Correlations between charge radii, E0 transitions and M1 strength

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Correlations between
- Charge radii and E0 transitions (Wood et al.)
- Charge radii and summed M1 strength (Heyde et al.)
- E0 transitions and summed M1 strength

Application in the rare-earth nuclei

2PTN-9, Padua, May 2018

Coexistence or collective?

Origin of E0 transitions in nuclei:

Mixing of coexisting configurations with different shapes (Heyde & Wood);

Between β-vibrational states in the geometric collective model (Reiner).

In a geometric framework E0 strength should rise in the transition from spherical to deformed.

Operators in the IBM

The charge radius operator:

$$\hat{T}(r^2) = \langle r^2 \rangle_{\mathrm{core}} + \alpha N_{\mathrm{b}} + \frac{\eta}{N_{\mathrm{b}}} \hat{n}_d$$

The E0 operator:

$$\hat{T}(\mathrm{E0}) = \left(e_{\mathrm{n}} N + e_{\mathrm{p}} Z\right) \frac{\eta}{N_{\mathrm{b}}} \hat{n}_d$$

The M1 operator:

$$\hat{T}(\mathrm{M1}) = \sqrt{\frac{3}{4\pi}} \left(g_\nu \hat{L}_\nu + g_\pi \hat{L}_\pi\right)$$

F. Iachello and A. Arima, The Interacting Boson Model

Application to rare-earth nuclei

Application to even-even nuclei with Z=58-74.

Procedure:

- *Determine IBM hamiltonian from spectra with special care to the spherical-to-deformed transitional region.*
- *Determine coefficients α and η in $T(r^2)$ from isotope and isomer shifts.*
- *Calculate $\rho(E0)$ values.*

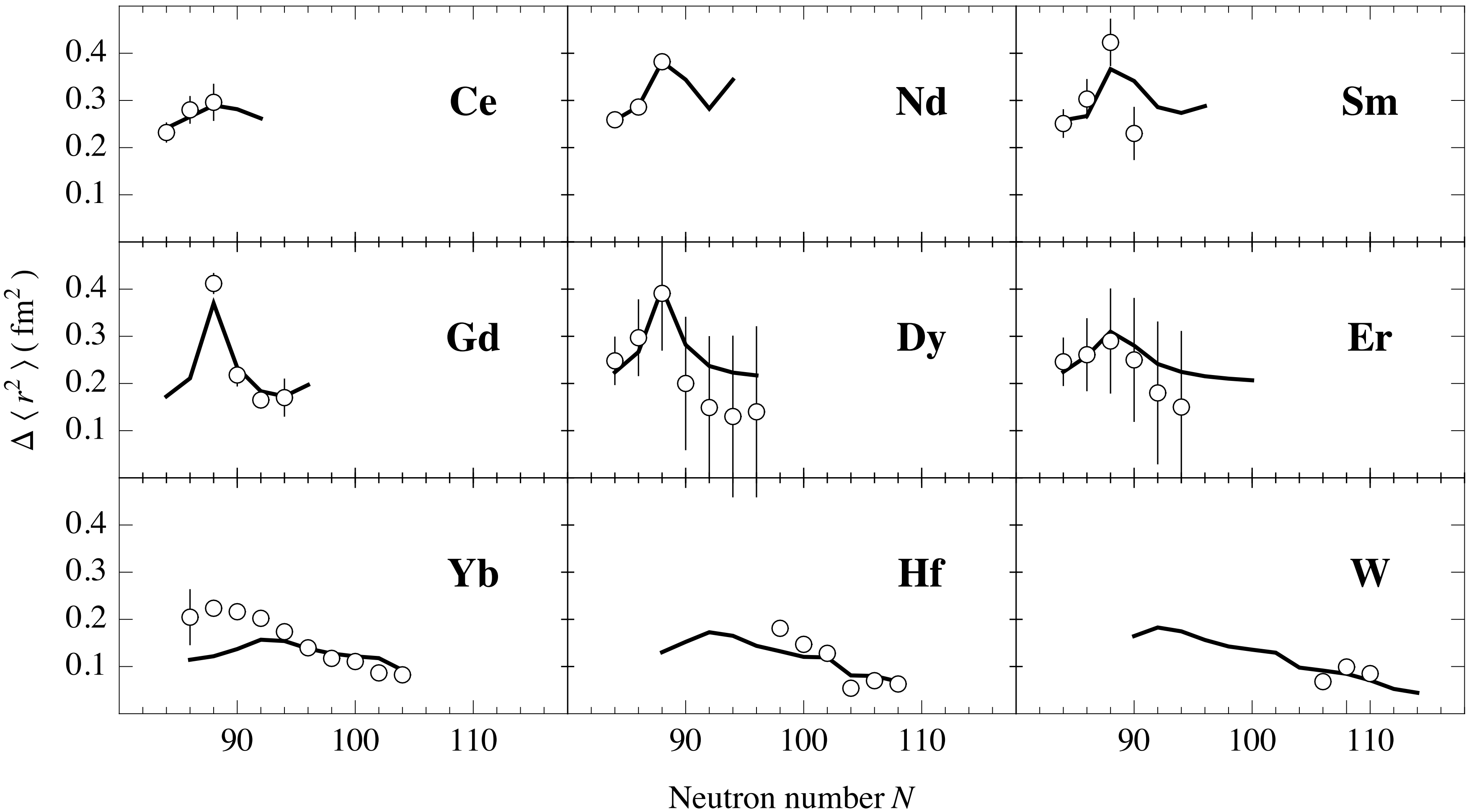
S. Zerguine *et al.*, Phys. Rev. Lett. **101** (2008) 022502
S. Zerguine *et al.*, Phys. Rev. C **85** (2012) 034331

