

Nonequilibrium dynamics of isolated many-body quantum systems



Lea F. Santos

Department of Physics, Yeshiva University, New York, NY, USA



E. Jonathan Torres-Herrera



Curro Pérez-Bernal



Marco Távora



Nonequilibrium dynamics of many-body quantum systems

How **fast** can isolated interacting quantum systems evolve?

How does the dynamics depend on the **time scale**?

Dynamics

How does the evolution depend on the initial state, **perturbation**?

Is the dynamics affected by **critical points**?

How does the dynamics depend on the **Hamiltonian**?
(interactions, chaos)

METAL-INSULATOR

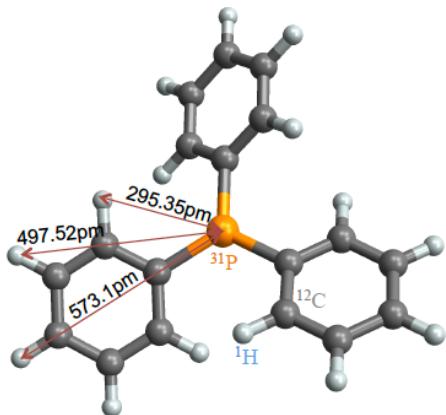
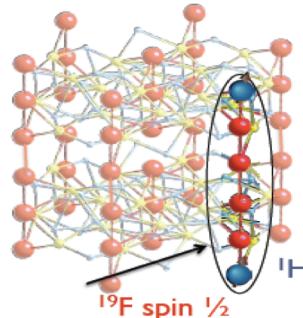
Coherent Evolution in Experiments (Quantum simulators)

NMR

They are collectively addressed with magnetic pulses;
Slow relaxation

Li, Fan, Wang, Zeng, Zhai, Peng, Jiangfeng
(Beijing, Anhui, Hubei)

Cory (Waterloo)
Cappellaro (MIT)

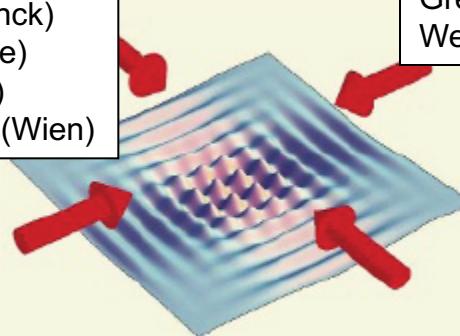


Ultracold Gases

Dynamics under designed potentials.

Bloch (Max Planck)
Fallani (Florence)
Esslinger (ETH)
Schmiedmayer (Wien)

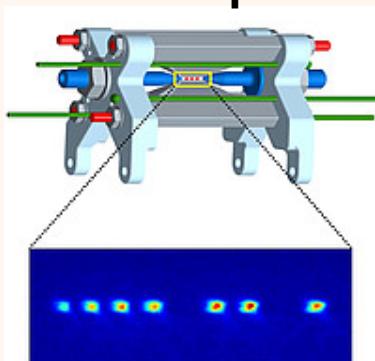
Greiner (Harvard)
Weiss (Penn State)



- highly controllable systems – interactions, level of disorder, 1,2,3D (simple models)
- quasi-isolated -- study evolution for very long time

Ion Traps

Ions trapped via electric and magnetic fields.
Laser used to induce couplings.
Isolated from an external environment.



Blatt (Innsbrück)

Monroe (Maryland)

SYSTEM MODELS

1D spin-1/2

Hardcore bosons, qubits

Integrable spin $\frac{1}{2}$ models

Noninteracting Integrable system:

XX model (1D)

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

Holstein-Primakoff

Map into hardcore bosons:

$$\sigma_J^+ = \hat{b}_J^\dagger \sqrt{1 - \hat{b}_J^\dagger \hat{b}_J}, \quad \sigma_J^- = \sqrt{1 - \hat{b}_J^\dagger \hat{b}_J} \hat{b}_J,$$

$$H_{XX} | \uparrow\downarrow \rangle = \frac{J}{2} | \downarrow\uparrow \rangle$$

$$H = -t \sum_{n=1}^{L-1} (b_n^\dagger b_{n+1} + b_{n+1}^\dagger b_n)$$

Integrable spin $\frac{1}{2}$ models

Interacting Integrable system:

XXZ model (1D)

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

Map into hardcore bosons:

Holstein-Primakoff

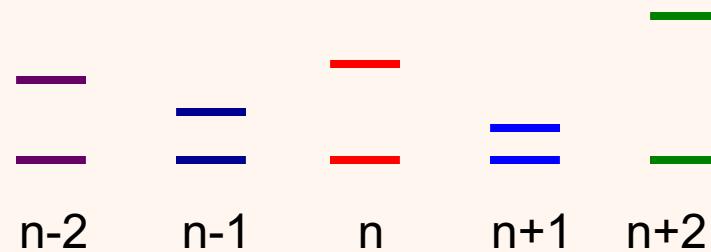
$$H = \sum_{n=1}^L \left[V \left(b_n^+ b_n - \frac{1}{2} \right) \left(b_{n+1}^+ b_{n+1} - \frac{1}{2} \right) - t \left(b_n^+ b_{n+1} + h.c. \right) \right]$$

$$\sigma_j^+ = \hat{b}_j^\dagger \sqrt{1 - \hat{b}_j^\dagger \hat{b}_j}, \quad \sigma_j^- = \sqrt{1 - \hat{b}_j^\dagger \hat{b}_j} \hat{b}_j,$$

$$\sigma_j^z = \hat{b}_j^\dagger \hat{b}_j - 1/2,$$

1D Disordered Spin-1/2 Model

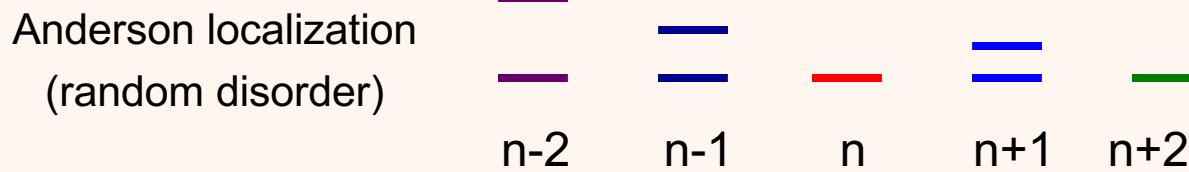
$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$



Disorder strength: h
[-h,h]

Anderson Localization

$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\boxed{\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y})$$

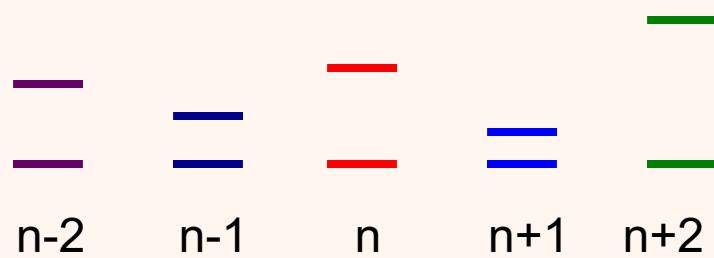


Disorder strength: h
[-h, h]

Chaotic Model

$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

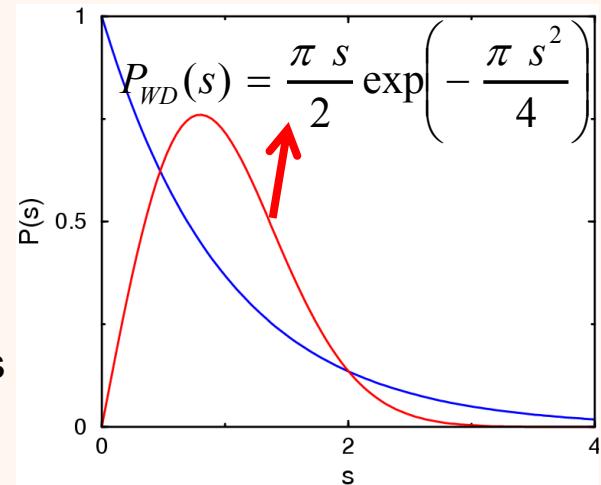
Many-body localization



Disorder strength: h
[-h, h]

Disorder strength $h \sim J$

Wigner-Dyson distribution
(time reversal symmetry)



Level repulsion
= quantum chaos

Chaotic Model

$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

PARTICIPATION RATIO

$$PR^{(\alpha)} = \frac{1}{\sum_{i=1}^D |c_i^{(\alpha)}|^4}$$

$PR \propto Dim$

$$\psi^{(\alpha)} = \sum_{i=1}^D c_i^{(\alpha)} \phi_i$$

BASIS
(site-basis vectors)

