

Topological Superfluid Phase with Repulsive Fermionic Atoms

Gerardo Ortiz

Department of Physics - Indiana University



QPTn-9 – May 24th 2018

**Is Landau theory the only possible framework
for (Quantum or Thermal) Phase Transitions ?**

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“There is life beyond the Landau paradigm”

– Anonymous

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Landau versus Topological orders

Landau orders

(Local) Order parameter

Broken (global) Symmetry

Topological orders

**String (non-local) Order /
Topological Invariants**

**Robustness against
local perturbations/
Bulk-boundary correspondence**

The Nobel Prize in Physics 2016



The Nobel Prize in Physics 2016



David J. Thouless
Prize share: 1/2



F. Duncan M. Haldane
Prize share: 1/4



J. Michael Kosterlitz
Prize share: 1/4

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**“For theoretical discoveries of topological phase transitions
and topological phases of matter”**

- Most of the work on Topological Quantum Matter is based on the Mean-Field quasi-particle picture
 - Paired Superfluids are particle-number non-conserving
 - Zero-energy modes are Majorana fermions (MFs) by default
 - MFs are blueprints for topological quantum computation

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 - Zero-energy modes are Majorana fermions (MFs) by default
 - MFs are blueprints for topological quantum computation
- But real “closed systems” are interacting particle-number conserving (pnc) systems
 - What is a topological superfluid in a pnc system?
 - What is a Majorana fermion in a pnc system?
 - How one detects Majorana fermions?
 - Can one braid MFs in pnc systems?

Some Answers

PRL 113, 267002 (2014); Ann.Phys. 372, 357 (2016); PRB 95, 201114(RC) (2017)

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What is a Particle-Conserving Topological Superfluid?

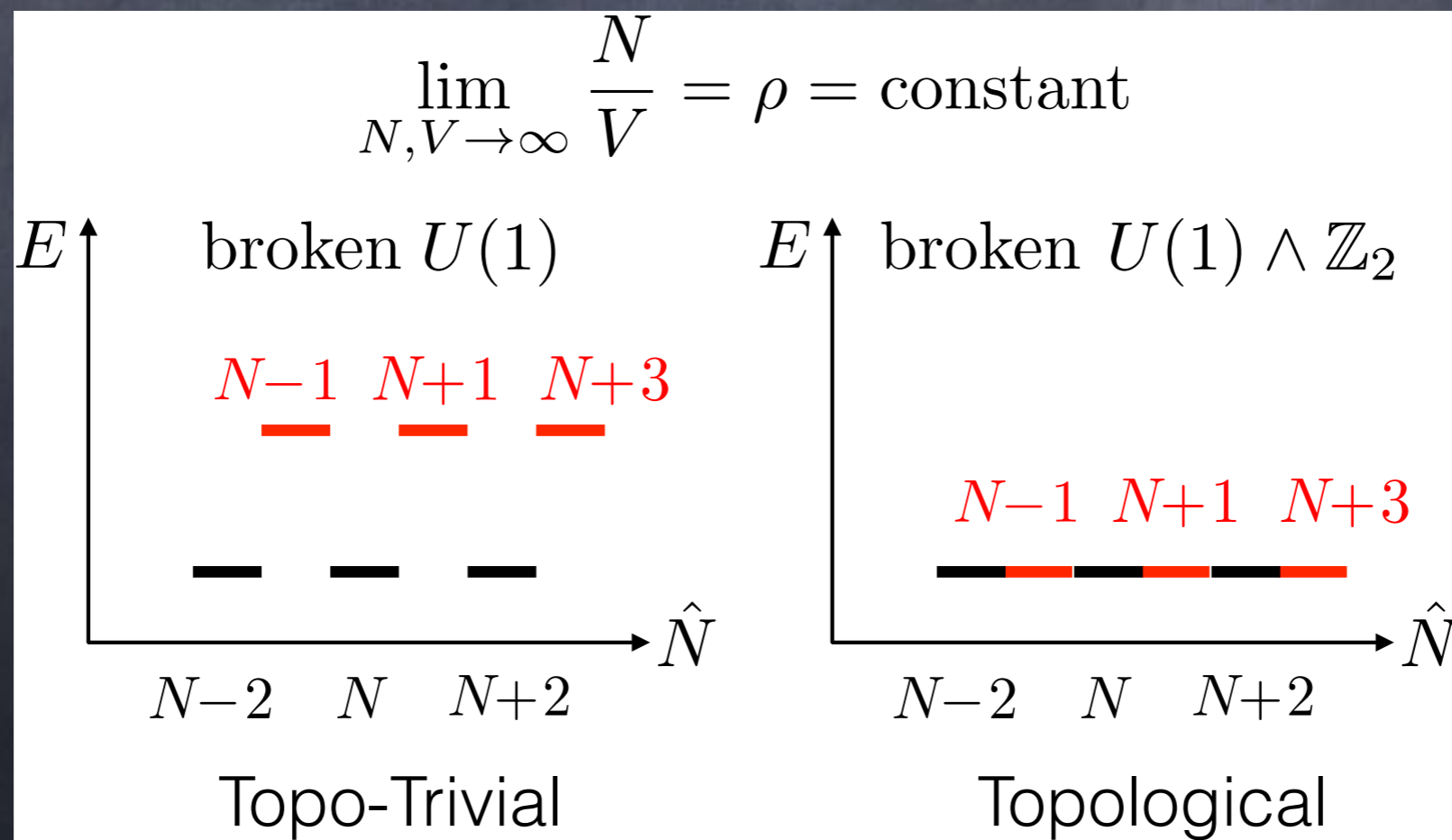
- Characterization of Topological Superfluidity in generic interacting many-body systems: **Fermion Parity Switches**

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 - Coherent superpositions of states with different # of particles
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Can one prepare/manipulate/braid **coherent** superpositions of states with a different number of particles?

INTERVIEW 8 May 2013

Nothing to see: The man who made a Majorana particle

By Lisa Grossman

Majorana in the News

Physicist Leo Kouwenhoven ended a 75-year hunt for the tricky Majorana fermion – a particle that is its own antiparticle – by creating one on a chip

The New York Times | <http://nyti.ms/1mfRwAV>

TECHNOLOGY

Microsoft Makes Bet Quantum Computing Is Next Breakthrough

By JOHN MARKOFF JUNE 23, 2014

That may change soon. The company has been spending heavily and is contributing to 10 of the roughly 20 academic research groups exploring a long-hypothesized class of subatomic particles known as Majorana fermions. Beyond being a scientific advance, proving the existence of the Majorana would mean that it was likely they could be used to form qubits for this new form of quantum computing.

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News > Science

'Angel particle' which is both matter and anti-matter discovered in 'landmark' quantum physics breakthrough

Scientists say they have found the first evidence that 'Majorana fermions' exist, 80 years after they were first suggested

Ian Johnston Science Correspondent | @montaukian | Thursday 20 July 2017 18:41 BST | 98 comments



Two angels painted in Byzantium between 395 and 1453AD Public domain image/St Catherine's Monastery, Mount Sinai

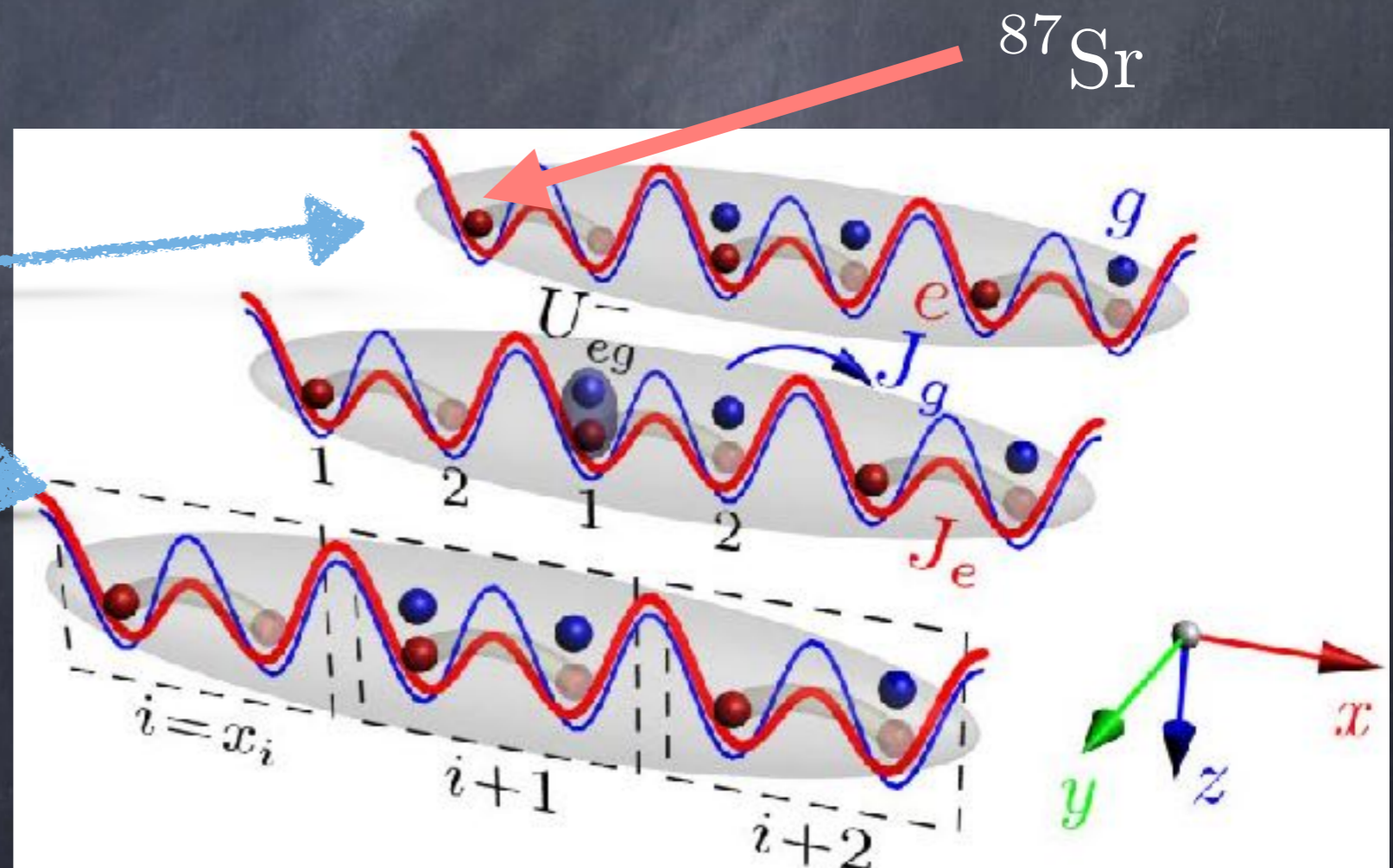
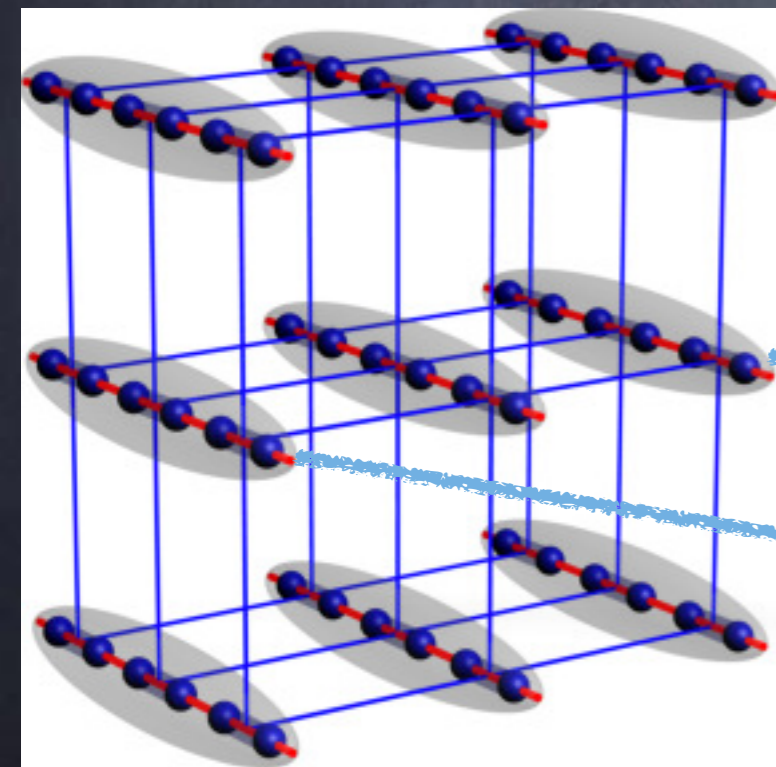
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- Engineering a number-conserving Topological Superfluid (TS) in Optical Lattices with repulsive fermionic atoms
 - New physical mechanism for emergent superfluidity
 - New magneto-electric phenomenon to detect TSs

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Collaborators:

Leonid Isaev: JILA, University of Colorado - Boulder



Ana Maria Rey: JILA, University of Colorado - Boulder



Adam Kaufman: JILA, University of Colorado - Boulder



Mostly: PRL 113, 267002 (2014)
and arXiv:1710.02768

Ann.Phys. 372, 357 (2016)

Outline

Motivation

- What and Why Topological Superfluidity?

