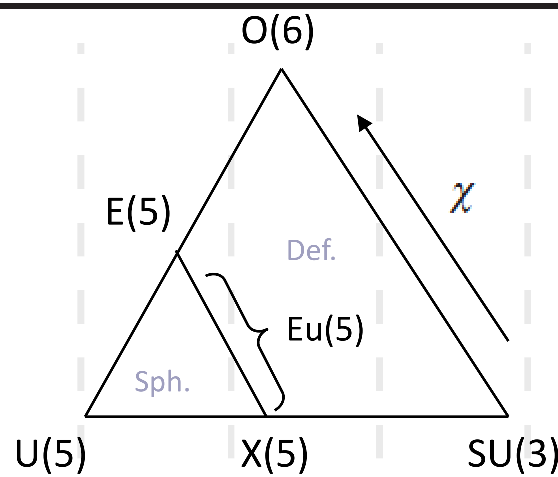
$$V(\beta) = 0, \quad \beta < 1$$

$$V(\beta) = \infty, \quad \beta \geq 1$$

1.7 Signals of the Eu(5) DS in experiments

Ratio	(χ, nucleus)				
	$(-0.2, ^{102}Pd)$	$(-0.4, ^{128}Xe)$	$(-0.9, ^{146}Ce)$	$(-1.2, ^{148}Ce)$	$(-1.3, ^{150}Nd)$
E_{4_1}/E_{2_1}	(2.26, 2.29)	(2.33, 2.33)	(2.57, 2.58)	(2.78, 2.86)	(2.89, 2.92)
E_{6_1}/E_{2_1}	(3.75, 3.79)	(3.94, 3.92)	(4.58, 4.53)	(5.13, 5.29)	(5.40, 5.53)
E_{8_1}/E_{2_1}	(5.46, 5.41)	(5.80, 5.67)	(6.95, 6.72)	(7.94, 8.14)	(8.44, 8.67)
E_{6_1}/E_{0_2}	(1.15, 1.32)	(1.12, 1.10)	(1.03, 1.12)	(0.98, 1.09)	(0.96, 1.07)
E_{14_1}/E_{0_2}	(3.64, 3.85)	(3.64, 2.92)	(3.64, -)	(3.64, 3.75)	(3.64, 3.97)
$\frac{B(E2;\,4_1\to 2_1)}{B(E2;\,2_1\to 0_1)}$	(1.66, 1.56)	(1.65, 1.43)	(1.62, -)	(1.61, -)	(1.60, 1.52)
$\frac{B(E2;\,0_2\to 2_1)}{B(E1;\,2_1\to 0_1)}$	(0.83, 1.60)	(0.79, 0.43)	(0.70, -)	(0.64, -)	(0.62, 0.39)

1.58



2.1 Nucleon-pair transfer reaction in the interacting boson model

Monopole-pair transfer $P^{(0)}_{+,\nu,0} = t_{a_\nu} s^\dagger A(\Omega_\nu, N_\nu), \quad P^{(0)}_{-,\nu,0} = t_{a_\nu} A(\Omega_\nu, N_\nu) s, \quad (1)$

Quadrupole-pair transfer $P^{(2)}_{+,\nu,\mu} = t_{b_\nu} d^\dagger_\mu A(\Omega_\nu, N_\nu), \quad P^{(2)}_{-,\nu,\mu} = t_{b_\nu} A(\Omega_\nu, N_\nu) \tilde{d}_\mu \quad (2)$

$$A(\Omega_\nu, N_\nu) = \left(\Omega_\nu - N_\nu - \frac{N_\nu}{N}\hat{n}_d\right)^{\frac{1}{2}} \left(\frac{N_\nu + 1}{N + 1}\right)^{\frac{1}{2}}$$

Arima, Iachello, PRC 16 (1977)

$$I^a(N+1, L' \to N, L) = \frac{1}{2L'+1} |\langle N, L \| P_- \| N+1, L' \rangle|^2$$