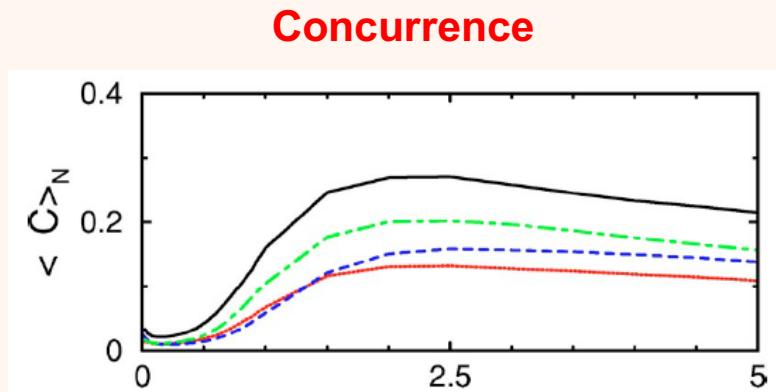
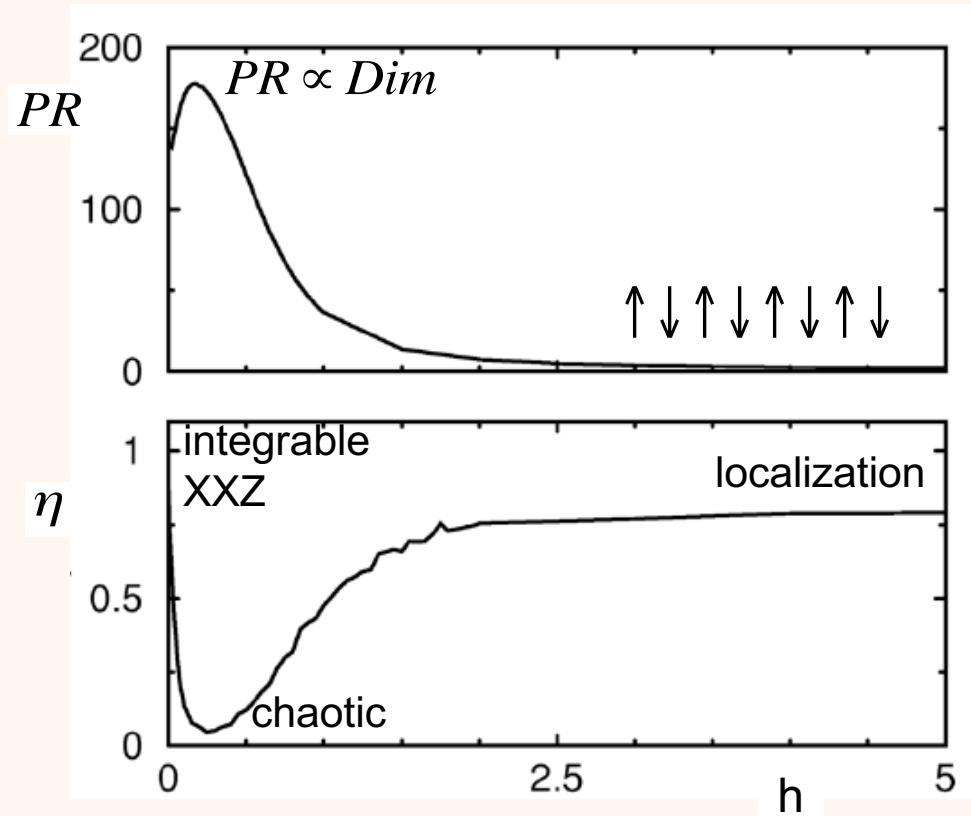
$$|\psi^{(\alpha)}\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \dots \end{pmatrix}$$