Repulsive Superfluidity in Optical Lattices

- Fermion Pairing from Repulsion
- Attraction from Local Fluctuations: New Mechanism
- Topological Superfluid State in a quasi-1D Lattice
- Topological Superfluidity in 2-D

Developing Probes in AMO Experiments

- Probing Topological Superfluidity

Attraction from Local Fluctuations

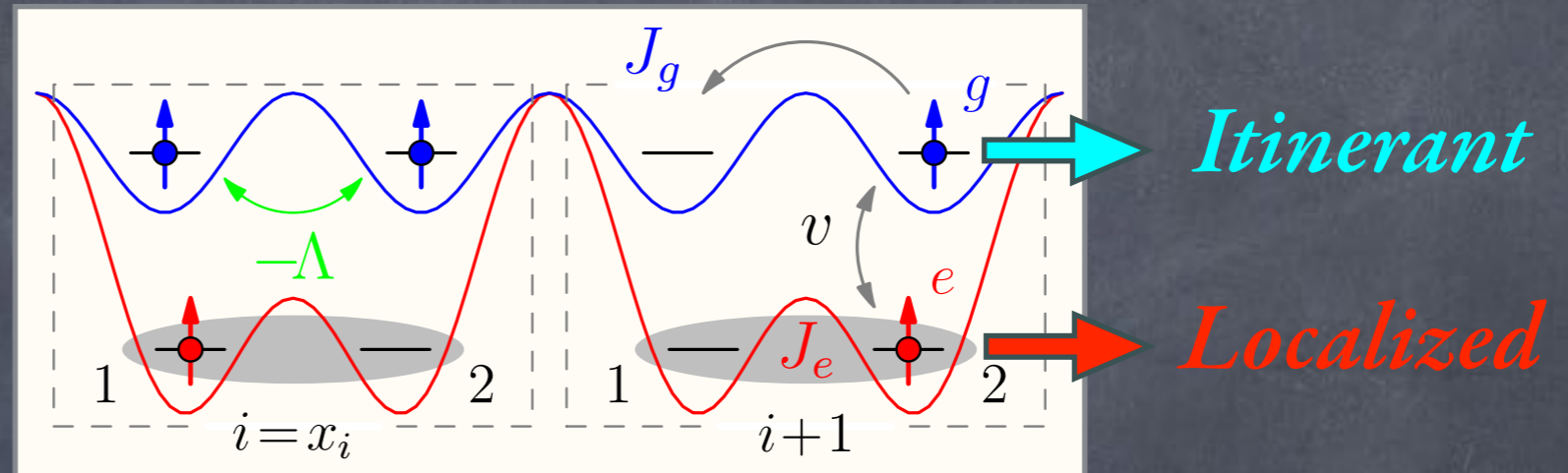
(Clock-State-Dependent Optical Superlattice)

Attraction from Local Fluctuations

(Clock-State-Dependent Optical Superlattice)

► Main ingredients

- Optical superlattice
- Spin-orbit coupling

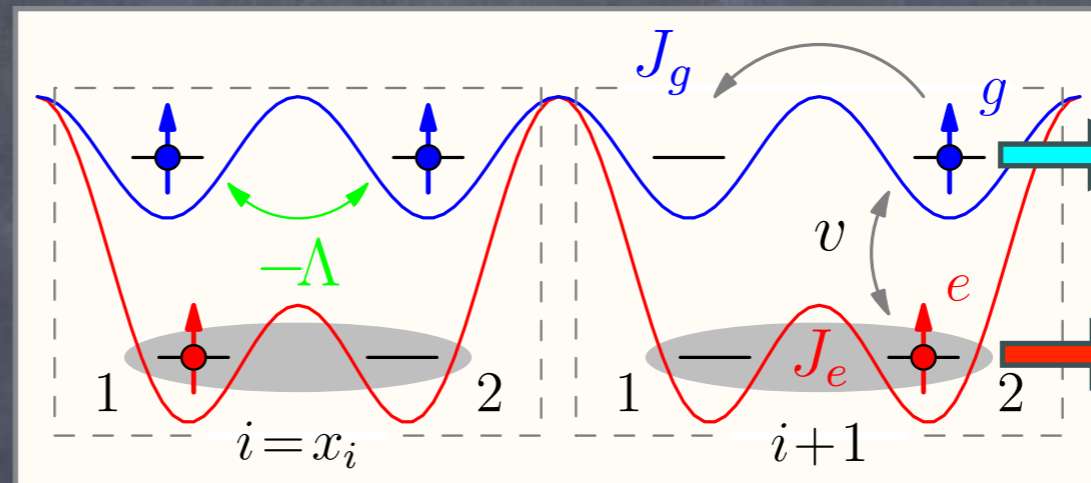


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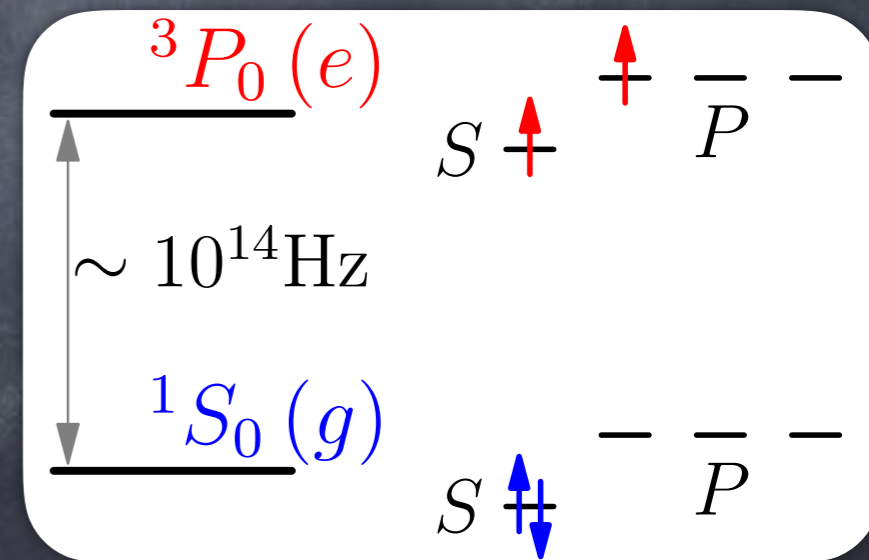
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► Alkaline-earth atoms ^{171}Yb , ^{173}Yb , ^{87}Sr

- Electronic states: 1S_0 , 3P_0 with $J = 0$
- Many nuclear spin states
- Nuclear spins decoupled from electrons

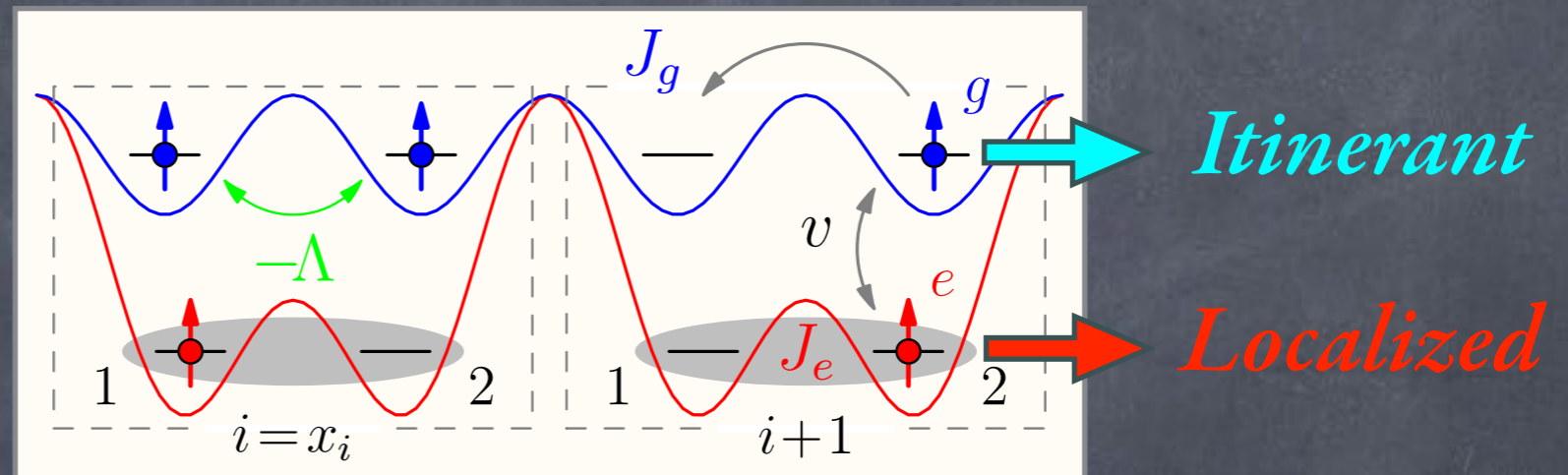


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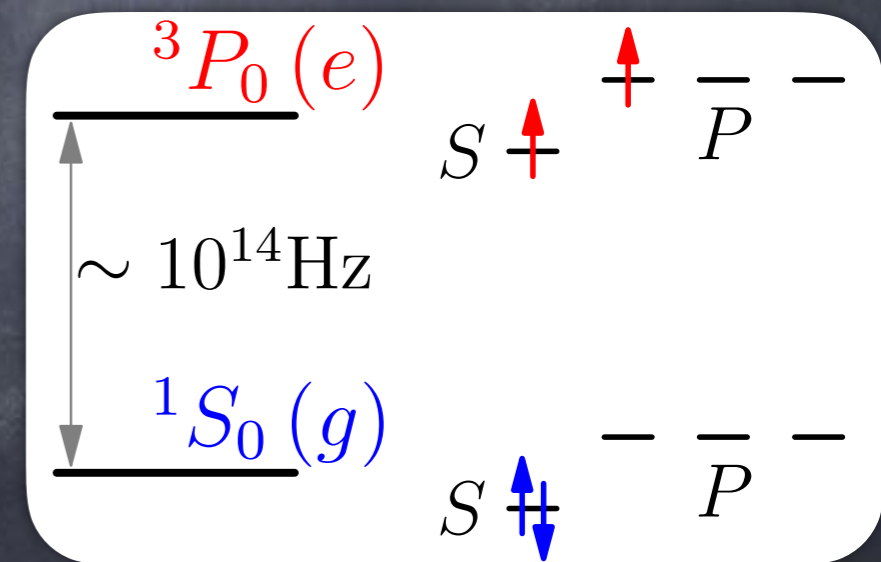
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► New Mechanism of Pairing: Engineer Subsystem Mediator