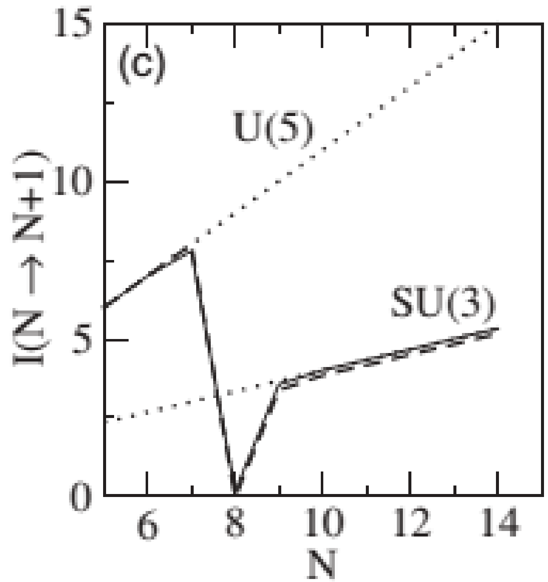
Iachello, Arima, The Interacting Boson Model (Cambridge England, 1987)

$$I^{U(5)}(N_\nu,\, 0_1^+ \to N_\nu + 1,\, 0_1^+) = t^2_{a_\nu}(N_\nu + 1)(\Omega_\nu - N_\nu)$$

$$I^{SU(3)}(N_\nu,\, 0_1^+ \to N_\nu + 1,\, 0_1^+) = t^2_{a_\nu}(N_\nu + 1)\frac{2N+3}{3(2N+1)}\left(\Omega_\nu - N_\nu - \frac{4(N-1)}{3(2N-1)}N_\nu\right)$$

$$I^{U(5)}(N_\nu,\, 0_1^+ \to N_\nu + 1,\, 2_1^+) = t^2_{\beta_\nu}(N_\nu + 1)\frac{5}{N+1}(\Omega_\nu - N_\nu)$$

$$I^{SU(3)}(N_\nu,\, 0_1^+ \to N_\nu + 1,\, 2_1^+) = t^2_{b_\nu}(N_\nu + 1)\frac{2(2N+3)(2N+5)}{3(2N+1)(2N+2)}\left(\Omega_\nu - N_\nu - \frac{4(N-1)}{3(2N-1)}N_\nu\right)$$



Fossion, Alonso, Arias, Fortunato, Vitturi PRC 76 (2007) 014316

2.2 Intrinsic matrix elements of boson operators

Iachello, Arima, The Interacting Boson Model (Cambridge England, 1987)
A. Leviatan, Z. Physics A 321（1985）467

$$|N+1;g'\rangle = \frac{1}{\sqrt{(N+1)!}}(B^\dagger_{g'})^{N+1}|0\rangle$$

$$|N+1;\beta'_v\rangle = \frac{1}{\sqrt{(N+1)}}(B^\dagger_{\beta'_v})B_{g'}|N+1;g'\rangle$$

$$|N+1;\gamma'_v\rangle = \frac{1}{\sqrt{(N+1)}}(B^\dagger_{\gamma'_v})B_{g'}|N+1;g'\rangle$$

$$|N+1;2\beta'_v\rangle = \frac{1}{\sqrt{2(N+1)N}}(B^\dagger_{\beta'_v}t)^2(B_{g'})^2|N+1;g'\rangle.$$

$$B^\dagger_{g'} = \frac{1}{\sqrt{1+\beta'^2}}\left[s^\dagger + \beta'\cos\gamma' d^\dagger_0 + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(d^\dagger_{-2}+d^\dagger_{+2})\right].$$

$$B^\dagger_{\beta'_v} = \frac{1}{\sqrt{1+\beta'^2}}\left[-\beta' s^\dagger + \cos\gamma' d^\dagger_0 + \frac{1}{\sqrt{2}}\sin\gamma'(d^\dagger_{-2}+d^\dagger_{+2})\right],$$

$$B^\dagger_{\gamma'_v} = \frac{1}{\sqrt{2}}\cos\gamma'(d^\dagger_{+2}+d^\dagger_{-2}) - \sin\gamma' d^\dagger_0.$$

For (t,p) or (p,t) transfer reactions between ground bands, one can derive

$$\langle N;g|s|N+1;g'\rangle = \langle N+1;g'|s^\dagger|N;g\rangle = \frac{\sqrt{N+1}}{\sqrt{1+\beta'^2}}\left[\frac{1+\beta\beta'\cos(\gamma-\gamma')}{\sqrt{(1+\beta'^2)(1+\beta^2)}}\right]^N,$$

$$\langle N;g|d_\mu|N+1;g'\rangle = \langle N+1;g'|d^\dagger_\mu|N;g\rangle = \frac{\sqrt{N+1}}{\sqrt{1+\beta'^2}}\left[\frac{1+\beta\beta'\cos(\gamma-\gamma')}{\sqrt{(1+\beta'^2)(1+\beta^2)}}\right]^N \times\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right].$$

