Smearing out of the ESQPT properties in the dissipative Lipkin-Meshkov-Glick (LMG) model and their restore by delayed feedback control

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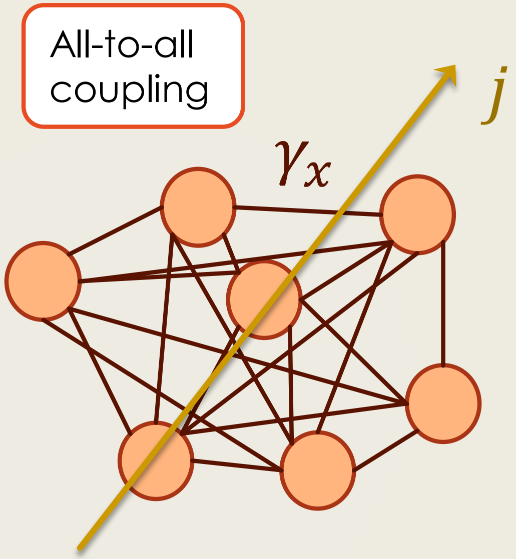
Lipkin-Meshkov-Glick Model (LMG) [1]

$$\hat{H}_{LMG} = -h\hat{J}_z - \frac{\gamma_x}{N}\hat{J}_x^2$$



- $\hat{J}_{x,y,z} = \frac{1}{2}\sum_{k=1}^{N}\hat{\sigma}_{x,y,z}^{(k)}$ collective spin operators
- γ_x – interaction strength
- N – number of atoms
- $\hat{J}^2$ – conserved
- restriction $j = N/2$

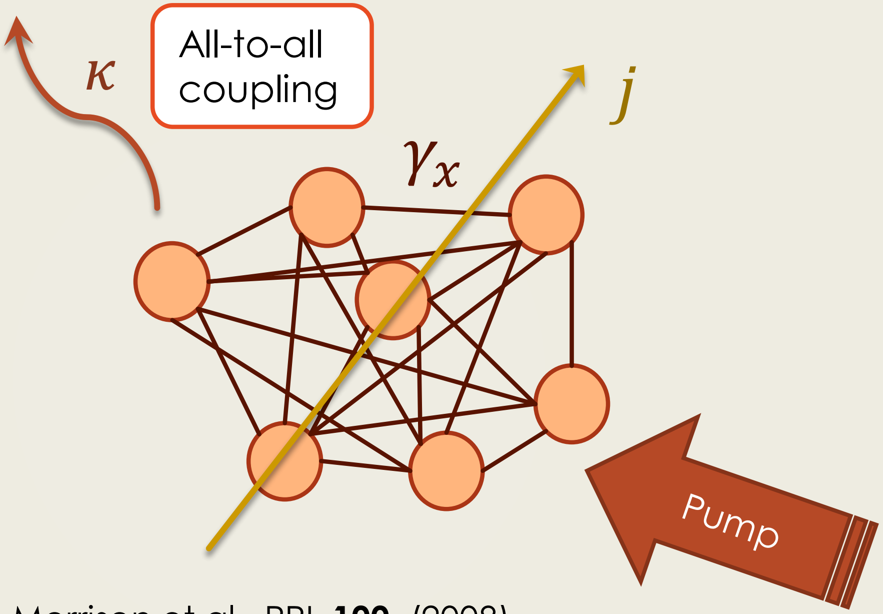
[1] H.J. Lipkin et al. Nucl. Phys. 62 (1965)

LMG: Dissipation [3]

$$\hat{H}_{LMG} = -h\hat{J}_z - \frac{\gamma_x}{N}\hat{J}_x^2$$

$$\frac{d}{dt}\hat{\rho}(t) = -i\left[\hat{H}_{LMG}, \hat{\rho}(t)\right] - \frac{\kappa}{2N}\, D[\hat{J}_+]\hat{\rho}(t)$$

$$D[\hat{J}^+]\hat{\rho}(t) = 2\hat{J}_+\hat{\rho}\hat{J}_- - \hat{J}_-\hat{J}_+\hat{\rho} - \hat{\rho}\,\hat{J}_-\hat{J}_+$$