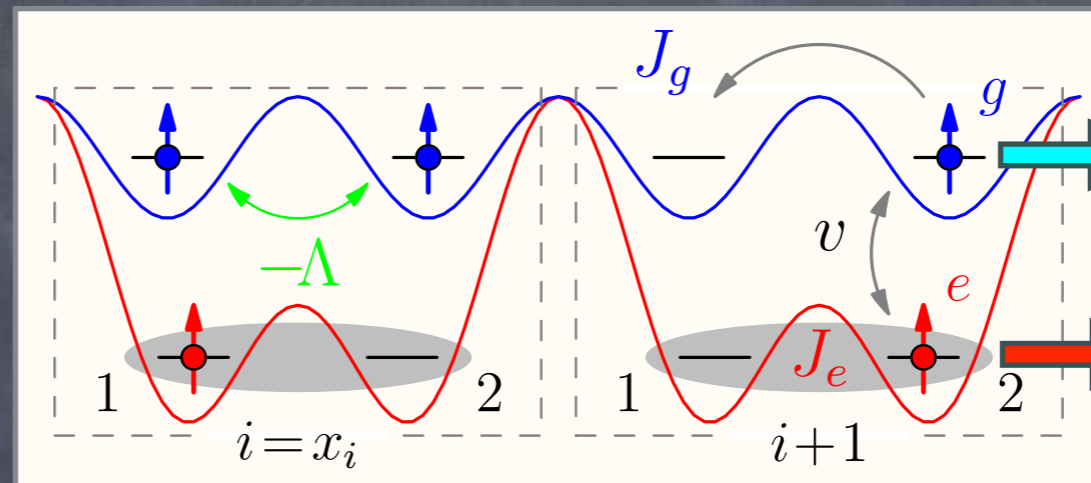
- Interplay between atomic repulsion and kinetic energy

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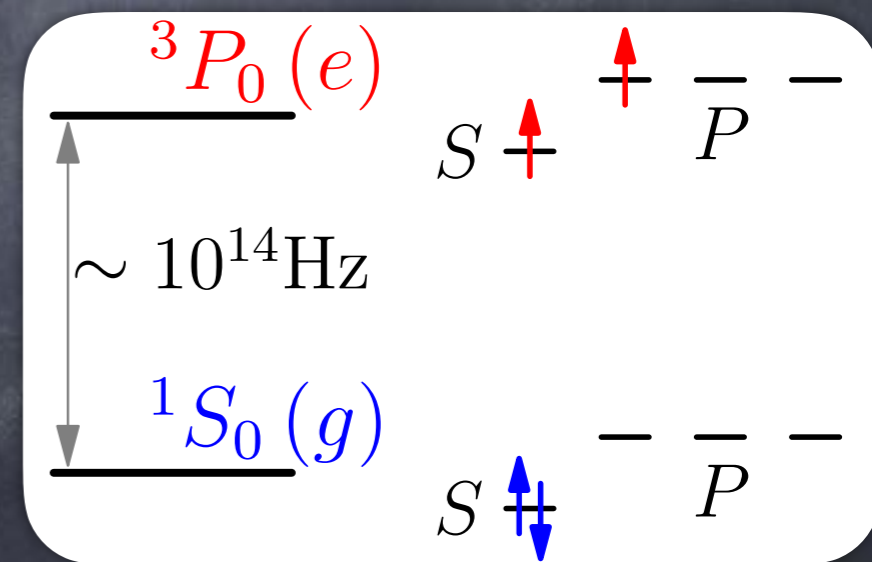
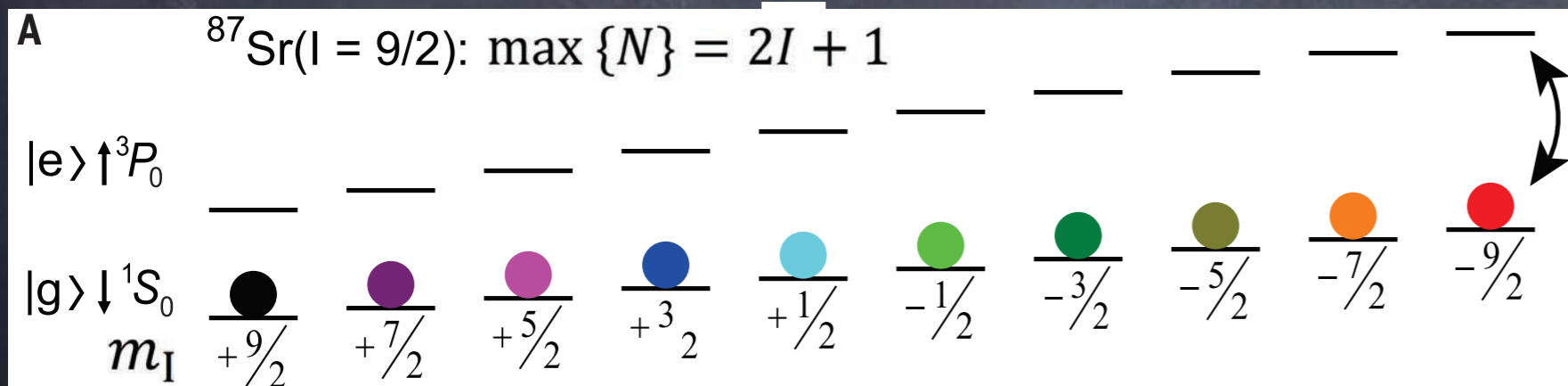
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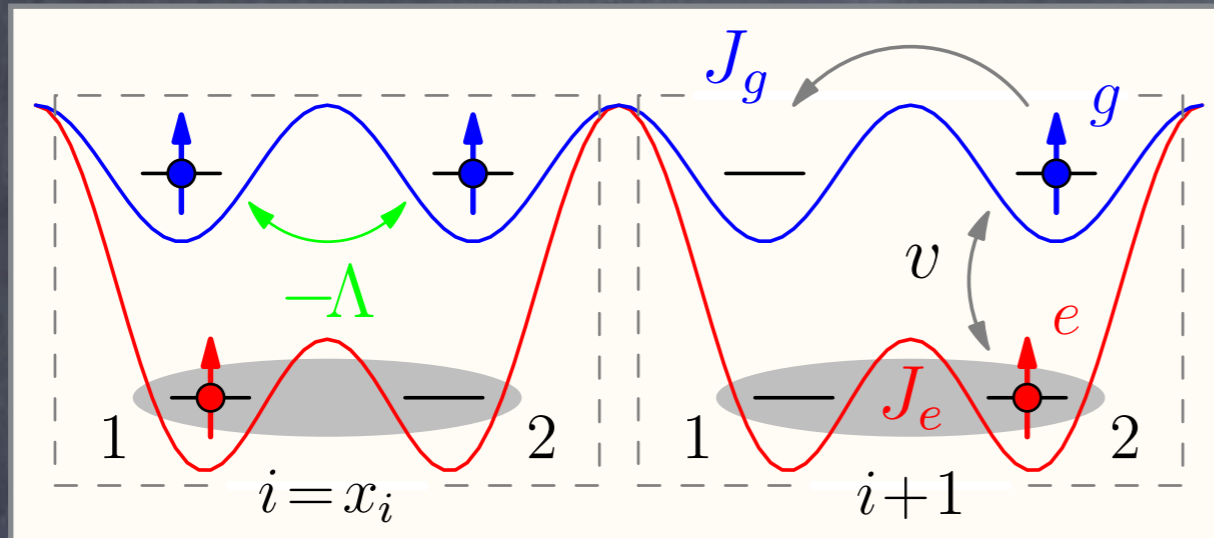
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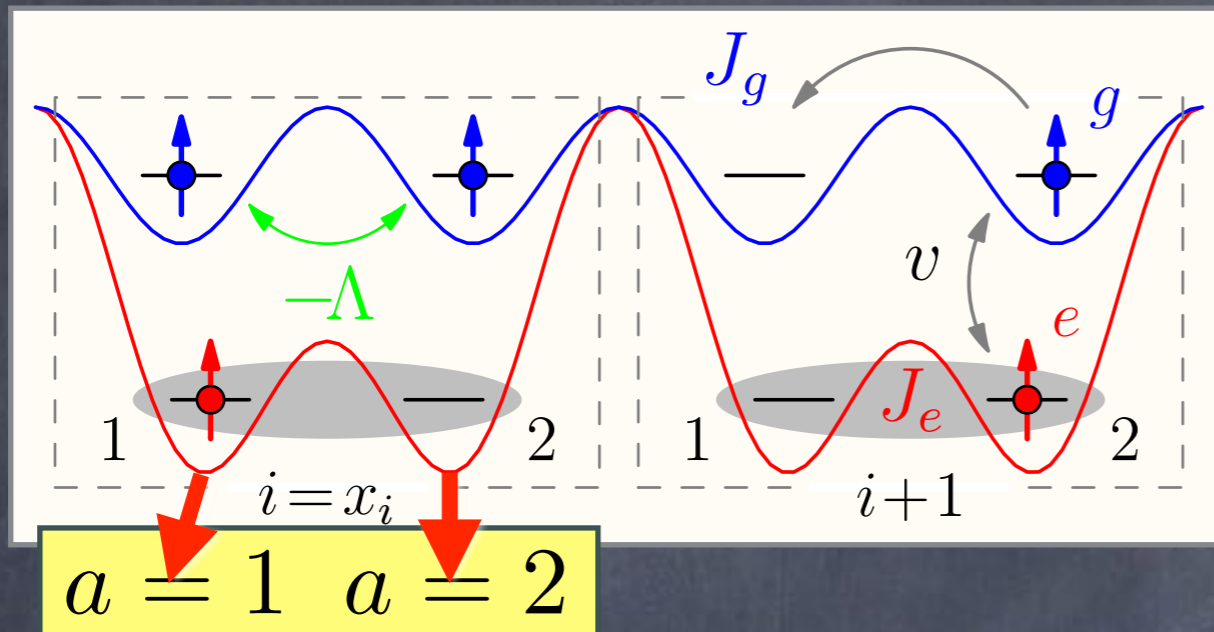
Superfluid State in a quasi-1D Lattice



