Shell Model, Level Density and Phase Transitions

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In collaboration with
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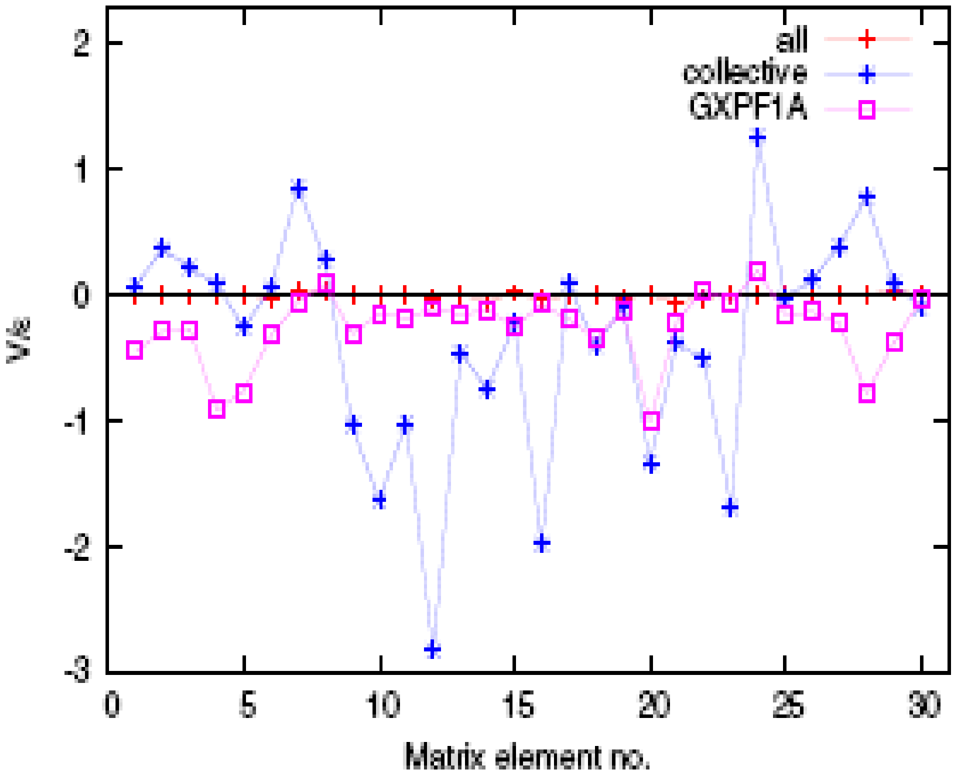
Work in progress

Strong interaction 4.0



	$\langle j_1 j_2 \vert V \vert j_3 j_4 \rangle (JT)$	Full average	Prolate average
1	$\langle ff \vert V \vert ff \rangle (10)$	0.021	0.078
2	$\langle ff \vert V \vert ff \rangle (30)$	0.012	0.374
3	$\langle ff \vert V \vert ff \rangle (50)$	-0.007	0.227
4	$\langle ff \vert V \vert ff \rangle (70)$	0.007	0.089
5	$\langle ff \vert V \vert ff \rangle (01)$	0.008	-0.252
6	$\langle ff \vert V \vert ff \rangle (21)$	-0.020	0.062
7	$\langle ff \vert V \vert ff \rangle (41)$	0.026	0.869
8	$\langle ff \vert V \vert ff \rangle (61)$	0.034	0.282
9	$\langle ff \vert V \vert pf \rangle (30)$	0.004	-1.033
10	$\langle ff \vert V \vert pf \rangle (50)$	0.022	-1.630
11	$\langle ff \vert V \vert pf \rangle (21)$	0.006	-1.010
12	$\langle ff \vert V \vert pf \rangle (41)$	-0.010	-2.826
13	$\langle ff \vert V \vert pp \rangle (10)$	0.014	-0.451
14	$\langle ff \vert V \vert pp \rangle (30)$	-0.043	-0.739
15	$\langle ff \vert V \vert pp \rangle (01)$	0.025	-0.223
16	$\langle ff \vert V \vert pp \rangle (21)$	-0.036	-1.977
17	$\langle pf \vert V \vert pf \rangle (20)$	0.007	0.088
18	$\langle pf \vert V \vert pf \rangle (30)$	0.010	-0.393
19	$\langle pf \vert V \vert pf \rangle (40)$	-0.018	-0.092
20	$\langle pf \vert V \vert pf \rangle (50)$	0.004	-1.328
21	$\langle pf \vert V \vert pf \rangle (21)$	-0.052	-0.376
22	$\langle pf \vert V \vert pf \rangle (31)$	-0.019	-0.507
23	$\langle pf \vert V \vert pf \rangle (41)$	0.011	-1.685
24	$\langle pf \vert V \vert pf \rangle (51)$	-0.003	1.276
25	$\langle pf \vert V \vert pp \rangle (30)$	0.007	-0.023
26	$\langle pf \vert V \vert pp \rangle (21)$	0.014	0.133
27	$\langle pp \vert V \vert pp \rangle (10)$	0.003	0.400
28	$\langle pp \vert V \vert pp \rangle (30)$	0.003	0.779
29	$\langle pp \vert V \vert pp \rangle (01)$	0.054	0.102
30	$\langle pp \vert V \vert pp \rangle (21)$	0.005	-0.092

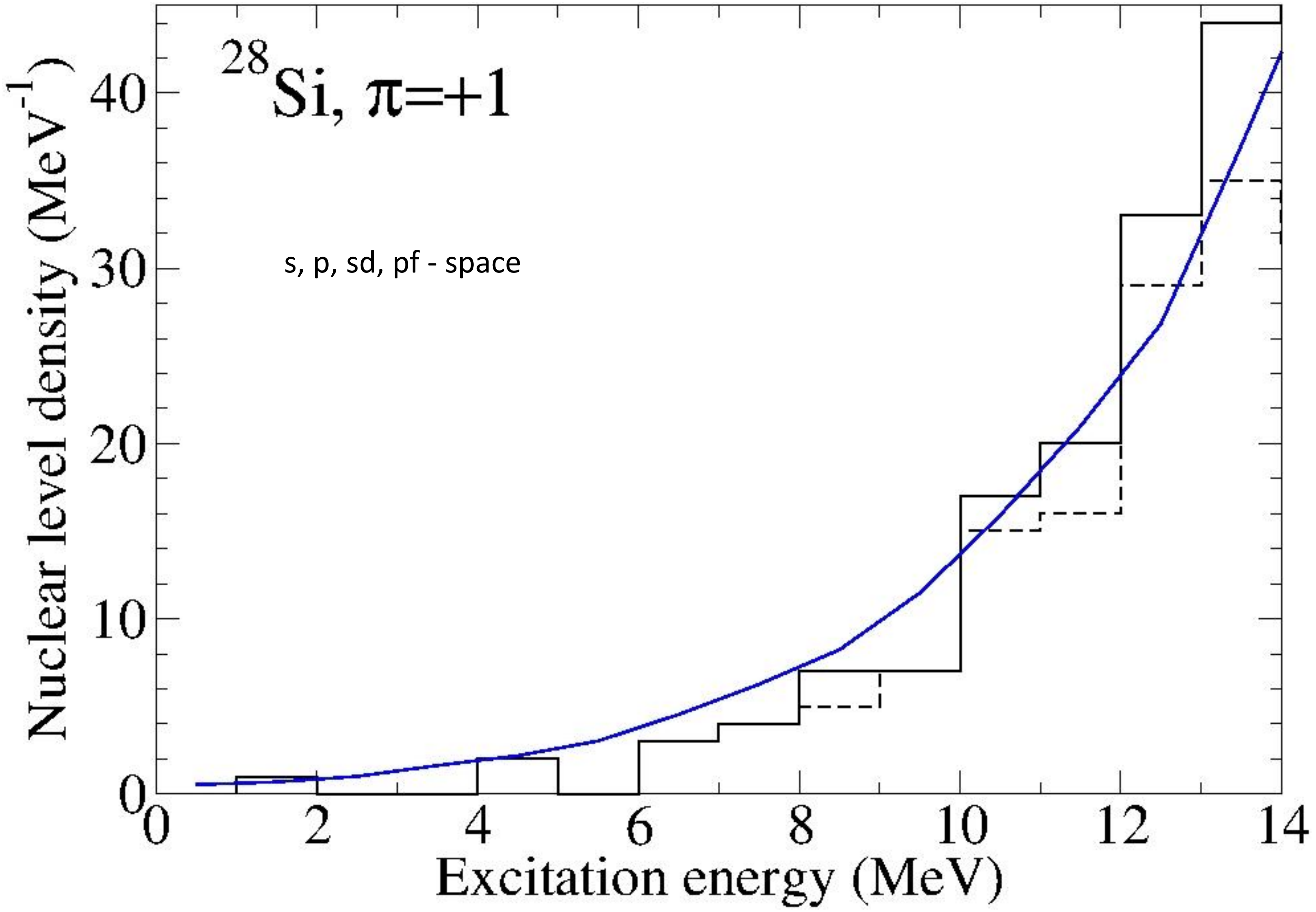
Matrix elements

9-12: pf mixing,

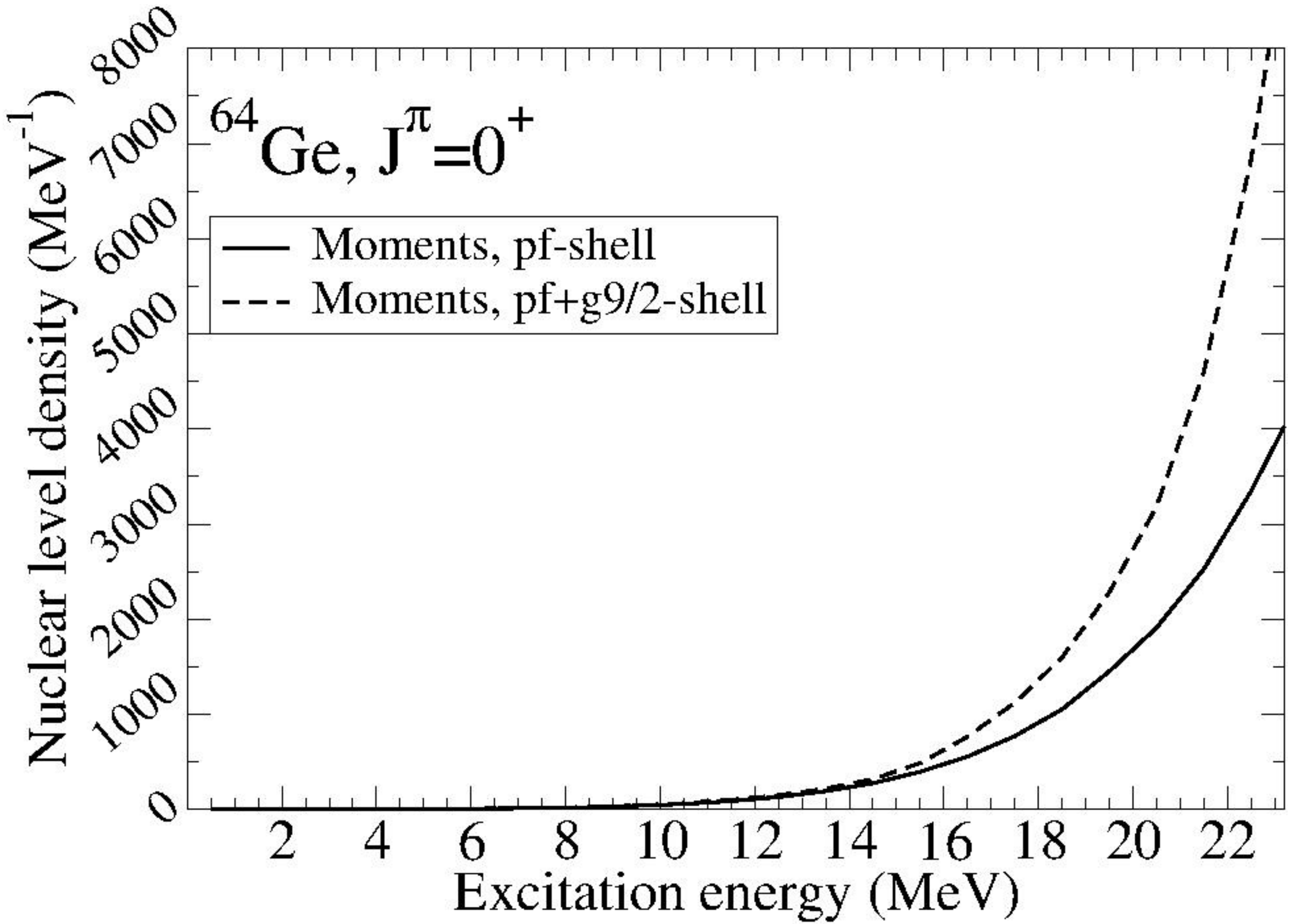
16 : quadrupole pair transfer,

20-24: quadrupole-quadrupole forces
in particle-hole channel = formation of the mean field