Isotope shifts

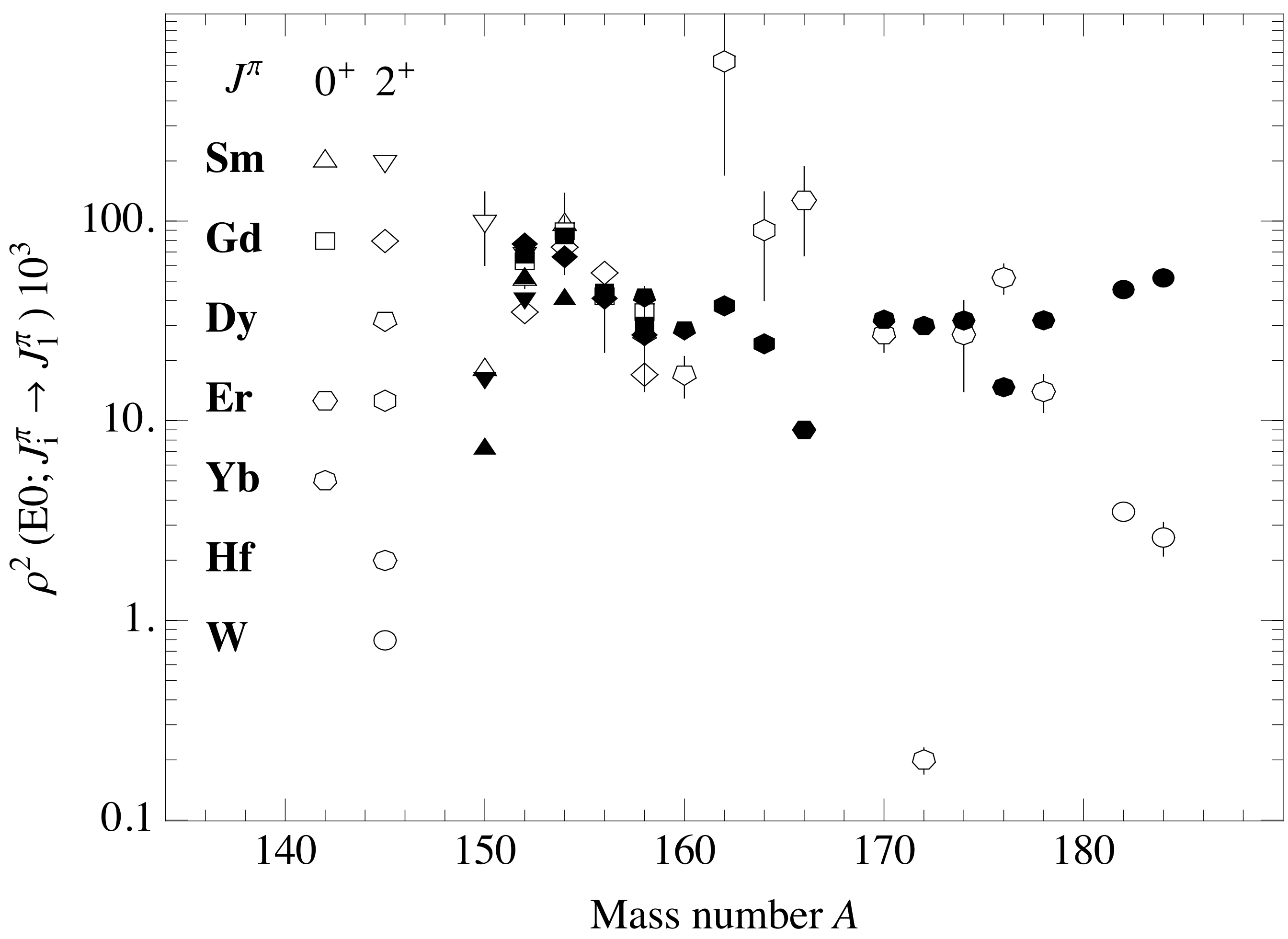
Isotopes shifts depend on the coefficients α and η:

$$\Delta\left\langle r^2\right\rangle \equiv \left\langle r^2\right\rangle_{0_1^+}^{(A+2)} - \left\langle r^2\right\rangle_{0_1^+}^{(A)} = |\alpha| + \frac{\eta}{N_\mathrm{b}}\left(\left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A+2)} - \left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A)}\right)$$

Isotope shifts



ρ^2 values



Data: Compilation B. Singh

ρ^2 values

Isotope	Transition	J	$\rho^2(E0) \times 10^3$				
			Th1[a]	Th2[b]	Th3[c]	Expt.[d]	
^{150}Sm	740 → 0	0	7	6		18	*2*
	1046 → 334	2	16	13		100	*40*
^{152}Sm	685 → 0	0	52	52	72	51	*5*
	811 → 122	2	41	41	77	69	*6*
	1023 → 366	4	29	29	84	88	*14*
	1083 → 0	0	2	2		0.7	*0.4*
	1083 → 685	0	47	47		22	*9*
^{154}Sm	1099 → 0	0	41	49		96	*42*
^{152}Gd	615 → 0	0	68	68		63	*14*
	931 → 344	2	77	77		35	*3*
^{154}Gd	681 → 0	0	84	102		89	*17*
	815 → 123	2	66	80		74	*9*
	1061 → 361	4	38	46		70	*7*
^{156}Gd	1049 → 0	0	44	64		42	*20*
	1129 → 89	2	41	59		55	*5*
^{158}Gd	1452 → 0	0	30	51		35	*12*
	1517 → 79	2	27	45		17	*3*
^{158}Dy	1086 → 99	2	42	70		27	*12*
^{160}Dy	1350 → 87	2	28	56		17	*4*
^{162}Er	1171 → 102	2	38	64		630	*460*
^{164}Er	1484 → 91	2	24	48		90	*50*
^{166}Er	1460 → 0	0	9	20		127	*60*
^{170}Yb	1229 → 0	0	32	72		27	*5*
^{172}Yb	1405 → 0	0	30	76		0.2	*0.03*
^{174}Hf	900 → 91	2	32	71		27	*13*
^{176}Hf	1227 → 89	2	15	38		52	*9*
^{178}Hf	1496 → 93	2	32	72		14	*3*
^{182}W	1257 → 100	2	45	77		3.5	*0.3*
^{184}W	1121 → 111	2	52	75		2.6	*0.5*