Y. Zhang and F. Iachello, PRC 95 (2017) 034306

For the (t,p) transfer reaction between ground bands and excited (e) bands, one can find

$$\langle N+1;\beta'_v|s^\dagger|N;g\rangle = [N\beta\cos(\gamma-\gamma') - (N+1)\beta' - \beta\beta'^2\cos(\gamma-\gamma')]\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}$$

$$\langle N+1;\beta'_v|d^\dagger_\mu|N;g\rangle = \frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}\Big\{[N\beta\beta'\cos\gamma\cos\gamma' - N\beta'^2 + 1 + \beta\beta'\cos(\gamma-\gamma$$
$$\times\left[\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] + N\beta\beta'\sin\gamma\sin\gamma'\cos\gamma'\Big\},$$

$$\langle N+1;\gamma'_v|s^\dagger|N;g\rangle = N\beta\sin(\gamma-\gamma')\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{[\sqrt{(1+\beta^2)(1+\beta'^2)}]^N},$$

$$\langle N+1;\gamma'_v|d^\dagger_\mu|N;g\rangle = \left[\frac{1+\beta\beta'\cos(\gamma-\gamma')}{\sqrt{(1+\beta^2)(1+\beta'^2)}}\right]^N\left[\frac{\cos\gamma'}{\sqrt{2}}(\delta_{\mu,2}+\delta_{\mu,-2}) - \sin\gamma'\delta_{\mu,0} + N\beta\sin(\gamma-\gamma')\frac{\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,-2}+\delta_{\mu,2})}{1+\beta\beta'\cos(\gamma-\gamma')}\right],$$

$$\langle N+1;2\beta'_v|s^\dagger|N;g\rangle = \sqrt{\frac{N}{2}}[\beta\cos(\gamma-\gamma')-\beta']\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}} \times\{(N-1)[\beta\cos(\gamma-\gamma')-\beta'] - 2\beta'[1+\beta\beta'\cos(\gamma-\gamma')]\},$$

$$\langle N+1;2\beta'_v|d^\dagger_\mu|N;g\rangle = \sqrt{\frac{N}{2}}[\beta\cos(\gamma-\gamma')-\beta']\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}}\left[\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] \times\{2[1+\beta\beta'\cos(\gamma-\gamma')] + (N-1)\beta'[\beta\cos(\gamma-\gamma')-\beta']\}.$$

(C) $\phi'_g(N+1)\to\phi_e(N)$

For the (p,t) transfer reaction between ground bands and excited bands, one can find

$$\langle N;\beta_v|s|N+1;g'\rangle = \frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}\sqrt{N(N+1)}[\beta'\cos(\gamma-\gamma')-\beta],$$

$$\langle N;\beta_v|d_\mu|N+1;g'\rangle = \sqrt{N(N+1)}\frac{(1+\beta\beta'\cos(\gamma-\gamma'))^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}[\beta'\cos(\gamma-\gamma')-\beta] \times\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{\beta'\sin\gamma'}{\sqrt{2}}(\delta_{\mu,2}+\delta_{\mu,-2})\right],$$

$$\langle N;\gamma_v|s|N+1;g'\rangle = \sqrt{N(N+1)}(\beta'+\beta^2\beta')\sin(\gamma'-\gamma)\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{[\sqrt{(1+\beta^2)(1+\beta'^2)}]^{N+1}},$$

$$\langle N;\gamma_v|d_\mu|N+1;g'\rangle = \sqrt{(N+1)N(1+\beta^2)}\beta'\sin(\gamma'-\gamma)\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] \times\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\frac{1}{(\sqrt{1+\beta'^2})^{N+1}},$$

$$\langle N;2\beta_v|s|N+1;g'\rangle = \sqrt{\frac{(N+1)N(N-1)}{2}}[\beta'\cos(\gamma-\gamma')-\beta]^2\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}},$$

$$\langle N;2\beta_v|d_\mu|N+1;g'\rangle = \sqrt{\frac{(N+1)N(N-1)}{2}}[\beta'\cos(\gamma-\gamma')-\beta]^2\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] \times\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}}.$$