$|\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\rangle$

$|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$

Localization and entanglement

$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

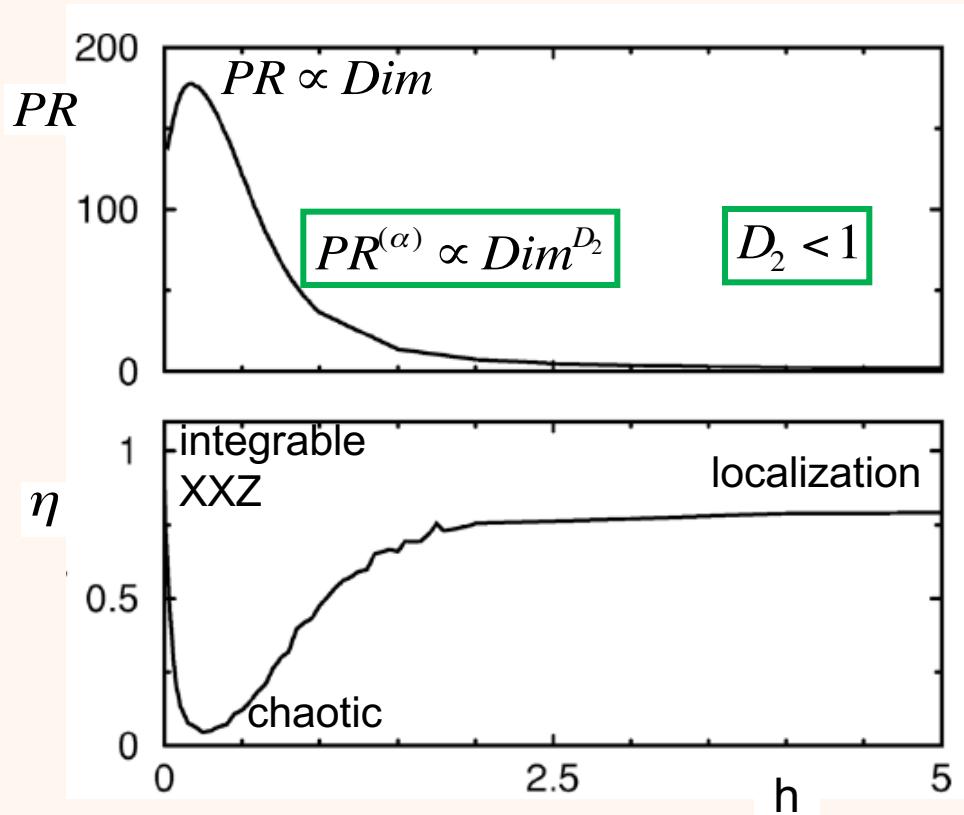


$$\eta = \frac{\int_0^{s_0} [P(s) - P_{WD}(s)] ds}{\int_0^{s_0} [P_p(s) - P_{WD}(s)] ds},$$

LFS, Rigolin, Escobar PRA (2004)

Integrable-chaos-integrable

$$H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$



Participation Ratio

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^D |c_i^{(\alpha)}|^4}$$

$$\psi^{(\alpha)} = \sum_{i=1}^D c_i^{(\alpha)} \phi_i$$

$$\eta = \frac{\int_0^{s_0} [P(s) - P_{WD}(s)] ds}{\int_0^{s_0} [P_P(s) - P_{WD}(s)] ds},$$

Torres & LFS,
 PRB **92**, 01420 (2015)
 Ann. Phys. **529**, 1600284 (2017)

DYNAMICS

Survival Probability (Fidelity)

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

$$H|\alpha\rangle = E_{\alpha}|\alpha\rangle$$

Eigenvalues and eigenstates
of the final Hamiltonian

Overlap between the initial state and the evolved state

$$F(t) = \left| \langle \Psi(0) | \Psi(t) \rangle \right|^2 = \left| \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 e^{-iE_{\alpha}t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} dE \right|^2$$



$$\rho_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 \delta(E - E_{\alpha})$$

Fourier transform

 of the weighted energy distribution of the initial state
of the L DOS (local density of states)

Strong Perturbation

$$\begin{array}{c} |\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\rangle \\ |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle \end{array}$$

$$\longrightarrow H = \sum_{n=1}^L \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