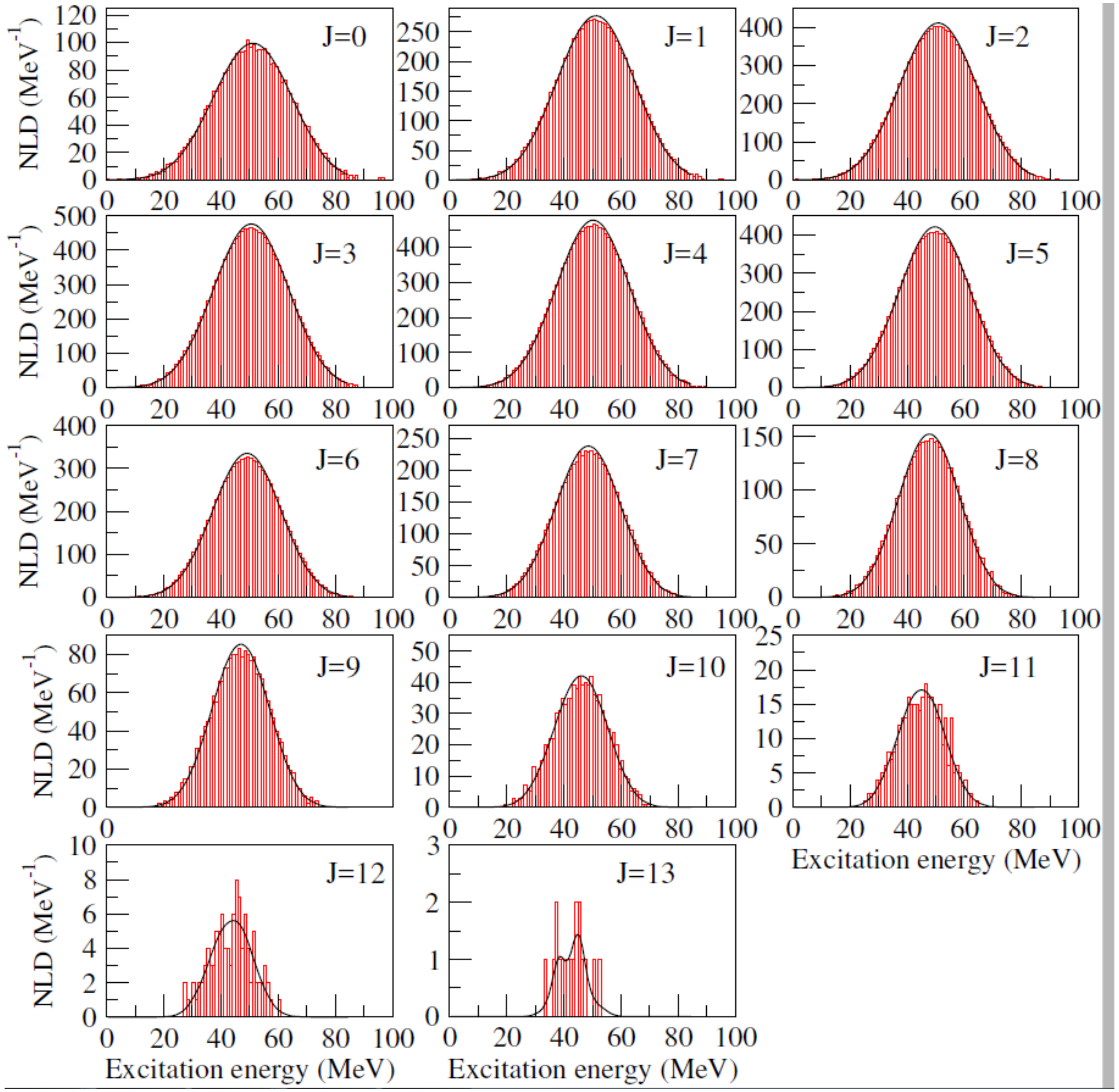
Large fluctuations of non-extensive nature (the same for 10 000 and 100 000 realizations)



^{28}Si, π=+1
s, p, sd, pf - space
Nuclear level density (MeV^{-1})
0
10
20
30
40
0
2
4
6
8
10
12
14
Excitation energy (MeV)



R.Sen’kov, V.Z.
PRC 93 (2016)

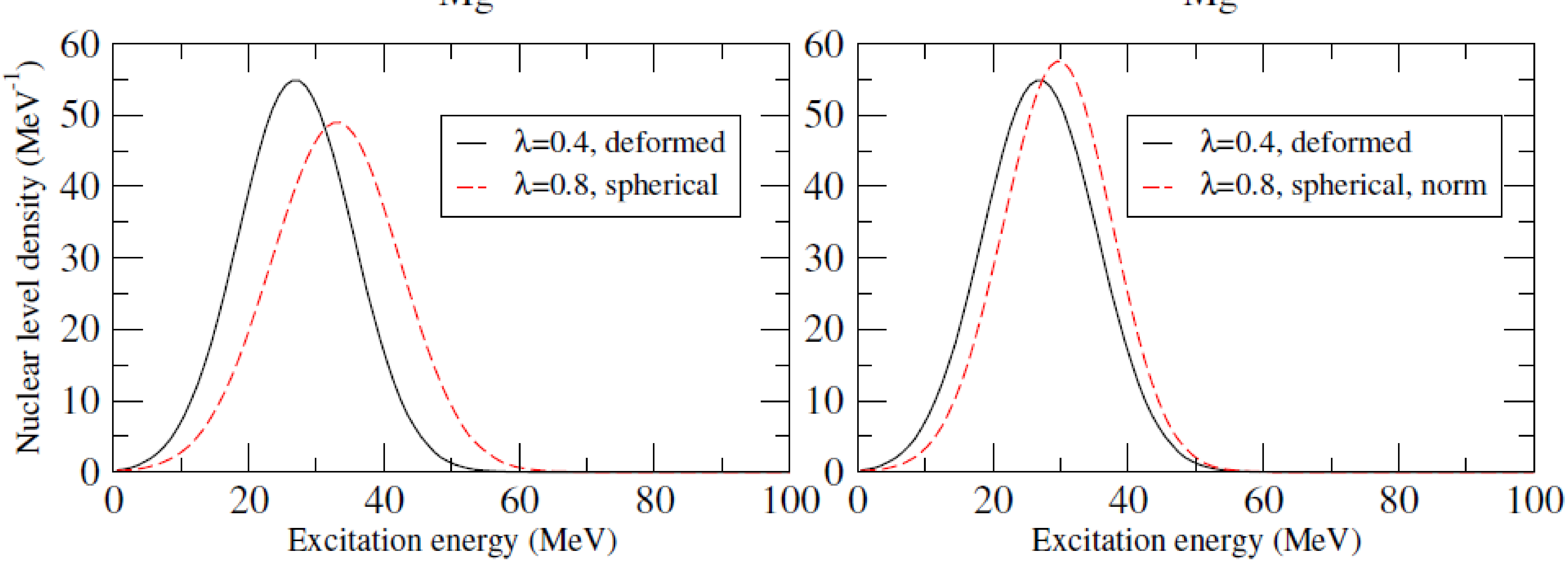


Generic shape (Gaussian)

Level density for different classes of states for 28Si

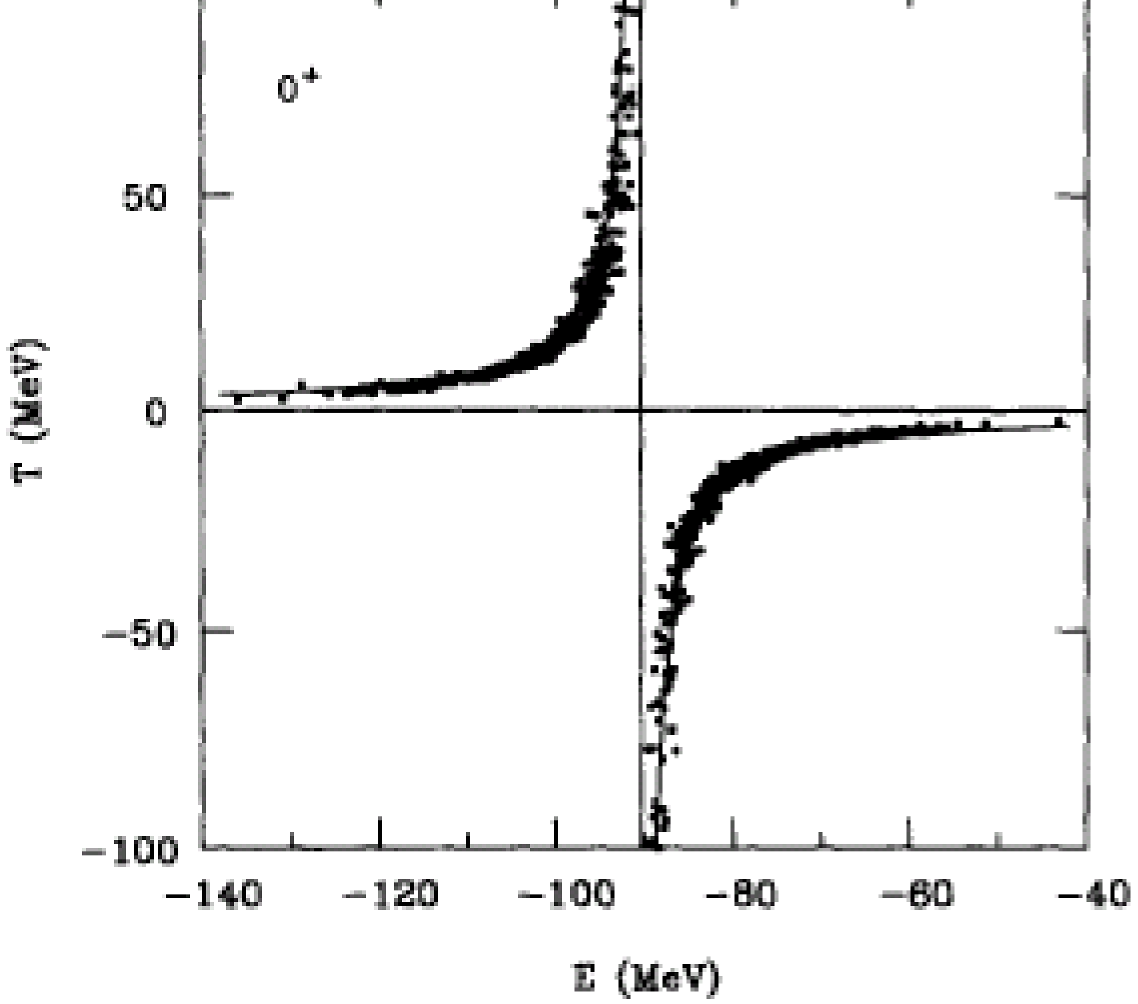
Full agreement between exact shell model and moments method

Problems: truncated orbital space, only positive parity in sd-model, …



Level density (0+) on two sides of deformation shape transition

/"collective enhancement"/



Gaussian level density

CENTROID E_0

WIDTH σ_E

Microcanonical temperature

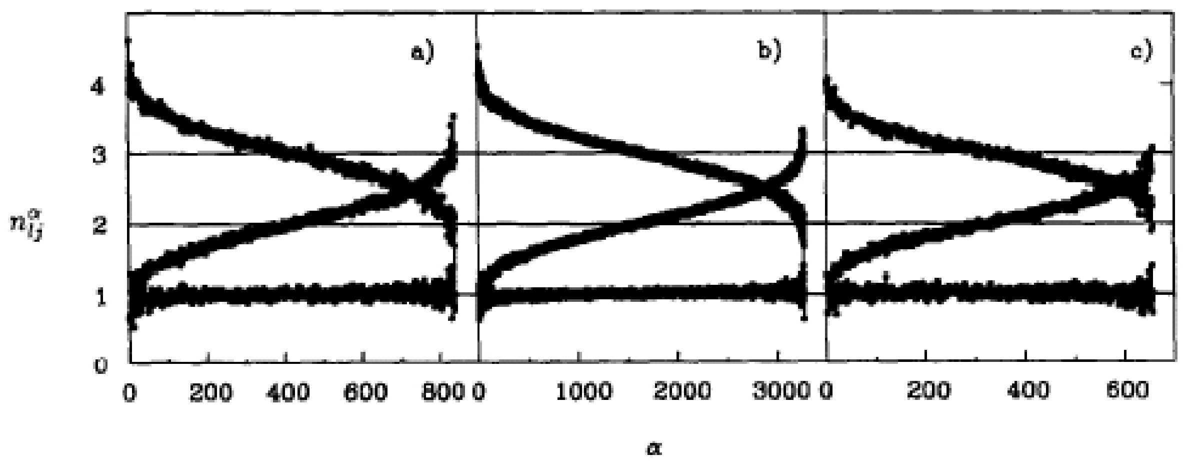
$$T_{\rm th} = \sigma_E^2/(E_0 - E)$$

839 states (28 Si) J=0

EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution (dots)
From thermodynamic entropy defined by level density (lines)