[3] S. Morrison et al., PRL **100** (2008)

Dissipative ESQPT Signal in LMG

$$\hat{H}_{LMG} = -h\hat{J}_z - \frac{\gamma_x}{N}\hat{J}_x^2$$

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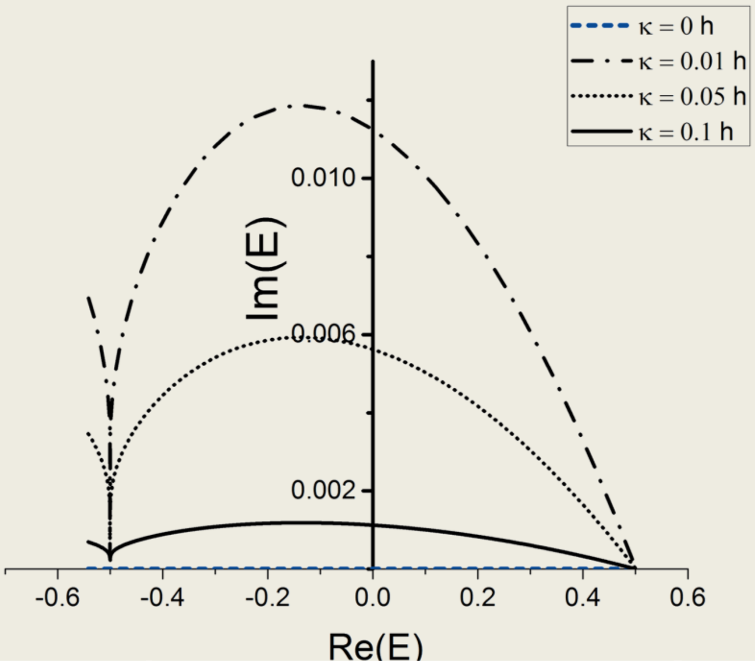
$$= -i\left[\hat{H}_{eff}, \hat{\rho}(t)\right] + \frac{\kappa}{N}\hat{J}^+\hat{\rho}(t)\hat{J}^-$$

- Hidden in the spectrum of effective Hamiltonian
- $\hat{H}_{eff} = \hat{H}_{LMG} - i\frac{\kappa}{2N}\hat{J}^-\hat{J}^+$



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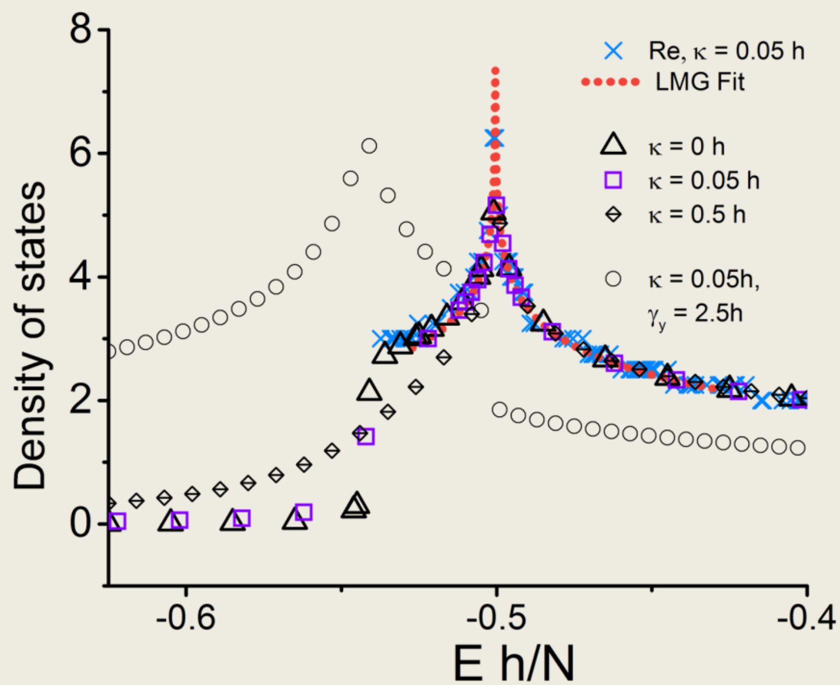
- Hidden in the spectrum of effective Hamiltonian
- $\hat{H}_{eff} = \hat{H}_{LMG} - i\frac{\kappa}{2N}\hat{J}^-\hat{J}^+$
- Still Logarithmic divergence at critical energy



