ESQPTs and quantum quench dynamics in an extended Dicke model

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Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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Forward quench protocols
Backward quench protocols
Summary
References
Backup

Quantum quench

An abrupt change of the initial Hamiltonian to a *new* final Hamiltonian $H_\mathrm{i} \to H_\mathrm{f}$.

- We follow the evolution of the initial eigenstate $H_\mathrm{i}|\psi_\mathrm{i}\rangle = E_\mathrm{i}|\psi_\mathrm{i}\rangle$ under H_f with eigenbasis $\{|\phi_{\mathrm{f}l}\rangle\}_{l=1}^{d}$ and energies $E_{\mathrm{f}l}$.

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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Figure: ‘Quantum’ tablecloth experiment

Question: *What determines the quench dynamics?*

Theoretical ingredients

Quantum quenching

Quantum quench

An abrupt change of the initial Hamiltonian to a *new* final Hamiltonian $H_{\rm i} \to H_{\rm f}$.

- We follow the evolution of the initial eigenstate $H_{\rm i}|\psi_{\rm i}\rangle = E_{\rm i}|\psi_{\rm i}\rangle$ under $H_{\rm f}$ with eigenbasis $\{|\phi_{{\rm f}l}\rangle\}_{l=1}^{d}$ and energies $E_{{\rm f}l}$.

Survival probability (Loschimdt echo)

$$P(t) = |\langle\psi_{\rm i}|e^{-iH_{\rm f}t}|\psi_{\rm i}\rangle|^2 = \left|\sum_{i=1}^{d} \underbrace{|\langle\psi_{\rm i}|\phi_{{\rm f}l}\rangle|^2}_{|s_l|^2} \mathrm{e}^{-iE_{{\rm f}l}t}\right|^2$$

either

or

$P(t) \approx 1$

$P(t) < 1$

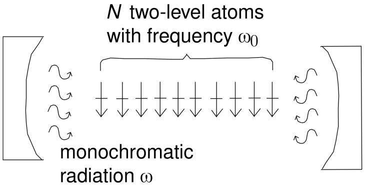
Theoretical ingredients

Extended Dicke model

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

The Hamiltonian

$$H(\lambda,\delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$$



- $b,\ b^\dagger \rightsquigarrow$ photon operators
- $J_z, J_+, J_- \rightsquigarrow$ collective pseudospin operators (atoms)
- $N = 2j \rightsquigarrow$ number of atoms
- $\lambda \in [0,\infty),\ \delta \in [0,1] \rightsquigarrow$ tunable parameters

Theoretical ingredients

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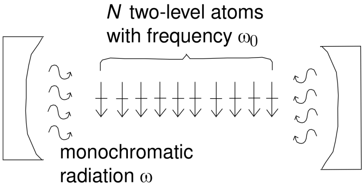

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A simple model but full of interesting physics...

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Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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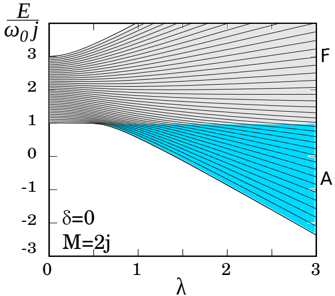
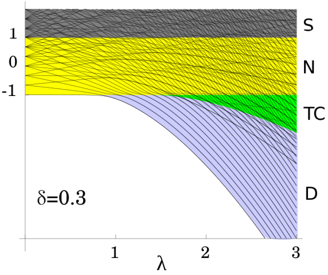
$\delta = 0$

- $M = b^\dagger b + J_z + j$ conserved, hence integrable regime
- Effectively $f = 1$ dof
- M-subspace with (ES)QPT

$\delta \neq 0$

- Partially chaotic, degree of chaoticity grows with δ
- $f = 2$
- QPT and ESQPTs





Forward quench protocols

$H(\lambda, \delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} \left(b J_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_- \right)$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

$$\delta \leftrightsquigarrow \text{ fixed}, \ H_\mathrm{i} \equiv H(\lambda_\mathrm{i}), \ H_\mathrm{f} \equiv H(\lambda_\mathrm{f})$$