$$H_{ini} = \sum_{n=1}^{L-1} \frac{J\Delta}{4} \sigma_n^z \sigma_{n+1}^z$$

$$|\Psi(0)\rangle = |ini\rangle$$

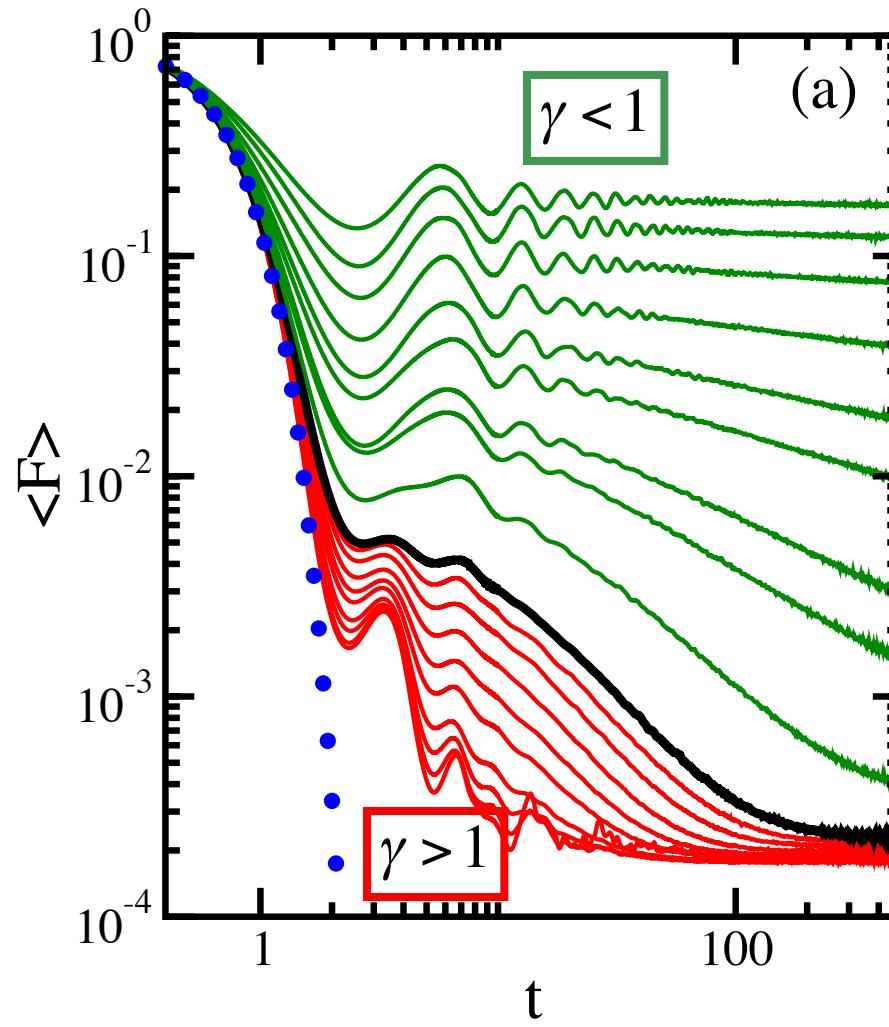
quench parameter

$\Delta \rightarrow \infty$ to $\Delta \rightarrow finite$

Very strong perturbation

Power-law exponent: energy bounds

$$t^{-\gamma}$$



$$PR^{(\alpha)} \propto Dim^{D_2}$$

$$h > J$$

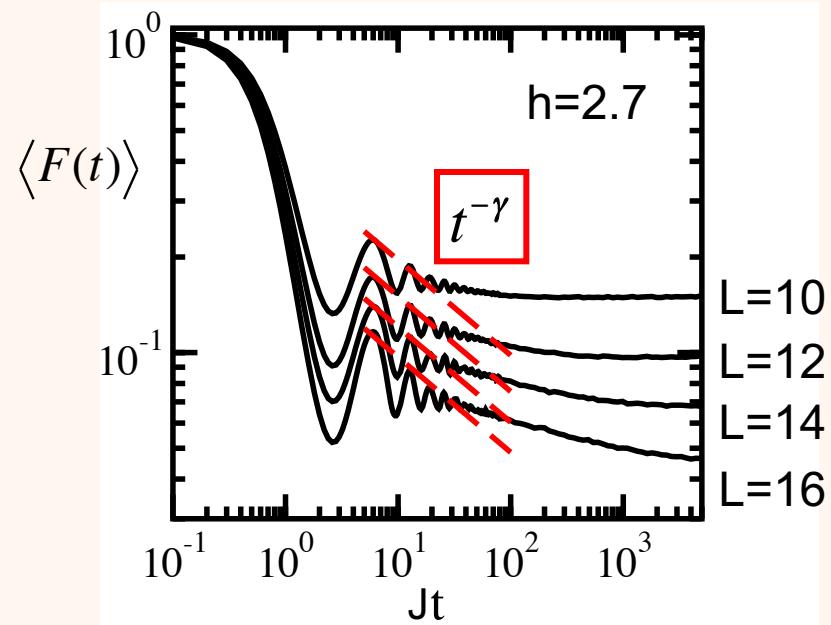
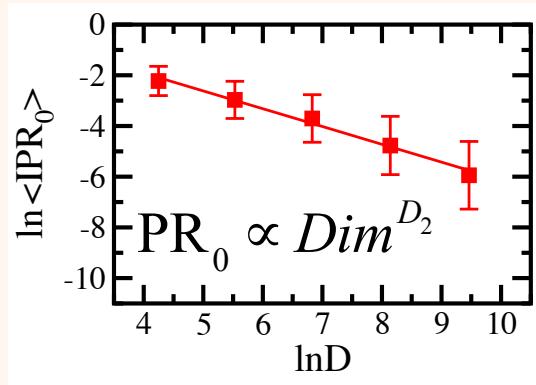
$$t^{-\gamma}$$

