QPTn9

Padova (Italy)
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2018

Quantum Phase Transitions in odd-A Nuclei: The Effect of the Odd Particle along the Critical Line

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Kırıkkale University, Turkey

collaboration with

Padova – Sevilla group

L. Fortunato, A. Vitturi – C. E. Alonso, J. M. Arias

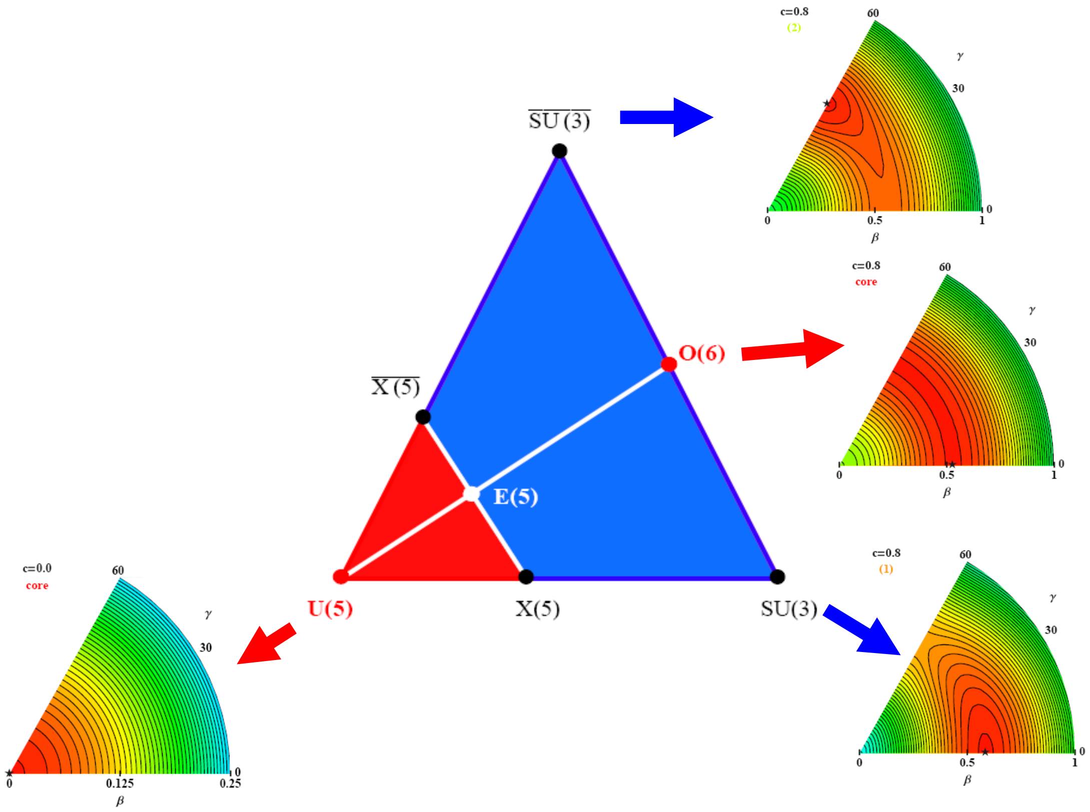
In this work...

- We consider
  - the case of a single-j fermion coupled to the bosonic core
  - that performs a transition along the $\mathrm{X(5)} - \mathrm{E(5)} - \overline{\mathrm{X(5)}}$ line varying the control parameters in the boson Hamiltonian.
- Our purpose is
  - to see the effect of the coupling of the single fermion,
  - to understand how the coupled single particle modifies geometry of the bosonic system
  - when the core shifts along the critical line of the extended symmetry triangle.
  - How the behavior of the **individual coupled states** at the transitional region.

  - ***This situation is described with the intrinsic frame formalism of the IBFM.***

Potential Energy Surphase (PES) V(β,γ):

- A useful way of looking at phase transitions is to apply the concept of intrinsic frame to associate Potential Energy Surphase to Hamiltonian depending on β and γ.
  - **The β and γ play a similar role to the intrinsic collective shape variables in the Bohr Hamiltonian.**
  - **The β variable measures the axial deviation from sphericity**
  - **The angle variable γ controls the departure from axial deformation.**



$$H^{B} = \boxed{(1-c)\hat{n}_d} - \boxed{\frac{c}{4N_B}\hat{Q}_B^{\chi}\cdot\hat{Q}_B^{\chi}}$$

Spherical

$$\hat{n}_d = \sum_{\mu} d_{\mu}^{+} d_{\mu}$$

Deformed

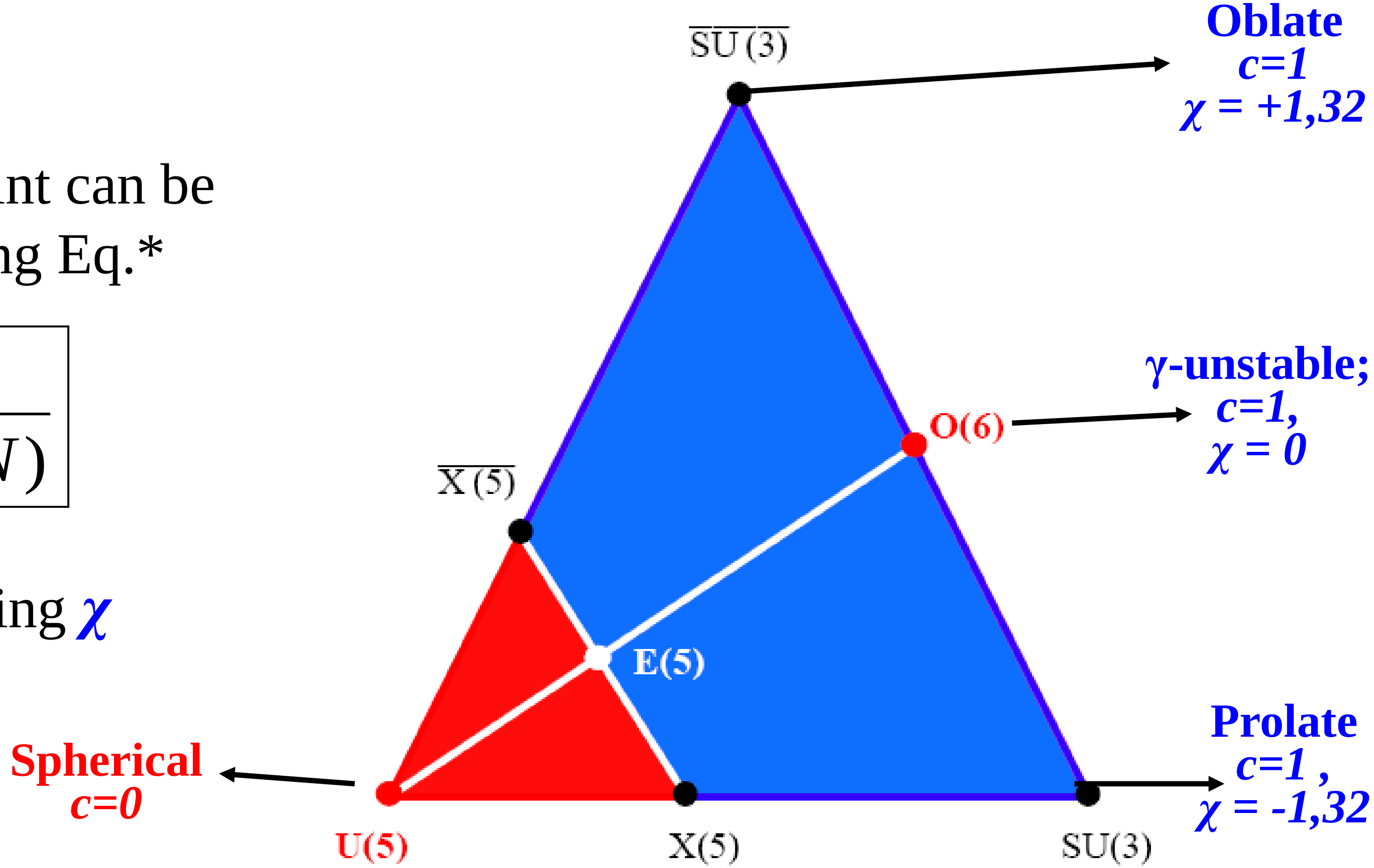
$$\boxed{\hat{Q}_B^{\chi} = (s^{+}\times\tilde{d} + d^{+}\times\tilde{s})^{(2)} + \chi(d^{+}\times\tilde{d})^{(2)}}$$

For the given Hamiltonian, the critical point can be calculated numerically within the following Eq.*

$$\boxed{c_{cr}(N,\chi) = \frac{28N}{56(N-1)+\chi^{2}(5+2N)}}$$

one can move along critical line by chancing χ

*by E. Williams, R. J. Casperson, V. Werner, Phys. Rev. C 82, 054308 (2010)



The Intrinsic Frame Formalism (within the IBM)

The intrinsic state for the g.s. band of an even-even nucleus

$$\boxed{\Phi_{gs}(\beta,\gamma)=\frac{1}{\sqrt{N!}}\left[b_{gs}^{+}(\beta,\gamma)\right]^{N}\left|0\right\rangle}$$

The boson creation operator is given by

$$\boxed{b_{gs}^{+}(\beta,\gamma)=\frac{1}{\sqrt{1+\beta^{2}}}\left[s^{+}+\beta\ \cos\gamma\ d_{0}^{+}+\frac{\beta}{\sqrt{2}}\sin\gamma\ (d_{2}^{+}+d_{-2}^{+})\right]\left|0\right\rangle}$$

The PES is obtained by calculating the expectation value of the H_B in the intrinsic state

$$\boxed{E_{gs}(\beta,\gamma)=\left\langle\Phi_{gs}(\beta,\gamma)\middle|H_{B}\middle|\Phi_{gs}(\beta,\gamma)\right\rangle}$$

Intrinsic Frame Formalism for odd-even nuclei

- Intrinsic frame states for **the mixed boson-fermion** system can **be constructed** by coupling the single particle states to the intrinsic states of the **boson core**.
- To obtain them, we **first construct the coupled states**

$$\boxed{\Psi_{jK}(\beta,\gamma)=\Phi_{gs}(\beta,\gamma)\otimes\left|\,jK\right\rangle}$$

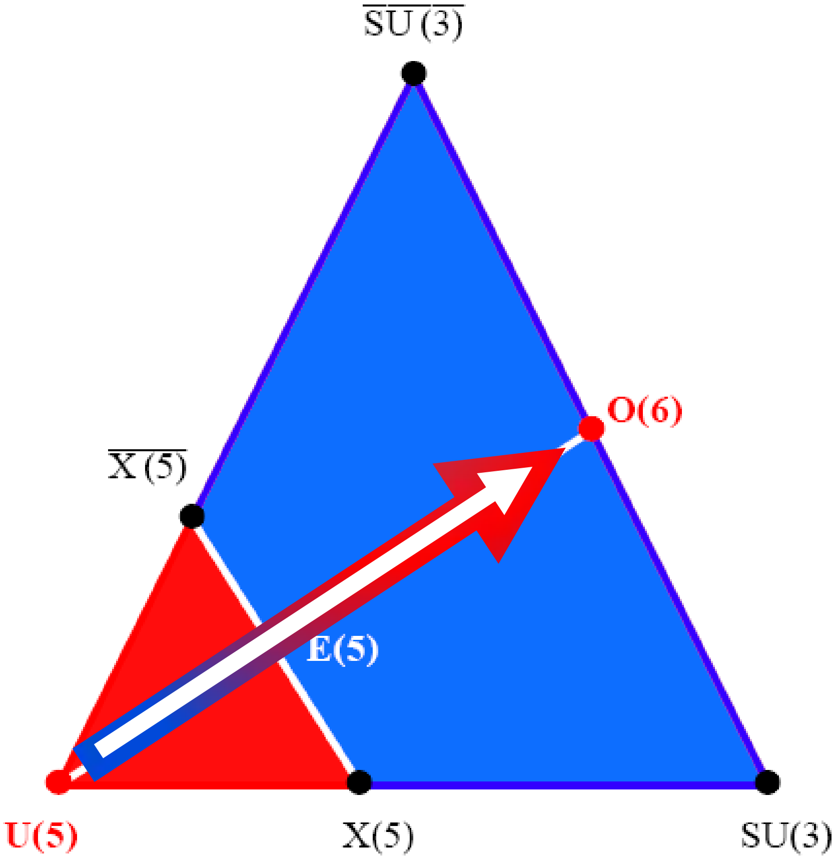
- **Then** diagonalize the total boson-fermion Hamiltonian in this basis, giving **a set of energy eigenvalues** $E_{\mathbf{n}}(\beta,\gamma)$.