d5/2, d3/2, s1/2



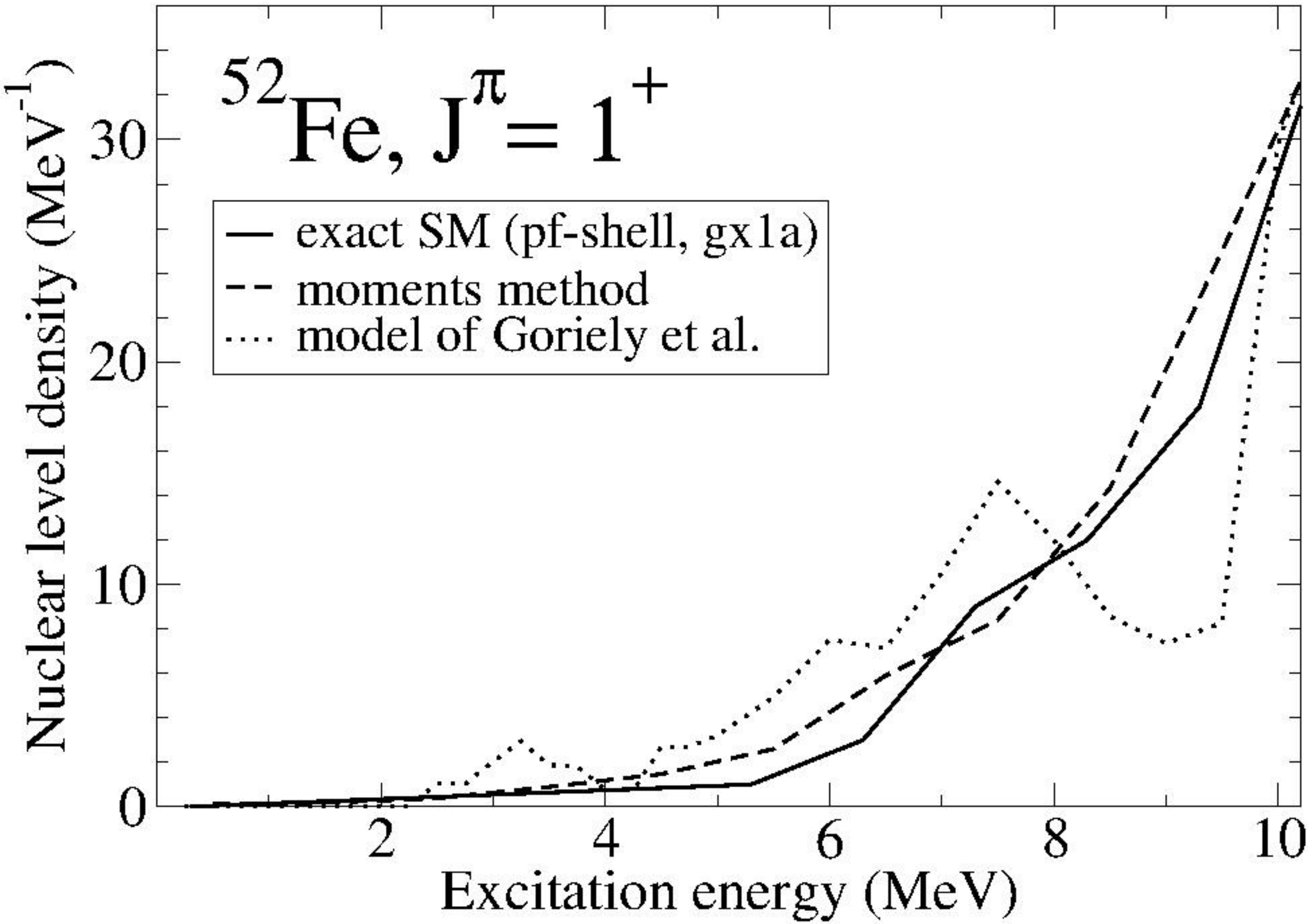
28 Si

J=0 *J=2* *J=9*

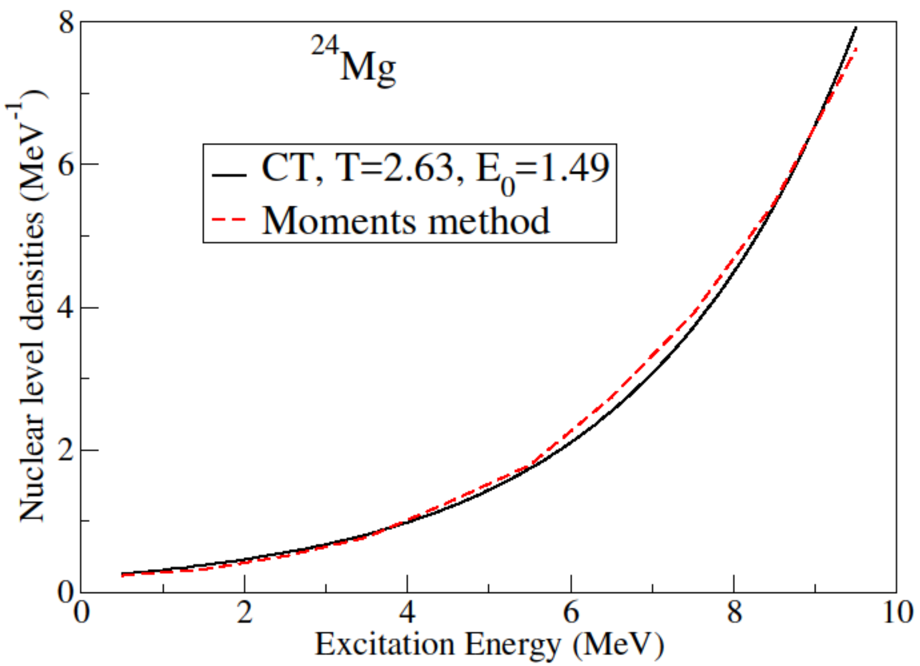
Single – particle occupation numbers

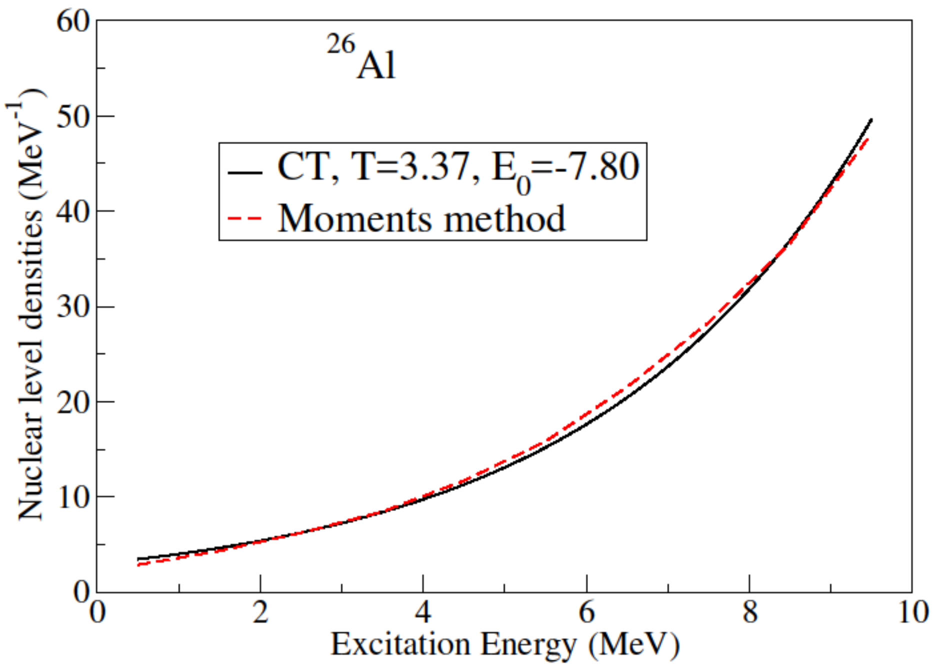
Thermodynamic behavior
identical in all symmetry classes

FERMI-LIQUID PICTURE



Role of incoherent processes (collision-like)





Level density and "**constant temperature**" fit

L.D.(E) = (const) exp[E/T] – melting pairing?

$$T_{\mathrm{t-d}} = \left(\frac{\partial S}{\partial E}\right)^{-1} = T\left(1 - e^{-E/T}\right)$$

Partition function = **Trace{exp[-H/T(t-d)]} diverges** at **T > T(t-d)**

Cumulative level number

N(E) = exp(S),

Entropy **S(E)= ln(N)**

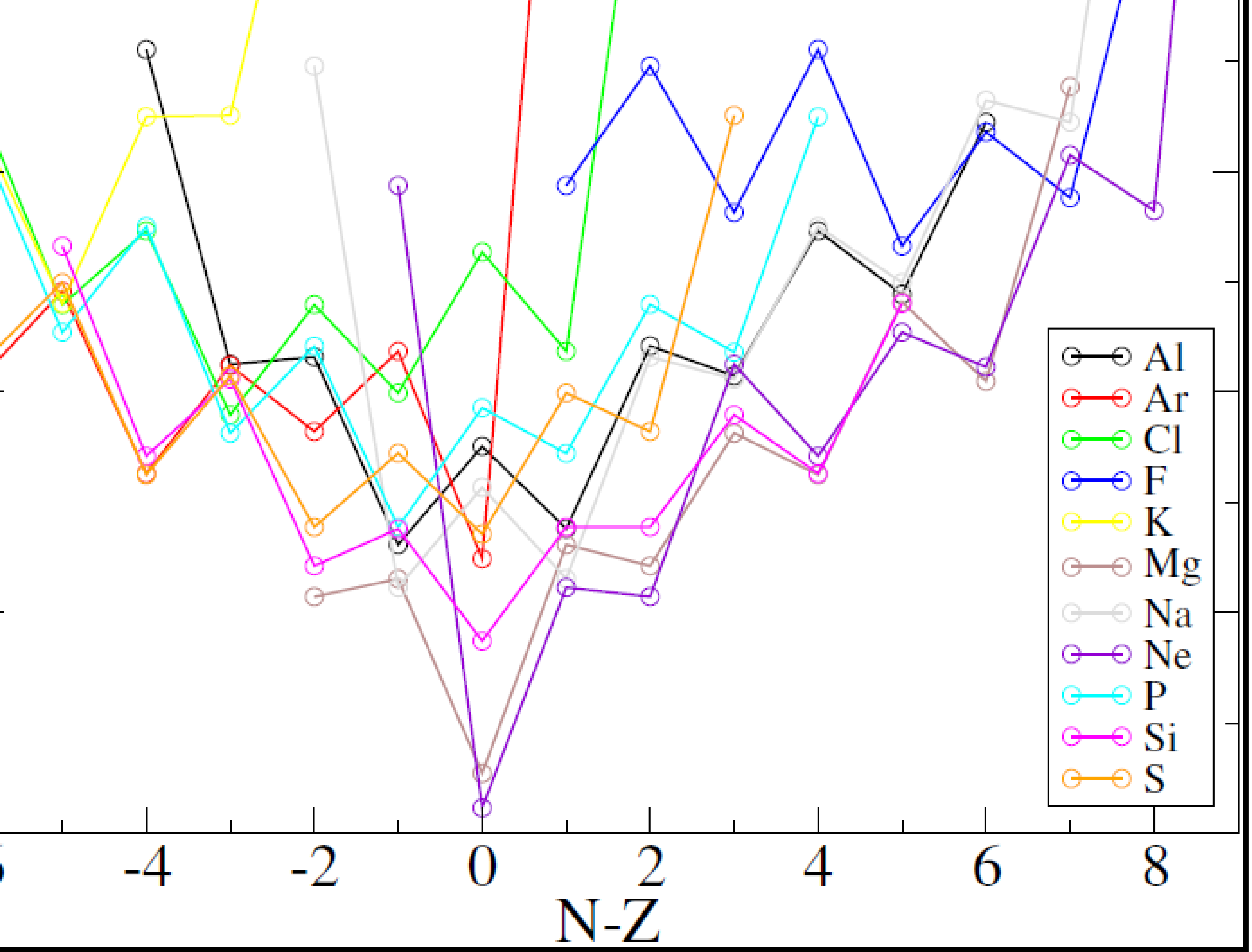
Thermodynamic temperature

T(t-d) = 1/[dS/dE] = T[1 – exp(- E/T)]

Parameter T is ***limiting temperature***

(***Hagedorn temperature*** in particle physics)

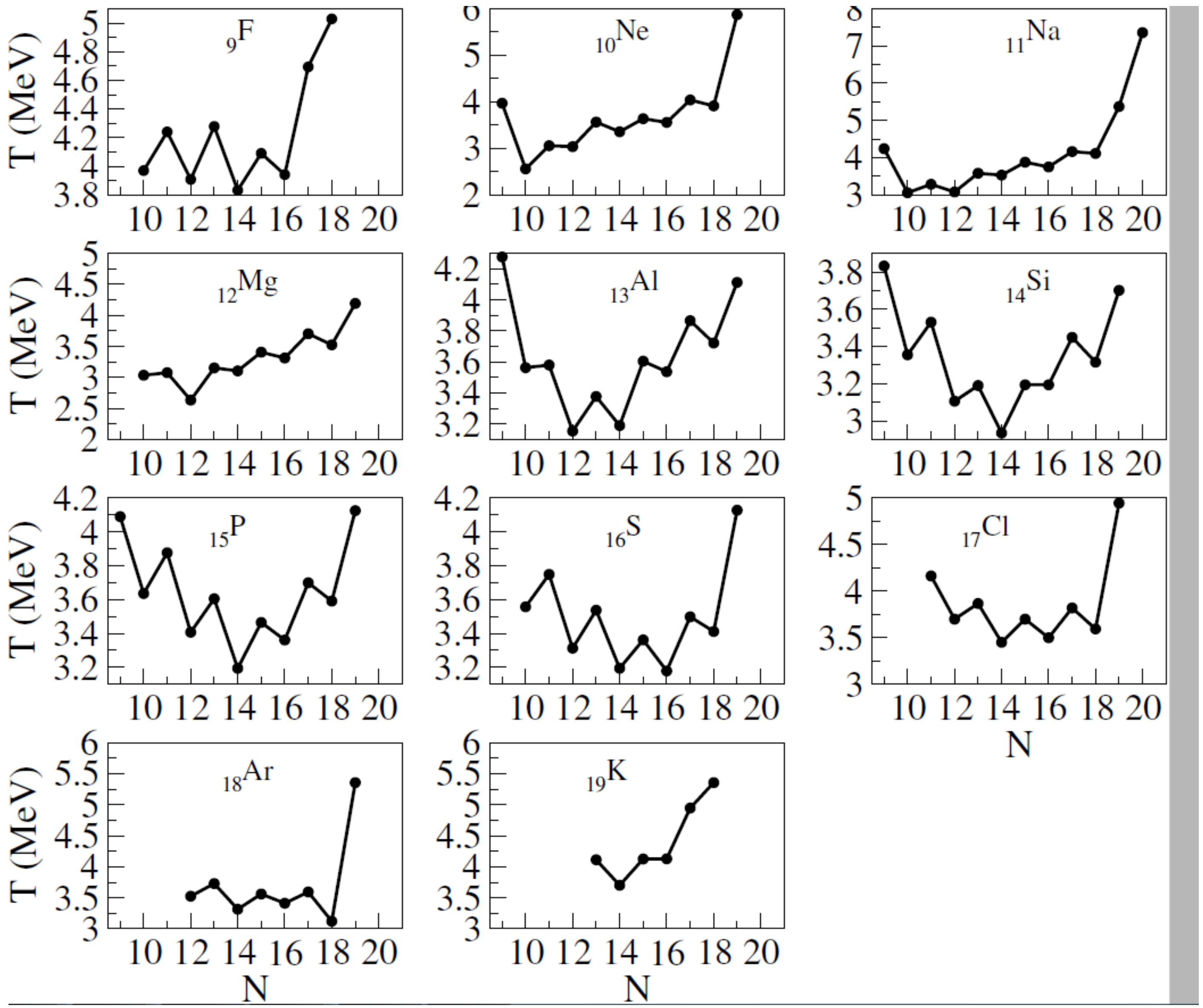
Pairing phase transition? (Moretto) - Chaotization



Effective temperature for the level density at low energy (up to 6 – 8 Mev)