$$h < J$$

$$PR^{(\alpha)} \propto Dim$$

Power-law exponent: correlations

$$PR^{(\alpha)} \propto Dim^{D_2} \quad D_2 < 1$$



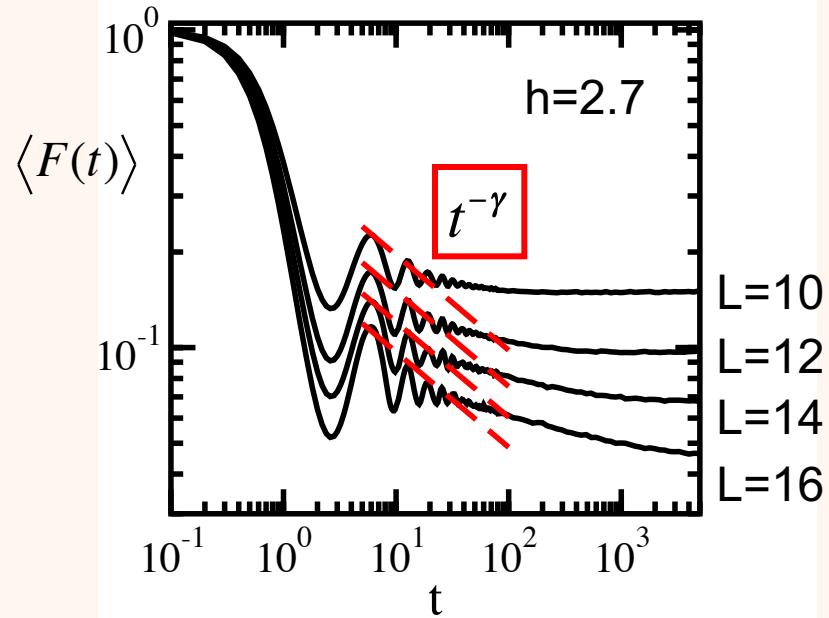
Power-law exponent: correlations

$$PR^{(\alpha)} \propto Dim^{D_2}$$

$$D_2 < 1$$

$$F(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

$$F(t) = \sum_{\alpha, \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t}$$



$$PR_q^{(\alpha)} \propto Dim^{(q-1)D_q}$$

Multifractality = nonlinear dependence of the generalized dimension on q

Lea F. Santos, Yeshiva University

Torres & LFS
PRB 92, 01420 (2015)

QPTN9, Padova, Italy 2018

Power-law exponent: correlations

$$PR^{(\alpha)} \propto Dim^{D_q}$$

$$D_2 < 1$$

$$F(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2$$

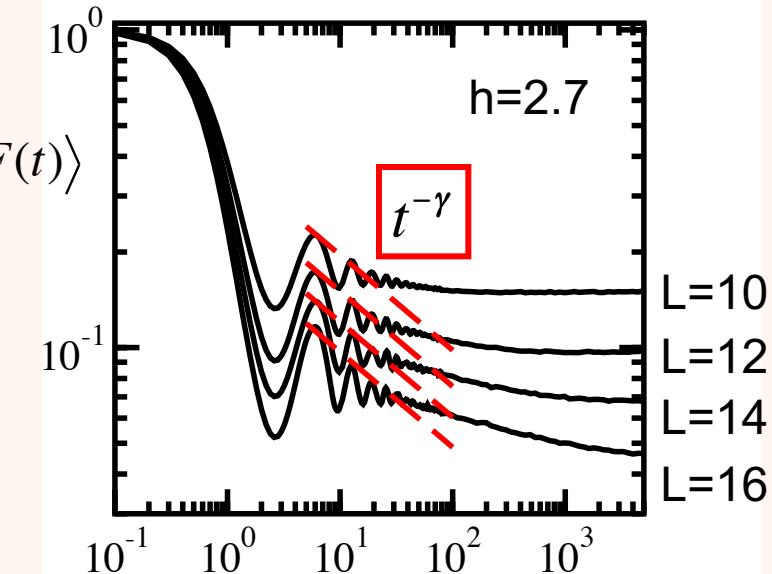
$$\begin{aligned} F(t) &= \sum_{\alpha, \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha} - E_{\beta})t} \\ &= \int G(E) e^{-iEt} dE \rightarrow t^{-\gamma} \end{aligned}$$

$$G(E) = \sum_{\alpha, \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 \delta(E - (E_{\alpha} - E_{\beta})) = |E|^{\gamma-1}$$

$$PR_q^{(\alpha)} \propto Dim^{(q-1)D_q}$$

Multifractality = nonlinear dependence of the generalized dimension on q

Lea F. Santos, Yeshiva University



Torres & LFS
PRB 92, 01420 (2015)

QPTN9, Padova, Italy 2018

POWER-LAW

ESQPT

Anderson model

$$H = J \sum_n (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) + \sum_n h_n \sigma_n^z$$

Aubry-André model

$$H = -J \sum_{n=1}^{L-1} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) + \lambda \sum_{n=1}^L \cos(2\pi\xi n) \sigma_n^z \quad \xi = (\sqrt{5} + 1)/2$$