Here n is an index to count solutions in the odd-even system.

First Application... U(5)→O(6)

“Shape phase transition in odd-even nuclei:
from *spherical to deformed γ-unstable* shapes”

M. Böyükata,
C. E. Alonso, J. M. Arias,
L. Fortunato, A. Vitturi,
Phys. Rev. C82, 014317 (2010)



Hamiltonian

- IBFM Hamiltonian is written as

$$\boxed{H = H_B + H_F + V_{BF}}$$

H_B is the bosonic part,
H_F is the fermionic part,
V_{BF} term couples the boson and fermion.

- The IBFM Hamiltonian is parametrized as follows **[U(5)→O(6)]** :

$$\boxed{H = (1-c)\hat{n}_d - \frac{c}{4N}\hat{Q}_{BF}^{\chi=0}\cdot\hat{Q}_{BF}^{\chi=0}} \qquad \boxed{\hat{n}_d = \sum_{\mu} d_{\mu}^{+} d_{\mu}} \qquad \boxed{\hat{Q}_{BF} = \hat{Q}_B + \hat{q}_F}$$

- The boson-fermion interaction

$$\boxed{\hat{V}_{BF} = -\frac{c}{2N}\hat{Q}_B \cdot \hat{q}_F} \qquad \boxed{\hat{Q}_B^{\chi=0} = (s^{+}\times\tilde{d} + d^{+}\times\tilde{s})^{(2)}} \cancel{+0\cdot(d^{+}\times\tilde{d})^{(2)}} \qquad \boxed{\hat{q}_F = t_j(a_j^{+}\times\tilde{a}_j)^{(2)}}$$

- For this Hamiltonian, the critical point is numerically found as

$$\boxed{c_{cr}(N, \chi = 0) = \frac{N}{2(N-1)}}$$

Hamiltonian

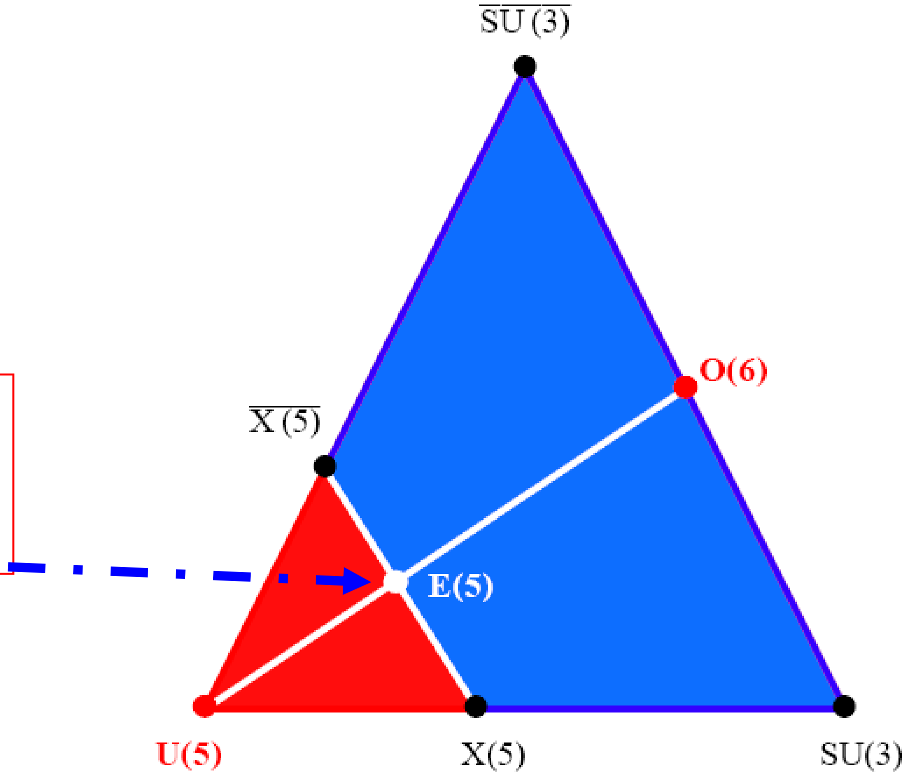
- The Boson Hamiltonian **[U(5); *c=0*** **and** **O(6); *c=1, χ=0.*]**

- By changing c between two limits,

a continuous ***(2nd-order)*** transition

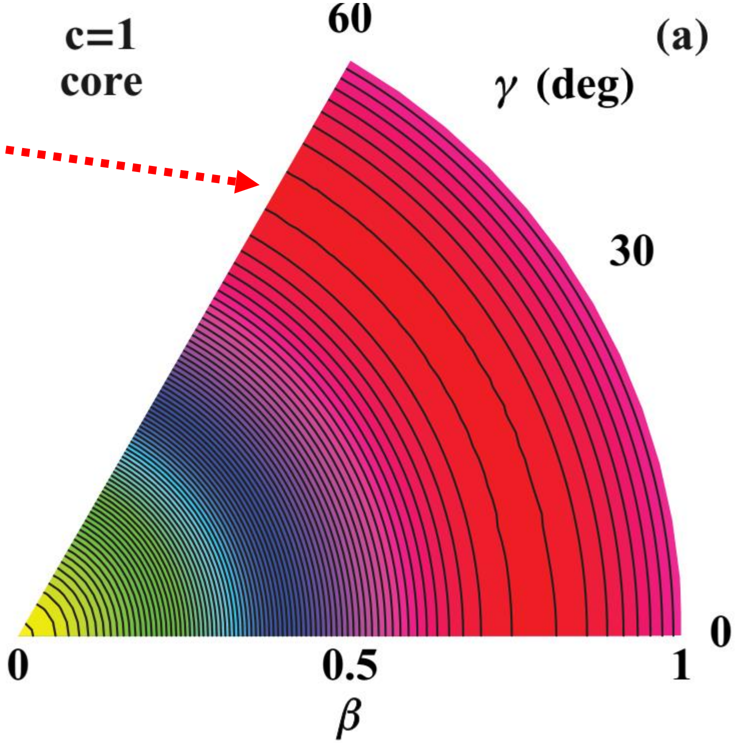
is observed with the critical point at

$$\boxed{c_{cr}(N,\chi=0)=\frac{N}{2(N-1)}}$$



Results of this application…

- Firstly, we consider the boson core in O(6) limit,
- For this, the control parameter **c=1, χ=0** in the H_B.
- The corresponding energy function is γ-independent
- In this case β = 0.78 for N=5.



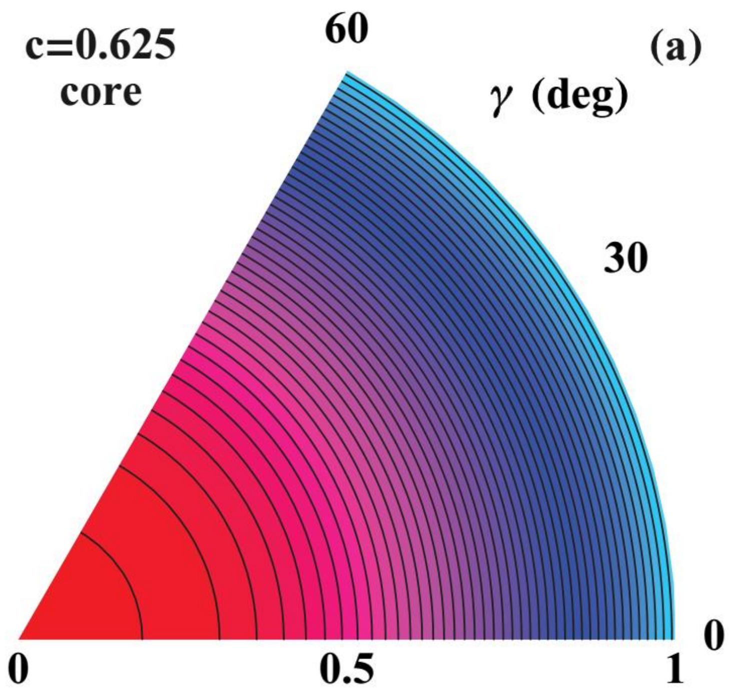
Coming to the situation at the critical point.

- For the case under study ($N_B = 5$) the critical point for the even-even system is located at c = 0.625.