Summed B(M1) strength

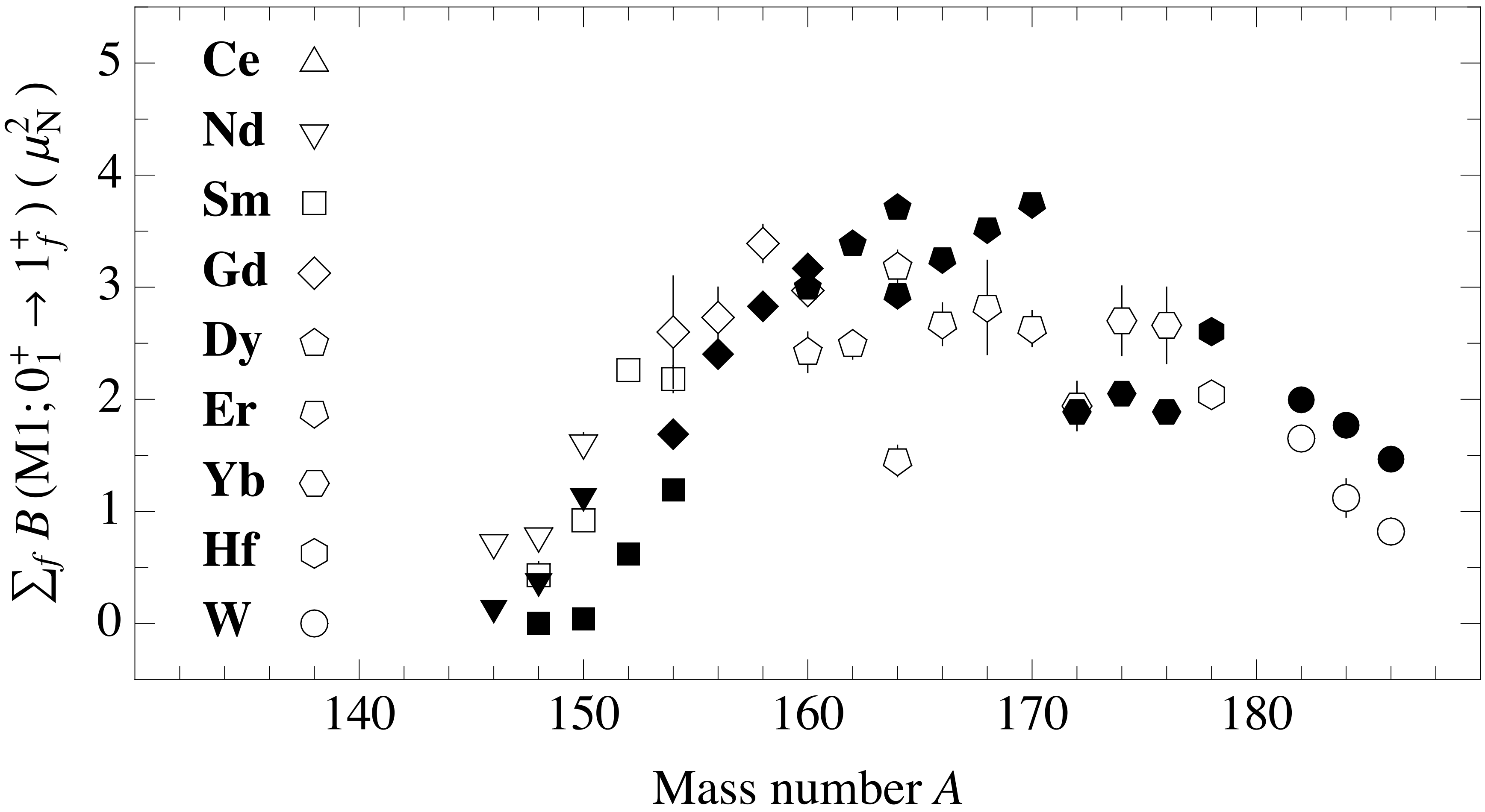
Ginocchio proved the following M1 sum rule:

$$\sum_f B\left(\mathrm{M1};0_1^+ \to 1_f^+\right) = \frac{3}{4\pi}\left(g_\nu - g_\pi\right)^2 \frac{6N_\nu N_\pi}{N_\mathrm{b}\left(N_\mathrm{b}-1\right)}\left\langle \hat{n}_d \right\rangle_{0_1^+}$$

Summed M1 strength to the scissors state is known in many rare-earth nuclei.

J.N. Ginocchio, Phys. Lett. B **265** (1991) 6

Summed B(M1) strength



N. Pietralla *et al.*, Phys. Rev. C **52** (1995) 2317(R)

Correlation S(M1)-$\langle r^2 \rangle$

Rewrite expressions for $\langle r^2 \rangle$ and S(M1):

$$\Delta\left\langle r^2\right\rangle - |\alpha| \equiv \frac{\eta}{N_\mathrm{b}}\left(\left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A+2)} - \left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A)}\right)$$

$$\tilde{S}(\mathrm{M1}) \equiv \frac{N_\mathrm{b}-1}{N_\nu}\sum_f B\left(\mathrm{M1};0_1^+ \to 1_f^+\right) = \frac{9}{2\pi}\left(g_\nu - g_\pi\right)^2 \frac{N_\pi}{N_\mathrm{b}}\left\langle \hat{n}_d\right\rangle_{0_1^+}$$