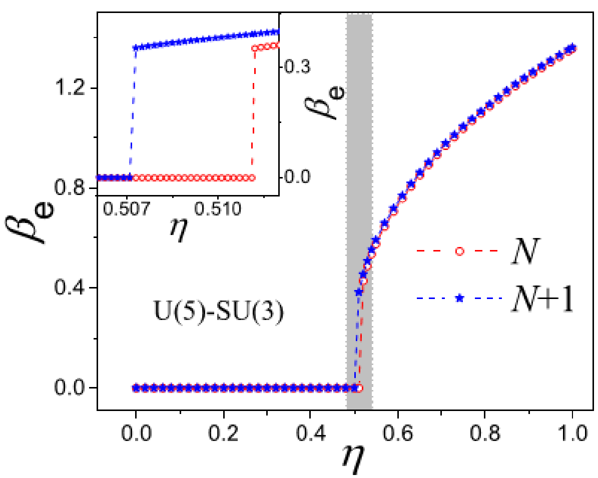
2.3 Classical nature of one-boson transfer amplitude

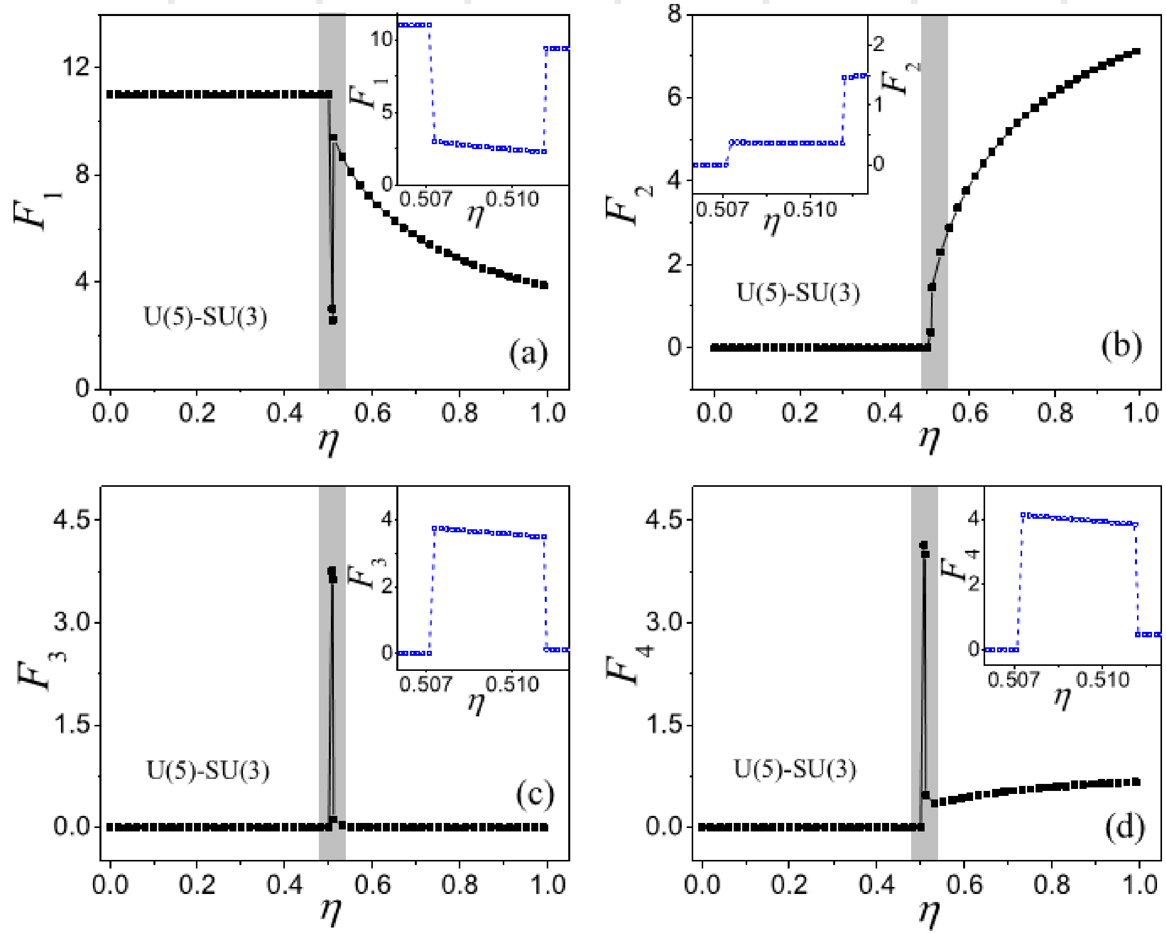
$$F_1 \equiv |\langle N; g|s|N+1; g'\rangle|^2 = |\langle N+1; g'|s^\dagger|N; g\rangle|^2,$$