$$c_{cr}(N, \chi = 0) = \frac{N}{2(N-1)}$$

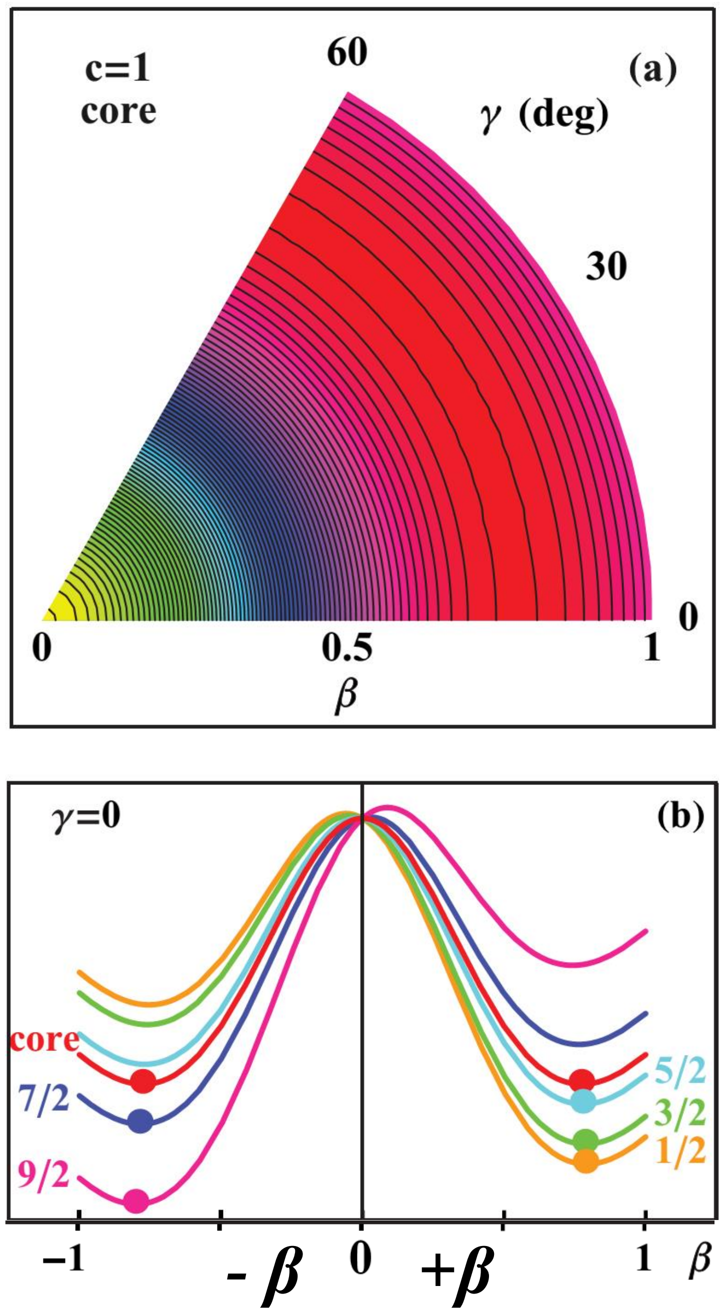
$$c_{cr}(N = 5, \chi = 0) = 0.625$$

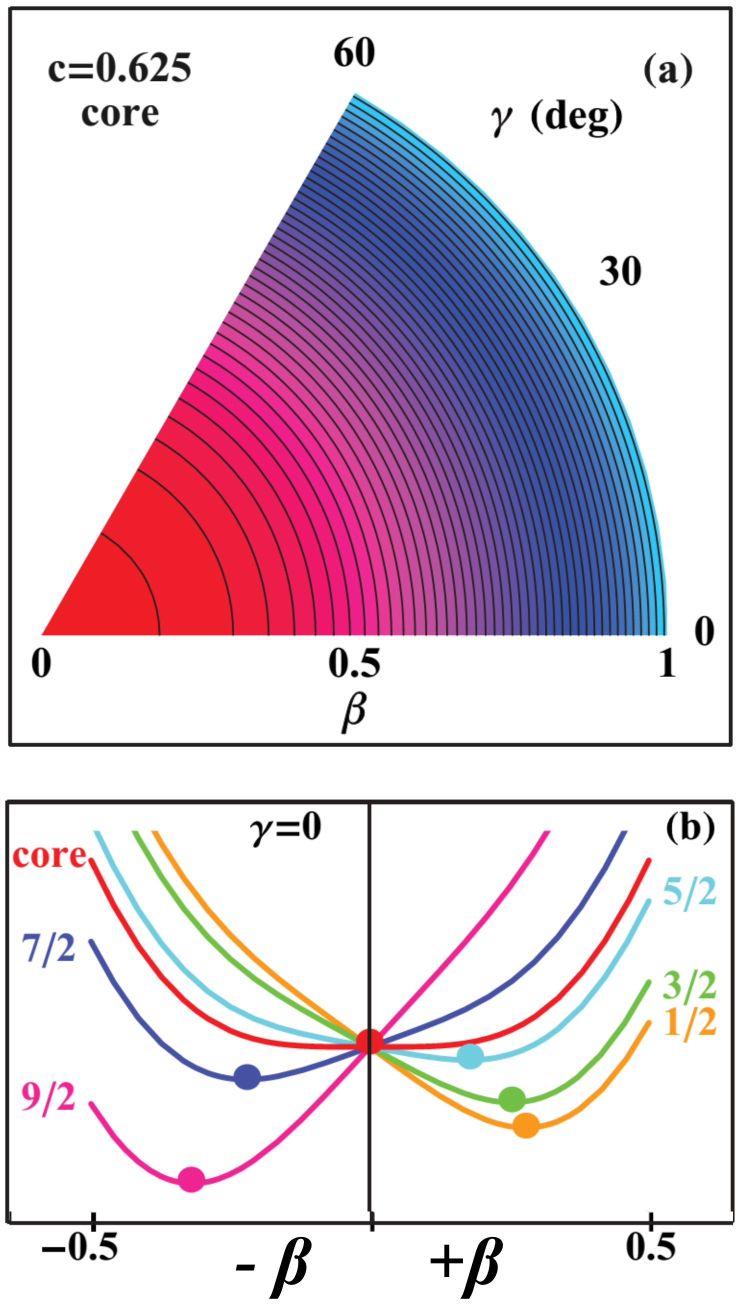
- The energy surface is plotted for the even-even core.
- It is seen that the even-even system is γ-unstable and has a spherical minimum.



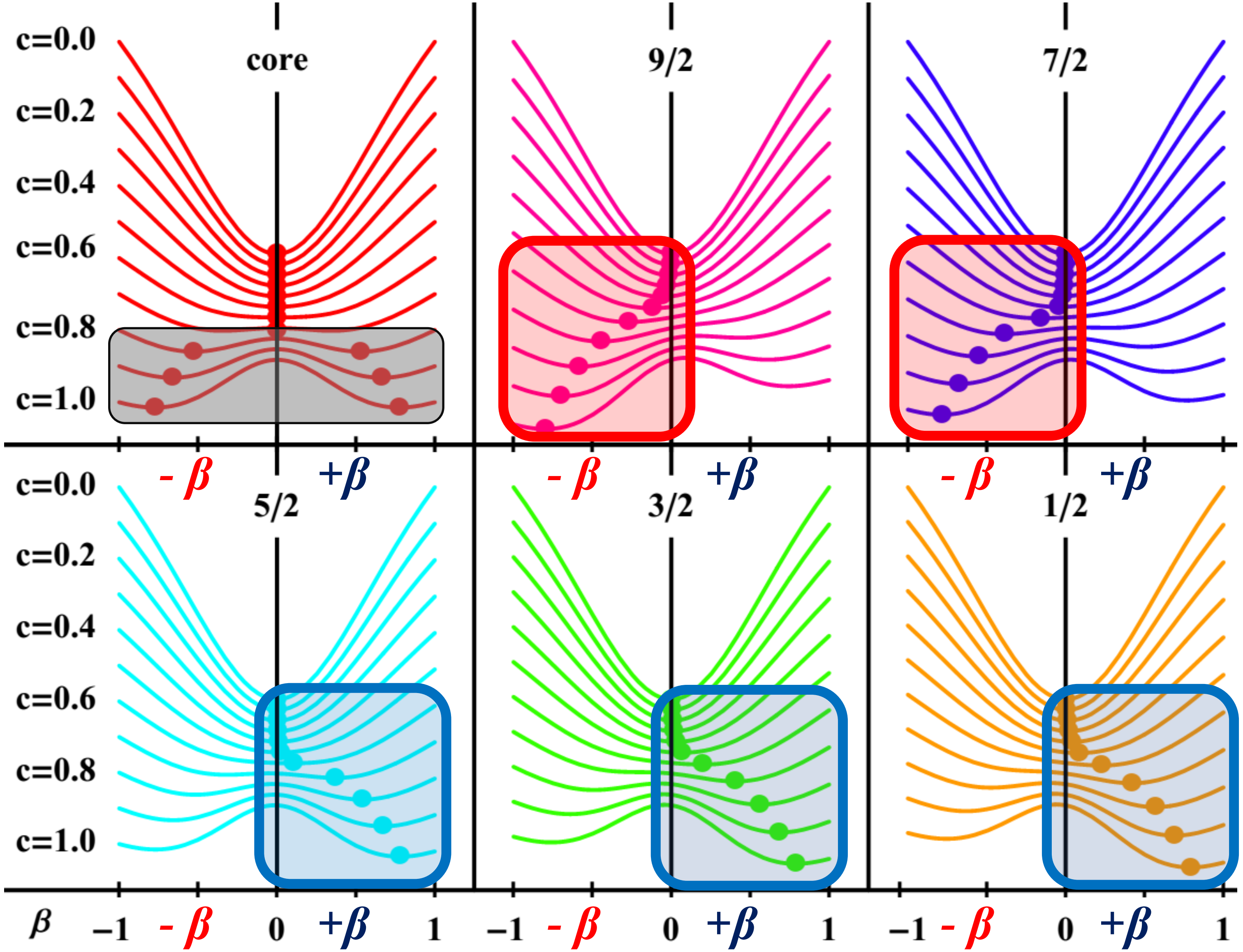
Bose-Fermi system

A single fermion with j=9/2 is coupled to boson core to consider in a mixed Bose-Fermi system.


c=1
core
60
(a)
γ (deg)
30
0
0
0.5
1
β
γ=0
(b)
core
7/2
9/2
5/2
3/2
1/2
−1
- β
0
+β
1
β


c=0.625
core
60
(a)
γ (deg)
30
0
0
0.5
1
β
γ=0
(b)
core
7/2
9/2
5/2
3/2
1/2
−0.5
- β
+β
0.5

Evolution of the energy surfaces for a set of **c**.

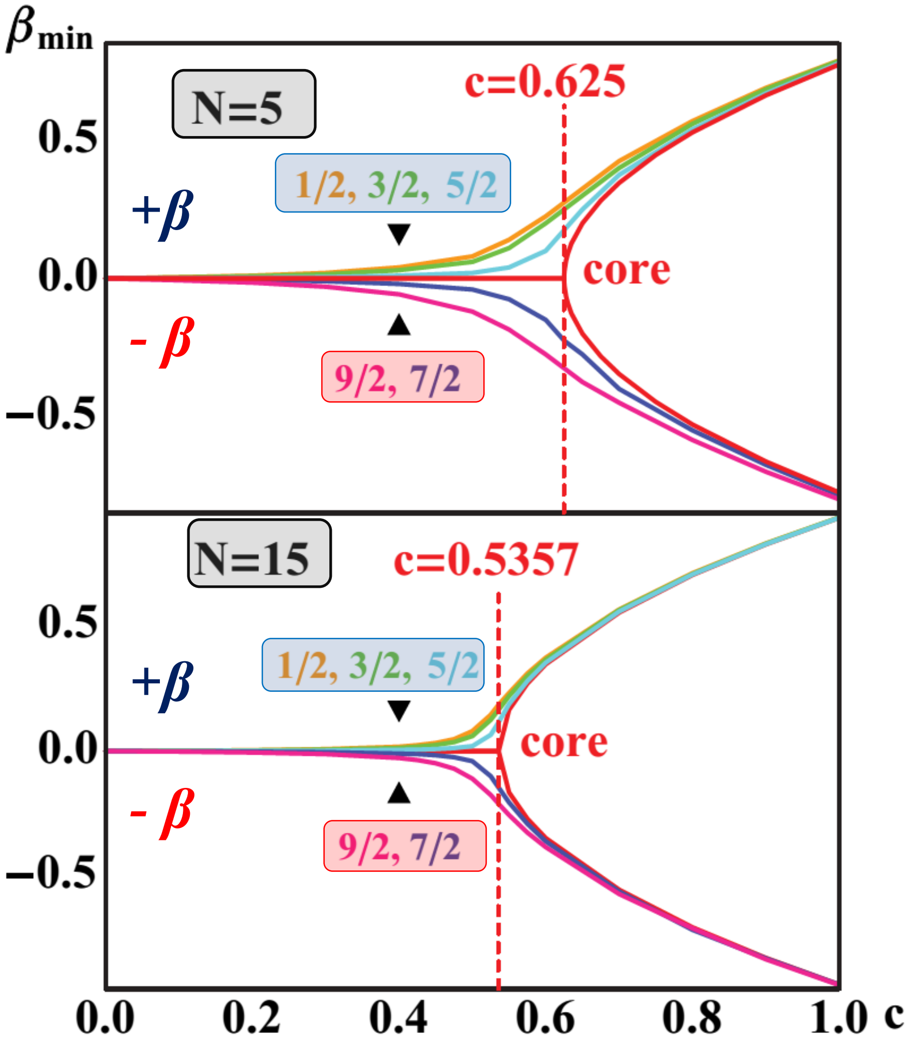


When moving
into the transitional region
(c is 0 → 1);

When the **core** is **γ-unstable**,
K=9/2,7/2 states are **oblate**,
K=5/2,3/2,1/2 are **prolate**

The overall results are summarized the minima in for the different odd-even states are plotted versus c.