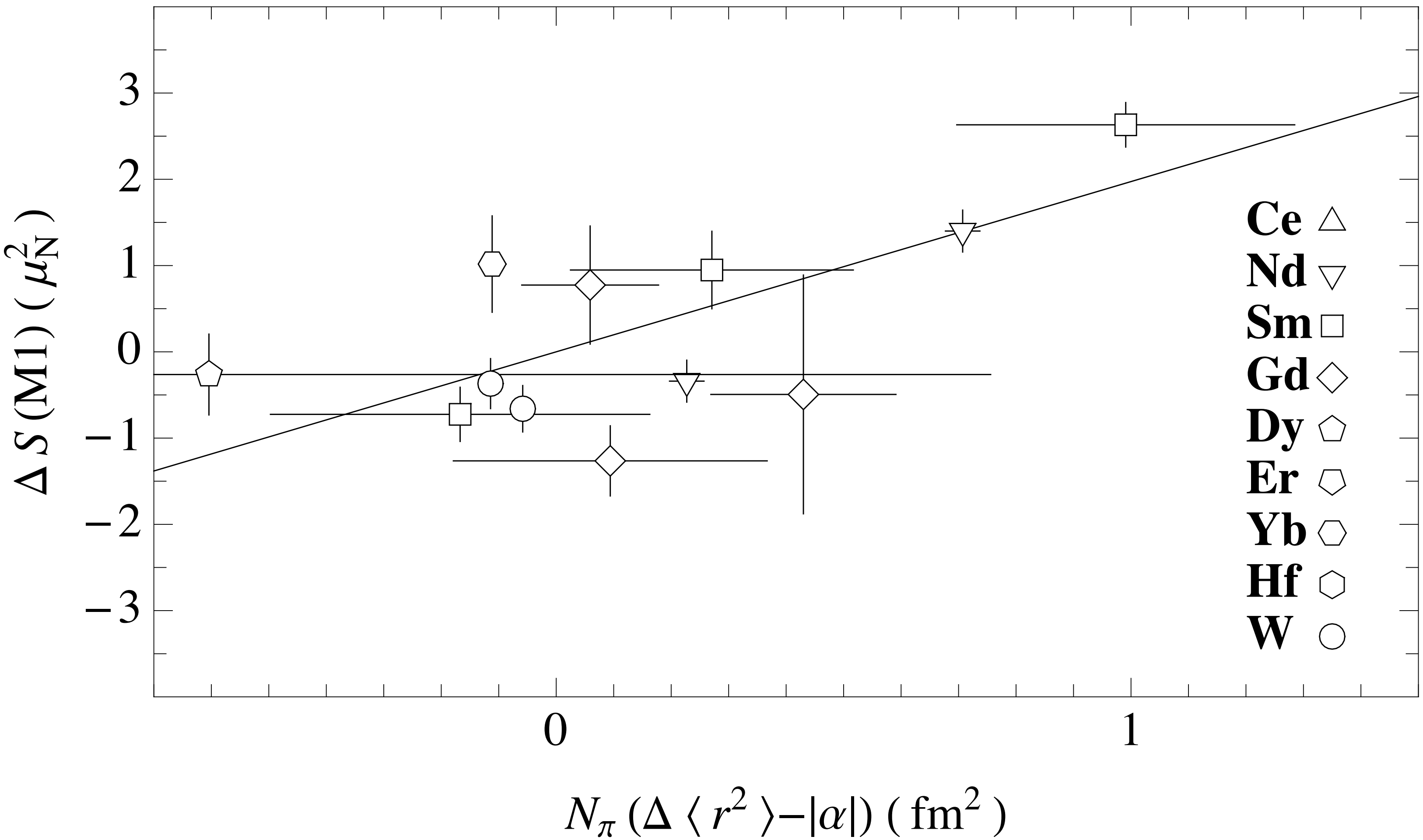
We obtain the relation

$$\Delta\tilde{S}(\mathrm{M1}) \equiv \tilde{S}(\mathrm{M1})^{(A+2)} - \tilde{S}(\mathrm{M1})^{(A)}$$

$$= \frac{9}{2\pi}\frac{\left(g_\nu - g_\pi\right)^2}{\eta} N_\pi\left(\Delta\left\langle r^2\right\rangle - |\alpha|\right)$$

K. Heyde *et al.*, Phys. Lett. B **312** (1993) 267

Correlation $\Delta S(M1) - \Delta \langle r^2 \rangle$



Correlation S(M1)- ρ (E0)

In well-deformed nuclei [SU(3)]:

$$\langle 0_1^+ | \hat{n}_d | 0_1^+ \rangle = \frac{4N_\mathrm{b}(N_\mathrm{b}-1)}{3(2N_\mathrm{b}-1)}$$

$$\left|\langle 0_\beta^+ | \hat{n}_d | 0_1^+ \rangle\right| = \left[\frac{8(N_\mathrm{b}-1)^2 N_\mathrm{b}(2N_\mathrm{b}+1)}{9(2N_\mathrm{b}-3)(2N_\mathrm{b}-1)^2}\right]^{1/2}$$

We obtain the relation (for large N_b)