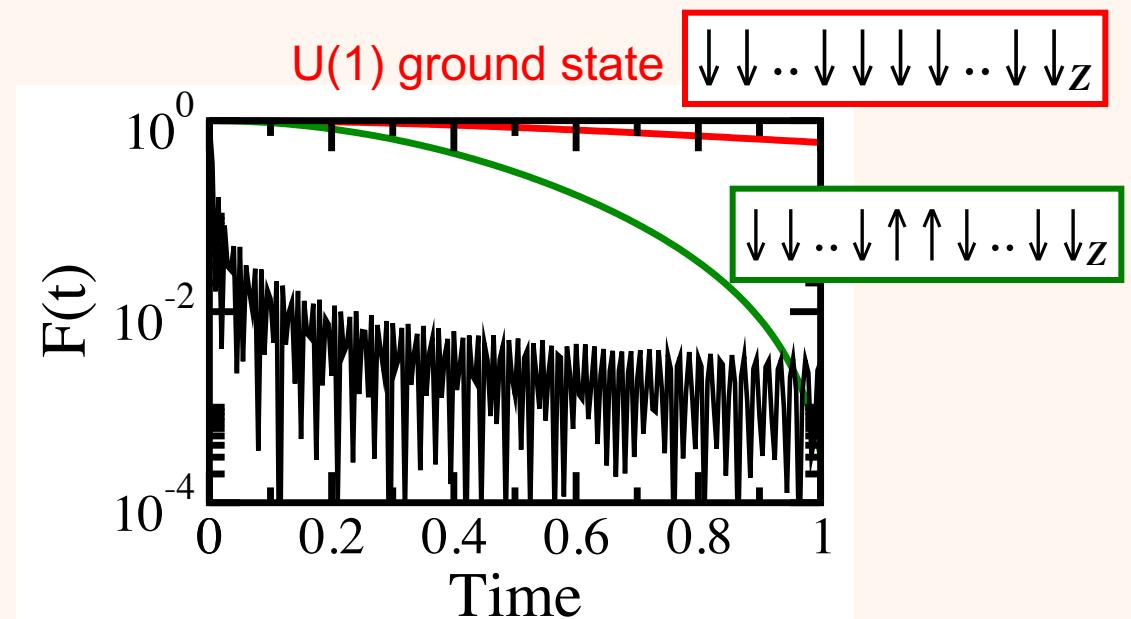
Fibonacci model

$$H = -J \sum_{n=1}^{L-1} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) + \sum_{n=1}^L V_n \sigma_n^z$$

Power-law decay and ESQPT

Lipkin model

$$|\Psi(0)\rangle = |S\ m_z\rangle \longrightarrow H_{U(2)} = (1 - \xi) \left(\frac{N}{2} + S_z \right) - \frac{4\xi}{N} S_x^2$$



LFS, Tavora, Bernal,
PRA **94**, 012113 (2016)

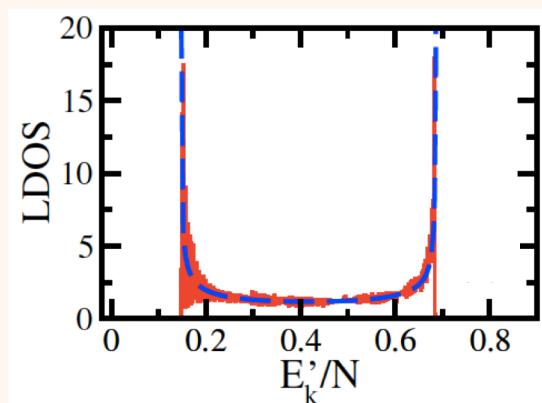
Power-law decay and ESQPT

Initial state $|\Psi(0)\rangle = |S\ m_z\rangle = \sum_k C_{s_z}^{(k)} |\psi_{U(2)}^{(k)}\rangle$