- Positive values correspond to **prolate** deformation, while negative ones mean **oblate** shapes.
- **The e-e case is plotted as a reference.**
  - All over the transition the states with **K=1/2,3/2,5/2** prefer to be **prolate** while **K=7/2,9/2** are **oblate**.
  - The odd surfaces tend to follow the behaviour of the bosonic core.
  - Smaller N;
    the deviations from the core are larger for all K's.
  - As N grows;
    the transition of o-e system gets closer to the core.
  - The γ-unstability of the core allows the odd states to drive either a prolate or a oblate shape.

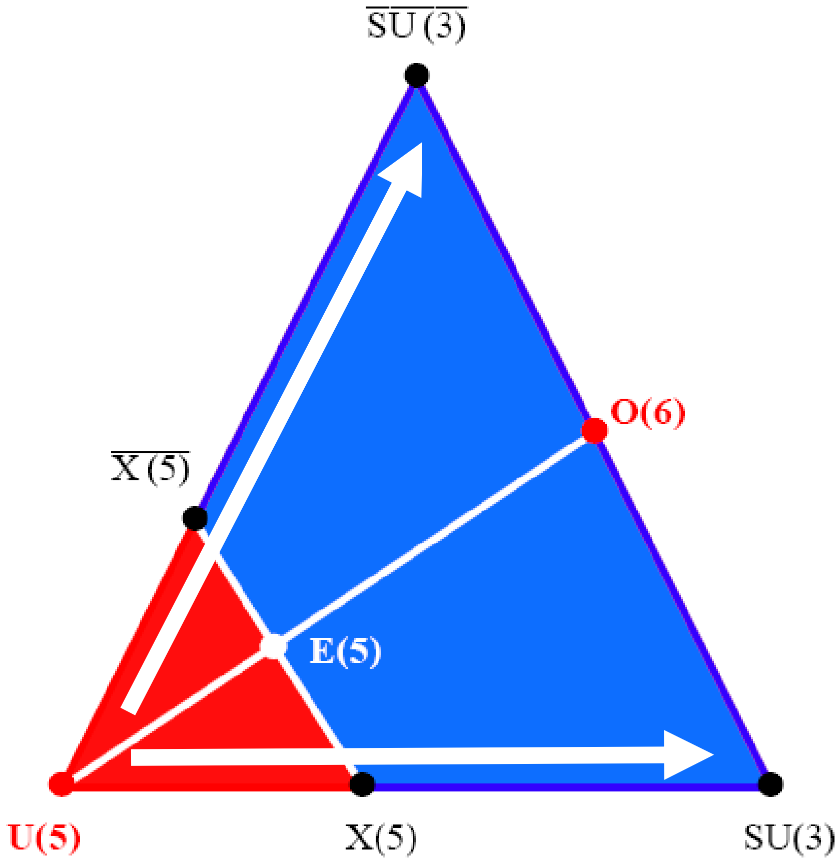


Application for... U(5)→SU(3) & U(5)→$\overline{\mathrm{SU(3)}}$

“Quantum Shape phase transition in odd-even nuclei:
from *spherical to deformed prolate* shapes, & from *spherical to deformed oblate* shapes
the effect of the odd particle around the critical point and along to transition lines”

M. Böyükata,
C. E. Alonso, J. M. Arias,
L. Fortunato, A. Vitturi,

EPJ WoC, 66, 02014, (2014).
&
IoP J. of Phys. Conf. Ser. 580, 012047 (2015).



Second Application : Transition from spherical to prolate

Transition from sph. → pro. [c=1, $\chi=-\sqrt{7}/2$] sph. → obl. [c=1, $\chi=+\sqrt{7}/2$]

$$H^{B} = \boxed{(1-c)\hat{n}_d} - \boxed{\frac{c}{4N_B}\hat{Q}_B^{\chi\neq0}\cdot\hat{Q}_B^{\chi\neq0}}$$

Spherical

$$\boxed{\hat{n}_d = \sum_{\mu} d^{+}_{\mu} d_{\mu}}$$

Deformed

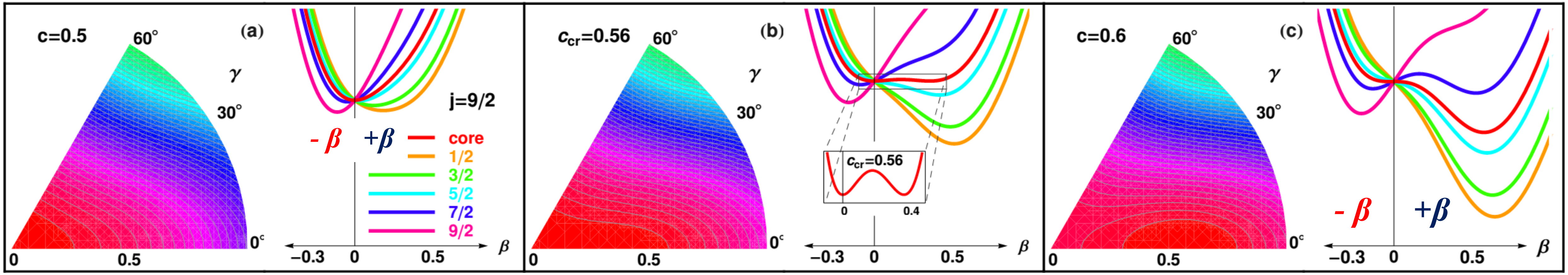
$$\boxed{\hat{Q}_B^{\chi\neq0} = (s^{+}\times\tilde{d} + d^{+}\times\tilde{s})^{(2)} \mp \frac{\sqrt{7}}{2}(d^{+}\times\tilde{d})^{(2)}}$$

- ***1st-order*** transition is observed with the critical point at

$$\boxed{c_{cr}(N,\chi) = \frac{28N}{56(N-1)+\chi^{2}(5+2N)}}$$

$$\boxed{c_{cr}(N,\chi=\mp\sqrt{7}/2) = \frac{16N}{(34N-27)}}$$

Transition from spherical to axially prolate



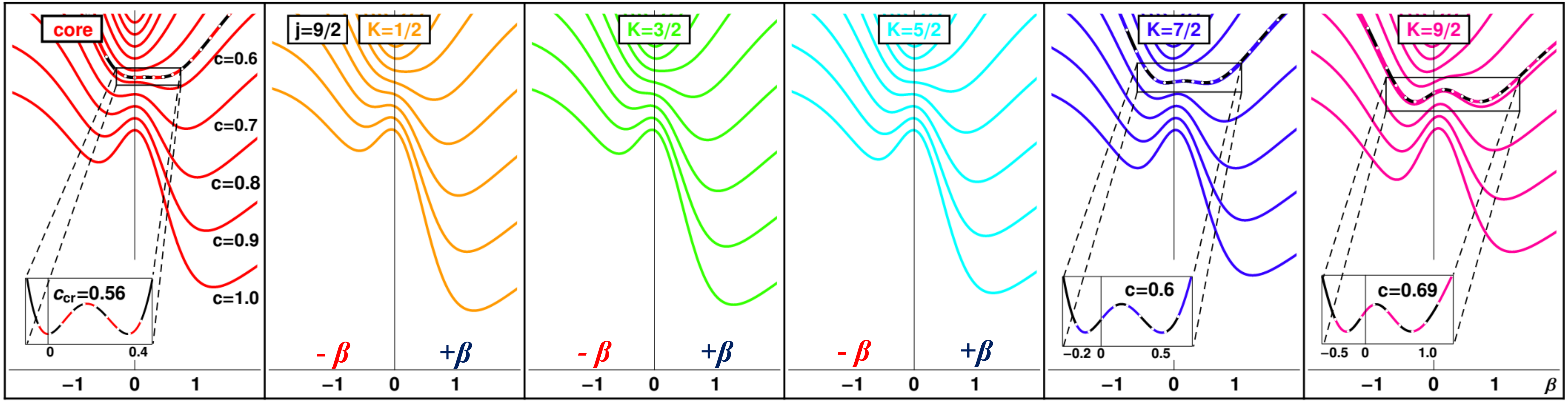
- Contour plots in the β-γ plane are for the e-e core with 5 bosons.
- Cuts of the energy surfaces as a function of β are for the o-e system (for each magnetic component of j = 9/2).

The effect of odd particle on the core around the critical point.

- Energy surfaces at c = 0.5,
- at the critical point c_{cr} = 0.56,
- at c = 0.6.