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$X = Tr(\hat{J}_x\,\hat{\rho})/j$

$Z = Tr(\hat{J}_z\hat{\rho})/j$

+ Mean-Field for $N \to \infty$

$\langle\hat{J}_\eta\hat{J}_\mu\rangle \approx \langle\hat{J}_\eta\rangle \cdot \langle\hat{J}_\mu\rangle$

Critical Point: $\gamma_x^{cr} = h + \frac{\kappa^2}{4h}$

Set of Mean-Field Eqs. :

$$\partial_t X = h \cdot Y - \kappa \cdot X \cdot Z,$$
$$\partial_t Y = -h \cdot X + 2\,\gamma_x \cdot X \cdot Z - \kappa \cdot Y \cdot Z,$$
$$\partial_t Z = -2\,\gamma_x \cdot X \cdot Y + \kappa(X^2 + Y^2).$$

$$H_{MF} = -hZ - \gamma_x X^2$$

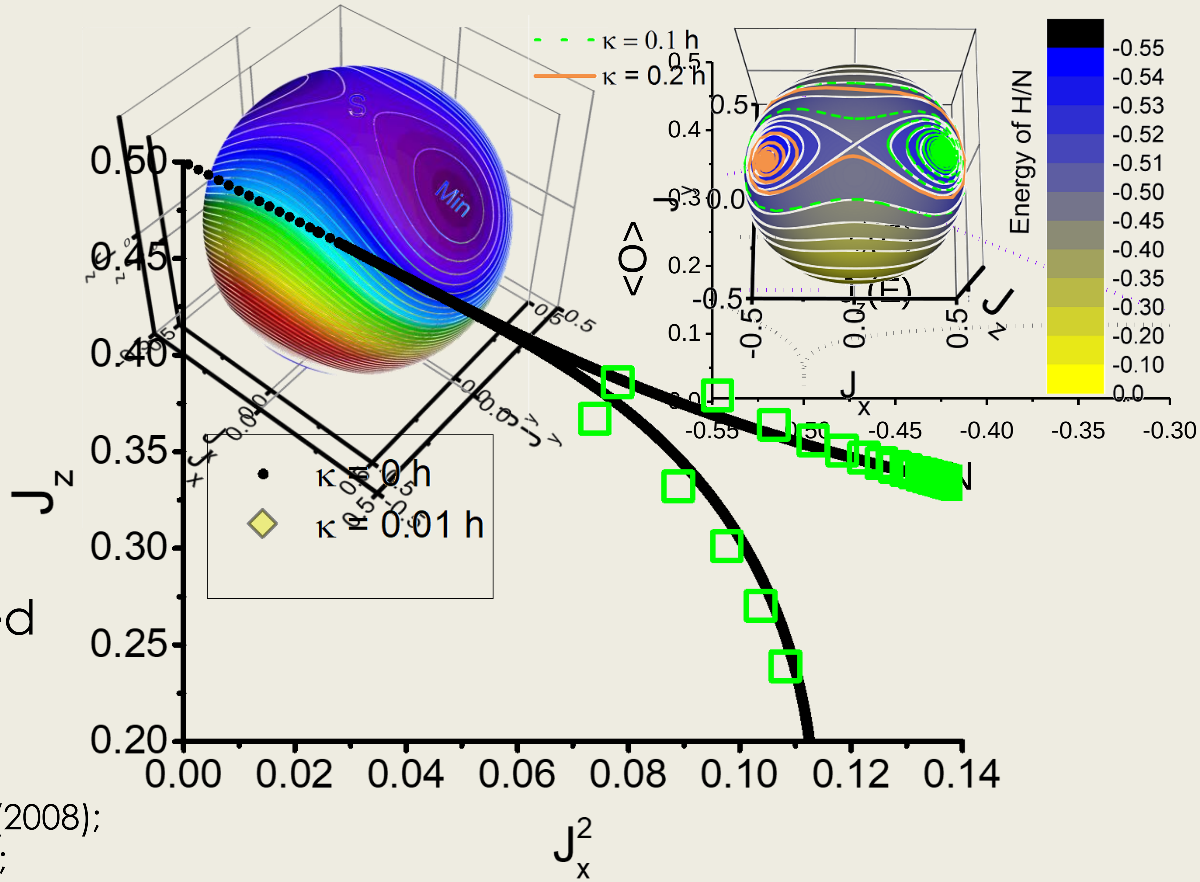
Spin coherent states $|\theta, \varphi\rangle$ are the closest to the classical one