- Mobile g -atoms
- e -atoms: 1 atom/dimer
- $i = 1 \dots N_d$, $v \sim a_{eg}^-$
- $m, n = \text{Nuclear spins}$

$$\begin{aligned}
 H = & -J_e \sum_{in} (e_{i,1n}^\dagger e_{i,2n} + \text{h.c.}) + U_{gg} \sum_{ia} n_{ia}^g (n_{ia}^g - 1) - \\
 & -J_g \sum_{in} (g_{i,1n}^\dagger g_{i,2n} + g_{i+1,1n}^\dagger g_{i,2n} + \text{h.c.}) + \\
 & + U_{eg} \sum_{ia} n_{ia}^e n_{ia}^g + V_{\text{ex}} \sum_{iamn} e_{i,an}^\dagger e_{i,am} g_{i,am}^\dagger g_{i,an}
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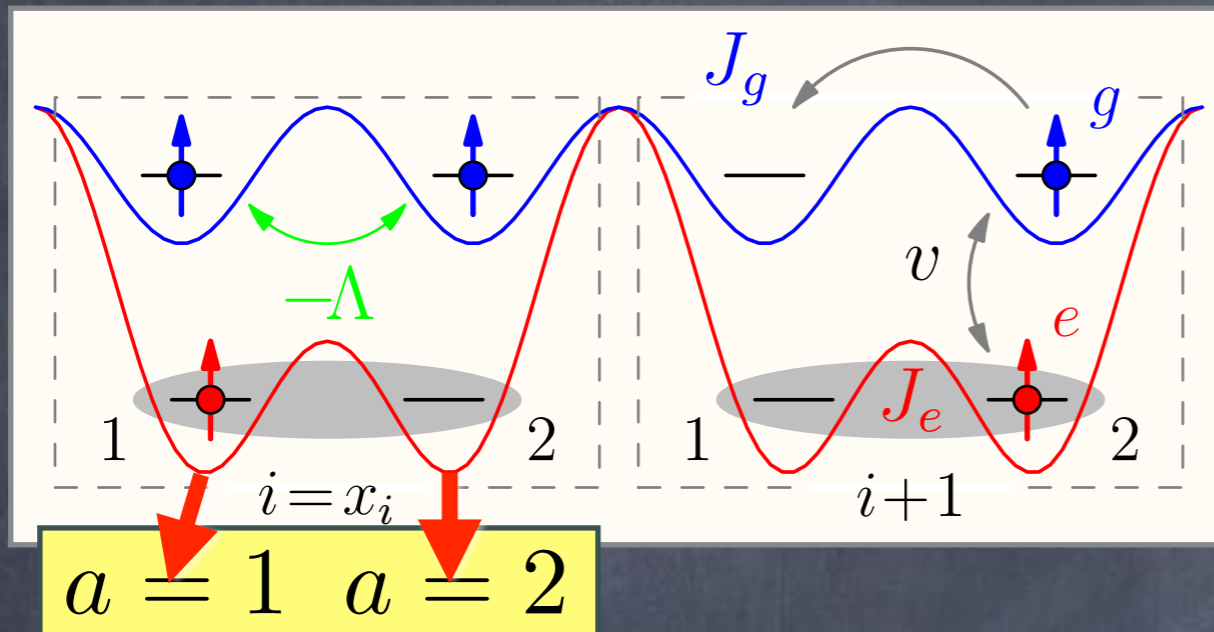
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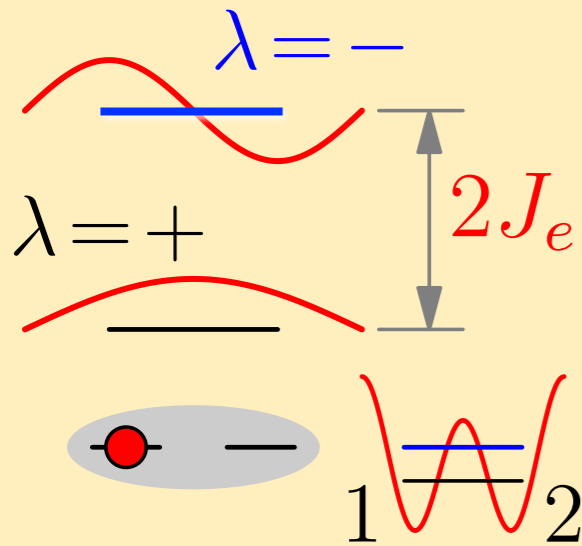
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 \end{aligned}$$

$$\underbrace{(U_{eg} + V_{\text{ex}})}_v \sum_i n_{ia\uparrow}^e n_{ia\uparrow}^g$$

Nuclear spin-polarized

Emergent Fermion pairing



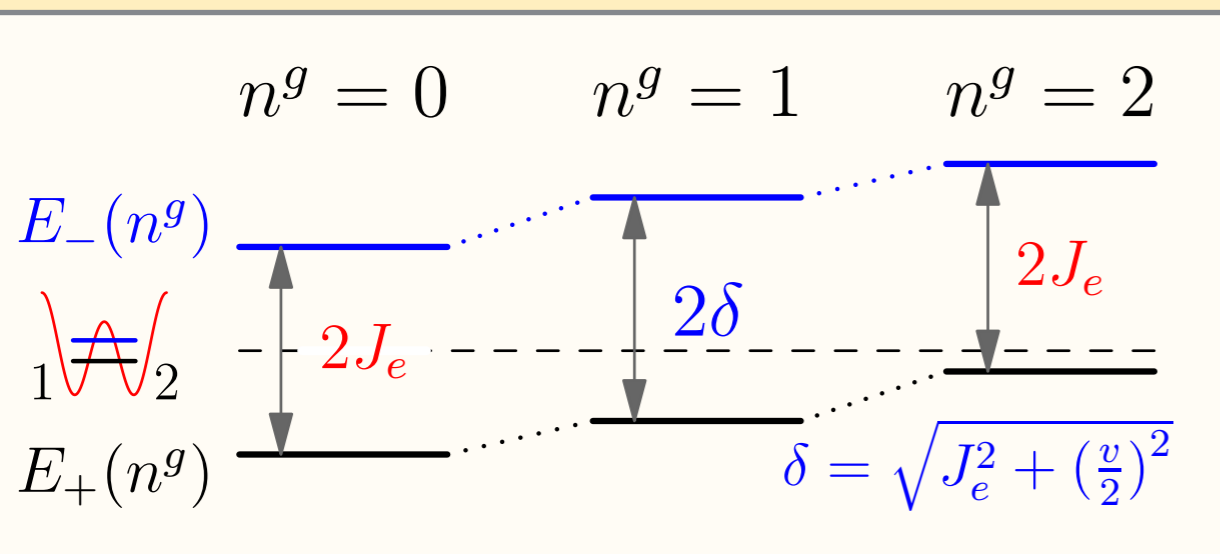
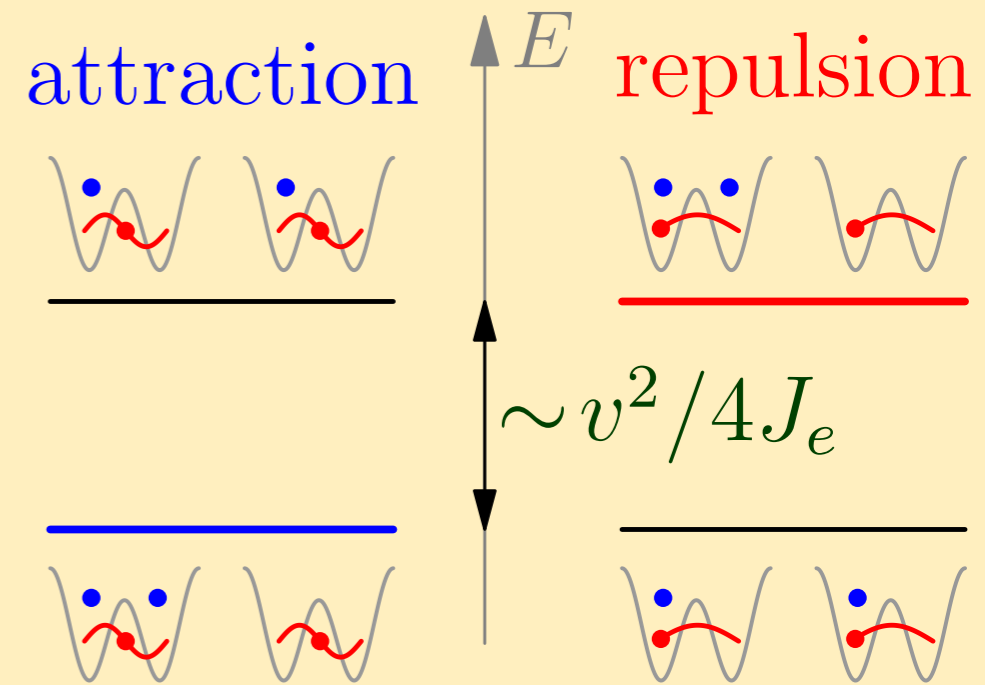
▶ *e*-atom quantum fluctuations

- Consider $J_e \gg J_g, v$
- Preparing *e*-atoms in a given motional state

▶ Local Pairing: $\Delta_{\pm} \approx \pm v^2 / 4J_e = \pm \Lambda$

- sym \rightarrow **repulsion** of *g*-atoms
- antisym \rightarrow **attraction** of *g*-atoms

- *e-g* interaction: $H_{eg} = v(n_1^e n_1^g + n_2^e n_2^g)$



- $\Delta_+ = E_+(2) + E_+(0) - 2E_+(1) \approx v^2 / 4J_e$
- $\Delta_- = E_-(2) + E_-(0) - 2E_-(1) \approx -v^2 / 4J_e$

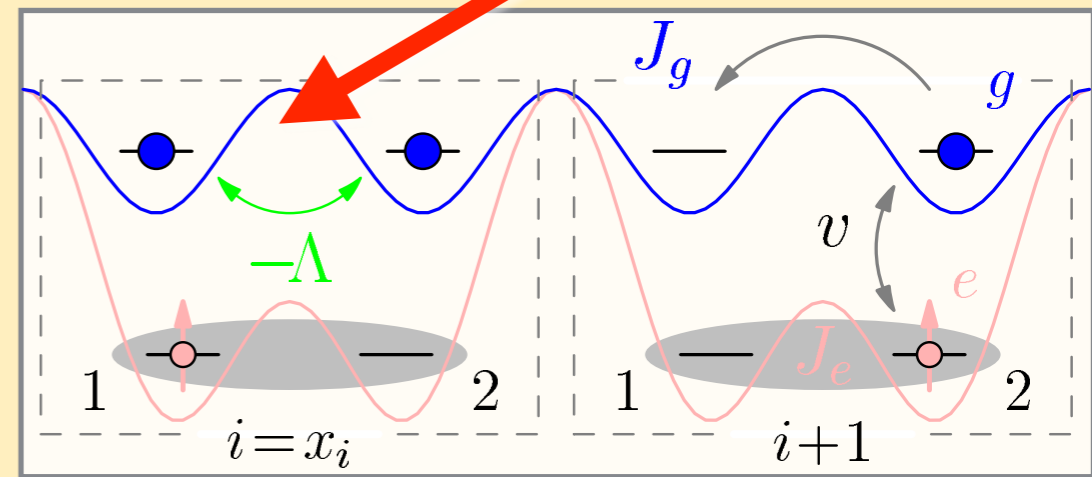
Effective Model (Schrieffer-Wolff Transformation)

$$H_{\text{ef}} = -J_g \sum_k (g_{k1}^\dagger \ g_{k2}^\dagger) \underbrace{\begin{pmatrix} 0 & 1 + e^{-ik} \\ 1 + e^{ik} & 0 \end{pmatrix}} \begin{pmatrix} g_{k1} \\ g_{k2} \end{pmatrix} - \frac{v^2}{4J_e} \sum_i n_{i,1}^g n_{i,2}^g$$