Forward quench protocols: $\lambda_\mathrm{i} = 0 < \lambda_\mathrm{f}$

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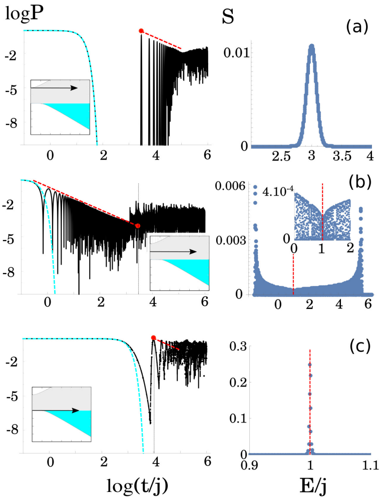

$\boxed{\delta = 0}$ case, $f = 1$

Strength function:

$$S(E) = \sum_l |s_l|^2 \delta(E - E_{\mathrm{f}l})$$

Fourier transform
$P(t) \leftrightarrow S(E)$

Forward quench protocols

$$H(\lambda,\delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$$

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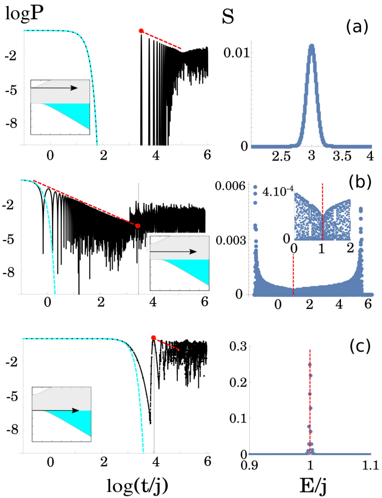

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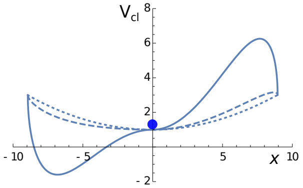

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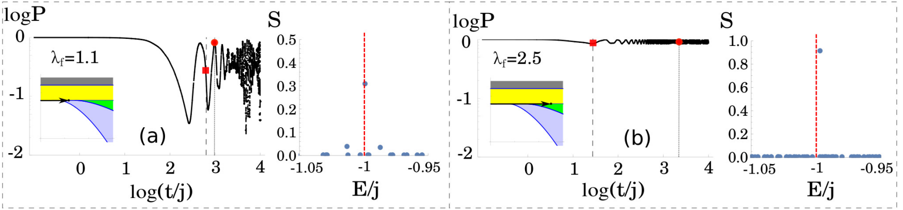
Ingredients
Forward quench protocols
Backward quench protocols
Summary
References

$\boxed{\delta = 0.3}$ case, $f = 2$



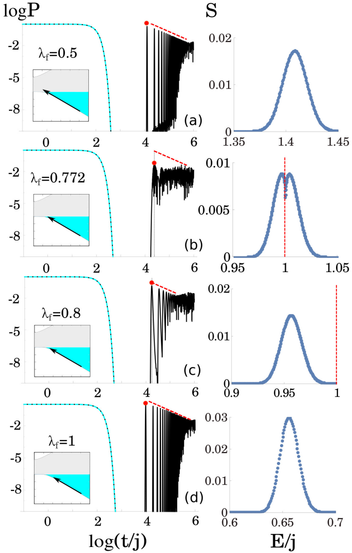
Panel (a): Quench to the ESQPT $(f, r) = (2, 1)$ (saddle point),
Panel (b): Quench to the ESQPT $(f, r) = (2, 2)$ (loc. maximum),

Backward quench protocols

$$H(\lambda,\delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

Backward quench protocols: $\lambda_{\rm f} < \lambda_{\rm i}$, $\boxed{\delta = 0}$ case, $f = 1$.



