Excited-state quantum phase transition and quantum speed limit

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Outline

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1. Introduction
2. Model and results
3. Summary

Quantum speed limit (QSL) time

- Isolated systems (unitary dynamics)
  1. Mandelstam-Tamm bound [J. Phys. (USSR) **9**, 249 (1945)]

$$\tau \geq \tau_{QSL} = \frac{\pi}{2}\frac{\hbar}{\Delta H}$$

[M. R. Frey, Quantum Inf. Process. **15**, 3919 (2016)]

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  2. Margolus-Levitin bound [Phys. D **120**, 188 (1998)]

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  3. Unified bound [S. Deffner and S. Campbell, J. Phys. A: Math. Theor. **50**, 453001 (2017)]

$$\tau_{QSL} = \max\left\{\frac{\pi}{2}\frac{\hbar}{\Delta H}, \frac{\pi}{2}\frac{\hbar}{\langle H \rangle - E_0}\right\}$$

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3. S. Deffner and E. Lutz, Phys. Rev. Lett. **111**, 010402 (2013)

$$\tau_{QSL} = [\sin^2 \mathcal{B}(\rho_0, \rho_\tau)] \max \left\{ \frac{1}{\Lambda_\tau^1}, \frac{1}{\Lambda_\tau^2}, \frac{1}{\Lambda_\tau^\infty} \right\}$$

$$\mathcal{B}(\rho_0, \rho_\tau) = \arccos \sqrt{\langle\psi_0|\rho_\tau|\psi_0\rangle}, \; \Lambda_\tau^p = (1/\tau) \int_0^\tau dt ||L(\rho_t)||_p, \; ||A||_p = \left(\sum_{j=1}^n \alpha_j^p \right)^{1/p}$$

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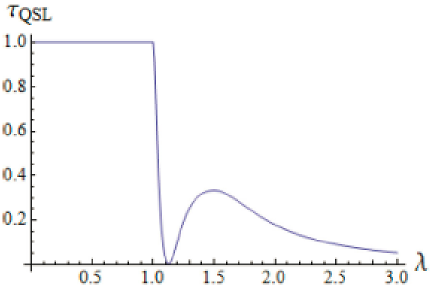
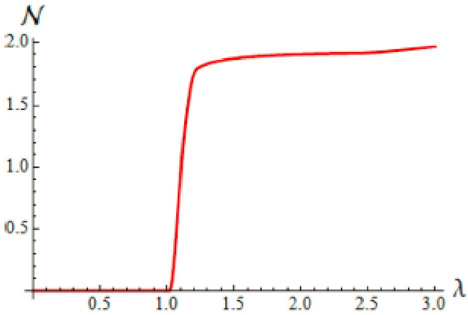
  4. A. D. Cimmarusti, Z. Yan, B. D. Patterson, L. P. Corcos, L. A. Orozco, and S. Deffner, Phys. Rev. Lett. **114**, 233602 (2015)

QSL time in Lipkin model

$$H = -2s_z - \frac{\lambda}{N}(J_+J_- + J_-J_+) - 2J_z - \lambda'(s_+J_- + s_-J_+)$$





[L. Hou, B. Shao, and J. Zou, Eur. Phys. J. D **70**, 35 (2016)]

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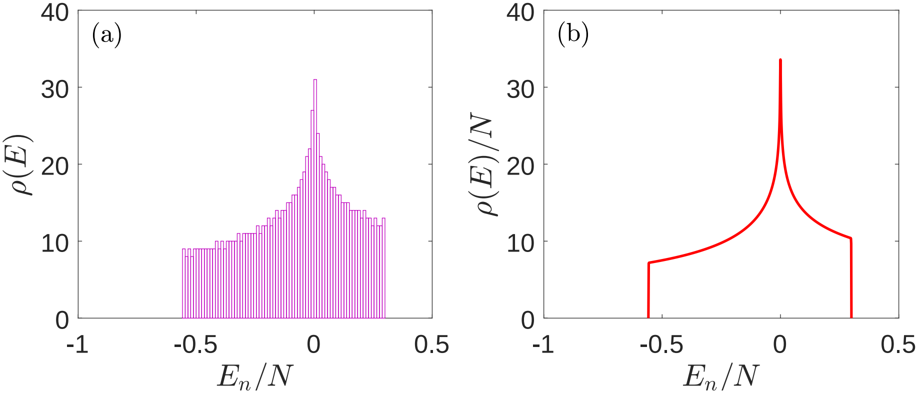
Model and QSL time

$$H = \sigma_z - \frac{4(1-\alpha)}{N} J_x^2 + \alpha \left(J_z + \frac{N}{2} \right) + \lambda \sigma_z J_z$$