$$F_2 \equiv |\langle N; g|d_0|N+1; g'\rangle|^2 = |\langle N+1; g'|d_0^\dagger|N; g\rangle|^2,$$

$$F_3 \equiv |\langle N; \beta_v|s|N+1; g'\rangle|^2,$$

$$F_4 \equiv |\langle N+1; \beta_v'|s^\dagger|N; g\rangle|^2$$

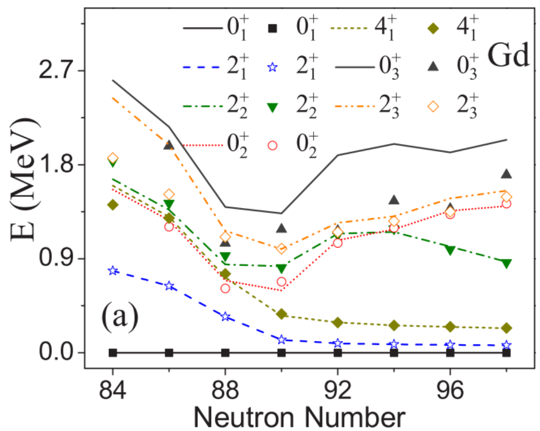




$$\hat{H}(\eta, \chi) = \varepsilon_0\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$

2.4 Comparison to experimental data in (p, t) reaction for Gd

$$\hat{H}(\eta,\chi) = \varepsilon_0\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$



$$I_1^a = I(N+1,0_1^+ \rightarrow N,0_1^+),$$

$$I_2^a = I(N+1,0_1^+ \rightarrow N,0_2^+),$$

$$I_3^a = I(N+1,0_1^+ \rightarrow N,0_3^+),$$

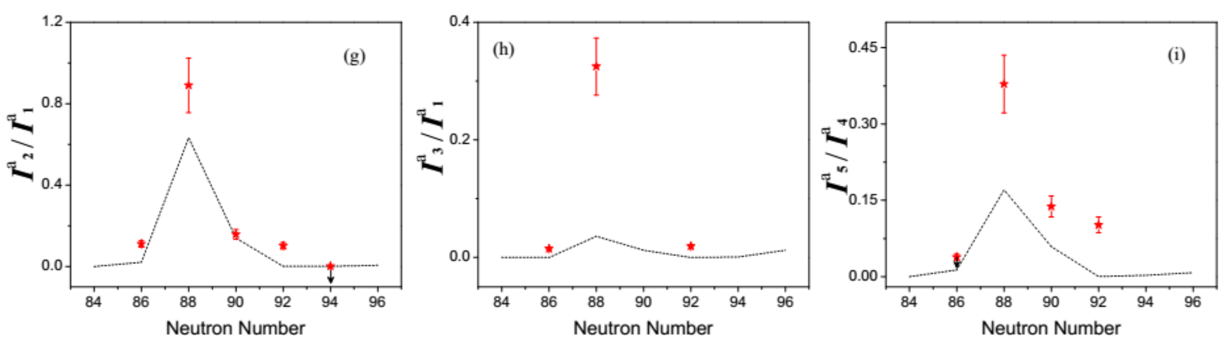
$$I_4^a = I(N+1,0_1^+ \rightarrow N,2_1^+),$$

$$I_5^a = I(N+1,0_1^+ \rightarrow N,2_2^+),$$

$$I_6^a = I(N+1,0_1^+ \rightarrow N,2_3^+),$$

for (p,t) reactions and

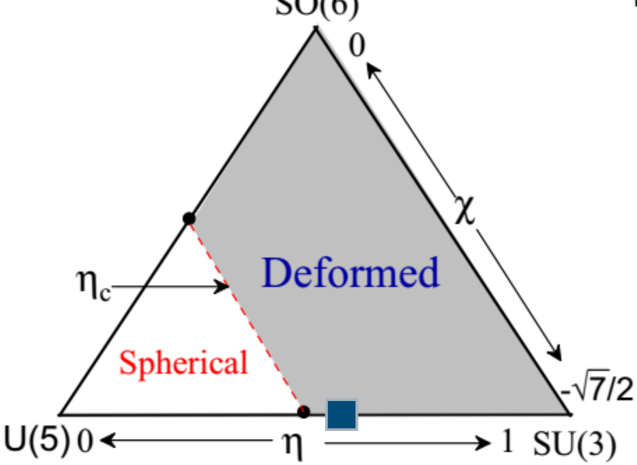
Hamiltonian parameter values taken from McCutchan, Zamfir, Casten PRC 69 (2004) 064306