$\lambda_{\rm i} = 2.5$

Red bullet in $P(t)$ evolution: **Heisenberg time** $t_{\rm H} = 2\pi/\langle \Delta E_{\rm f}\rangle_{\rm i}$ (inverse mean level spacing)

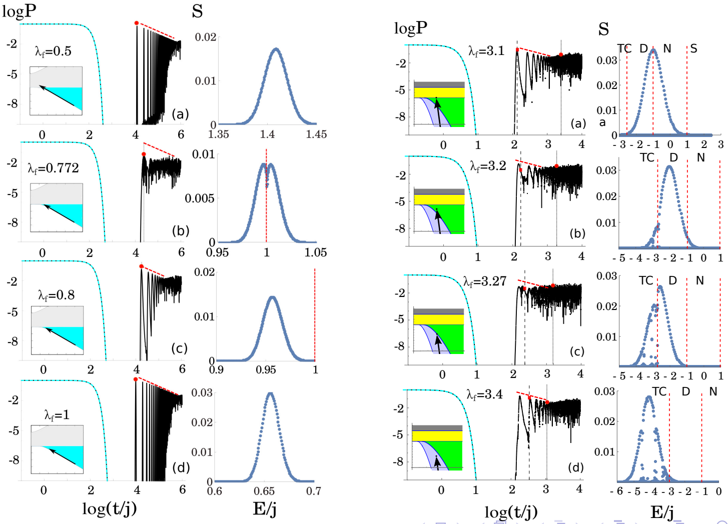
Red dashed lines in $P(t)$ evolution: power-law dependence $1/t$

S. Lerma-Hernández *et al.*, arXiv:1710.05937 [quant-ph] (2017)

Backward quench protocols

$H(\lambda,\delta)=\omega b^\dagger b+\omega_0 J_z+\frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$

$\boxed{\delta=0}$ case, $f=1$ ($\lambda_i=2.5$) versus $\boxed{\delta=0.3}$ case, $f=2$ ($\lambda_i=6$)



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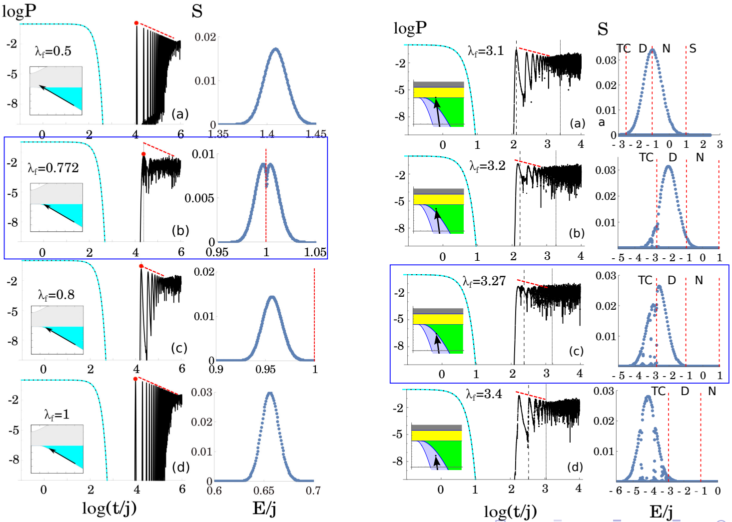

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

Backward quench protocols

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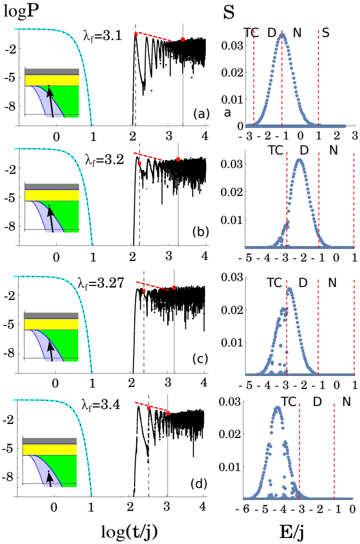