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- Attraction is intra-dimer

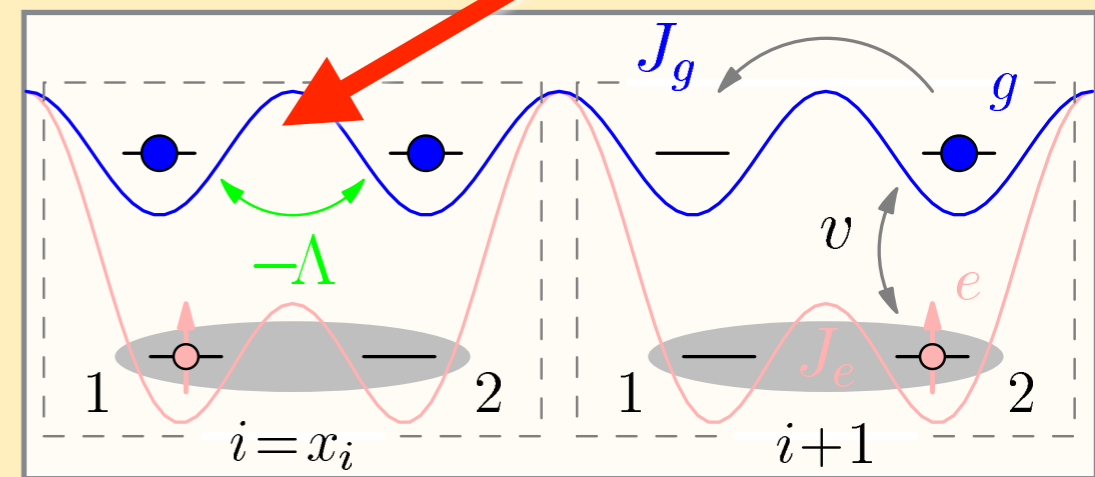


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$$H_{\text{SOI}} = (1 + \cos k)\sigma^x + \sin k \sigma^y$$

- Attraction is intra-dimer
 - Effective **spin-orbit interaction**
- due to doubling of unit cell



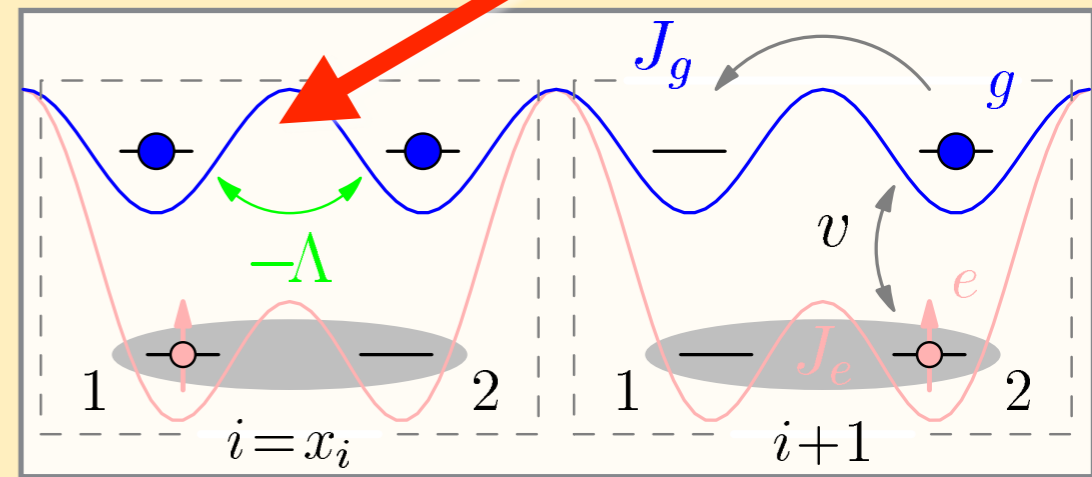
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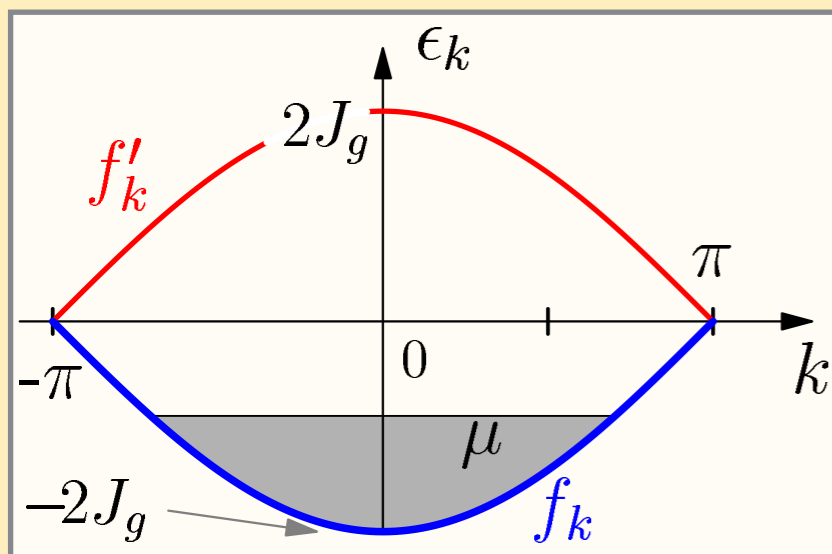
► Non-interacting band-structure

- Quasiparticle states

$$\begin{pmatrix} g_{k1} \\ g_{k2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} f_k + f'_k \\ \frac{1+e^{ik}}{\sqrt{2(1+\cos k)}} [f_k - f'_k] \end{pmatrix}$$

- Energies:

$$E_k = \pm J_g \sqrt{2(1 + \cos k)}$$



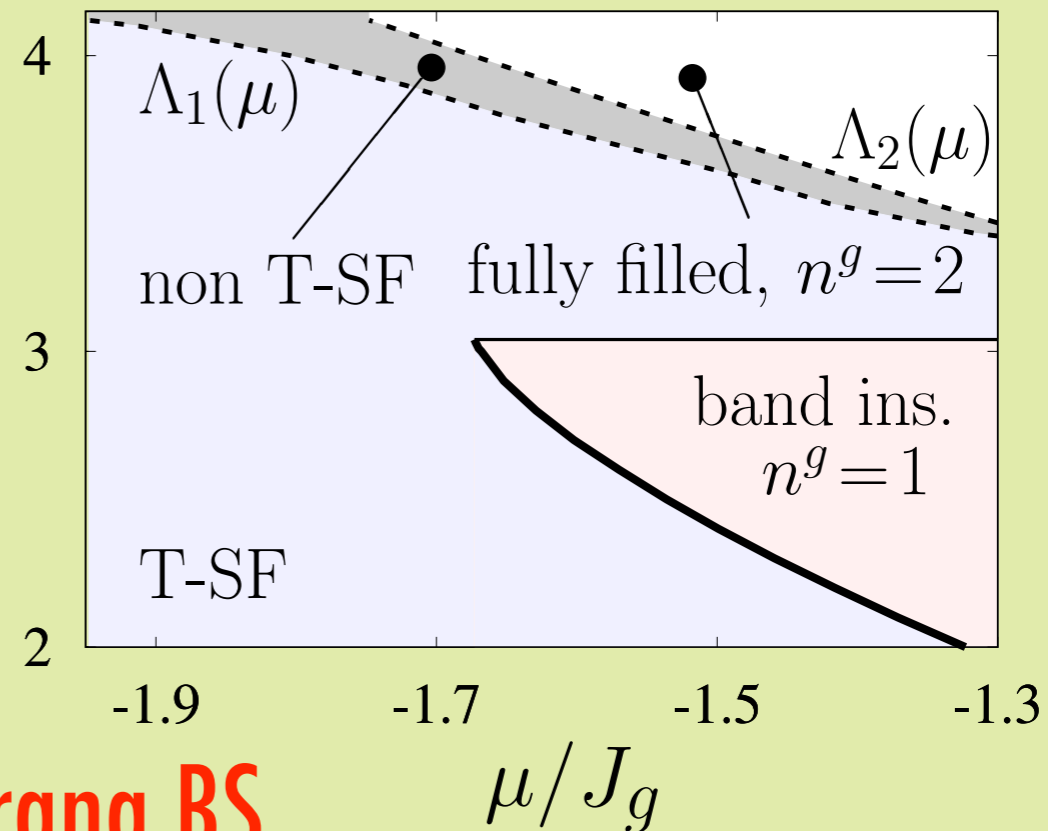
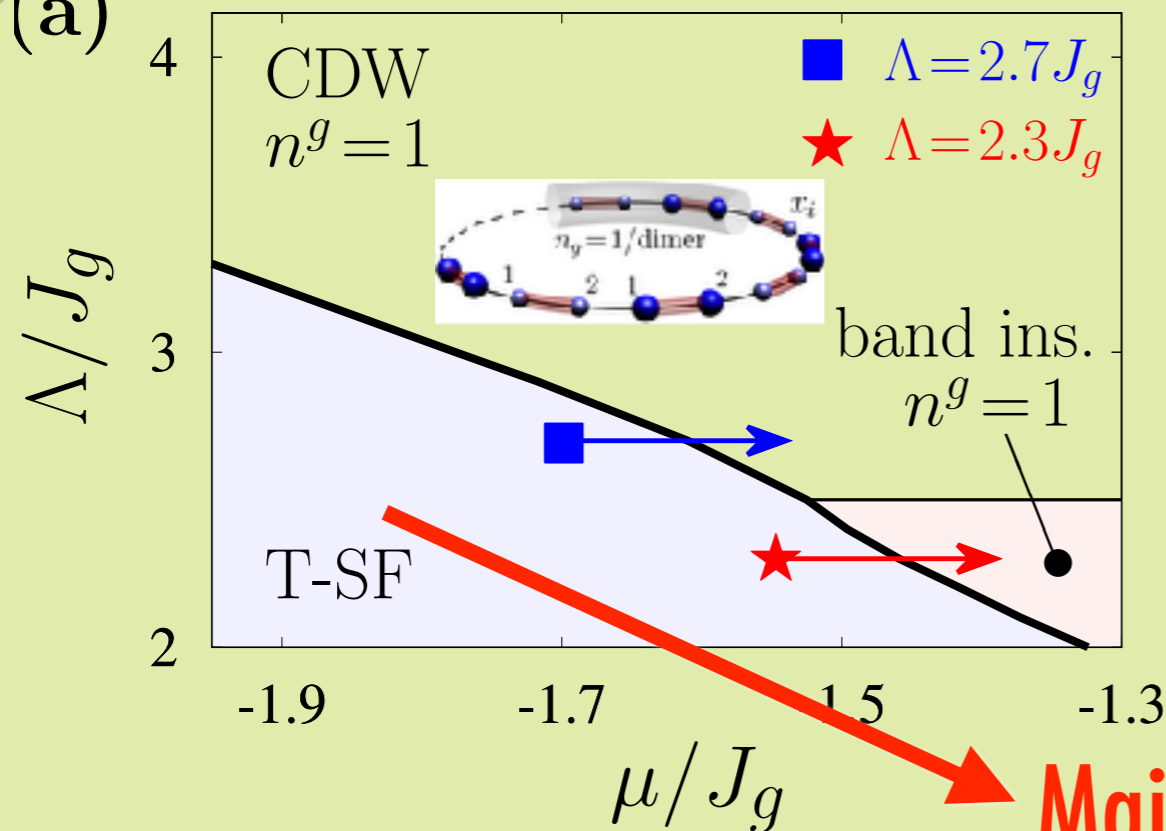
p-wave symmetry