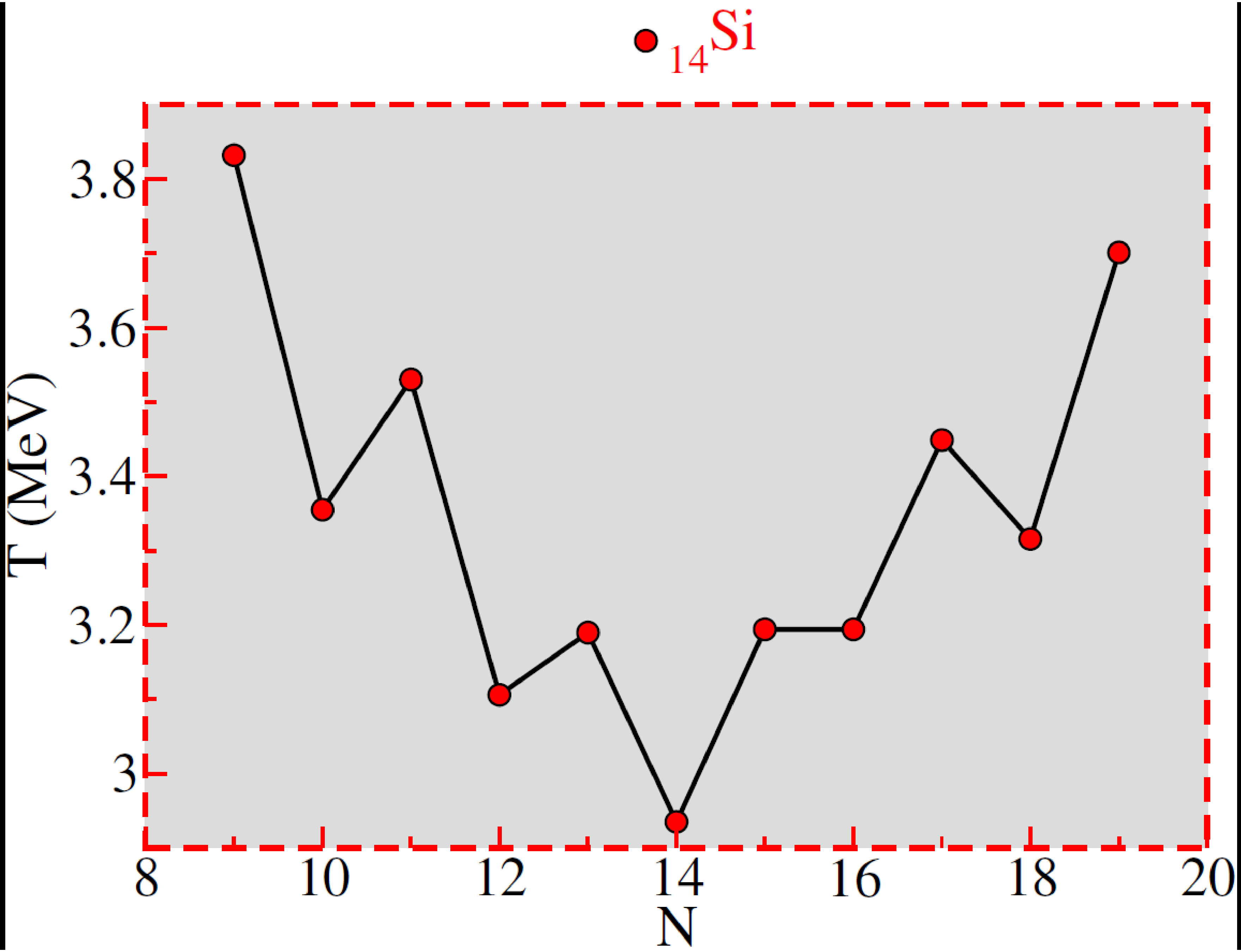
Even-odd staggering

Clear minima in the vicinity of N=Z

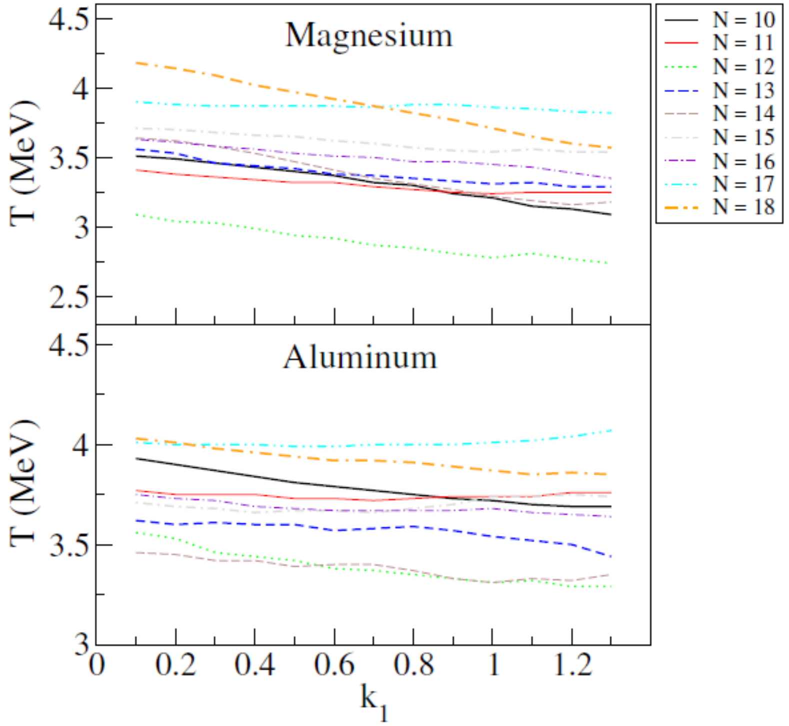


Effective temperature T

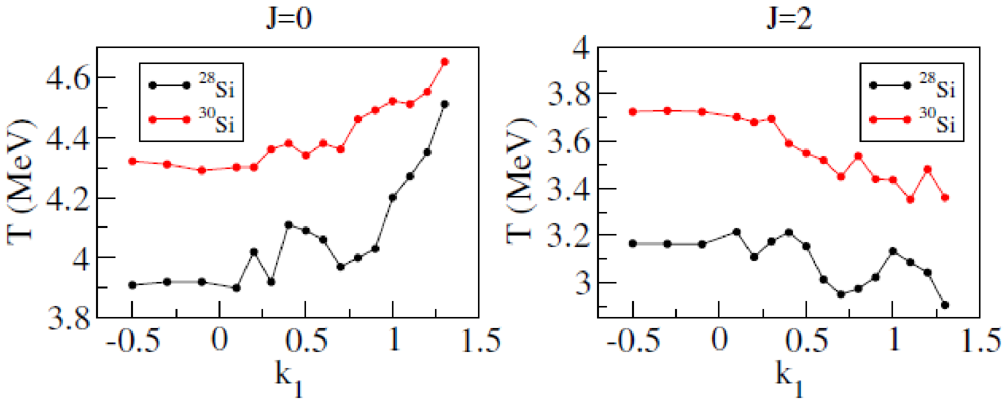
for (sd) - nuclei

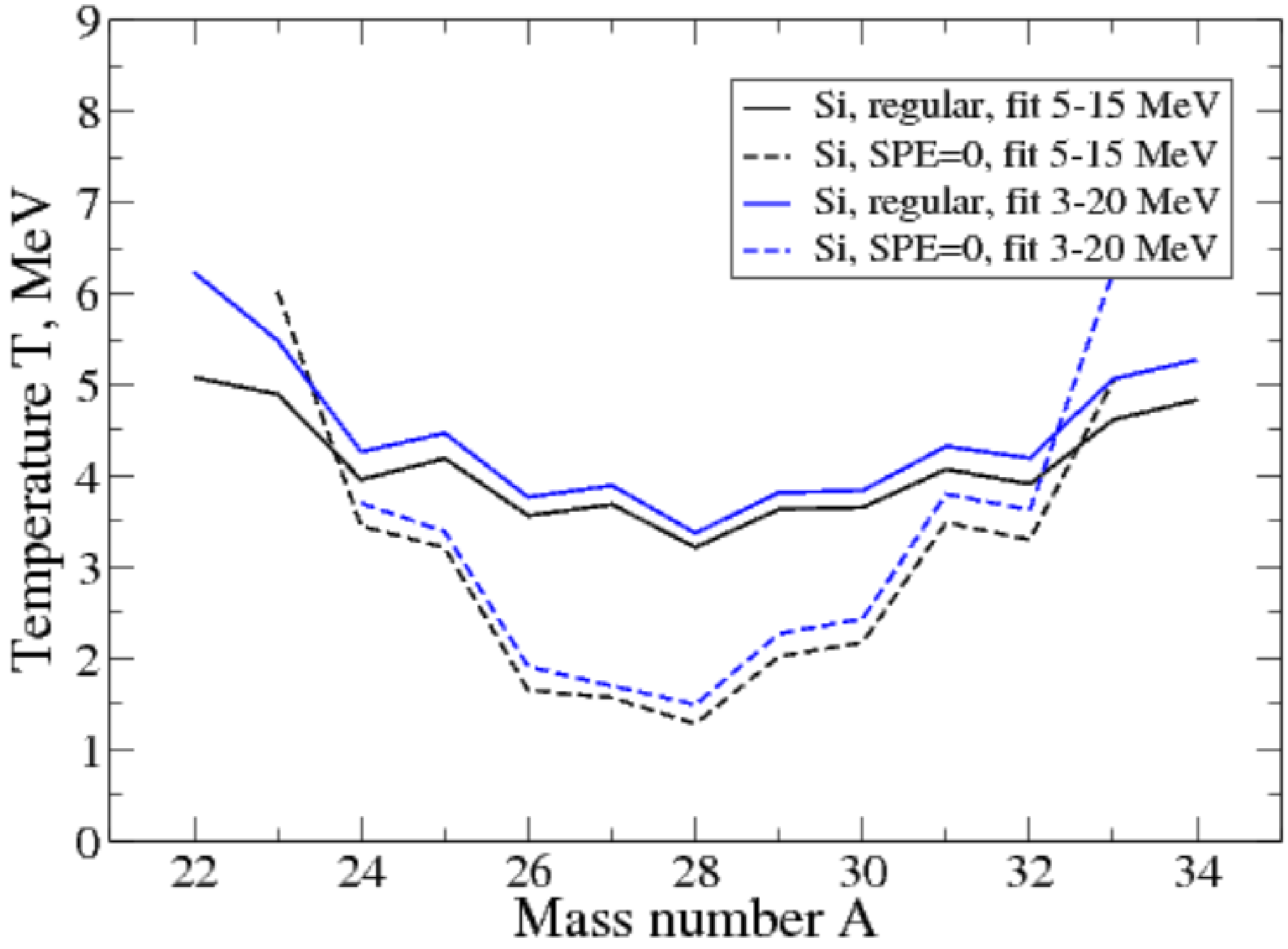


$_{14}$Si
3.8
3.6
3.4
3.2
3
T (MeV)
8
10
12
14
16
18
20
N

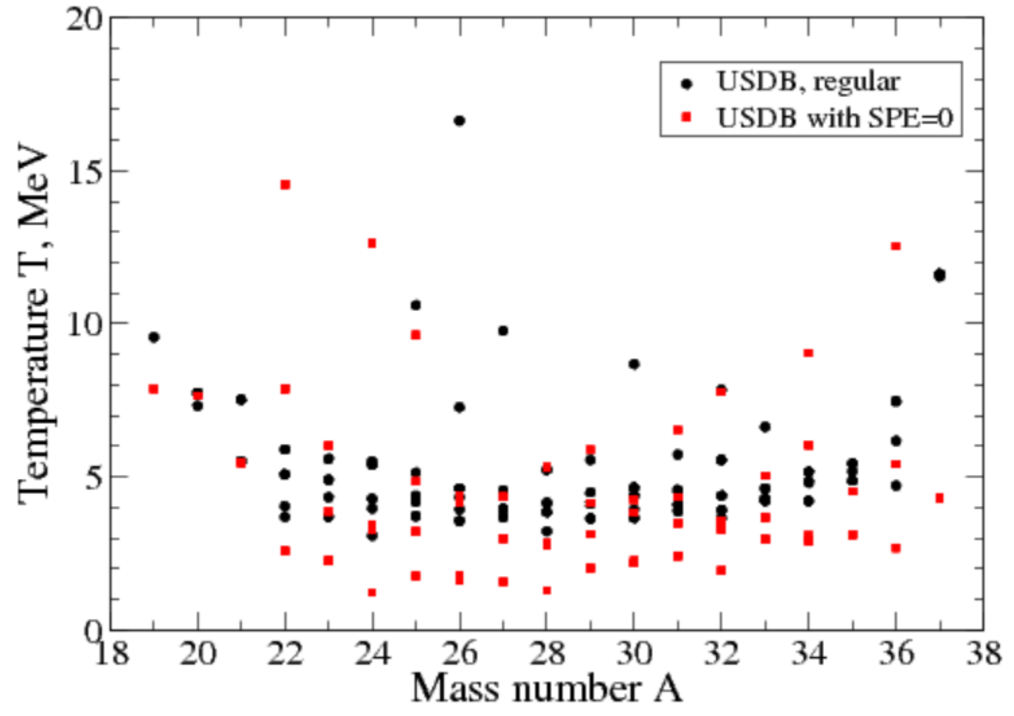


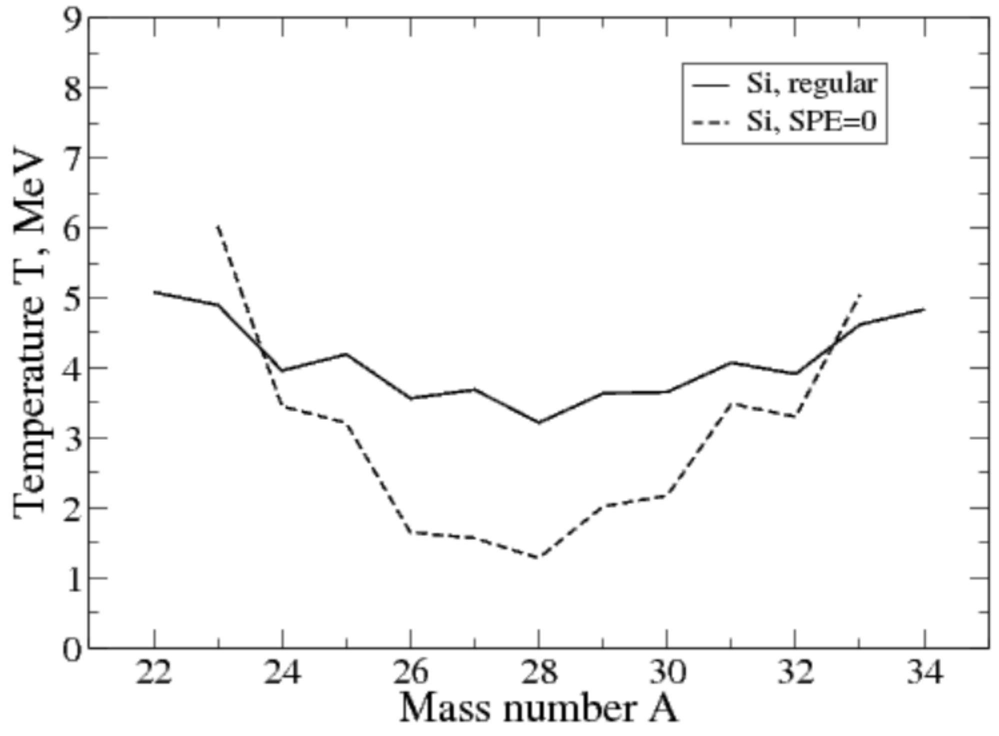
Eliminating pairing interaction



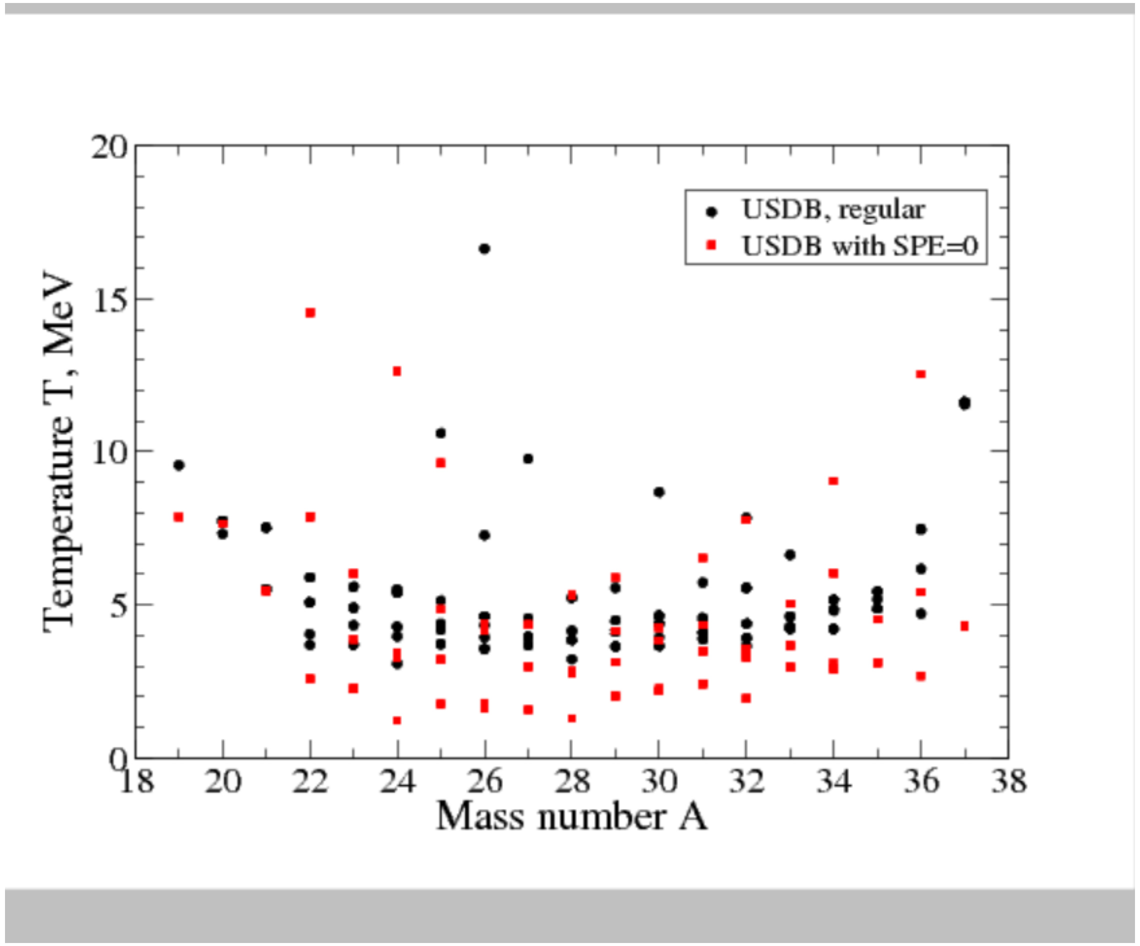


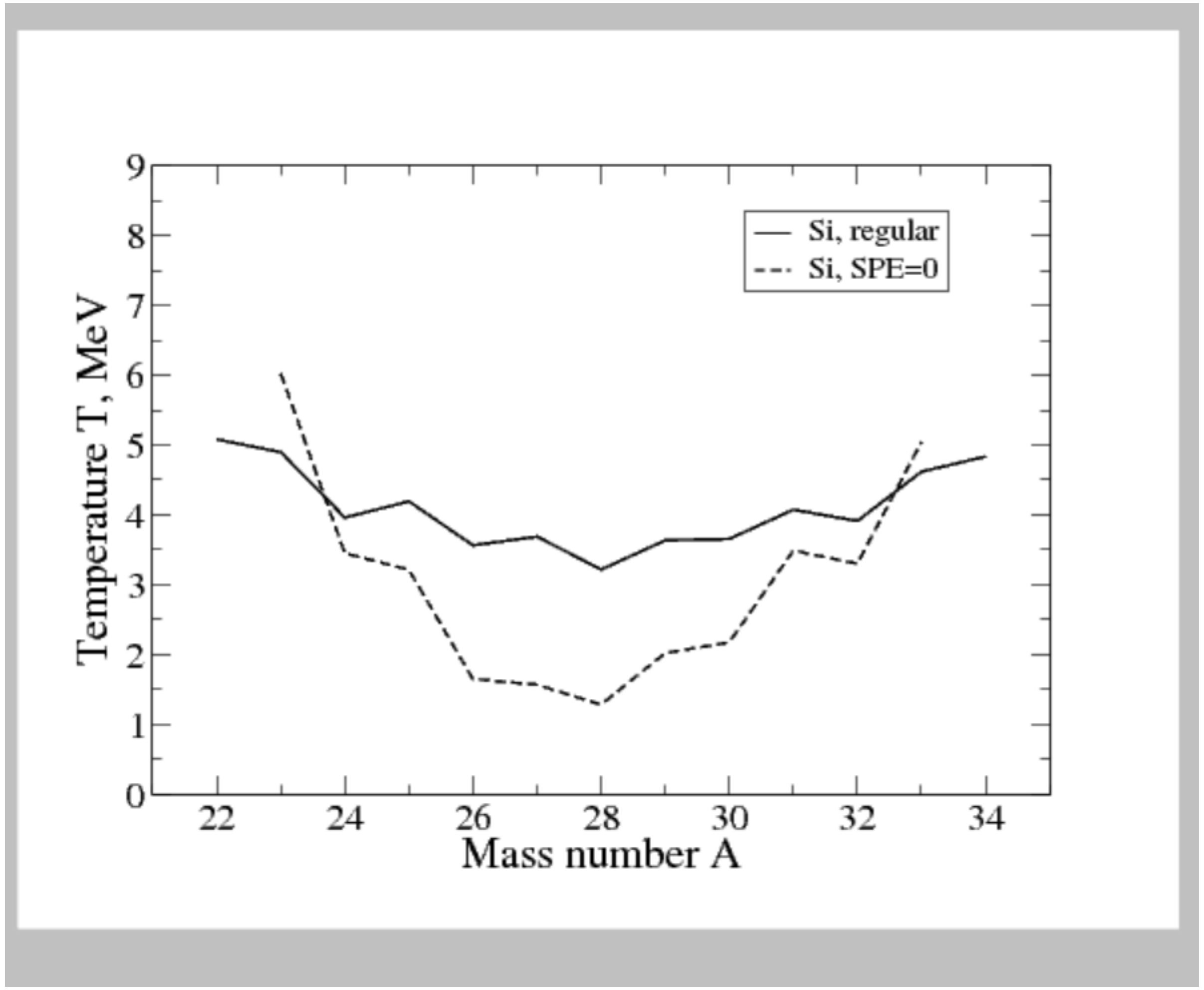
Sensitivity to the fit interval





Degenerate single-particle levels – smaller T (faster chaotization)