left: $\boxed{\delta=0.3}$ case, $f=2$ ($\lambda_i=6$)
TC-D $(f,r)=(2,1)$, D-N $(f,r)=(2,2)$, N-S $(f,r)=(2,2)$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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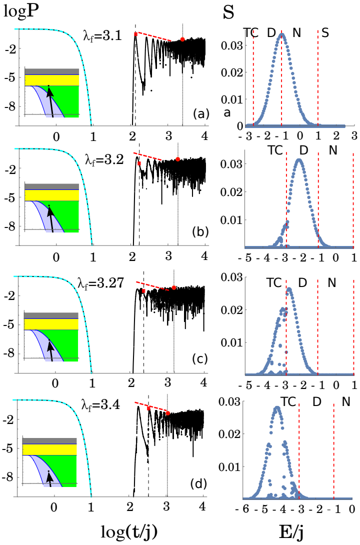
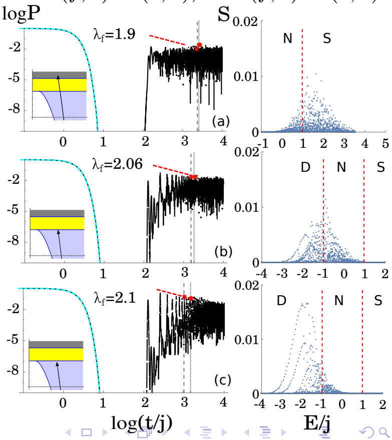
$$H(\lambda, \delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} \left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_- \right)$$



left: $\boxed{\delta = 0.3}$ case, $f = 2$ ($\lambda_i = 6$)
TC-D $(f, r) = (2, 1)$, D-N $(f, r) = (2, 2)$, N-S $(f, r) = (2, 2)$

bottom: $\boxed{\delta = 1}$ case, $f = 2$ ($\lambda_i = 4$)
D-N $(f, r) = (2, 1)$, N-S $(f, r) = (2, 2)$

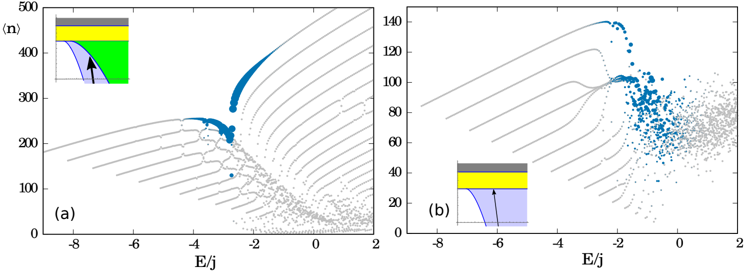


Influence of chaos

$H(\lambda,\delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

Peres lattice: quantum expectation value $\langle\phi_{fl}|\bullet|\phi_{fl}\rangle$ in individual energy eigenstates *vs.* the energy E_{fl}



Distribution of the initial state in the final eigenbasis (strength function) encoded in the size of blue dots.

Influence of chaos

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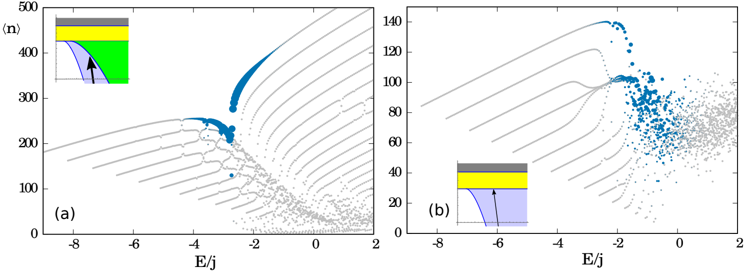

Distribution of the initial state in the final eigenbasis (strength function) encoded in the size of blue dots.

$\rightsquigarrow$ Chaos suppresses the effects of ESQPTs...

Influence of chaos

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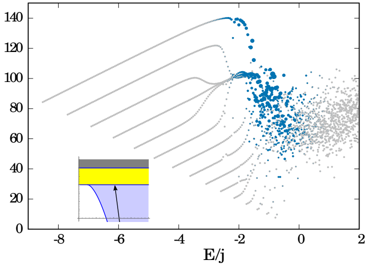
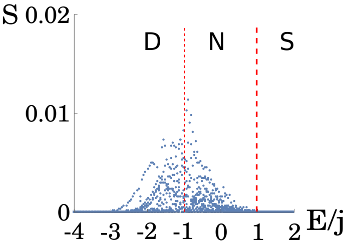
Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

However, if one searches thoroughly...