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2. Initial state

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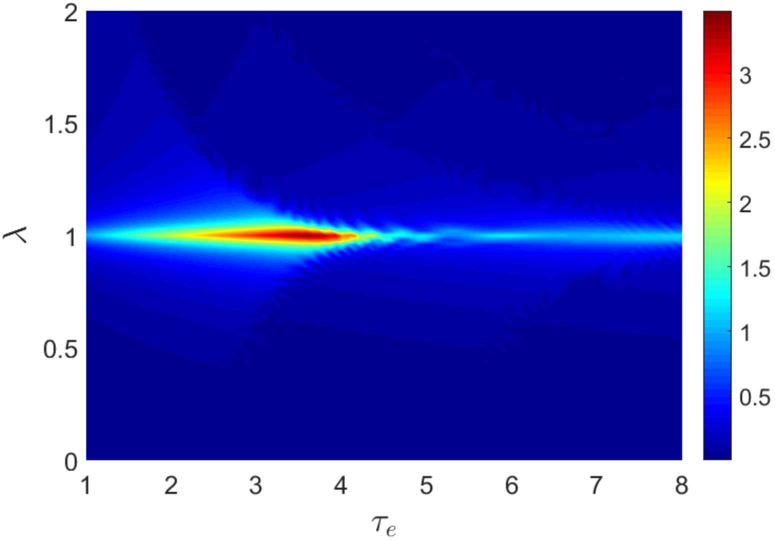
3. QSL time

$$\tau_{\mathrm{QSL}} = \frac{\sin(\theta)\{1 - \mathcal{R}[\mathcal{M}(\tau_e)]\}}{(1/\tau_e)\int_0^{\tau_e} dt|\partial_t \mathcal{M}(t)|}$$

$$\mathcal{M}(t) = \langle\Psi^0_{\mathcal{E}}(G)|e^{-iH^1_{\mathcal{E}}}|\Psi^0_{\mathcal{E}}(G)\rangle$$

Numerical results



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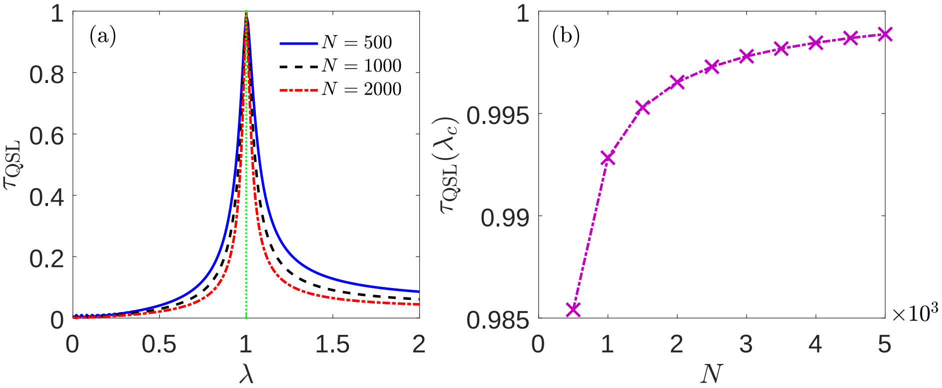
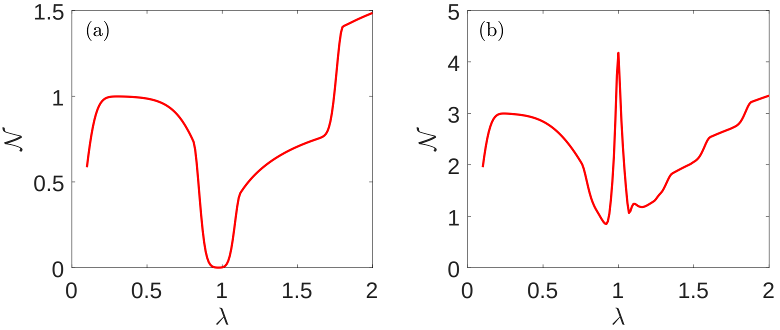


Non-Markovianity

$$\left.\begin{aligned}&\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\eta>0} \eta[t, \rho_{1,2}(0)]dt\\ &\eta[t, \rho_{1,2}(0)] = \partial_t D[\rho_1(t), \rho_2(t)]\\ &D[\rho_1(t), \rho_2(t)] = \mathrm{Tr}|\rho_1(t) - \rho_2(t)|/2\end{aligned}\right\} \Longrightarrow \begin{cases}\mathcal{N} = \frac{1}{2}\left[\int_0^{\tau_e} |\partial_t D(t)|dt + |\mathcal{M}(\tau_e)| - 1\right]\\ D(t) = |\mathcal{M}(t)|\end{cases}$$



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Summary

- The QSL time exhibits a sharp peak at the critical point of ESQPT;
- At the critical point, increase the size of environment will lead to the QSL time approach the actual evoultion time;
- In contrast to the ground state QPT, the critical behavior of QSL time in ESQPT case can not be interpreted via the Markovian nature of the environment.

Thank you!