$$\rho_{ini}(E) = \sum_{\alpha} |C_{\alpha}^{ini}|^2 \delta(E - E_{\alpha})$$

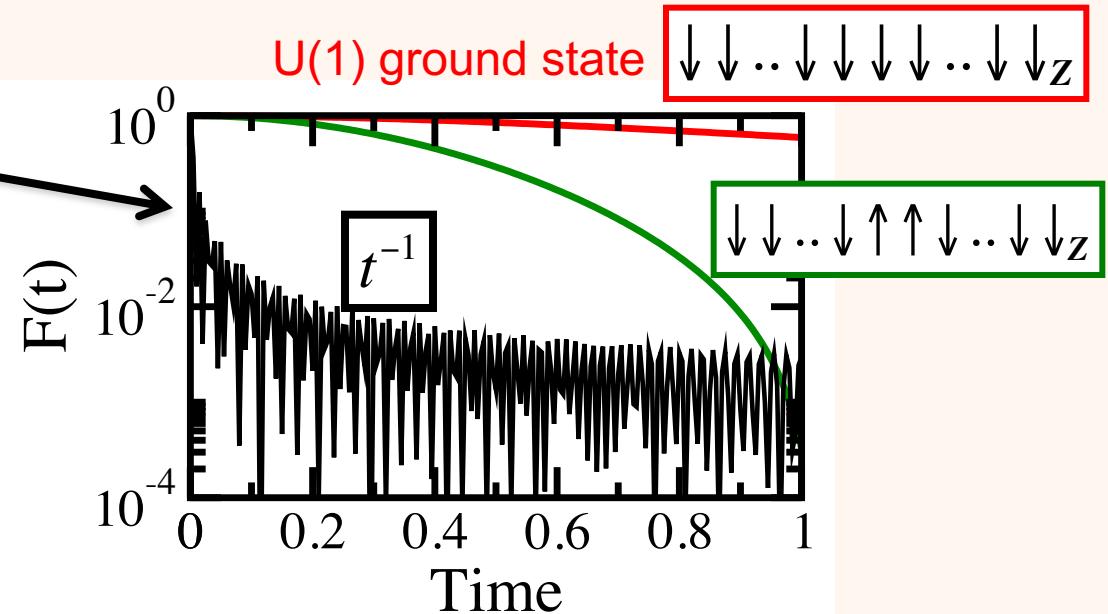
Survival Probability

$$F(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2 = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha}t} \right|^2 \approx \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} dE \right|^2$$



$\xi = 0.6$

$N=1000$



Open Questions

- Power-law decay vs ESQPT

LFS, Távora, Bernal,
PRA **94**, 012113 (2016)



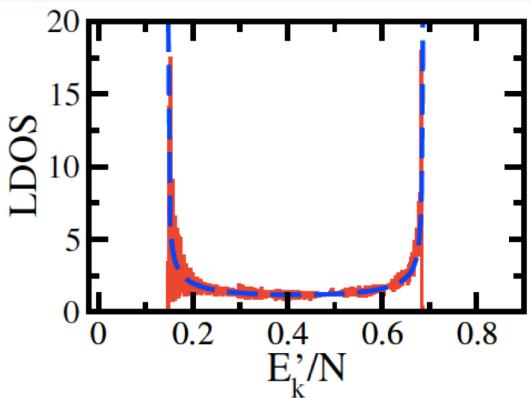
LMG model vs XX model

LMG model (infinite-range interaction)

$$|\Psi(0)\rangle = |S\ m_z\rangle = \sum_k C_{s_z}^{(k)} |\psi_{U(2)}^{(k)}\rangle$$

$$H_{U(2)} = (1 - \xi) \left(\frac{N}{2} + \mathcal{S}_z \right) - \frac{4\xi}{N} \mathcal{S}_x^2$$

LDOS for **delocalized** state at ESQPT



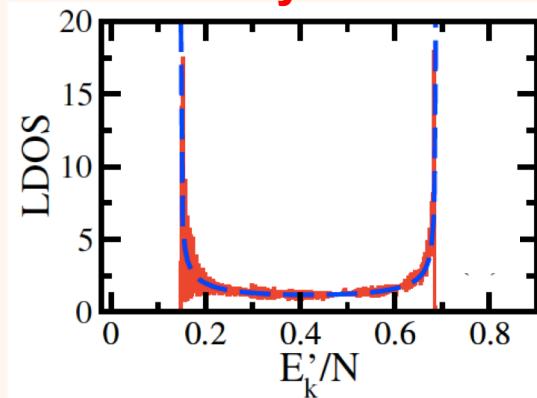
$$\rho_{ini}(E) = \frac{1}{\pi \sqrt{J^2 - E^2}}$$

XX model (nearest-neighbor coupling)
with 1 excitation

$$|\Psi(0)\rangle = |\uparrow\downarrow\downarrow\downarrow\dots\rangle$$

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

LDOS for **any** initial state in z



Open Questions

- Power-law decay vs ESQPT
- Power-law decay: short vs long-range couplings

LFS, Távora, Bernal,
PRA **94**, 012113 (2016)



Open Questions

- Power-law decay vs ESQPT

LFS, Távora, Bernal,
PRA **94**, 012113 (2016)



- Power-law decay: short vs long-range couplings

- Power-law decay: few vs many-body

Schiulaz, Távora, LFS
arXiv:1802.08691

Open Questions

- Power-law decay vs ESQPT

LFS, Távora, Bernal,
PRA **94**, 012113 (2016)



- Power-law decay: short vs long-range couplings

- Power-law decay: few vs many-body

Schiulaz, Távora, LFS
arXiv:1802.08691

Many-body quantum systems close to the metal-insulator transition

- Power-law decay caused by correlations between the eigenstates (fractal)

Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. **529**, 1600284 (2017)