Phase Diagram in quasi-1D

Unconstrained

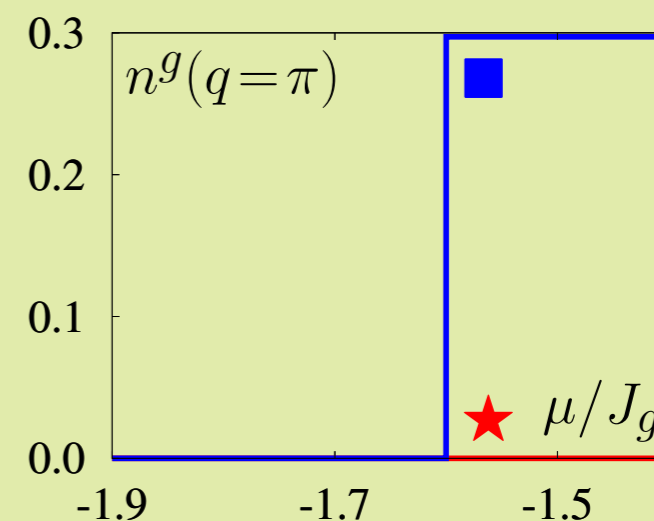
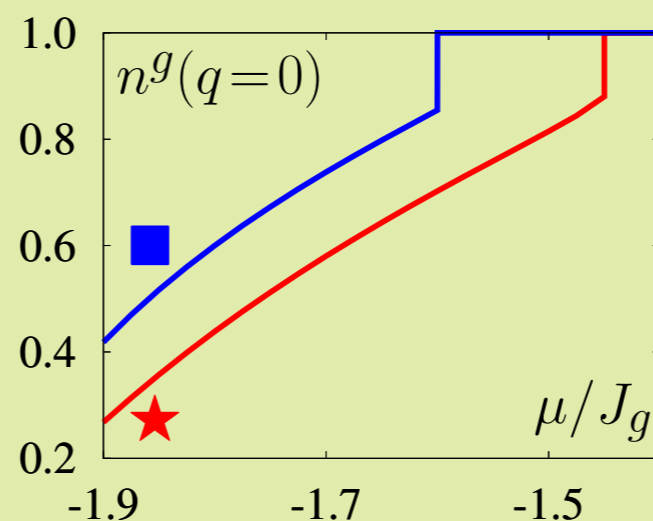
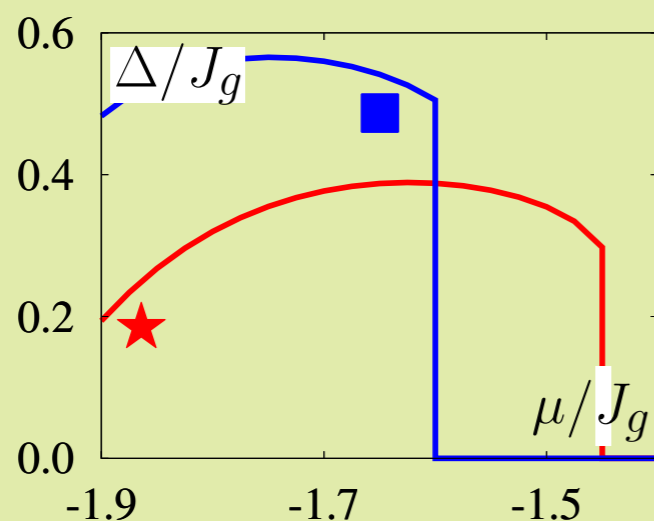
Single-site unit cell

(a)



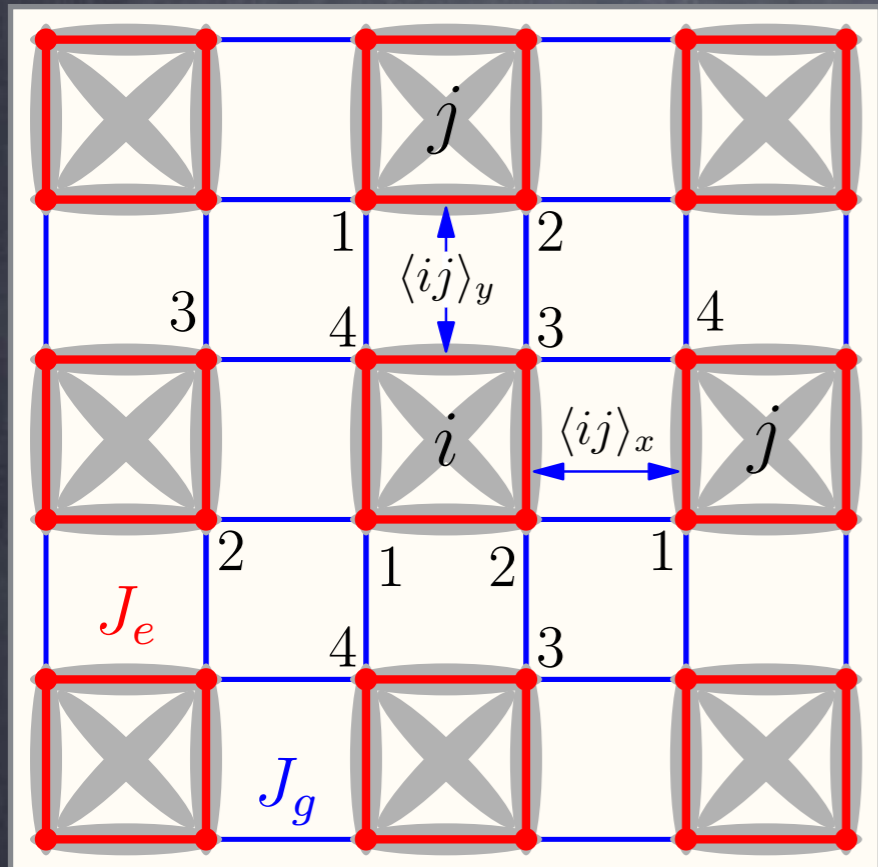
Majorana BS

(b)



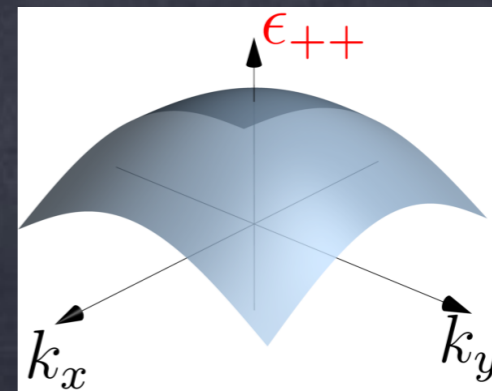
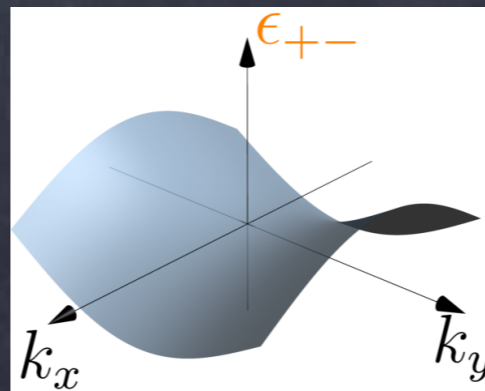
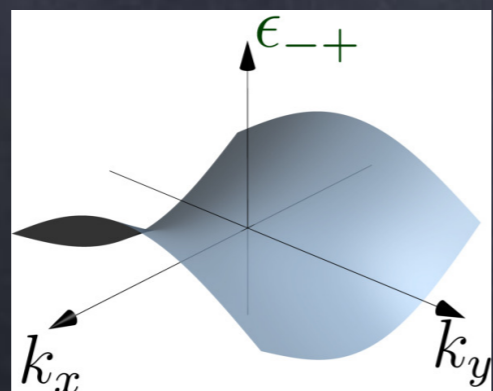
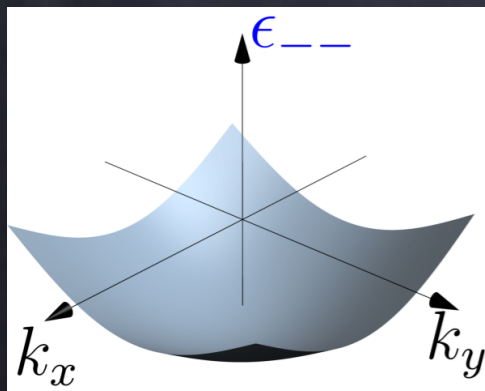
Topological Superfluidity in two-dimensions

$(p_x + ip_y)$ -wave symmetry



$$\begin{aligned}
 H = & - J_e \sum_{i, \square_{ab}} (e_{ia}^\dagger e_{ib} + \text{h.c.}) + v \sum_{ia} n_{ia}^e n_{ia}^g - \\
 & - J_g \sum_{i, \square_{ab}} (g_{ia}^\dagger g_{ib} + \text{h.c.}) - \\
 & - J_g \sum_{\langle ij \rangle_x} (g_{i,2}^\dagger g_{j,1} + g_{i,3}^\dagger g_{j,4} + \text{h.c.}) - \\
 & - J_g \sum_{\langle ij \rangle_y} (g_{i,4}^\dagger g_{j,1} + g_{i,3}^\dagger g_{j,2} + \text{h.c.})
 \end{aligned}$$

► g -atom dispersion: $\epsilon_{n_x n_y} = 2J_g \left(n_x \cos \frac{k_x}{2} + n_y \cos \frac{k_y}{2} \right)$, $n_{x,y} = \pm 1$



Outline

Motivation

- What and Why Topological Superfluidity?

Repulsive Superfluidity in Optical Lattices

- Fermion Pairing from Repulsion
- Attraction from Local Fluctuations: New Mechanism
- Topological Superfluid State in a quasi-1D Lattice
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Developing Probes in AMO Experiments (^{87}Sr)

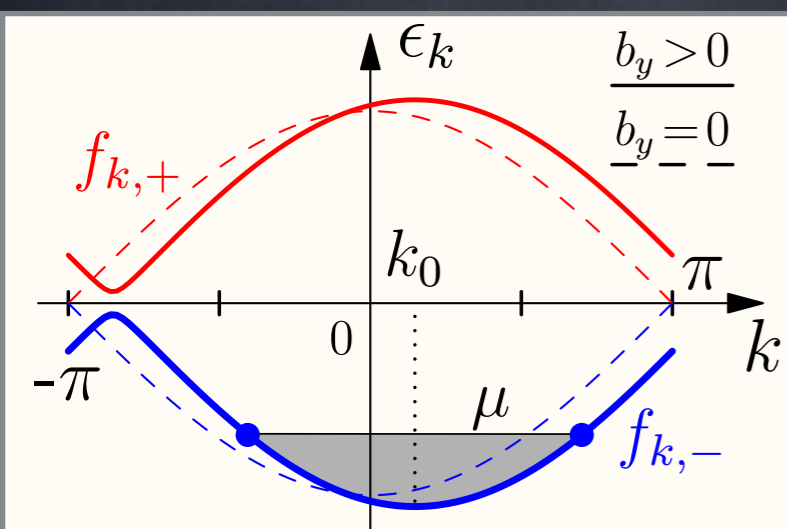
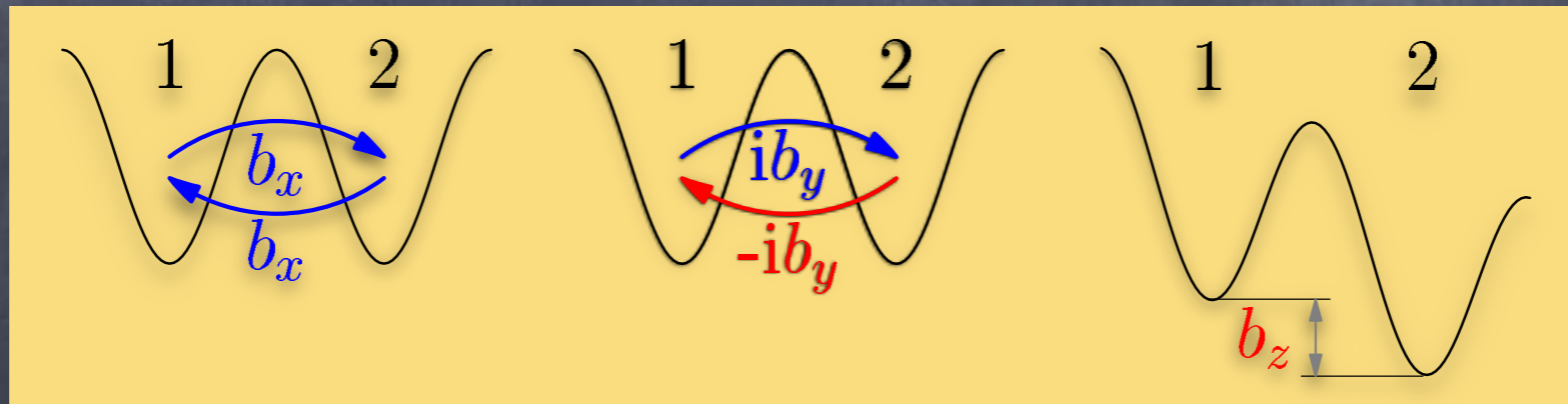
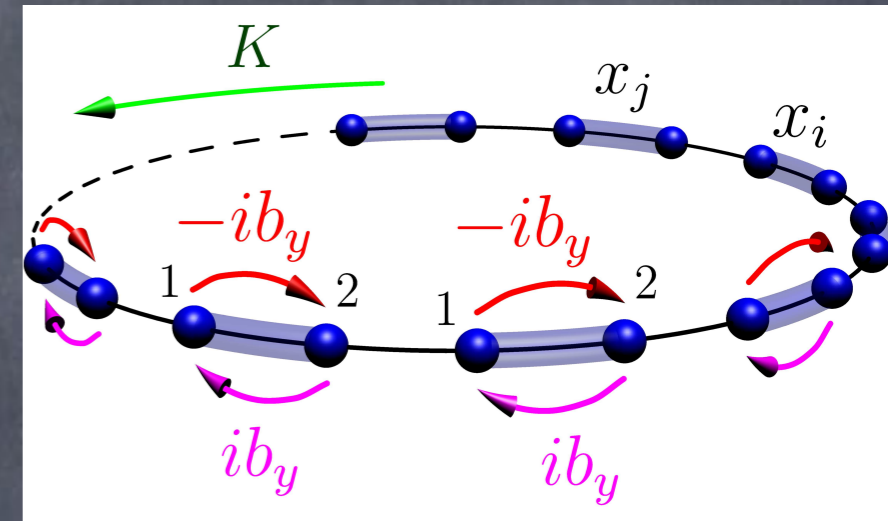
- Probing Topological Superfluidity