[3] S. Morrison et al., PRL **100** (2008)
P. Ribeiro et al., PRE **78** (2008);

LMG: Mean-Field and Observables[4]

- Semiclassical Energy landscape
  $$H_{MF} = -hZ - \gamma_x X^2$$
- Critical Point
  $$\gamma_x^{cr} = h + \frac{\kappa^2}{4h}$$
- Dissipation: $\kappa > 0$
  - Solve MF Eq.
  - Steady State
  - ESQPT is smoothened

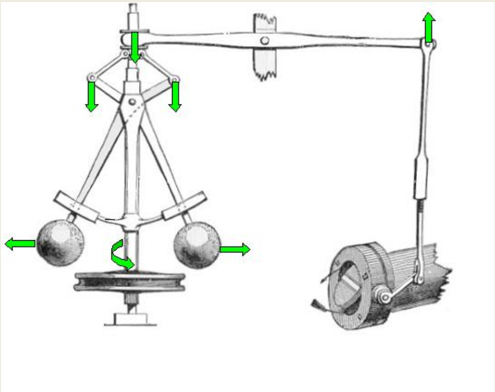
$\gamma_x < \gamma_x^{cr}$ $\gamma_x > \gamma_x^{cr}$



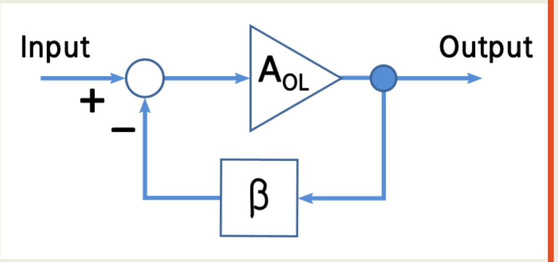
[4] P. Ribeiro et al., PRE **78** (2008);
M. Caprio et al., Ann.Phys. **323** (2008);
S. Morrison et al., PRL **100** (2008);
G. Engelhardt et al., PRA **91** (2015)

Control ESQPT?

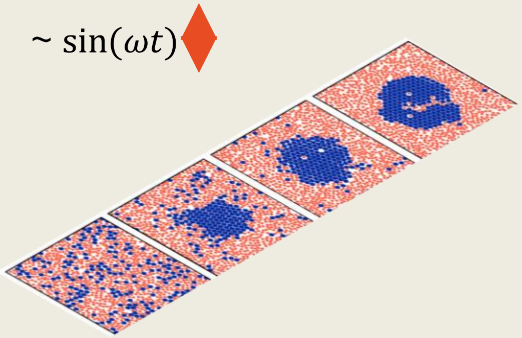
Centrifugal governor (James Clerk Maxwell, 1868)



Feedback Amplifier



Grain segregation [5]