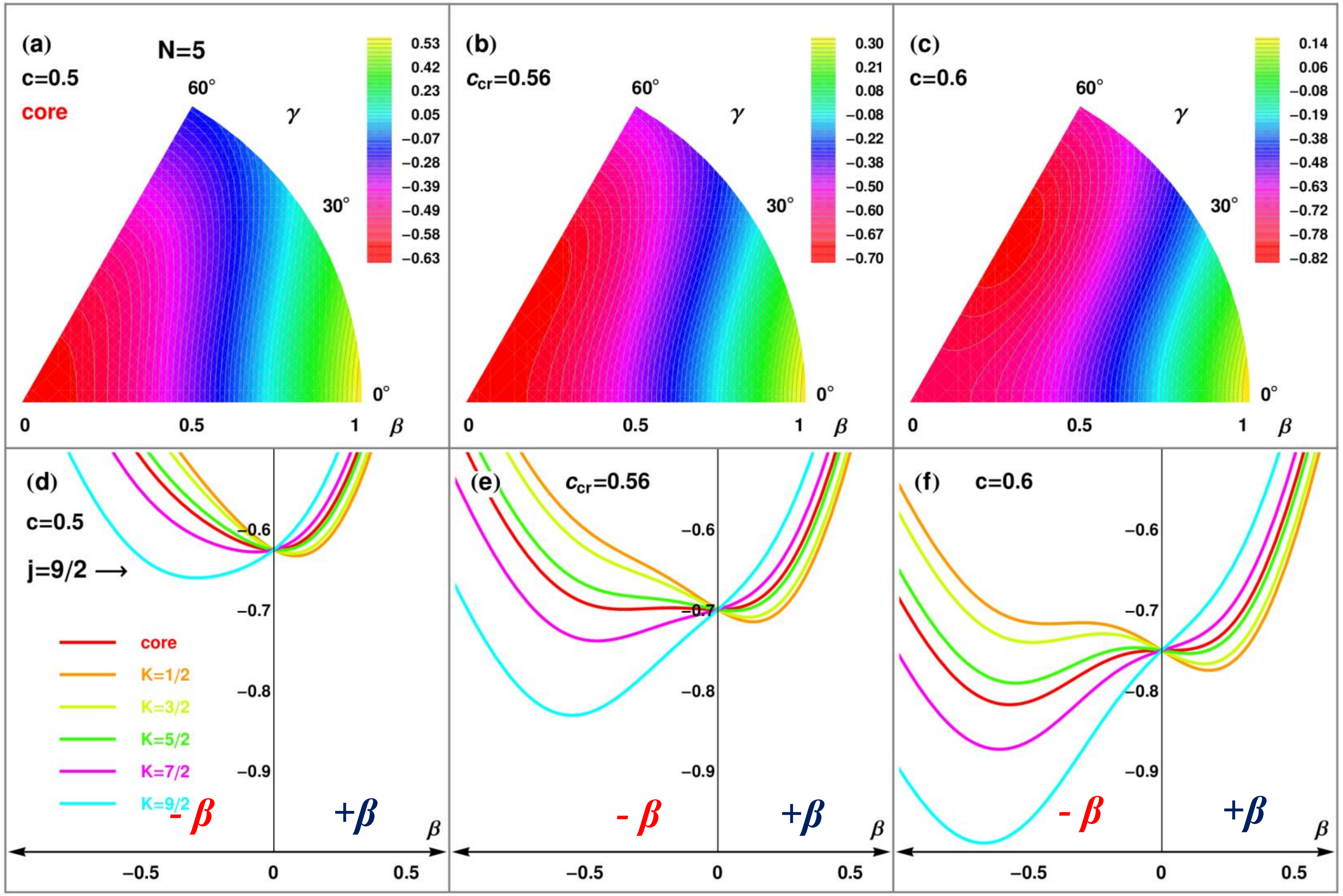
$$c_{cr}(N, \chi = -\sqrt{7}/2) = 16N/(34N - 27)$$

$$c_{cr}(N = 5, \chi = -\sqrt{7}/2) = 0.56$$

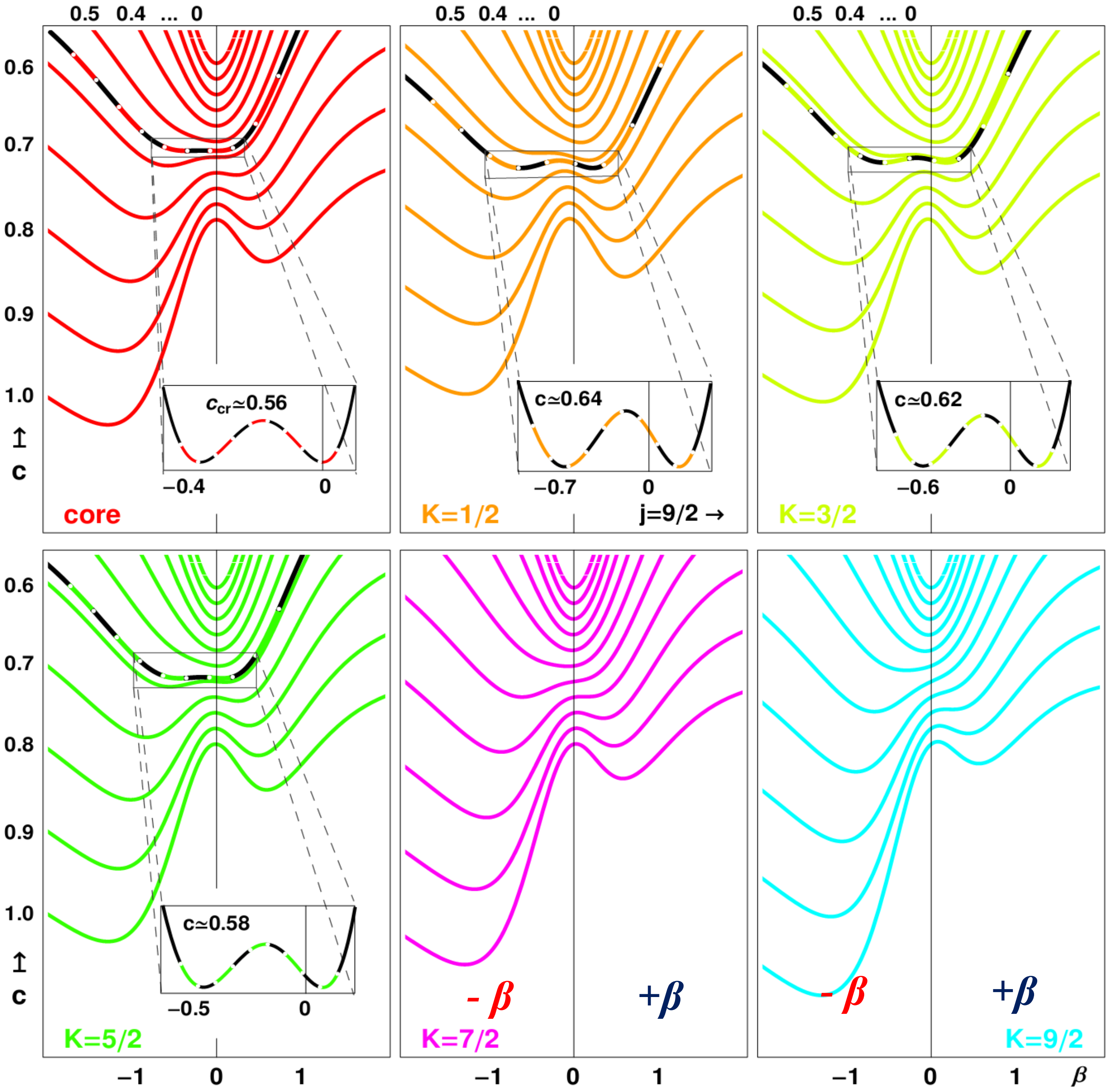


- The evolution of the energy surfaces along the shape phase transition for the boson core with 5 magnetic components K.
- States with K= 1/2, 3/2, 5/2 always favor prolate shapes,

  while states with K= 7/2, 9/2 are oblate up to $c_{(7/2)} \simeq 0.6$ and $c_{(9/2)} \simeq 0.7$, after these points, they suddenly change to prolate shapes.
- Therefore we can suggest that both states, K=7/2, 9/2, show 1st order transitions, since they have two absolute minima.

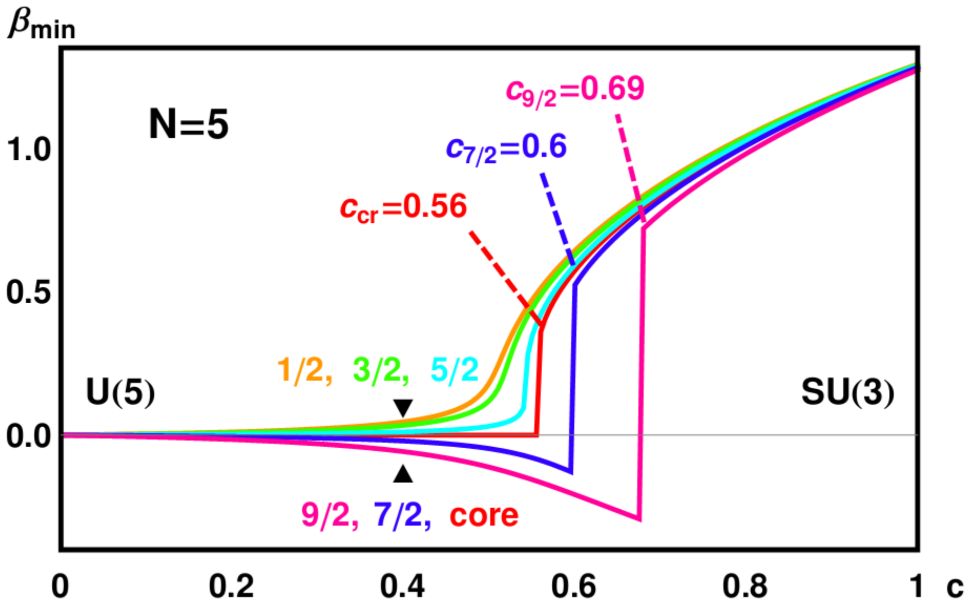


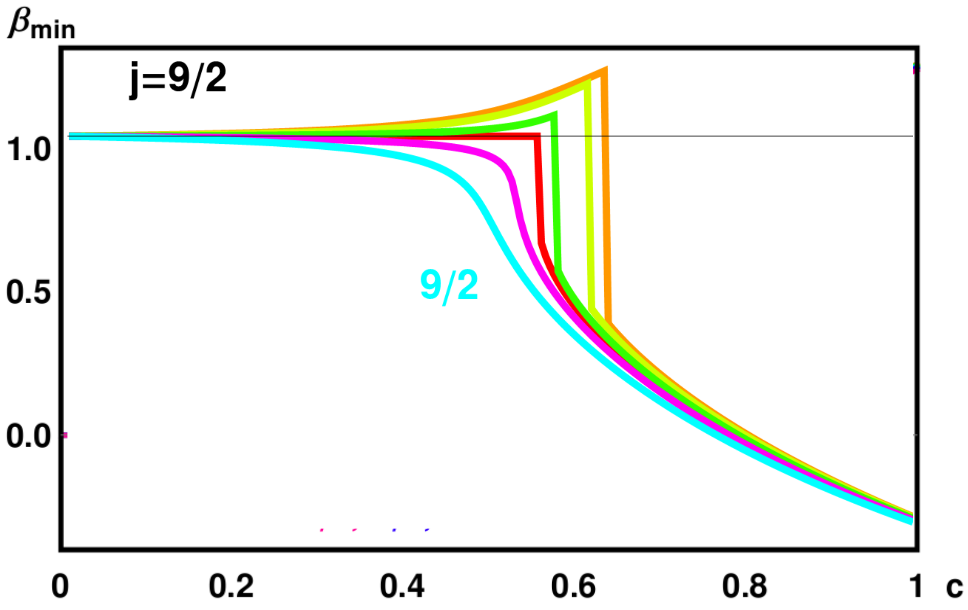
- The effect of odd particle on the core around the critical point.
    - Energy surfaces at c = 0.5, at the critical point c_{cr} = 0.56, at c = 0.6.
- Contour plots in the β-γ plane are for the e-e core with 5 bosons.
- Energy surfaces as a function of β are for the o-e system (for each magnetic component of j = 9/2).



- The evolution of the energy surfaces along the shape phase transition for the boson core with 5 magnetic components K.
- States with K= 7/2, 9/2 always favor oblate shapes, while states with K= 1/2, 3/2, 5/2 are prolate up to $c_{(1/2)}$≃0.64, $c_{(3/2)}$≃0.62 and $c_{(5/2)}$≃0.58.
- After these points, they suddenly change to oblate shapes.
- Therefore we can suggest that both states, 1/2, 3/2, 5/2 , show 1st order transitions, since they have two absolute minima.