Synthetic **Magneto-Electric** Effect (Spin-Galvanic Effect)

► Laser-induced Magnetic Field

$$\delta H_M = - \sum_{i,a,b} (\mathbf{b} \cdot \boldsymbol{\sigma}_{ab}) g_{ia}^\dagger g_{ib}$$

- $b_x / b_y \leftrightarrow$ intra-dimer real/**complex hopping**
- $b_z =$ relative energy shift of dimer sites

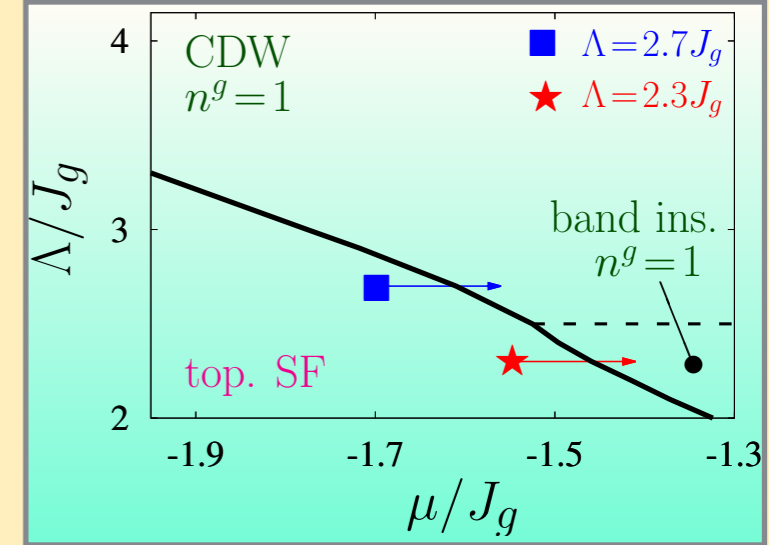
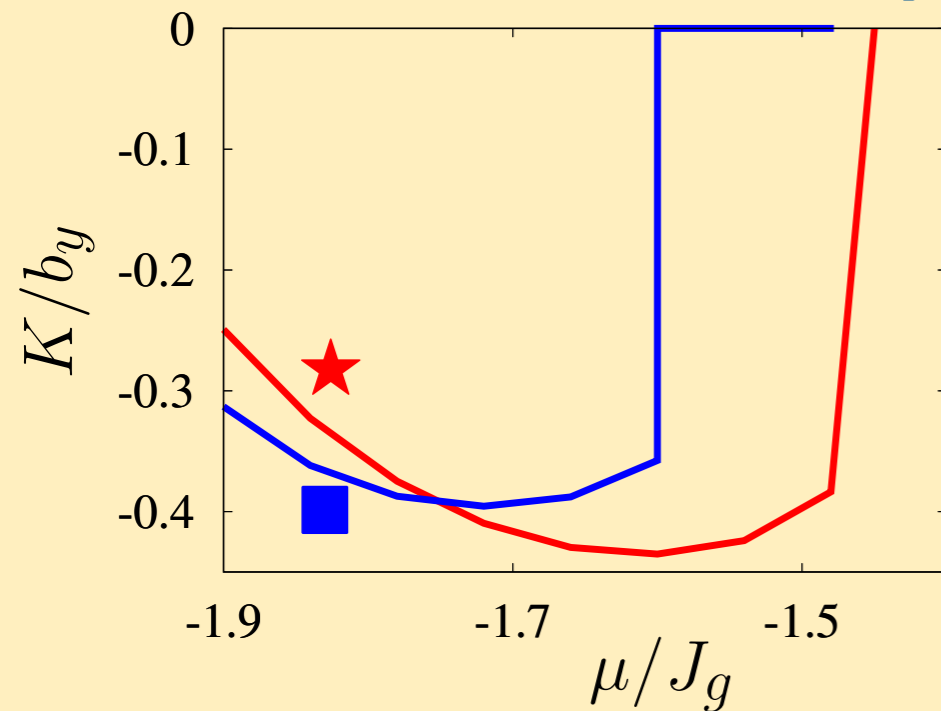


► Time-reversal Symmetry \mathcal{T}

- $g_{k,a} \rightarrow g_{-k,a}$ and a c-number $c \rightarrow c^*$
- Only b_y breaks \mathcal{T}
- We consider $\mathbf{b} = (0, b_y, 0)$

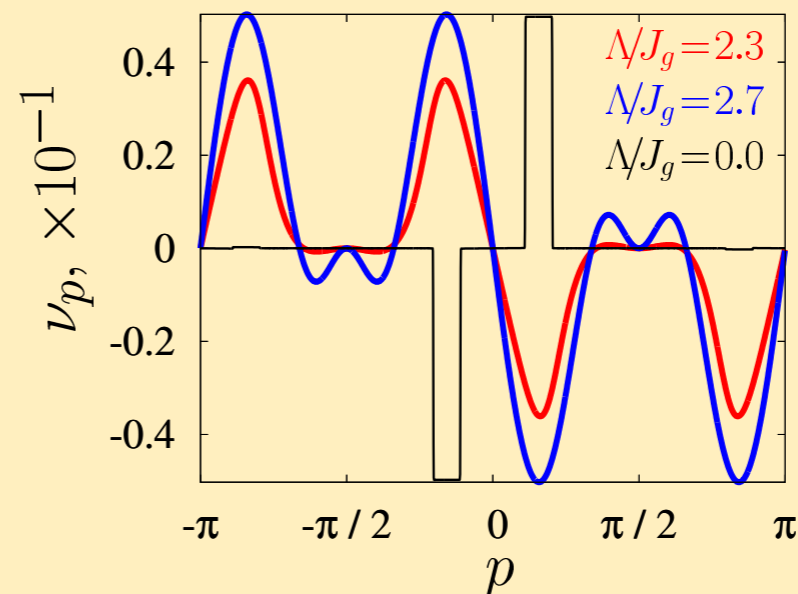
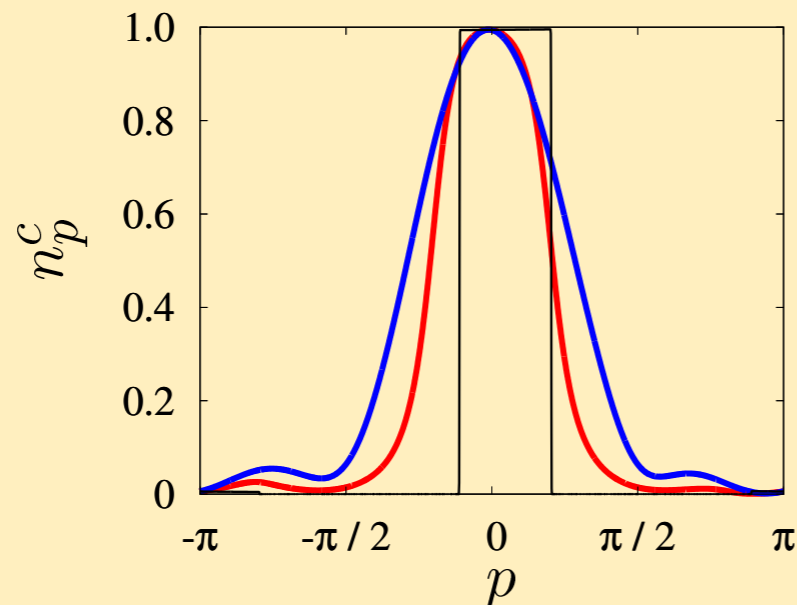
Unveiling the Superfluid State

Linear Mass Current Response



- $b_y \ll |\Delta|$ to avoid FFLO states
- Supercurrent K is a fingerprint of a SF

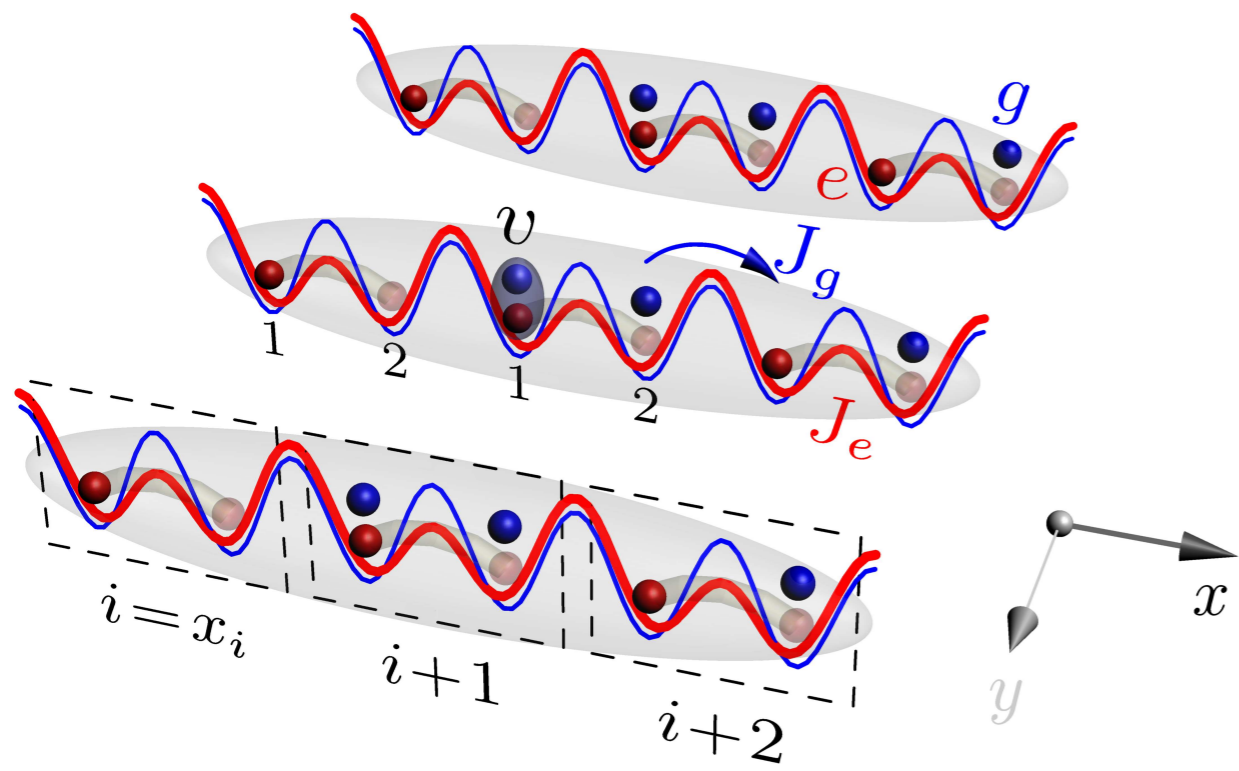
Momentum Distribution Asymmetry [$\nu_p = J_g(n_p^c - n_{-p}^c)/b_y$]