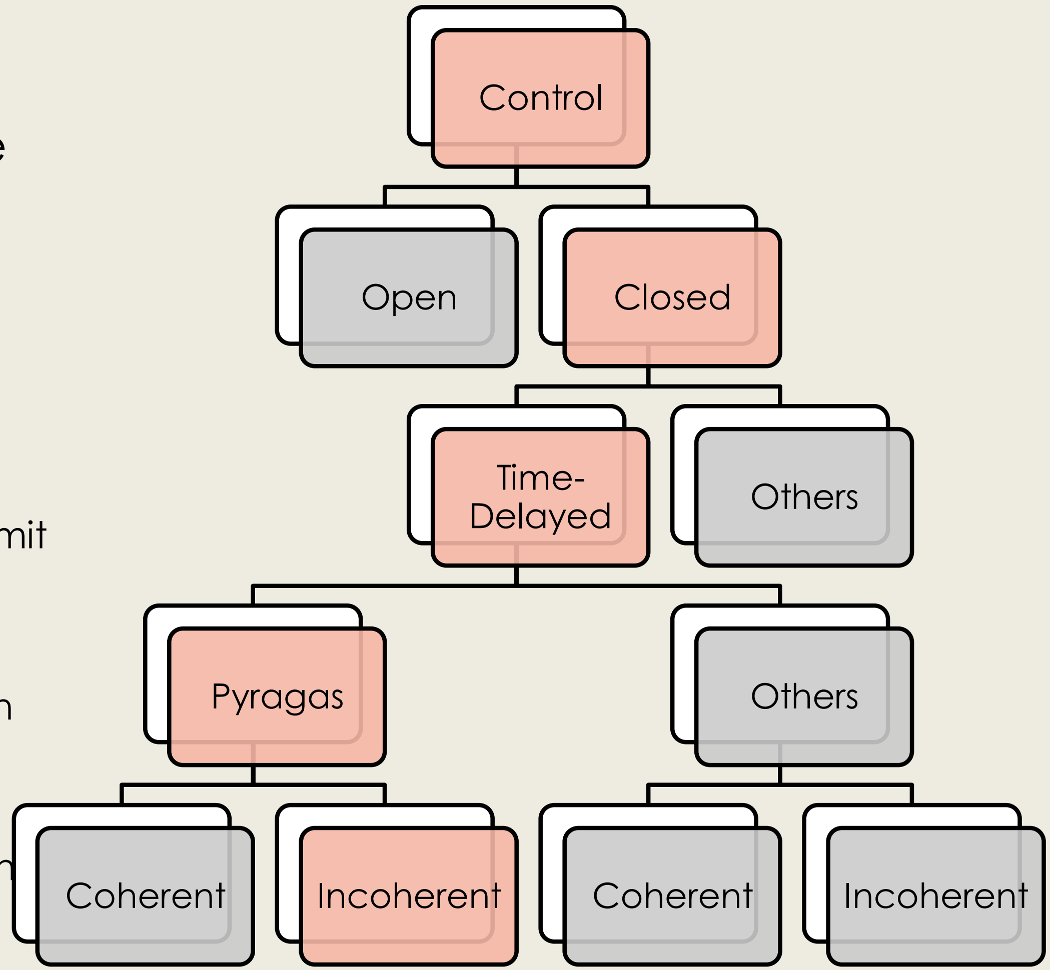


[5] N. Rivas et al, New J. Phys. **13**, 055018 (2011)

Overview: Control Loops

- Delay τ in control signal proceeding
- Wanted and/or unavoidable effects
- Pyragas Control [6]
  - Form of Control term

    $\sim \lambda(X(t-\tau) - X(t))$
  - Invasive
  - Stabilizes fixed points, unstable limit cycles
- Now
  - Apply to a many-body systems in QM
  - use it for creation of new phases
  - Use it to influence the QPT and th ESQPT



[6] K. Pyragas, Physics Letters A **170**, 421-428 (1992)

LMG: Feedback Implementation

Various Control Schemes possible [7]

- Periodic modulation of coupling constant [7]
  ⇨ New Phases
- Coherent Pyragas Feedback [8]
  ⇨ Convergence Speedup
- Linear instantaneous increase of coupling constant [9]
  ⇨ Influence on Squeezing
- Modification of ESQPT divergence
- etc.