Influence of chaos

$H(\lambda, \delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} \left(b J_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_- \right)$

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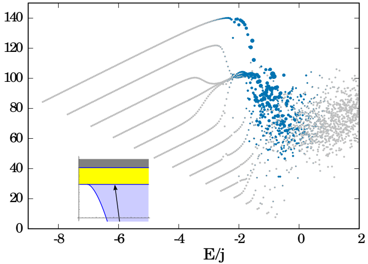

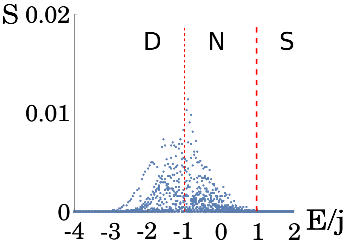

⇝ The fingerprints are hidden in the splitting of the strength function

Influence of chaos

$H(\lambda, \delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} \left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_- \right)$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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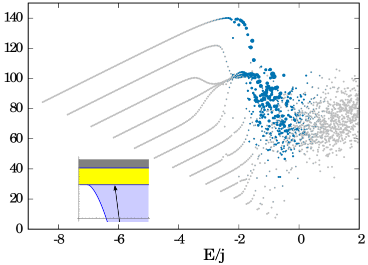
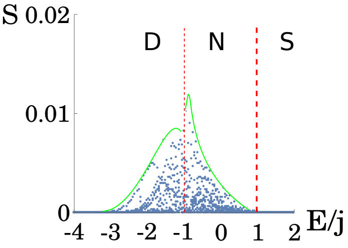




$\rightsquigarrow$ The fingerprints are hidden in the splitting of the strength function

Observables

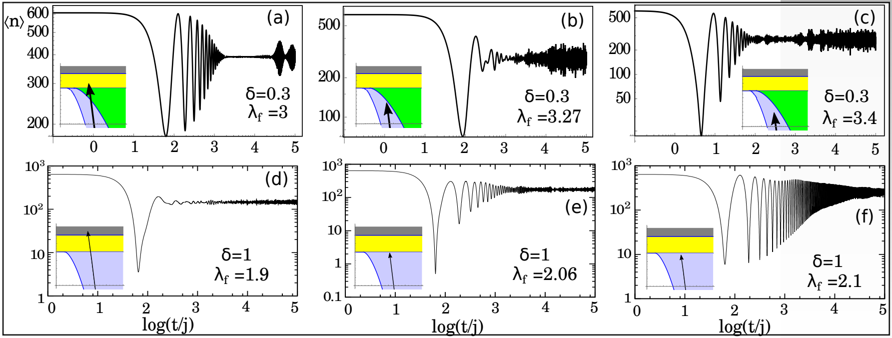
$H(\lambda,\delta) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(bJ_+ + b^\dagger J_- + \delta b^\dagger J_+ + \delta b J_-\right)$

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

$\langle n \rangle$ ⇝ average number of photons in the cavity.

$$\langle n \rangle = \sum_l |s_l|^2 n_{ll} + 2\sum_{l>l'} \mathrm{Re}[s_l s_{l'}^* e^{i\omega_{ll'}t}] n_{ll'},$$

$\omega_{ll'} = E_{fl} - E_{fl'}$, and $n_{ll'} = \langle \phi_{fl} | \hat{n} | \phi_{fl'} \rangle$.



Summary

- In some cases the ESQPT induces a **stabilization** of the initial state. **Conditions:** Forward quenches of the ground state across the QPT to the ESQPT region in both integrable and non-integrable regimes. **Surprise:** In the non-integrable case the stabilization is stronger for longer quenches to the TC phase and weaker for the shorter quenches to the D phase.
- In some other cases, in contrast, the ESQPT induces a **faster onset of the saturation regime** in the survival probability. **Conditions:** Backward quenches from the ground state to the ESQPT regions of different types. In non-integrable systems this effect competes with the effect of chaos.
- Qualitatively similar effect seen in the evolution of the observable. This may suggest a possible detection of some ESQPT-induced effects.

Thank you and References

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

- This talk based on
  M. Kloc, P. Stránský, and P. Cejnar, arXiv: 1805.06285 [quant-ph] (2018)
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  - P. Pérez-Fernández *et al.*, Phys. Rev. A **83**, 033802 (2011).
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Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

Back-up slides

Criticality in a single subspace

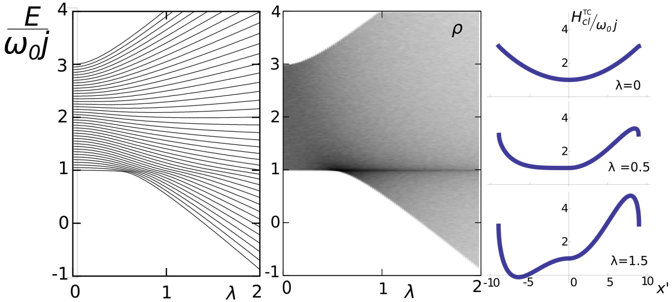

Figure: Energy spectrum and semiclassical level density for a **critical** $M = 2j = 40$ invariant subspace. Other parameters $\omega = 2, \omega_0 = 1$.