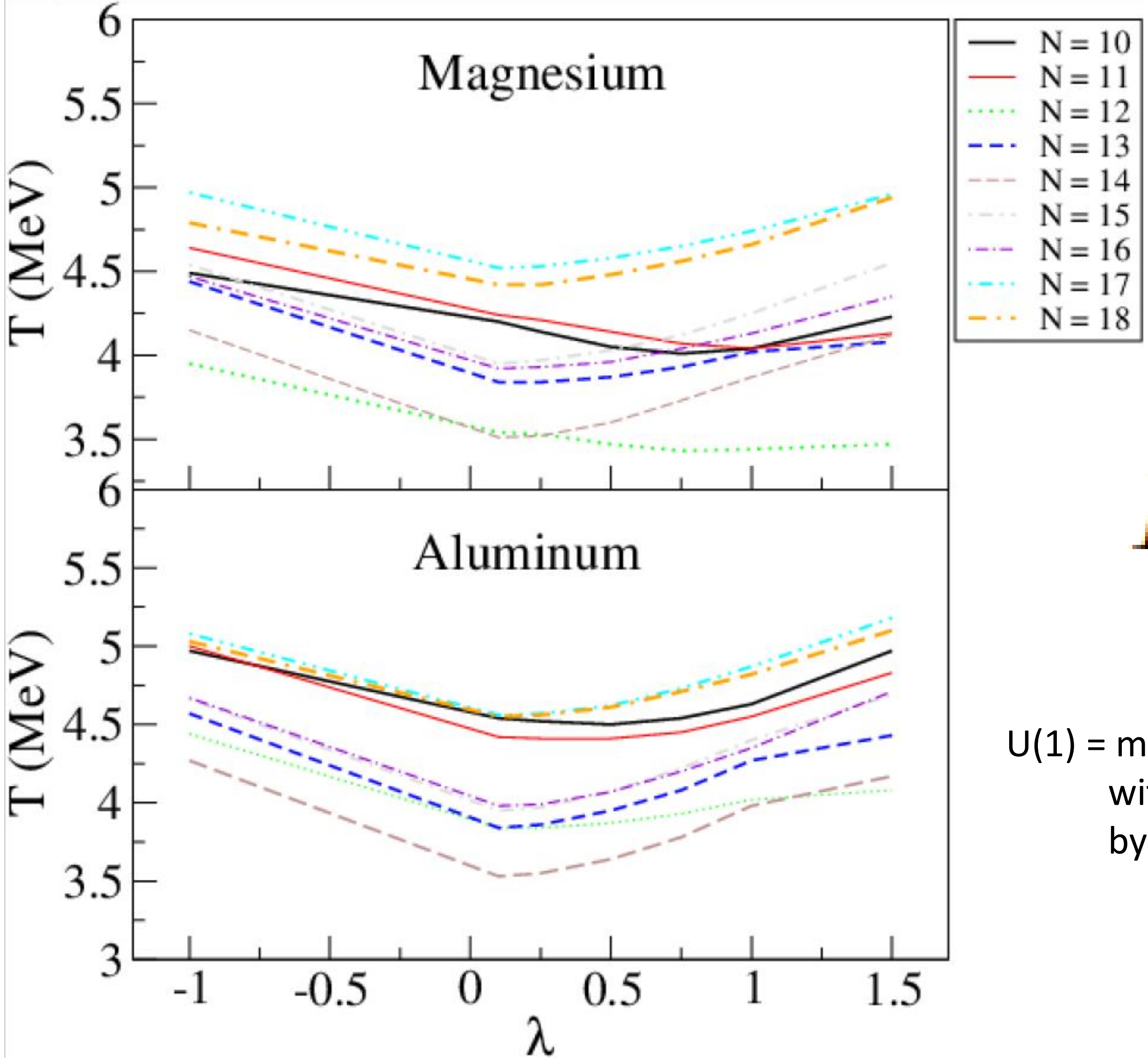




Degenerate single-particle levels – smaller T (faster chaotization)

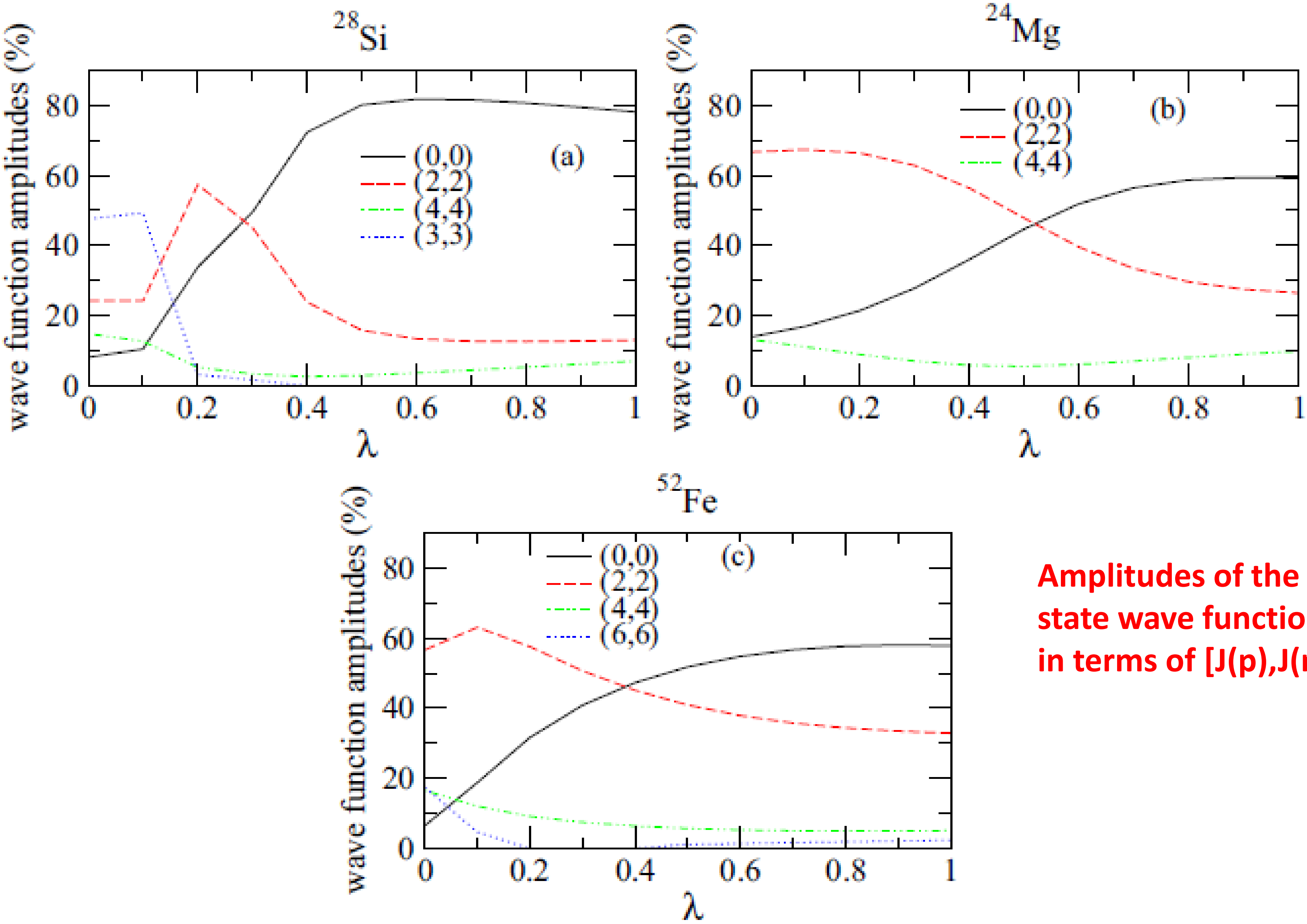
ARTIFICIAL NOISE AS THEORETICAL TOOL

* *Add random noise to the dynamics*
* *Construct the density matrix by averaging for any individual wave function*
* *Calculate the corresponding entropy*
  - *measurement of sensitivity of eigenstates*
  - *quantum phase transitions*
  - *basis-independent criteria*



$$H = h + \lambda U_1 + U_2.$$

U(1) = matrix elements of the two-body interaction with change of orbital momentum of one particle by 2 units (the same parity) – way to deformation