[7] V.M. Bastidas, PRL 108, 043003 (2012)
A. Grimsmo et al., NJP 16, 065004 (2014)
O. Acevedo et al., NJP 17, 093005 (2015)
V.M. Bastidas et al., PRA 90, 063628 (2014)
V. M. Bastidas et al. PRL 112, 140408 (2014)
G. Engelhard et al, PRE 87 052110 (2013)

LMG

$$\gamma_x \to \gamma_x(t) = \gamma + \frac{\lambda}{N^2}\left(\langle \hat{J}_z \rangle^2 (t-\tau) - \langle \hat{J}_z \rangle^2 (t)\right)$$

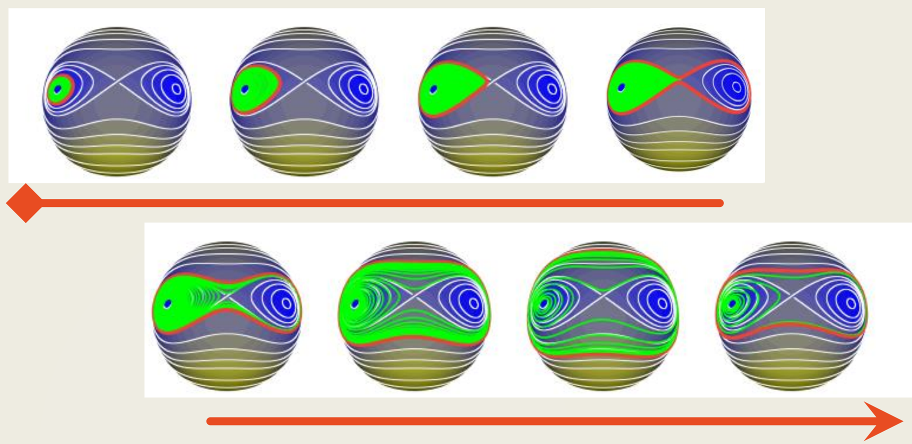
- Pyragas feedback with time delay τ [8]
- Coupling λ is conditioned on the difference of the J_z^2 average at two different times
- Mean-Field-Eq. [9]:

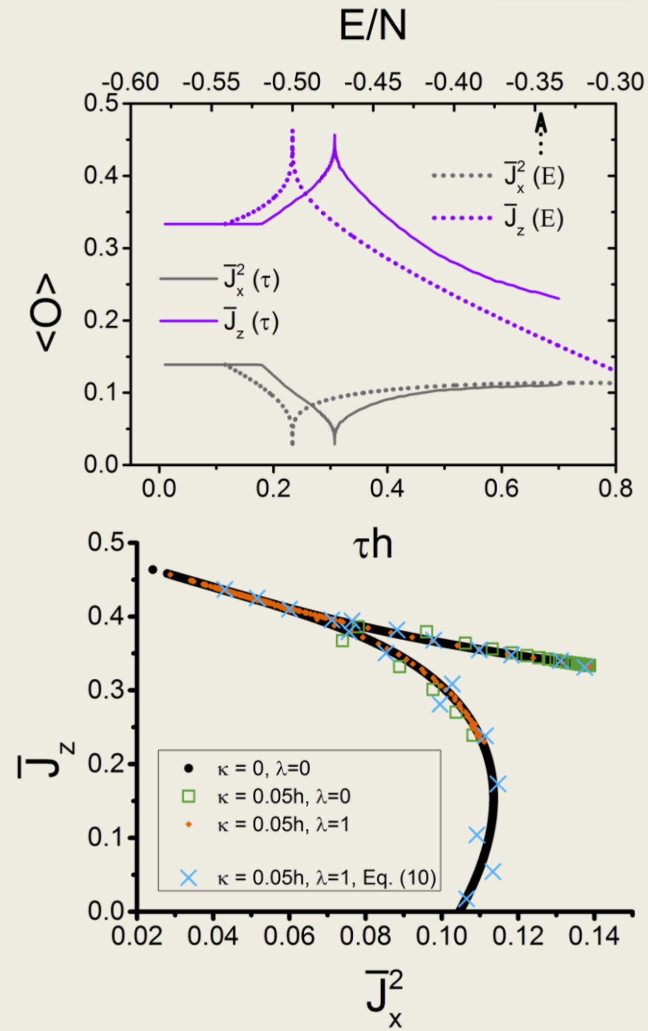
$$\partial_t X = h \cdot Y - \kappa \cdot X \cdot Z,$$
$$\partial_t Y = -h \cdot X + 2\,\gamma_x(t) \cdot X \cdot Z - \kappa \cdot Y \cdot Z,$$
$$\partial_t Z = -2\,\gamma_x(t) \cdot X \cdot Y + \kappa(X^2 + Y^2).$$