Excited-state quantum phase transitions

‘Phase diagram’ of an Extended Dicke model

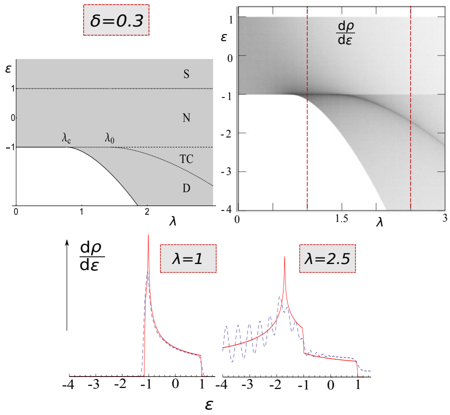

Figure: The derivative of semiclassical level density ρ with respect to $\varepsilon \equiv \frac{E}{\omega_0 j}$

Singularities in level density as a generalization of QPTs → **Excited-state quantum phase transitions** (ESQPTs)

$$\lambda_c = \frac{\sqrt{\omega\omega_0}}{1+\delta}\ ,\ \lambda_0 = \frac{\sqrt{\omega\omega_0}}{1-\delta}$$

‘Phase diagram’ - - →

D - Dicke phase
TC - Tavis-Cummings phase
N - Normal phase
S - Saturated phase

Quenching schemes

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

- Schematic shorthand notation $\rightsquigarrow H^{(1,2)} = H_0 + \lambda^{(1,2)} V$
- $H^{(2)} = H^{(1)} + (\lambda^{(2)} - \lambda^{(1)}) V \equiv H^{(1)} + \Delta\lambda \, V$

Quenching schemes

Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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- $H^{(2)} = H^{(1)} + (\lambda^{(2)} - \lambda^{(1)}) V \equiv H^{(1)} + \Delta\lambda\, V$
- Consider $H^{(1)} |\psi_n^{(1)}\rangle = E_n^{(1)} |\psi_n^{(1)}\rangle$
- Recall *Feynman-Hellmann theorem*

$$\frac{\mathrm{d}E(\lambda)}{\mathrm{d}\lambda} = \langle \frac{\mathrm{d}H(\lambda)}{\mathrm{d}\lambda} \rangle$$

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Ingredients
Forward quench protocols
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Summary
References
Backup

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$$\frac{\mathrm{d}E(\lambda)}{\mathrm{d}\lambda} = \langle \frac{\mathrm{d}H(\lambda)}{\mathrm{d}\lambda} \rangle$$

$$\langle H^{(2)} \rangle_{\psi_n^{(1)}} = E_n^{(1)} + \Delta\lambda \langle V \rangle_{\psi_n^{(1)}} = E_n^{(1)} + \Delta\lambda \frac{\mathrm{d}E_n}{\mathrm{d}\lambda}(\lambda = \lambda^{(1)})$$

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Ingredients
Forward quench protocols
Backward quench protocols
Summary
References
Backup

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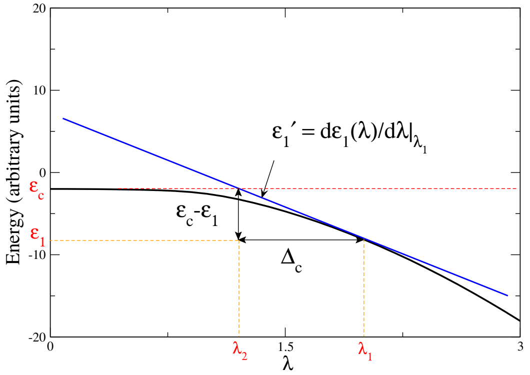

Figure: Visualization of a quench