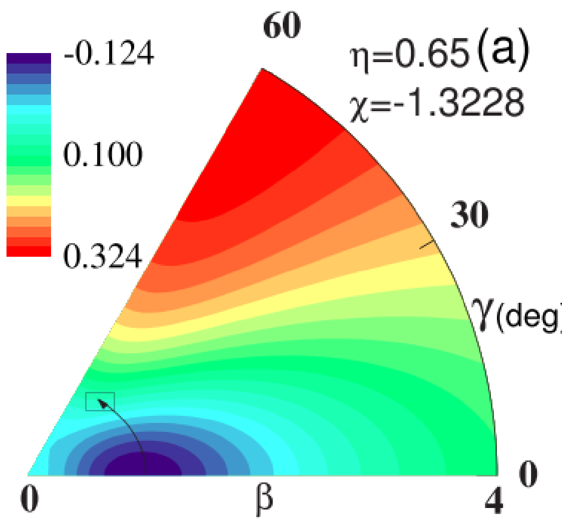


Data taken from Fleming, et al PRC 8 (1973)

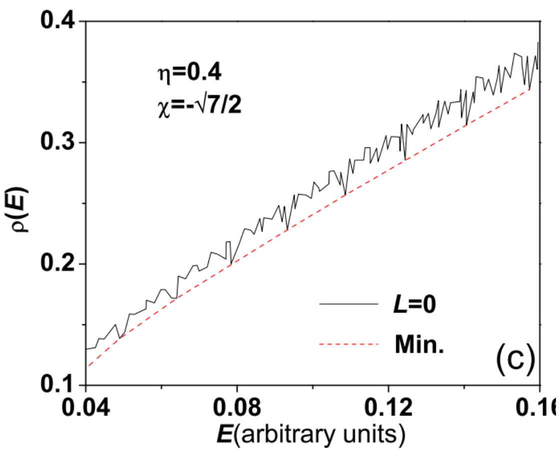
3.1 Jacobi-type transition: the gamma-rigid to gamma-soft transition in excited states

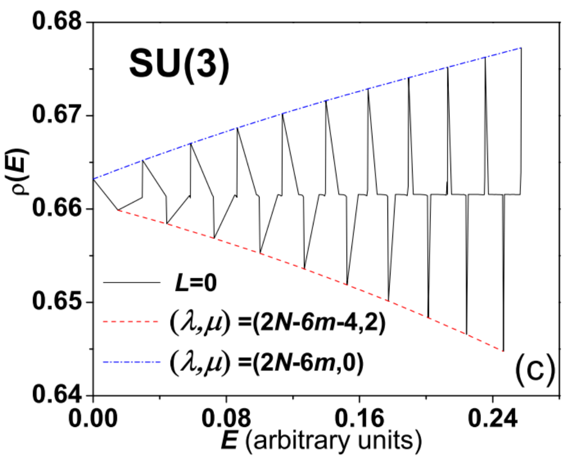
$$\hat{H}(\eta,\chi) = \varepsilon_0\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$