Amplitudes of the ground state wave functions in terms of [J(p),J(n)]

Invariant correlational entropy as signature of phase transitions

$$|\alpha(\lambda)\rangle = \sum_k C_k^\alpha(\lambda)|k\rangle.$$

Eigenstates in an arbitrary basis
(Hamiltonian with random parameters)

$$\rho_{kk'}^\alpha(\lambda) = \overline{C_k^\alpha C_{k'}^{\alpha *}}$$

Density matrix of a given state
(averaged over the ensemble)

$$S^\alpha(\lambda) = -\mathrm{Tr}\{\rho^\alpha \ln(\rho^\alpha)\}$$

$$\lambda \in [\lambda, \lambda + \delta]$$

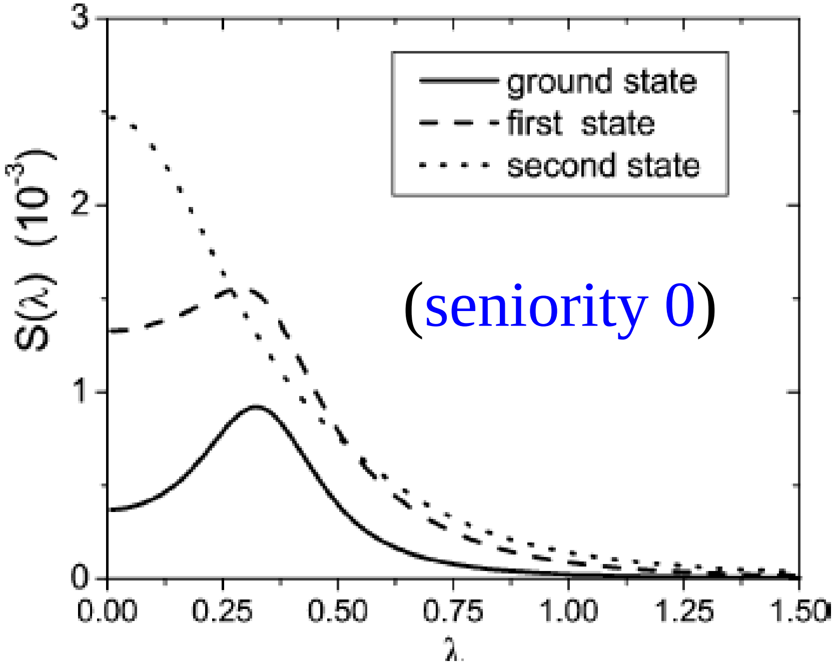
Correlational entropy
has clear maximum
at phase transition
(extreme sensitivity)

Pure state: eigenvalues of the density matrix are **1 (one) and 0 (N-1),**
S=0

Mixed state: between 0 and 1, **S up to ln N**

For two discrete points

$$r_\pm^\alpha = \frac{(1 \pm |\langle\alpha(\lambda)|\alpha(\lambda')\rangle|)}{2}$$



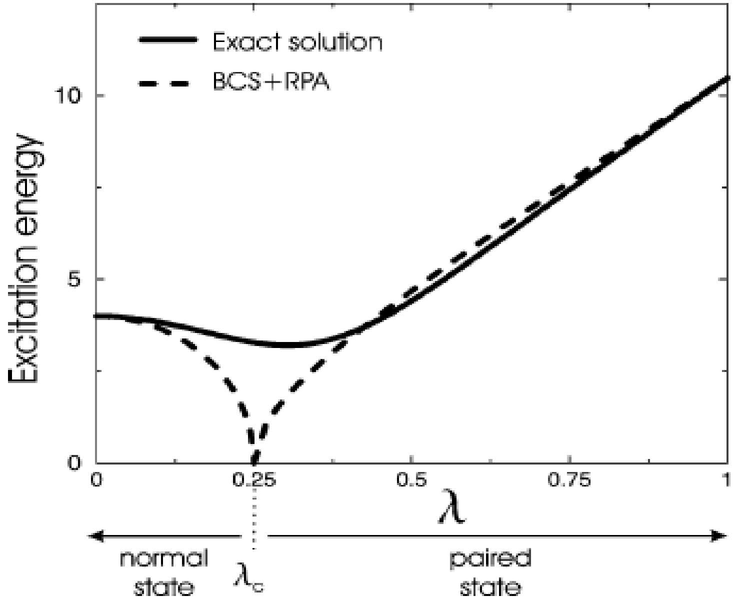
Model of two levels with pair transfer

Capacity 16 + 16, N=16

Critical value 0.3

(in BCS ¼)

Averaging interval 0.01

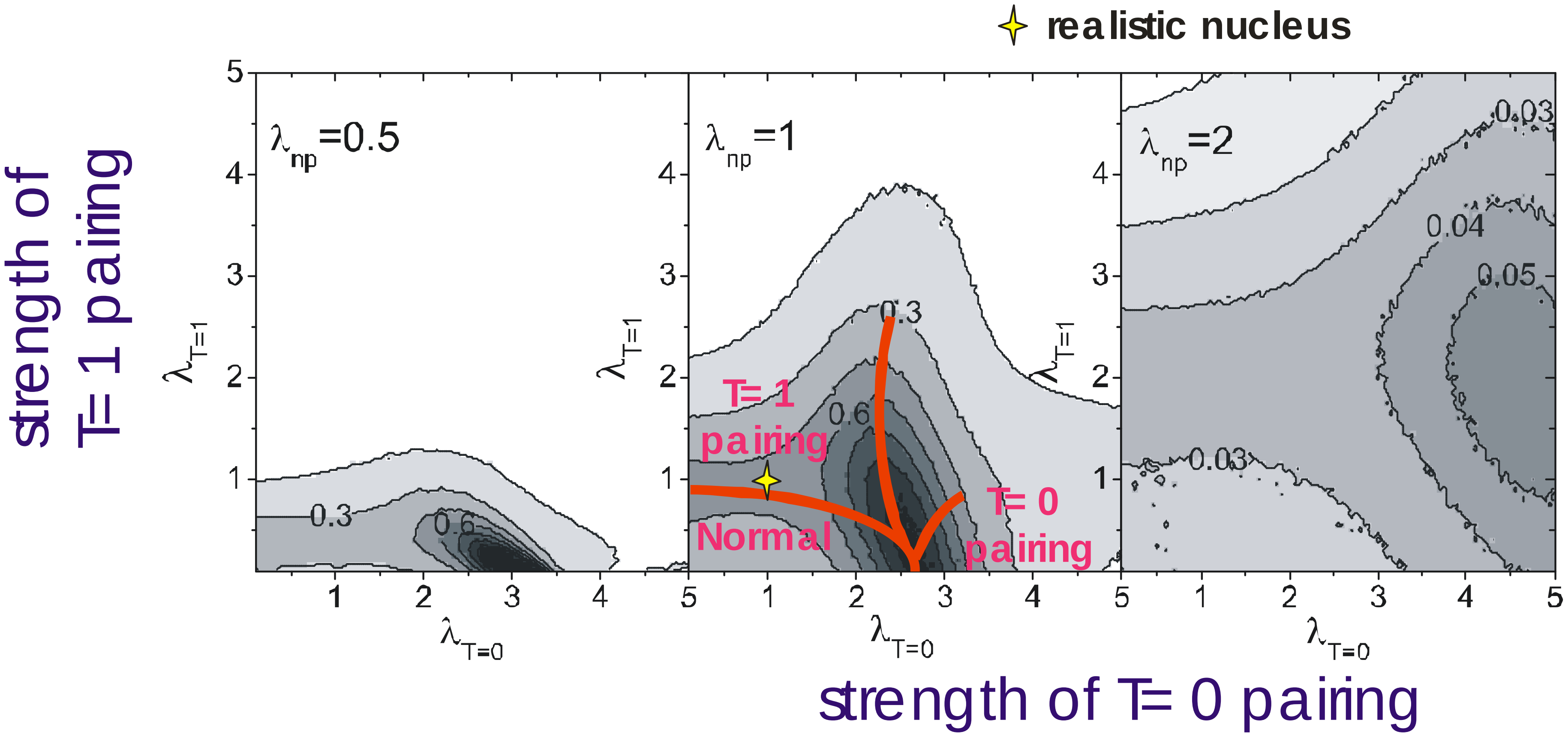


First excited state "pair vibration"

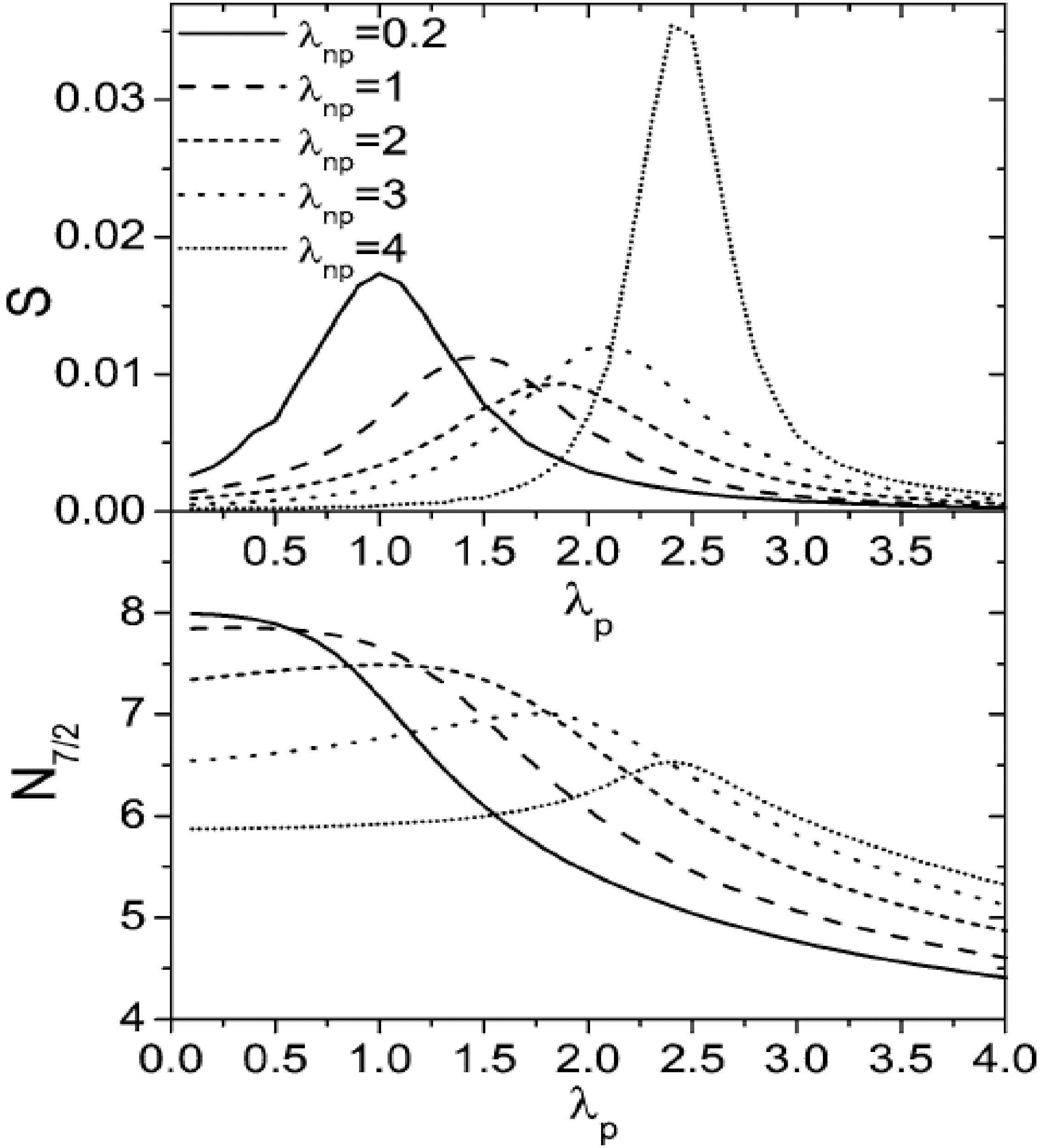
No instability in the exact solution

Softening at the same point 0.3

^{24}Mg phase diagram



Contour plot of invariant correlational entropy showing a phase diagram as a function of T=1 pairing ($\lambda_{T=1}$) and T=0 pairing ($\lambda_{T=0}$); three plots indicate phase diagram as a function of non-pairing matrix elements (λ_{np}) . Realistic case is $\lambda_{T=1}$=$\lambda_{T=0}$ =λ_{np}=1