Evolution of βmin as a function of **c: transition from sph. to pro. & sph. to obl.**





Before the critical point of core;

The states with larger K values move to their preferred oblate deformation, while the states with smaller K ones are prolate

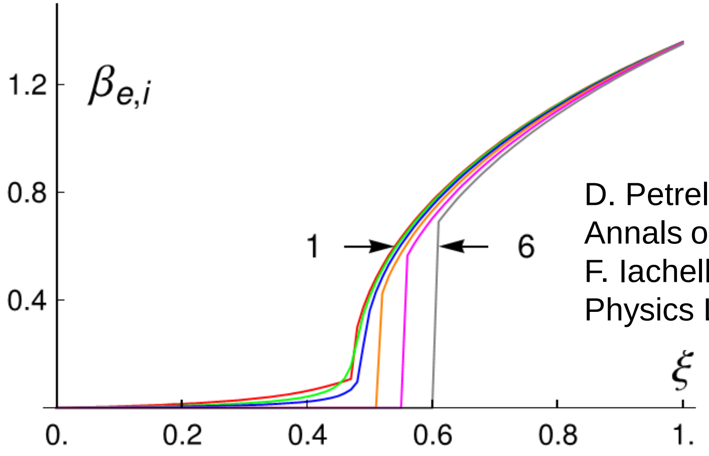
The states with larger K values move to their preferred oblate deformation, while the states with smaller K values are oblate

After the bosonic critical point;

each oblate state will shift to prolate shape at some specific value of the control parameter.

each oblate state will shift to oblate shape at some specific value of the control parameter.

- **In the pure deformed region the dominant action of the strongly prolate/oblate core drives all odd states into the prolate/oblate side and they have approximately the same βmin as the core.**

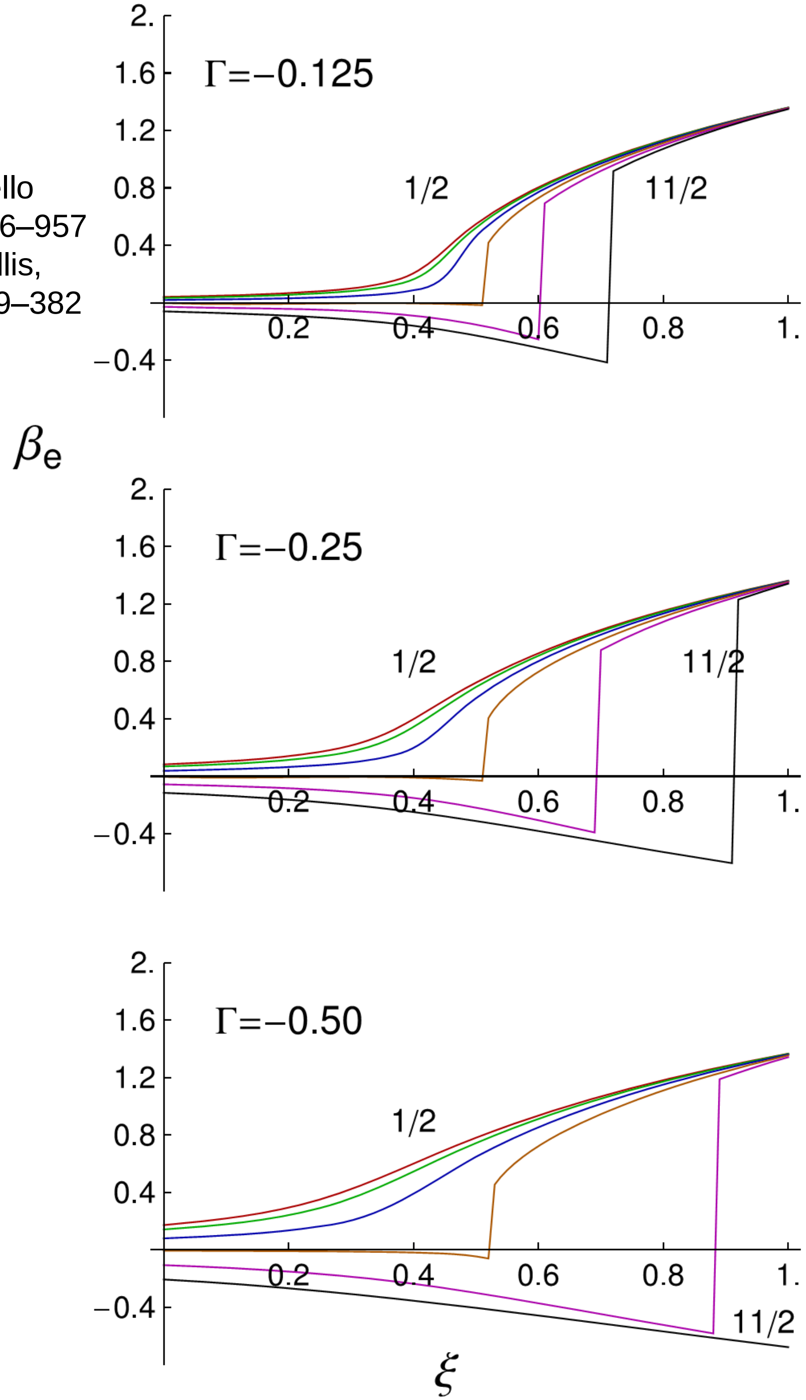


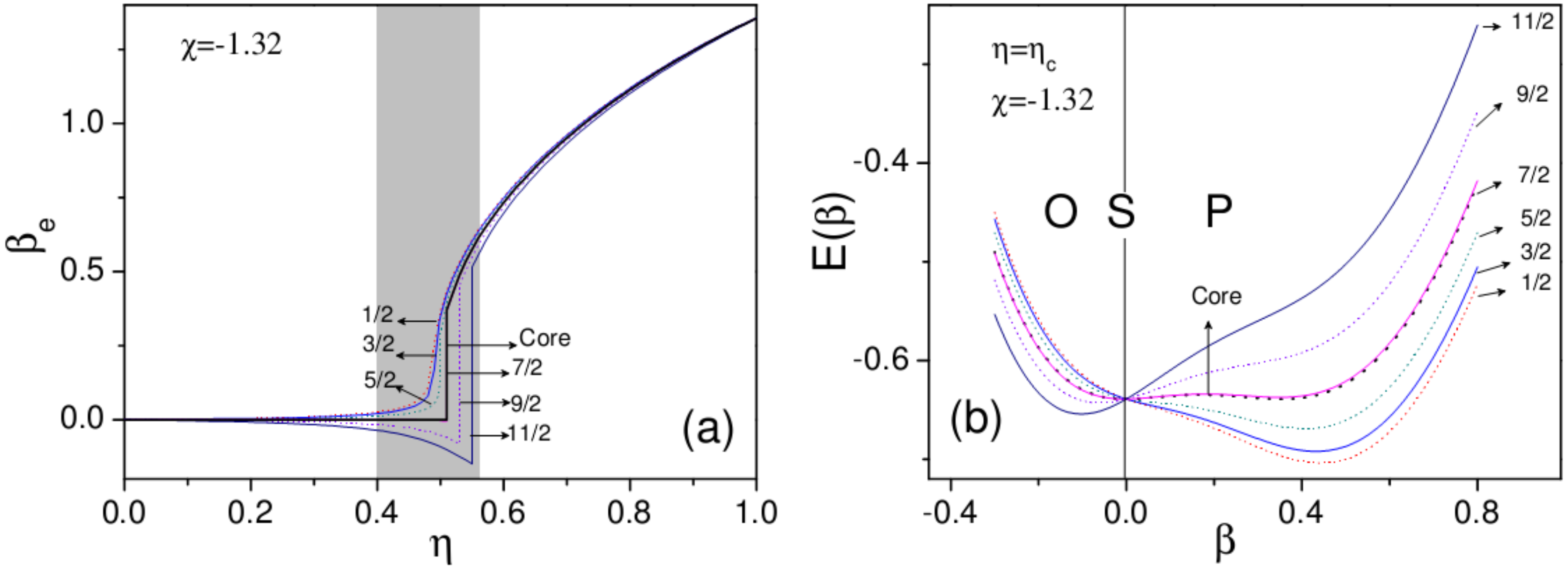
D. Petrellis, A. Leviatan, F. Iachello
Annals of Physics 326 (2011) 926–957
F. Iachello, A. Leviatan, D. Petrellis,
Physics Letters B 705 (2011) 379–382

$$H_B = \varepsilon_0\left[(1-\xi)\hat{n}_d - \frac{\xi}{4N}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi}\right],$$

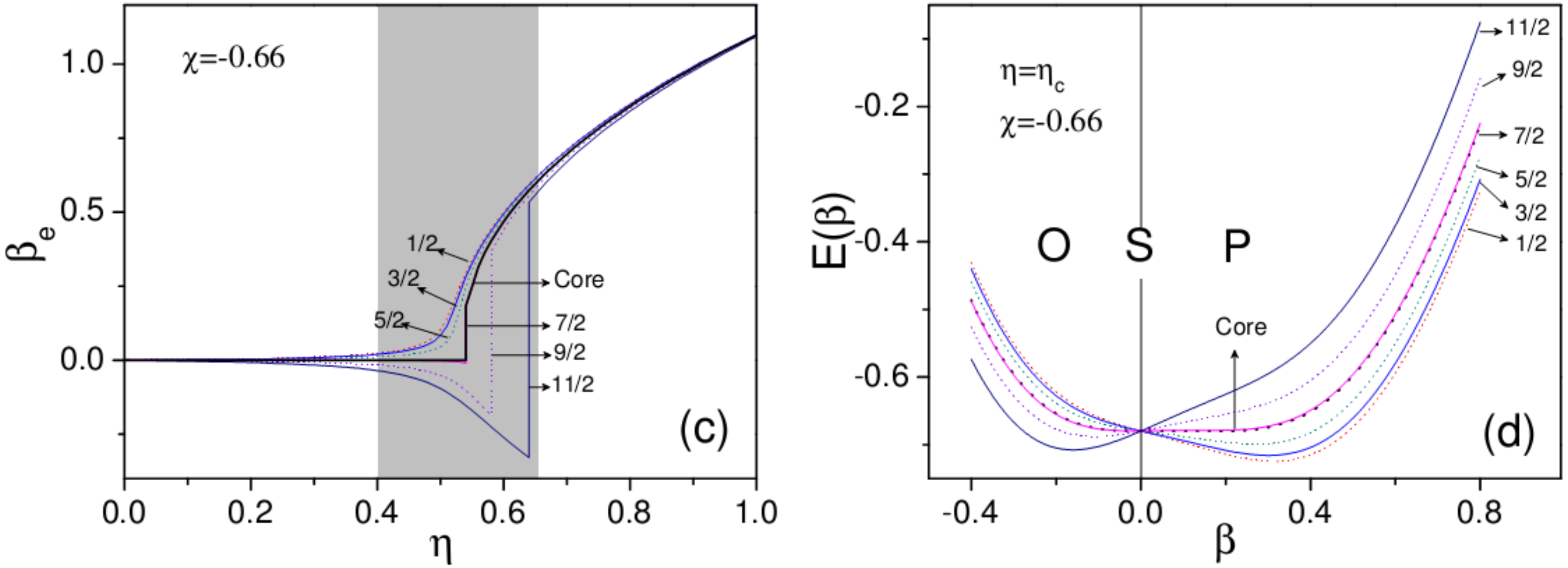
$$H_F = \varepsilon_j,$$

$$V_{BF} = \Gamma\hat{Q}^{\chi}\cdot\hat{q}.$$





$$\chi = -1.32$$



$$\chi = -0.66$$

Y. Zhang, W. Dong, H. Jiang, EPJ WoC 178, 05004 (2018)
"Shape phase transition and shape coexistence in the Bose-Fermi system".