$$\rho = \frac{\langle 0^+|\hat{n}_d|0^+\rangle}{N}$$





P. H. Regan et al., PRL 90 (2003) 152302

$$R = \frac{E_\gamma(L \to L-2)}{L}$$

$R \propto \frac{1}{L}$ in the U(5) limit

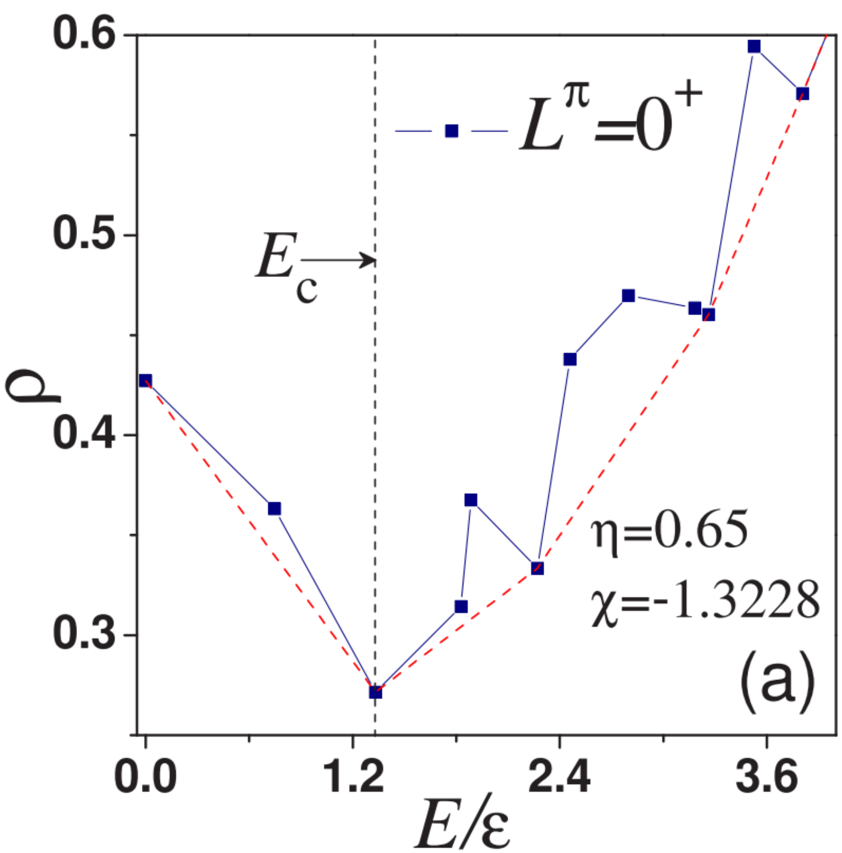
$R \propto (4 - \frac{2}{L})$ in the SU(3) limit

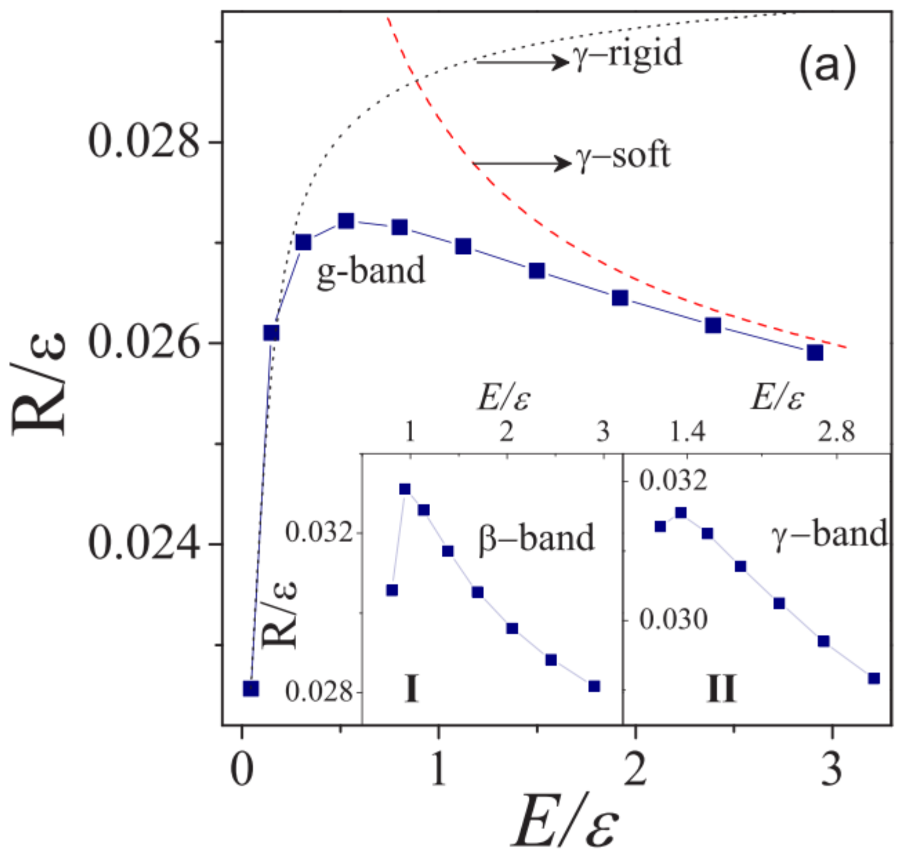
3.2 Changes in monotonicity of ρ and R as functions of excitation energy