Shell model 48Ca

Ground state

invariant entropy;

phase transition

depends on

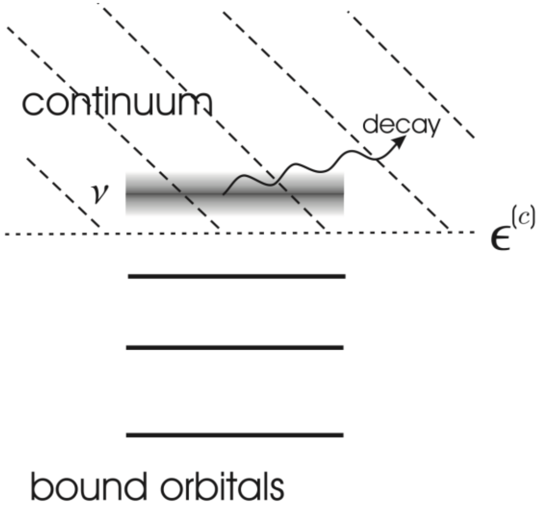
non-pairing

interactions

Occupancy of f7/2 shell

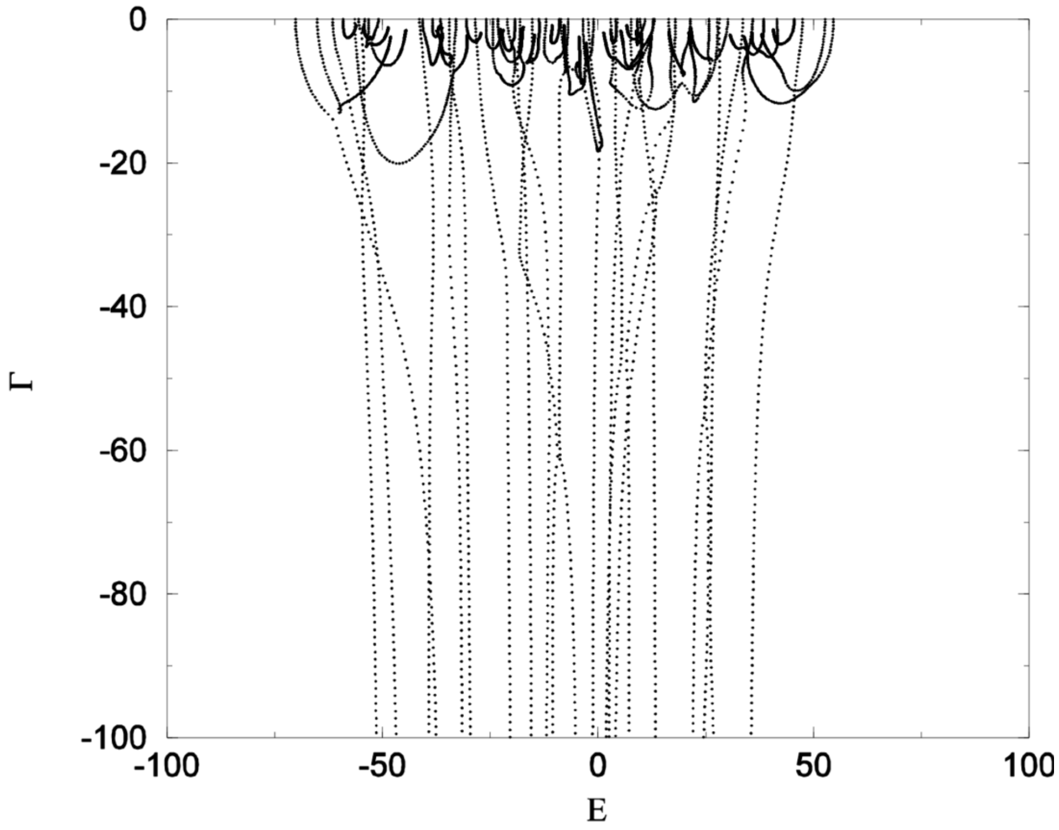
Correlation energy ~ 2 MeV

Single-particle decay in many-body system



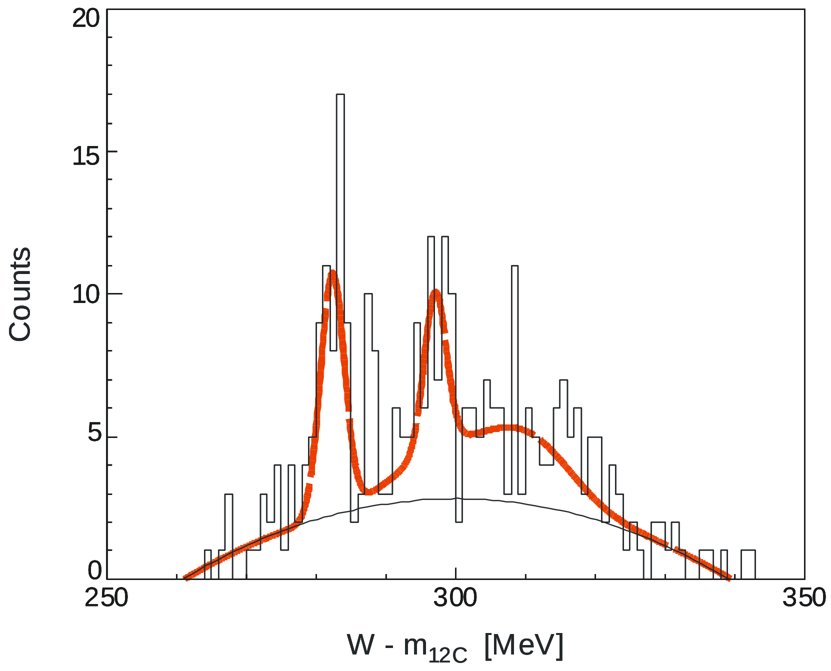
- 8 s.p. levels, 3 particles
- One s.p. level in continuum

Evolution of complex energies

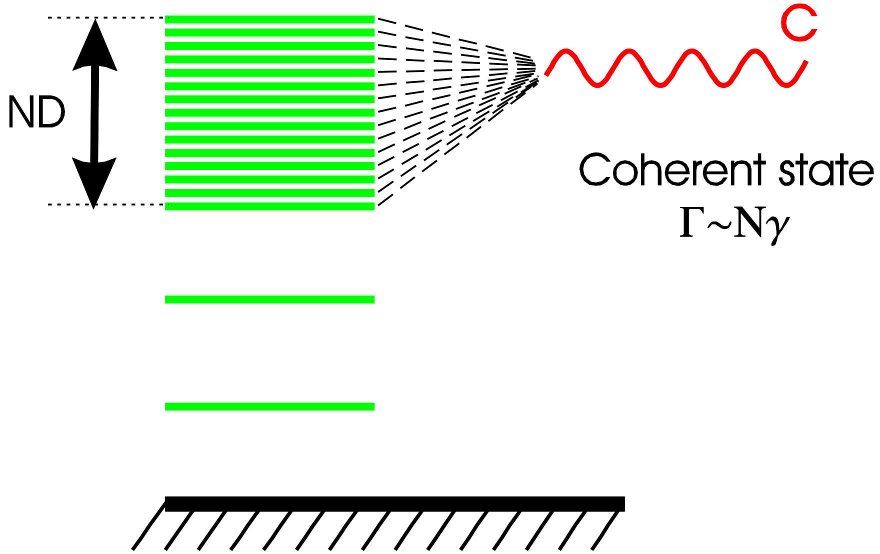


Total states 8!/(3! 5!)=56; states that decay fast 7!/(2! 5!)=21 – **superradiant** doorways

Examples of superradiance



Mechanism of superradiance
Interaction via continuum
Trapped states - self-organization



Narrow resonances and broad superradiant state in ^{12}C in the region of Delta

- Optics
- Molecules
- Microwave cavities
- Nuclei
- Hadrons
- Quantum computing
- Measurement theory

Bartsch *et.al.* Eur. Phys. J. A 4, 209 (1999)
N. Auerbach, V.Z.. Phys. Lett. B590, 45 (2004)

Strong coupling, $\kappa > 1$

k open channels $\Rightarrow$ k nonzero eigenvalues of W.

<u>*Doorway representation*</u>:

$$k = 1 \Rightarrow \Gamma_d = \mathrm{Trace}\, W$$

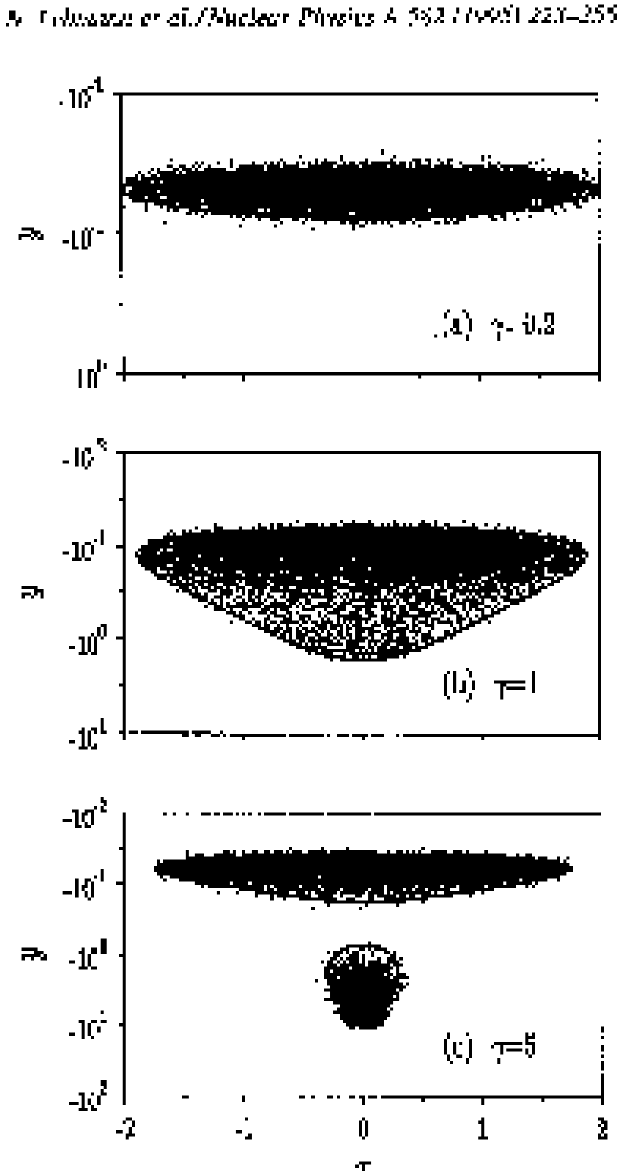
$$\mathcal{H} = \begin{pmatrix} \tilde{\epsilon}_1 - (i/2)\Gamma_d & h_2 & h_3 \\ h_2 & \tilde{\epsilon}_2 & 0 \\ h_3 & 0 & \tilde{\epsilon}_3 \end{pmatrix}$$

Width collectivization:

broad *super-radiant* state $\Gamma_1 \approx \Gamma_d[1 - O(\kappa^{-2})]$,

narrow (*trapped*) states $\Gamma_{2,3} \sim \Gamma_d/[(N-1)\kappa^2]$

Dynamics is determined by alignment
to open decay channels



Super-radiant transition

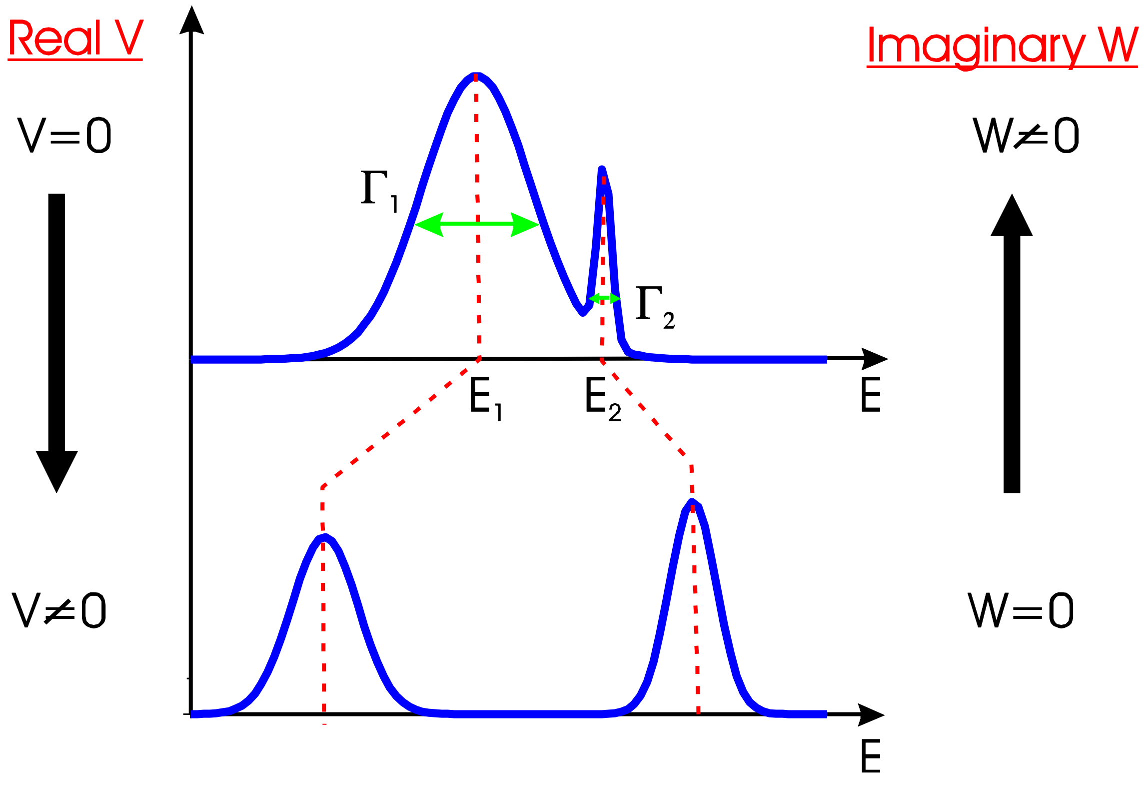
in Random Matrix Ensemble

N= 1000, m=M/N=0.25

Interaction between resonances

- Real V
  - Energy repulsion
  - Width attraction
- Imaginary W
  - Energy attraction
  - Width repulsion