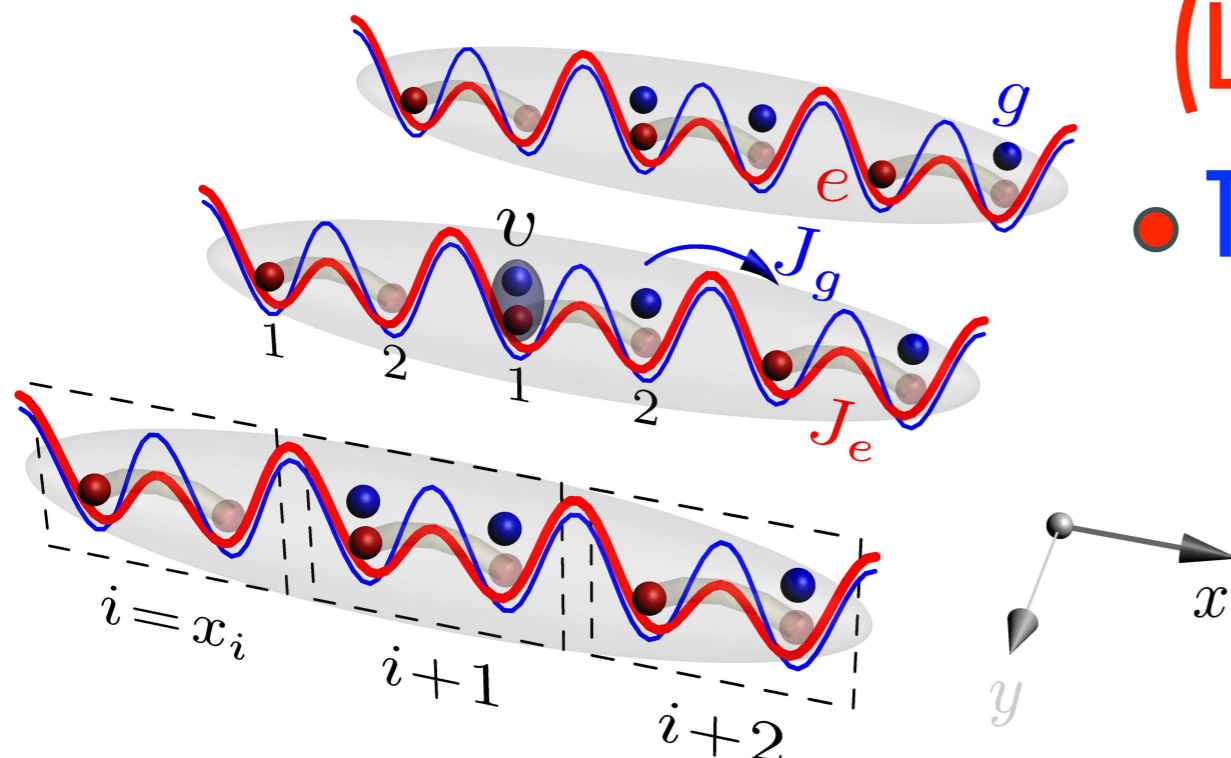
- Time-of-flight measurement

- Fingerprint of a SF

Conclusions

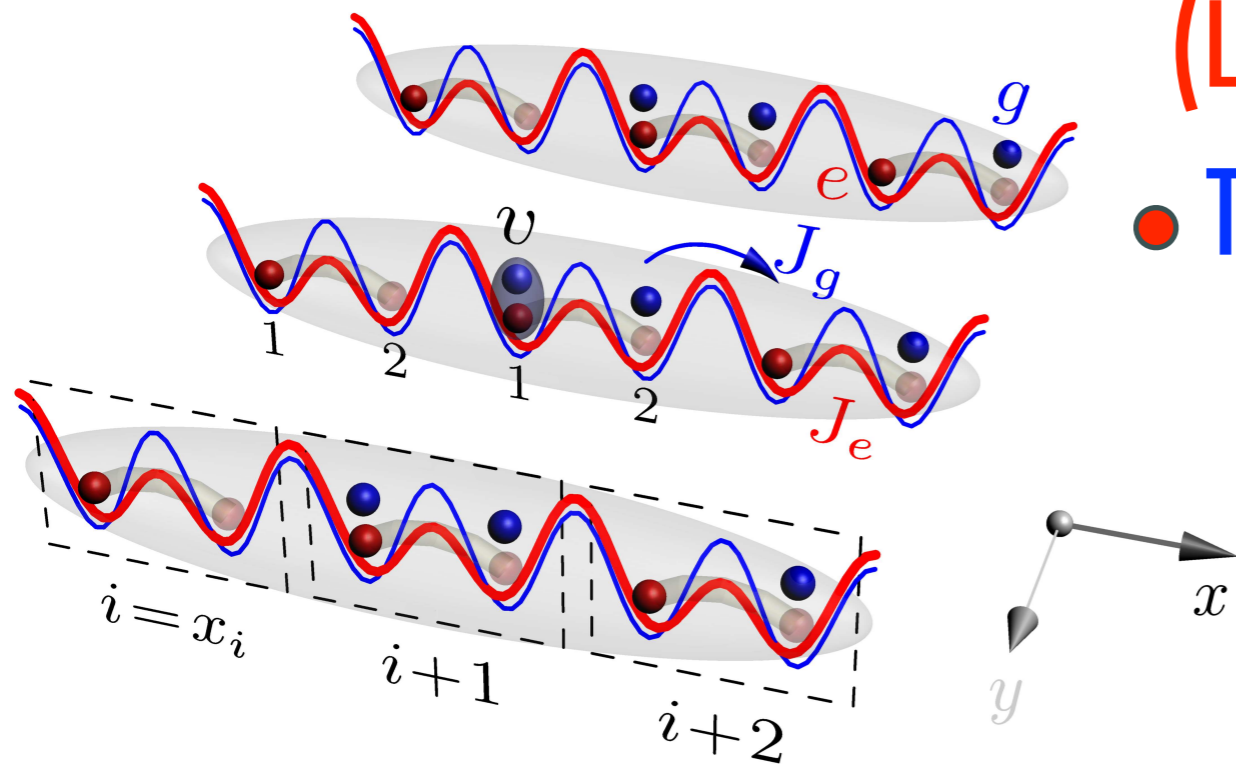


Conclusions



- **New Mechanism for Repulsive Fermionic SF**
(Local kinetic-energy Fluctuations)
- **Topological p-wave Superfluidity**
(Due to Emergent SOC)
- **New Magneto-Electric Effect**
and Experimental Probes

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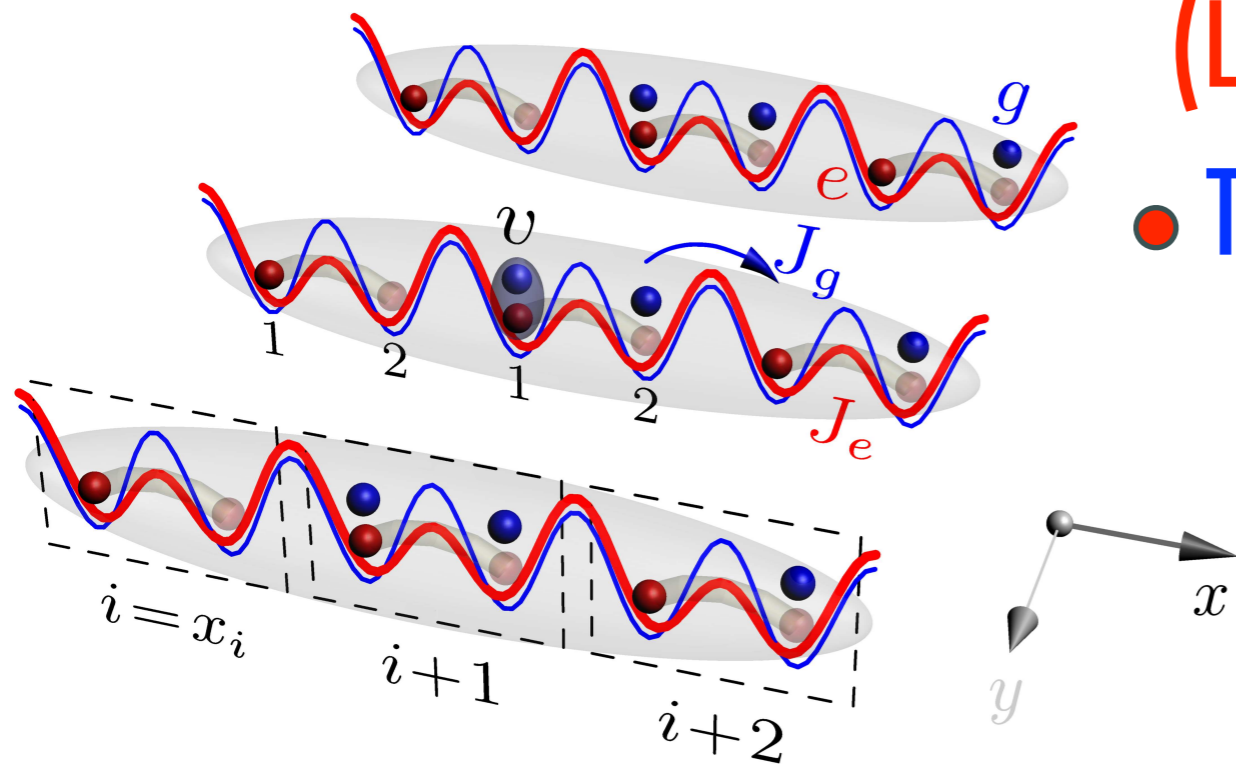
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Conclusions

Chi l'ha visto?



Ettore Majorana, ordinario di fisica teorica all'Università di Napoli, è misteriosamente scomparso dagli ultimi di marzo. Di anni 31, alto metri 1,70, snello, con capelli neri, occhi scuri, una lunga cicatrice sul dorso di una mano. Chi ne sapesse qualcosa è pregato di scrivere al R. P. E. Maria-necci, Viale Regina Margherita 66 - Roma.

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Majorana: The Mystery Must Be Solved