Y. Zhang and F. Iachello, PRC 95 (2017) 061304(R)

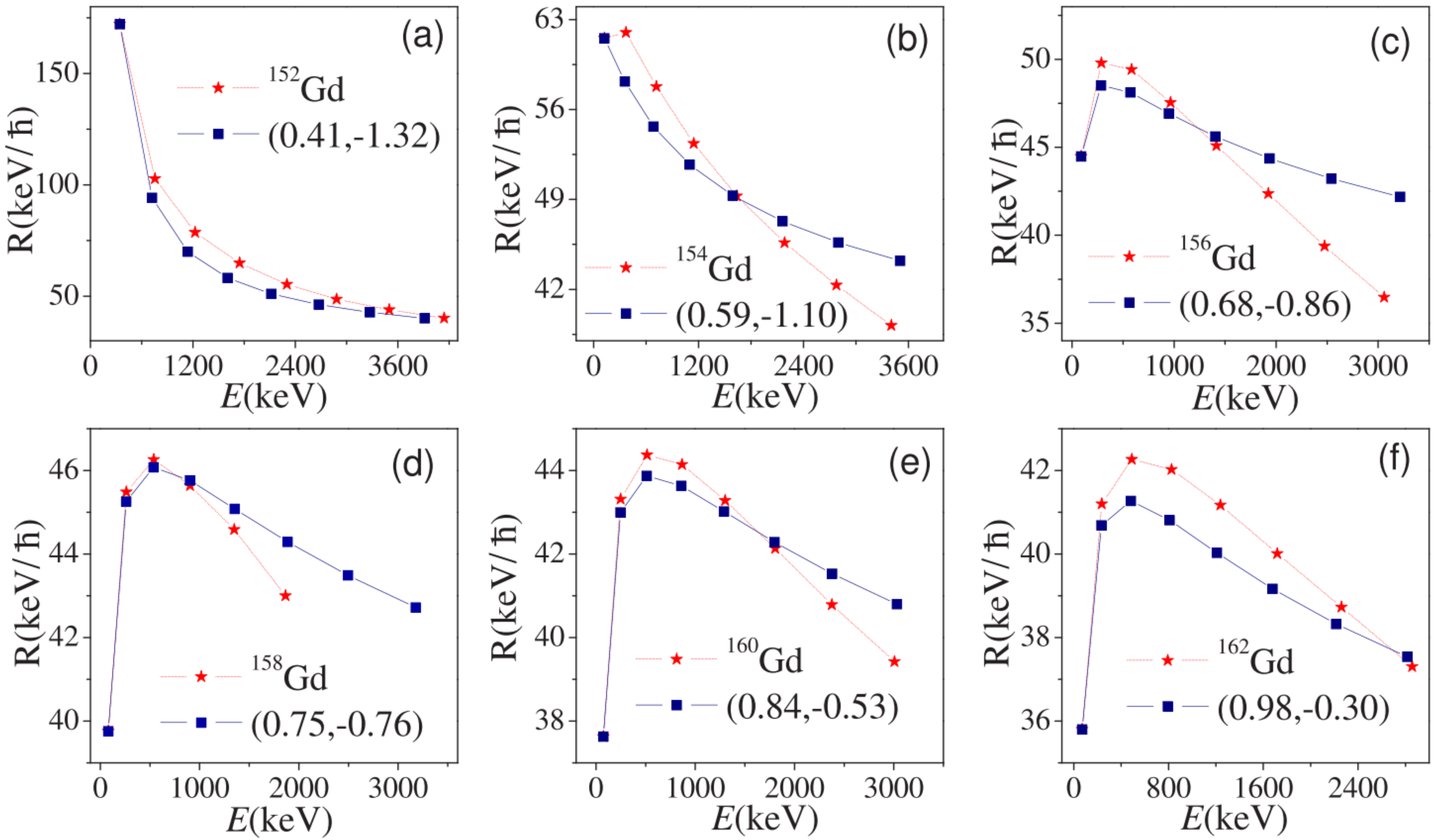
$$\rho = \frac{\langle 0^+|\hat{n}_d|0^+\rangle}{N}$$

$$R = \frac{E_\gamma(L\to L-2)}{L}$$





3.3 Evidences of Jacobi-type tranistions in the Gd isotopes



Hamiltonian parameter values taken from McCutchan, Zamfir, Casten PRC 69 (2004) 064306

Conclusion:

1) Euclidan dynamical symmetry as the critical symmetry is dominant but hidden in the whole critical region of the SPT.

2) Monopole- and quadrupole-pair transfer reactions provide alternative ways to “see” shape phase transitions in nuclei.

3) Signatures of Jacobi-type transitions can be used to indicate the shape phase transitions and, may be, the associated excited-state quantum phase transitions in nuclei.

Thank you for your attention !