Application... SU(3) → O(6)→$\overline{\text{SU(3)}}$

“Quantum phase transitions in odd-A nuclei:
The effect of the odd particle from *prolate to oblate* shapes”

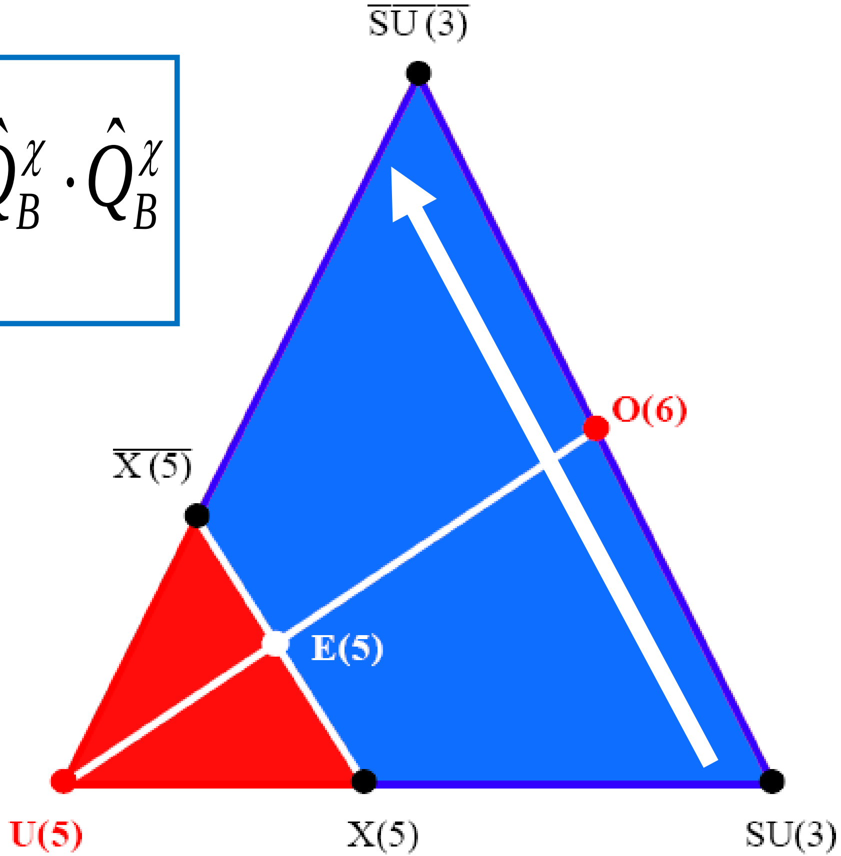
$$H^B = \cancel{(1-c)}\hat{n}_d \boxed{-\frac{c}{4N_B}\hat{Q}_B^\chi\cdot\hat{Q}_B^\chi}$$