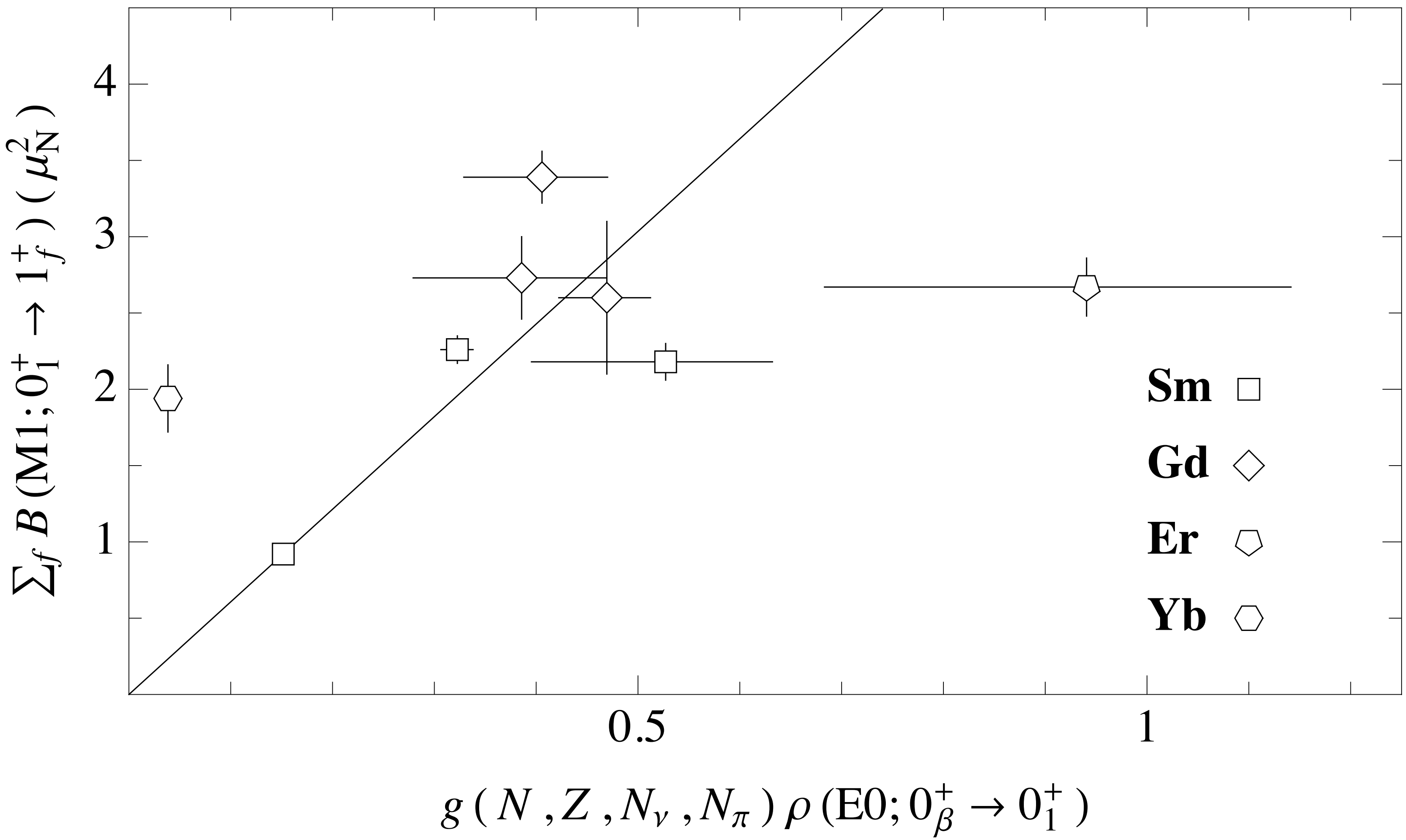
$$B(\mathrm{M1};0_1^+ \to 1_1^+) \approx \frac{9}{\pi}(g_\nu - g_\pi)^2 \frac{r_0^2}{\eta} g(N,Z,N_\nu,N_\pi)\rho(\mathrm{E0};0_\beta^+ \to 0_1^+)$$

with

$$g(N,Z,N_\nu,N_\pi) = \frac{e(N+Z)^{2/3}}{e_\mathrm{n}N + e_\mathrm{p}Z}\frac{N_\nu N_\pi}{\sqrt{2N_\mathrm{b}}}$$

P. Van Isacker, Nucl. Data Sheets **120** (2014) 119

Correlation S(M1)- ρ (E0)



Conclusions

Consistent treatment of charge radii and E0 transitions assuming the same effective charges.

An additional correlation exists with summed M1 strength, which can be related to charge radii and E0 transitions.

Outlook:

S(M1)-ρ(E0) correlation for transitional nuclei.

Need for $\Delta\langle r^2\rangle$ through shape transition.