$\mathcal{H}$=H^0+V - iW/2



P. Von Brentano

Interplay of collectivities

Definitions

n - labels particle-hole state

ε_n – excitation energy of state n

d_n - dipole operator

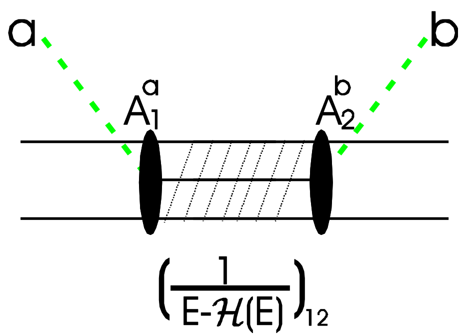
A_n – decay amplitude of n

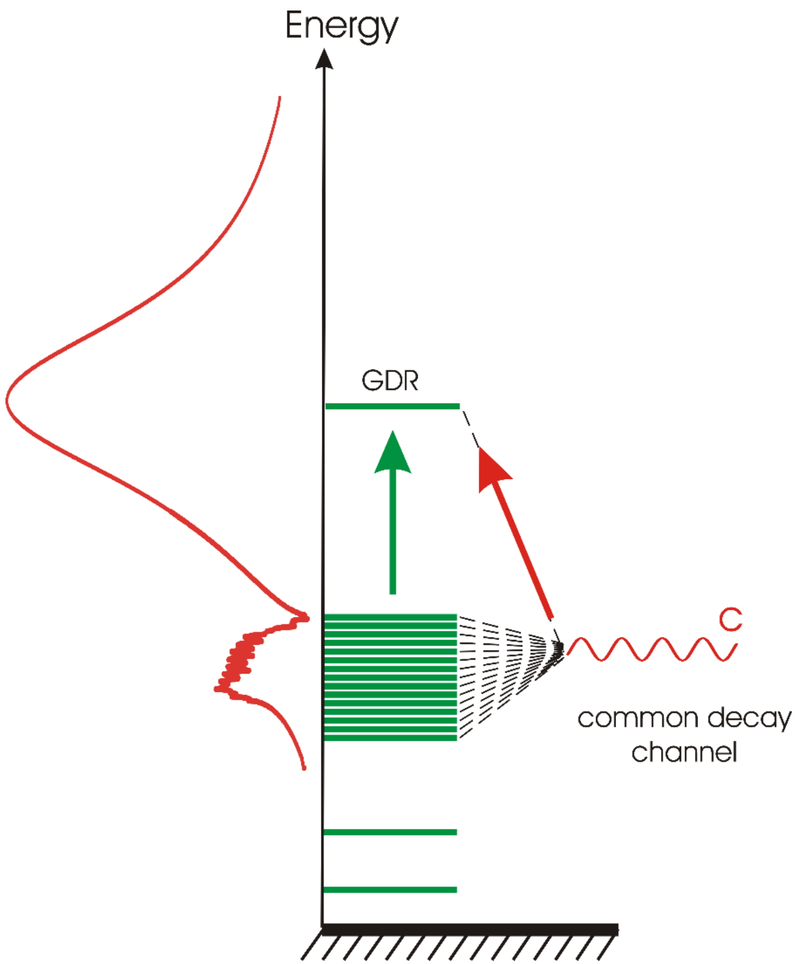
Model Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

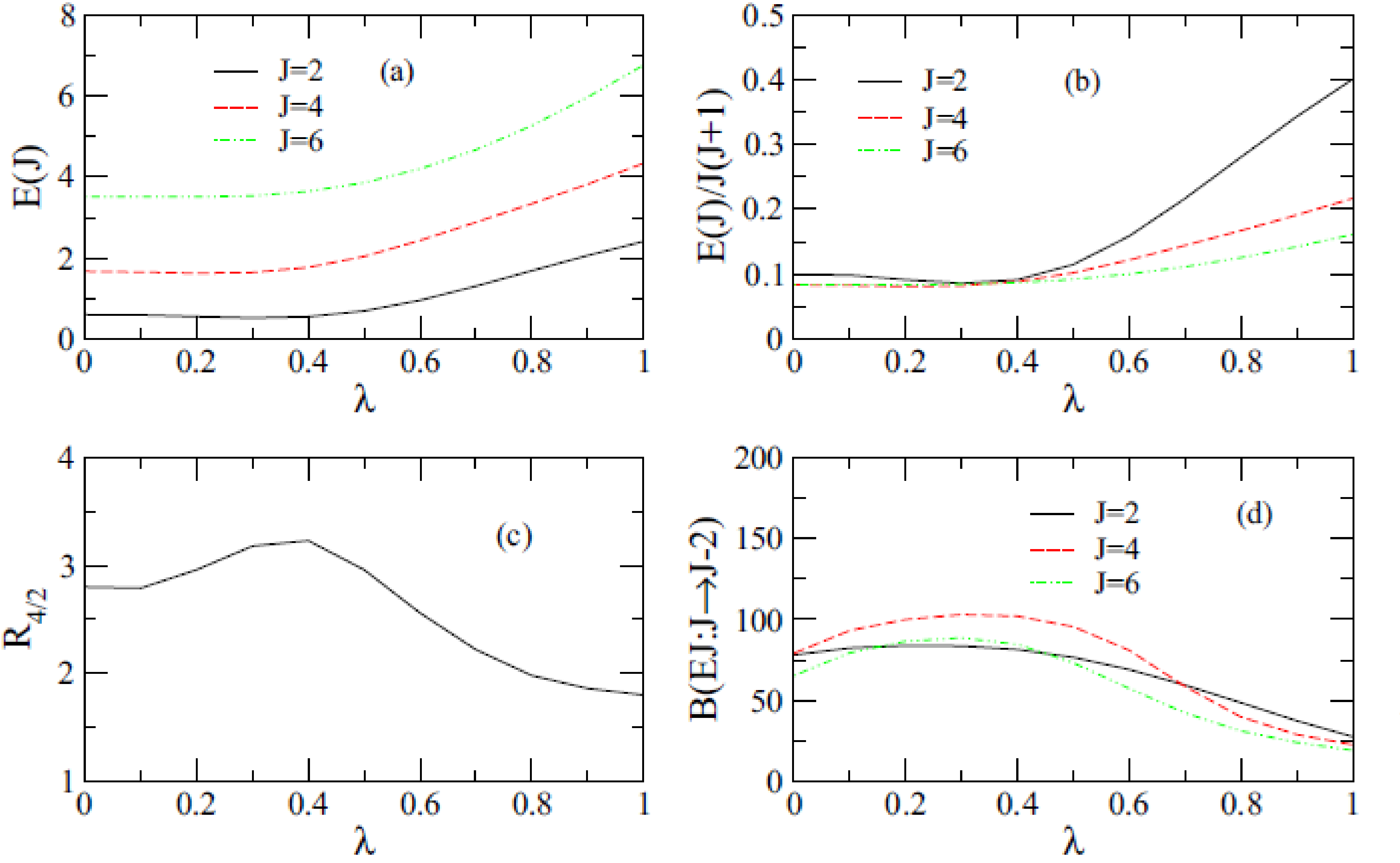
Driving GDR externally

(doing scattering)





Everything depends on angle between multi dimensional vectors **A** and **d**



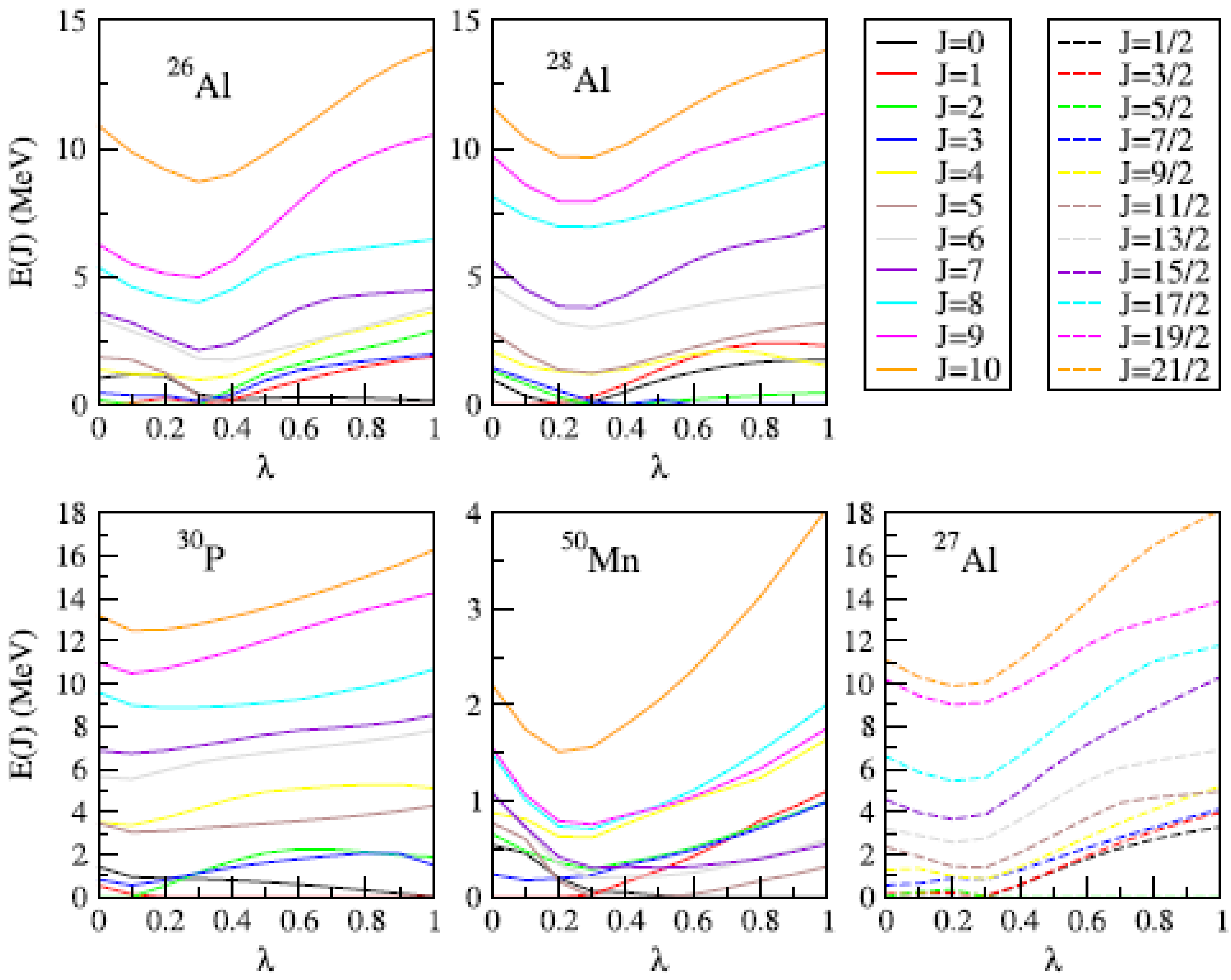
24 Mg

Low-lying levels
in absolute (a)
and rotational (b)
units;

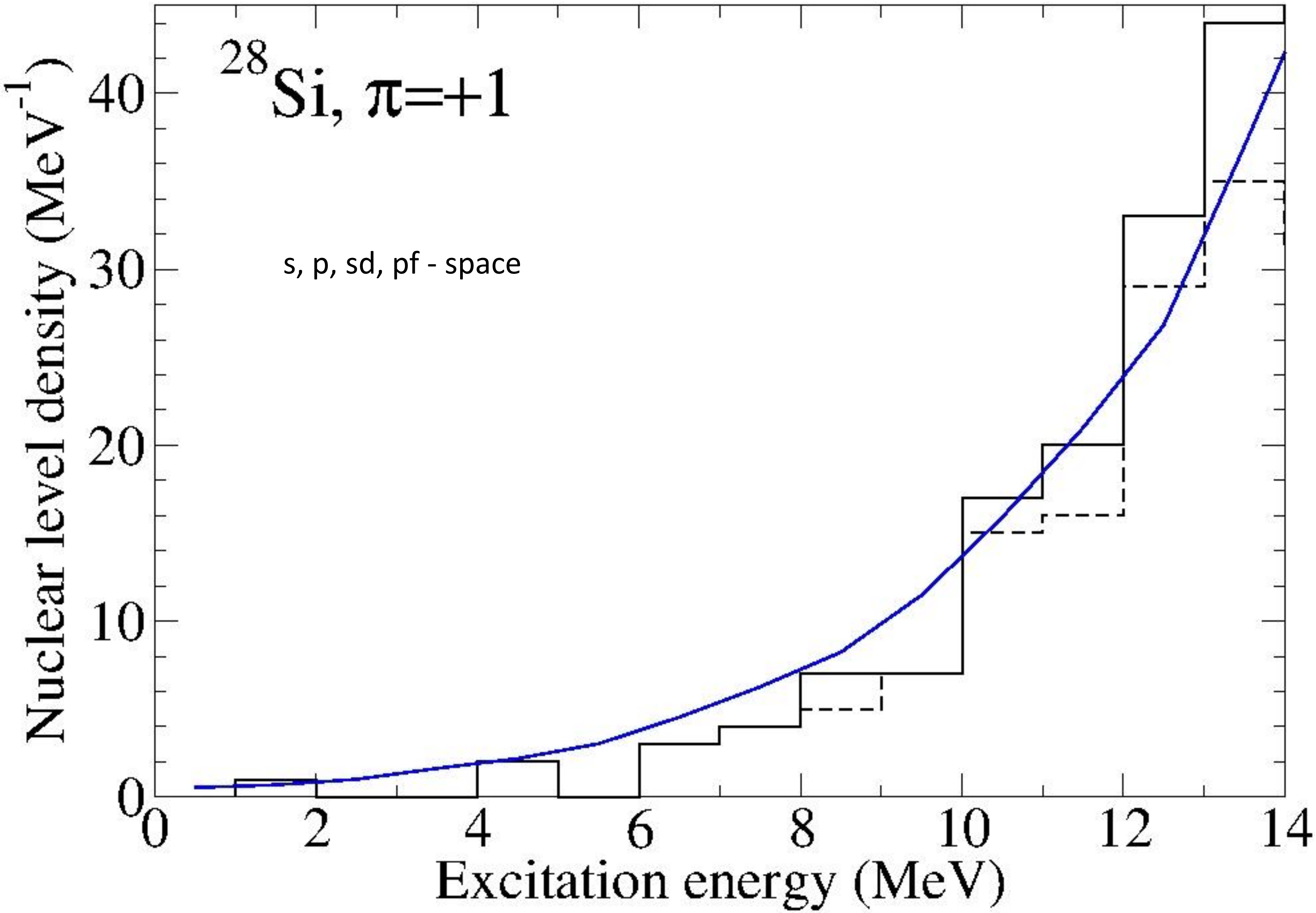
Ratio E(4)/E(2) (c)

Transition rates (d)

$$H = h + (1-\lambda)V_1 + \lambda V_2$$



$$H = h + (1-\lambda)V_1 + \lambda V_2,$$



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Excitation energy (MeV)
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