Analytical description of the excited state phase transition to octupole deformed shape in alternating parity bands

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In nuclei with strong quadrupole deformation, yrast negative parity states 1^-, 3^-, 5^- ... together with the members of the ground state positive parity band form the unified band. However, in most even-even nuclei, at low angular momenta, the negative parity states are shifted up with respect to the positions which they would have in a unified alternating parity band of molecular type. This shift denoted by the parity splitting point out that at low angular momenta nuclei perform the vibrational dynamics in reflection-asymmetric degree of freedom.

The analysis of the experimental data shows that the behavior of the parity splitting and transitional dipole moment with angular momentum looks quite universal. In other words, basing on the general ideas about the reflection-asymmetric mode one can propose simple analytical description of the angular momentum dependence of these quantitites containing a small number of the parameters having a clear physical meaning.

$$H_I = -\frac{\hbar^2}{2B}\frac{d^2}{d\beta_{30}^2} + V_I(<\beta_{20}>, \beta_{30}),$$

$$\xi_I = \sqrt{\frac{B\omega}{\hbar}}\beta_m(I),$$

$$\Psi^{(\pm)}(x) = \frac{\xi^{1/2}}{\beta_m^{1/2}\pi^{1/4}}\frac{1}{2\sqrt{1 \pm \exp\left(-\xi^2\right)}} \times \left(\exp\left[-\frac{1}{2}\xi^2(x+1)^2\right] \pm \exp\left[-\frac{1}{2}\xi^2(x-1)^2\right]\right).$$

$$
\begin{aligned}
H_I &= \hbar\omega h(\xi_I), \\
h(\xi) &= -\frac{1}{2\xi^2}\frac{d^2}{dx^2} + v_\xi(x), \\
v_\xi(x) &= \frac{1}{2}(\xi^2 - 1) + \frac{1}{2}\xi^2 x^2 - \xi^2 x \tanh(\xi^2 x).
\end{aligned} \tag{1}
$$

From the equation above it follows that the parity splitting can be parametrized as

$$
\Delta E(I) \equiv E_I^{(-)} - E_I^{(+)} = \hbar\omega f(\xi_I), \tag{2}
$$

Both limits of small and large ξ are reproduced by one general expression

$$f(\xi) = \frac{\xi^2 e^{-\xi^2}}{2\left[1 + (1 - e^{-\alpha\xi})\frac{\sqrt{\pi}}{4}\xi\right]} \coth\left(\frac{\xi^2}{2}\right), \quad \text{for} \quad \alpha = 0.056, \tag{3}$$

where the value of the parameter α has been obtained by fitting the numerical results for the $f(\xi)$.

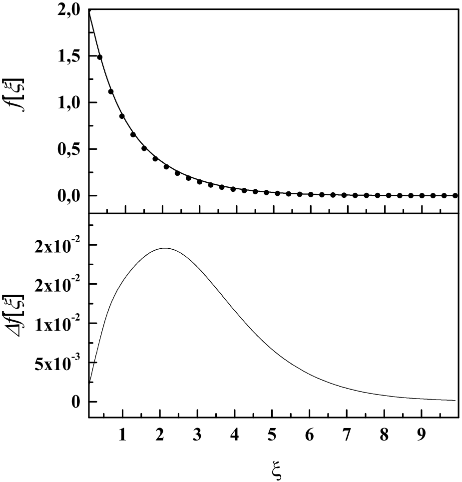

Figure : Upper part: The function $f(\xi)$ obtained by the numerical diagonalization of the Hamiltonian h_I (dots) and with an approximation (solid line). Lower part: difference between an exact and approximated values of function $f(\xi)$.

The angular momentum dependence of the dipole reduced transition probabilitiy has the form

$$B(E1, i \to f) = B(E1, 0^+ \to 1^-) \frac{\xi^2 e^{2\xi^2}}{e^{2\xi^2} - 1}, \qquad (4)$$

where $\xi = \sqrt{\xi_i \xi_f}$.

$$\xi(I) \;=\; cI. \qquad (5)$$

$$cI_{crit} = \frac{1}{\sqrt{2}}. \qquad (6)$$

$$\Delta E(I) = \Delta E_{exp}(0) f \left[\frac{I}{\sqrt{2} I_{crit}} \right]. \qquad (7)$$

$$\Delta E(0) = \hbar\omega, \qquad (8)$$

Table : Calculated (calc.) and experimental (exp.) energies (in keV) of the members of the lowest positive and negative parity bands in ^{240}Pu.

Nucleus	$\Delta E(0)$	c	I_{crit}	Nucleus	$\Delta E(0)$	c	I_{crit}
^{222}Ra	0.209	0.252	2.81	^{238}Pu	0.584	0.053	13.32
^{224}Ra	0.192	0.210	3.37	^{240}Pu	0.585	0.058	12.10
^{226}Ra	0.235	0.150	4.70	^{242}Pu	0.767	0.060	11.77
^{228}Ra	0.456	0.094	7.53	^{244}Pu	0.888	0.047	14.94
^{224}Th	0.226	0.247	2.86	^{230}U	0.351	0.063	11.21
^{226}Th	0.209	0.149	4.88	^{232}U	0.548	0.044	16.20
^{228}Th	0.311	0.094	7.54	^{234}U	0.772	0.031	22.90
^{230}Th	0.492	0.069	10.21	^{236}U	0.674	0.046	15.39
^{232}Th	0.699	0.049	14.50	^{238}U	0.669	0.056	12.67
^{234}Th	0.685	0.060	11.77	^{240}U	0.789	0.058	12.12

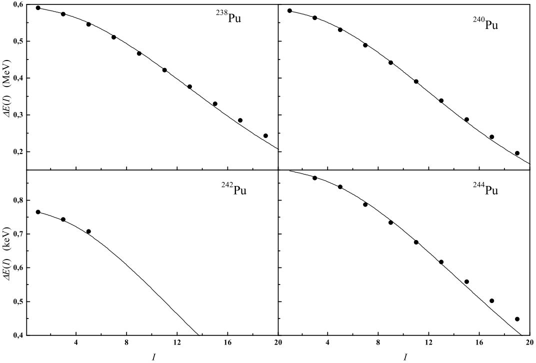

Figure : Parity splitting as a function of angular momentum for various Pu isotopes.

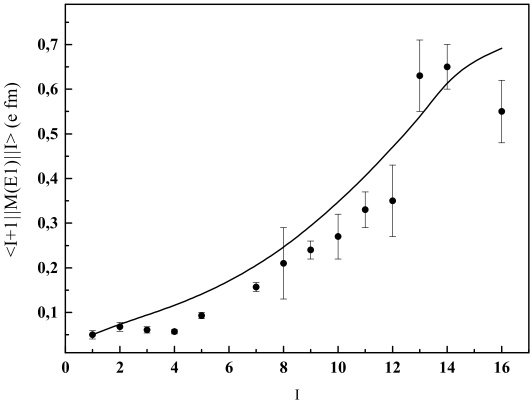

Figure : Dependence of the calculated and experimental values of the transitional dipole moment on the angular momentum obtained for ^{240}Pu.

Summary

Basing on the general ideas about the reflection-asymmetric mode a simple analytical description of the angular momentum dependence of the parity splitting and E1 transition probabilities is proposed containing a small number of the parameters having a clear physical meaning.