[8] K. Pyragas, Physics Letters A **170**, 421-428 (1992)
[9] S. Morrison et al., PRL **100** (2008)

LMG: ESQPT Control

$$\gamma_x \rightarrow \gamma_x(t) = \gamma + \frac{\lambda}{N^2}\left(\langle \hat{J}_z \rangle (t-\tau) - \langle \hat{J}_z \rangle (t)\right)$$

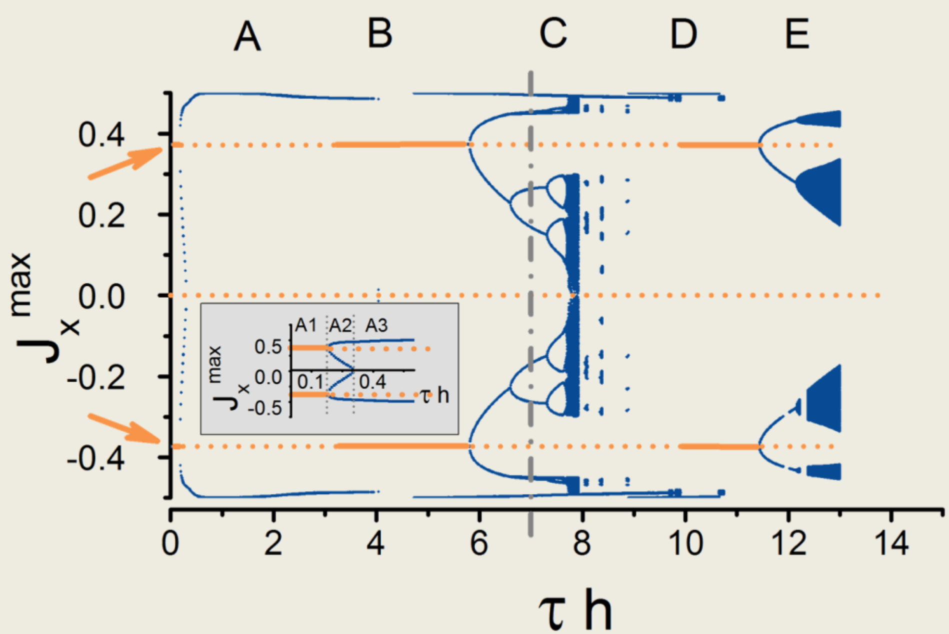




- ➢ Average in stationary state over a period
- ➢ ESQPT signal is restored by feedback

LMG with Control – Bifurcation Diagram

Region	Dynamics
A	Hopf-Bifurcation
B	Limit-Cycle and Fixed Point coexistence
C	Period-Doubling Structure
D	Limit-Cycle and Fixed Point coexistence
E	Period-Doubling Structure



Summary

ESQPT	Dissipation	ESQPT-Signal vanishing
Mean-Field	Pyragas	ESQPT-Signal Restore

W. Kopylov and T. Brandes, NJP **17** (2015)

Beyond Mean-Field

[10] A. Grimsmo, PRL 115, 060402
S. Hein, PRA 91, 052321

Thank You for Your Attention!