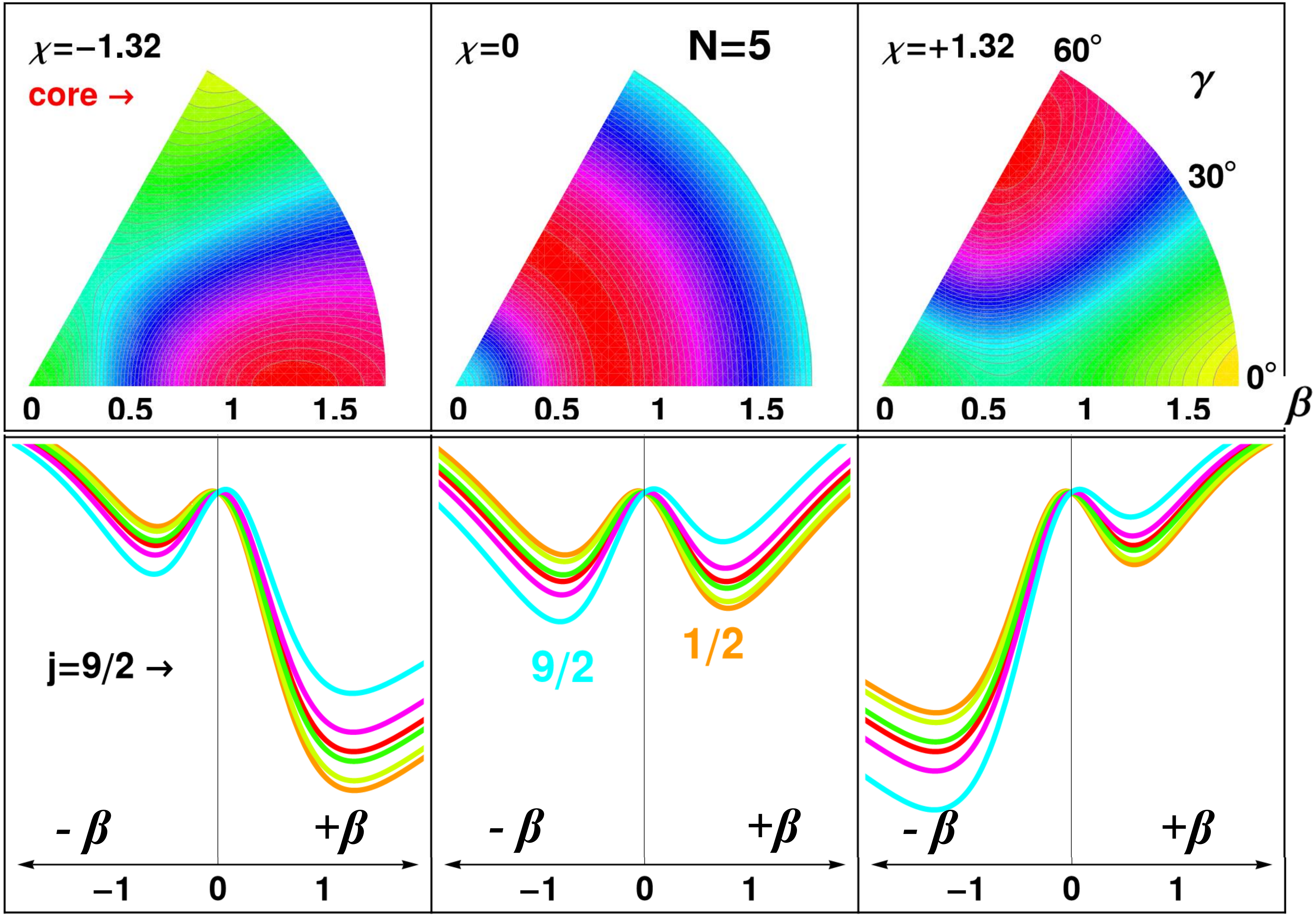
Deformed

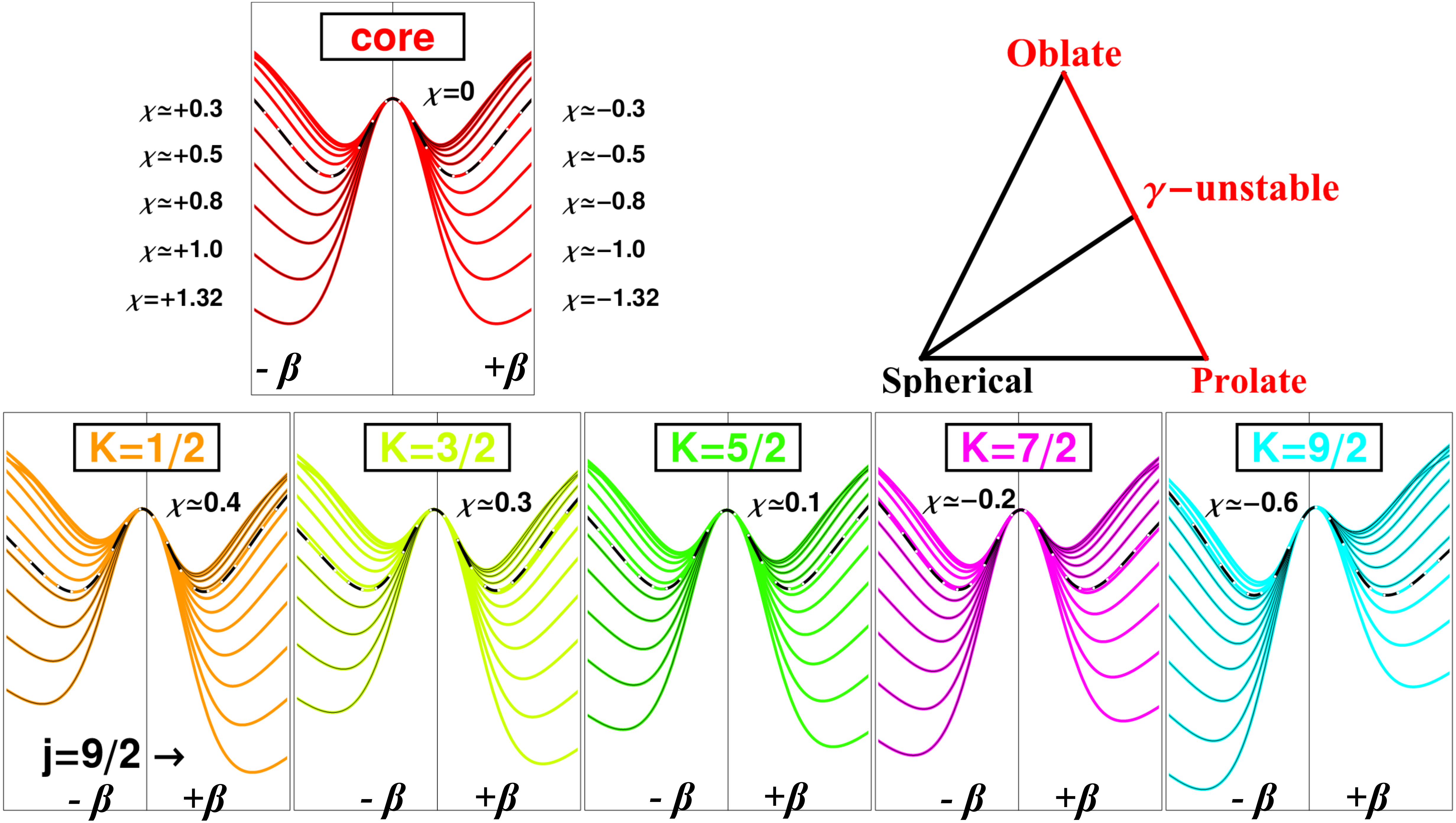
Spherical

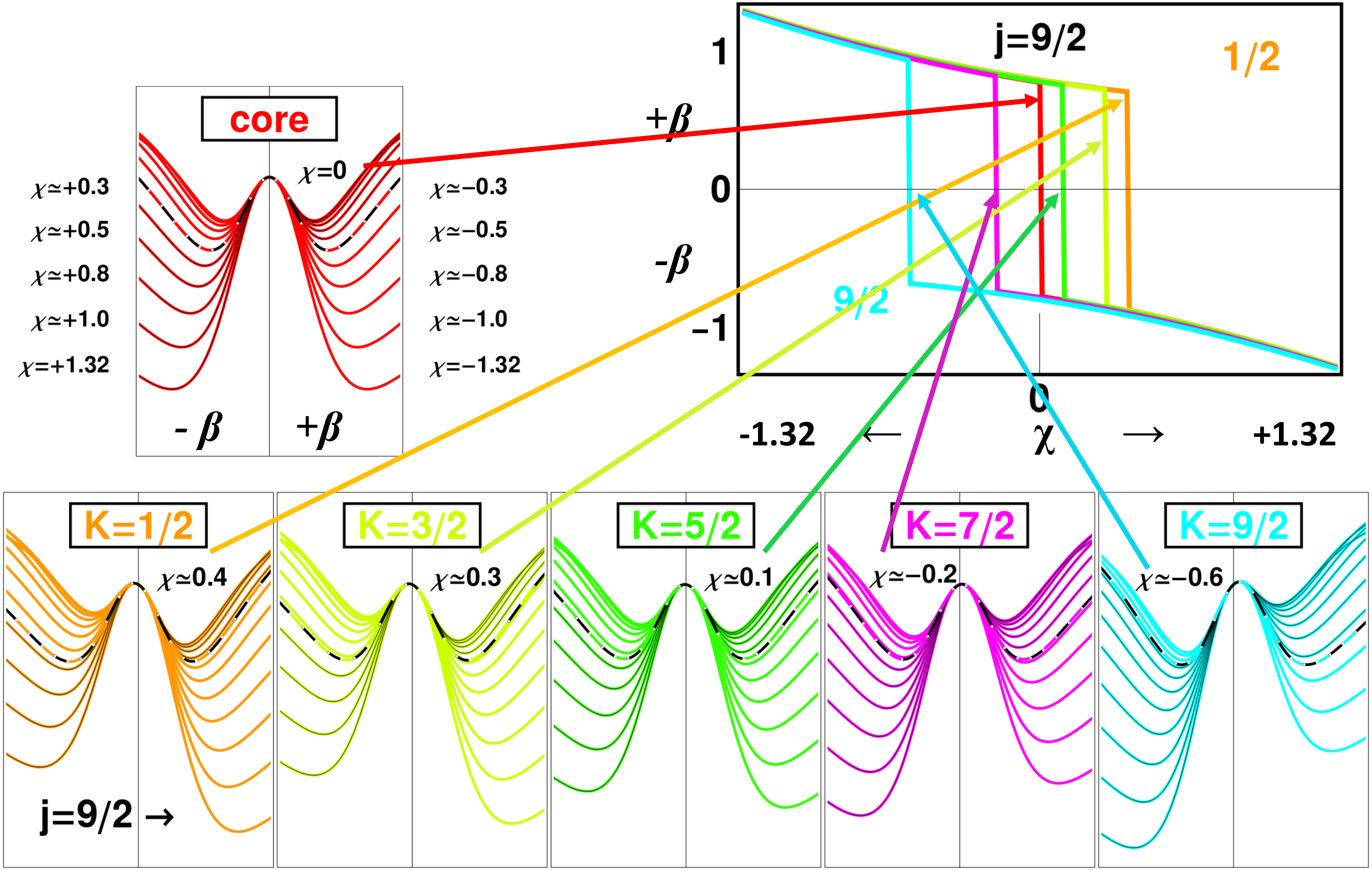
$\boxed{\mathbf{c=1}}$

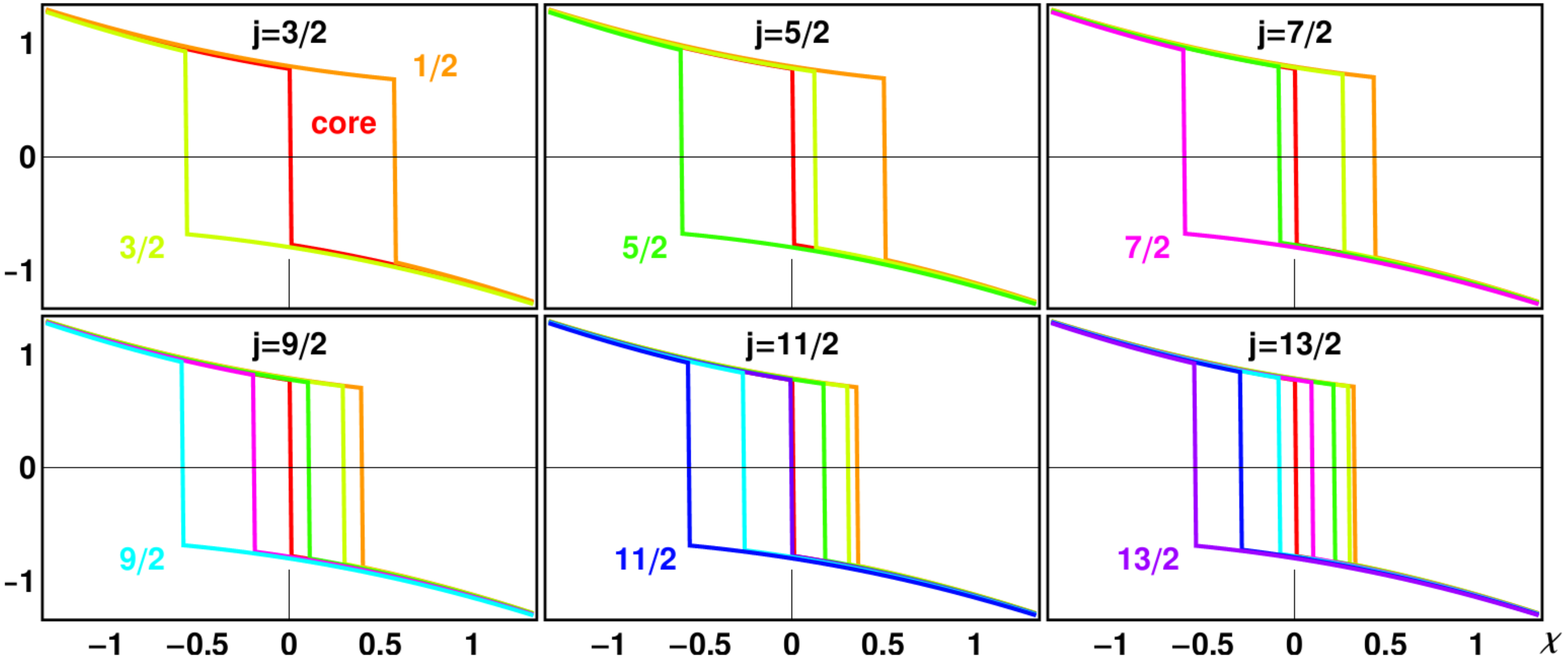
$$\boxed{\begin{gathered}H^B = (1-c)\hat{n}_d - \frac{c}{4N_B}\hat{Q}_B^\chi\cdot\hat{Q}_B^\chi \\ -\frac{\sqrt{7}}{2} \le \chi \le \frac{\sqrt{7}}{2}\end{gathered}}$$




χ=−1.32
core →
χ=0
N=5
χ=+1.32
60°
γ
30°
0°
β
0
0.5
1
1.5
j=9/2 →
9/2
1/2
- β
+β
−1
0
1


core
χ=0
$\chi\simeq$+0.3
$\chi\simeq$+0.5
$\chi\simeq$+0.8
$\chi\simeq$+1.0
χ=+1.32
$\chi\simeq-0.3$
$\chi\simeq-0.5$
$\chi\simeq-0.8$
$\chi\simeq-1.0$
$\chi=-1.32$
- β
+β
Oblate
γ−unstable
Spherical
Prolate
K=1/2
$\chi\simeq$0.4
K=3/2
$\chi\simeq$0.3
K=5/2
$\chi\simeq$0.1
K=7/2
$\chi\simeq-0.2$
K=9/2
$\chi\simeq-0.6$
j=9/2 →
- β
+β


core
χ=0
χ≃+0.3
χ≃+0.5
χ≃+0.8
χ≃+1.0
χ=+1.32
χ≃−0.3
χ≃−0.5
χ≃−0.8
χ≃−1.0
χ=−1.32
- β
+β
j=9/2
1/2
1
+β
0
-β
9/2
−1
0
-1.32
←
χ
→
+1.32
K=1/2
χ≃0.4
K=3/2
χ≃0.3
K=5/2
χ≃0.1
K=7/2
χ≃−0.2
K=9/2
χ≃−0.6
j=9/2 →


j=3/2
1/2
core
3/2
j=5/2
5/2
j=7/2
7/2
j=9/2
9/2
j=11/2
11/2
j=13/2
13/2
1
0
-1
1
0
-1
−1
−0.5
0
0.5
1
−1
−0.5
0
0.5
1
−1
−0.5
0
0.5
1
χ

28.05.2018

Application... X(5) → E(5)→$\overline{\text{X(5)}}$

“Quantum phase transitions in odd-A nuclei:
The effect of the odd particle along to critical line”

Deformed

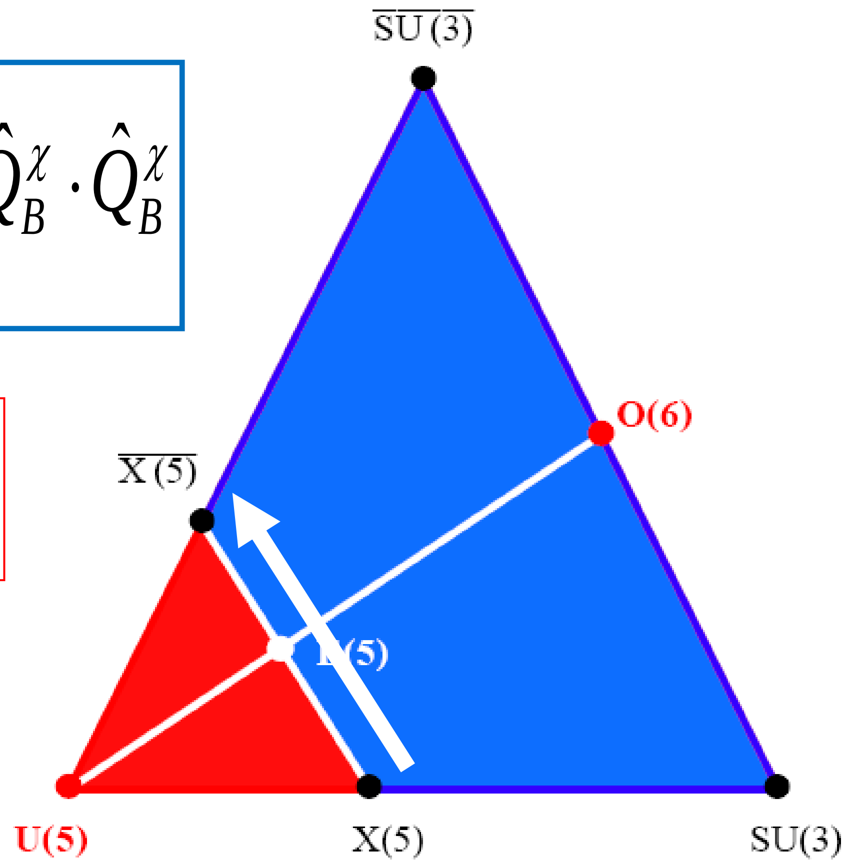
$$H^B = (1-c)\hat{n}_d \boxed{-\frac{c}{4N_B}\hat{Q}_B^{\chi}\cdot\hat{Q}_B^{\chi}}$$

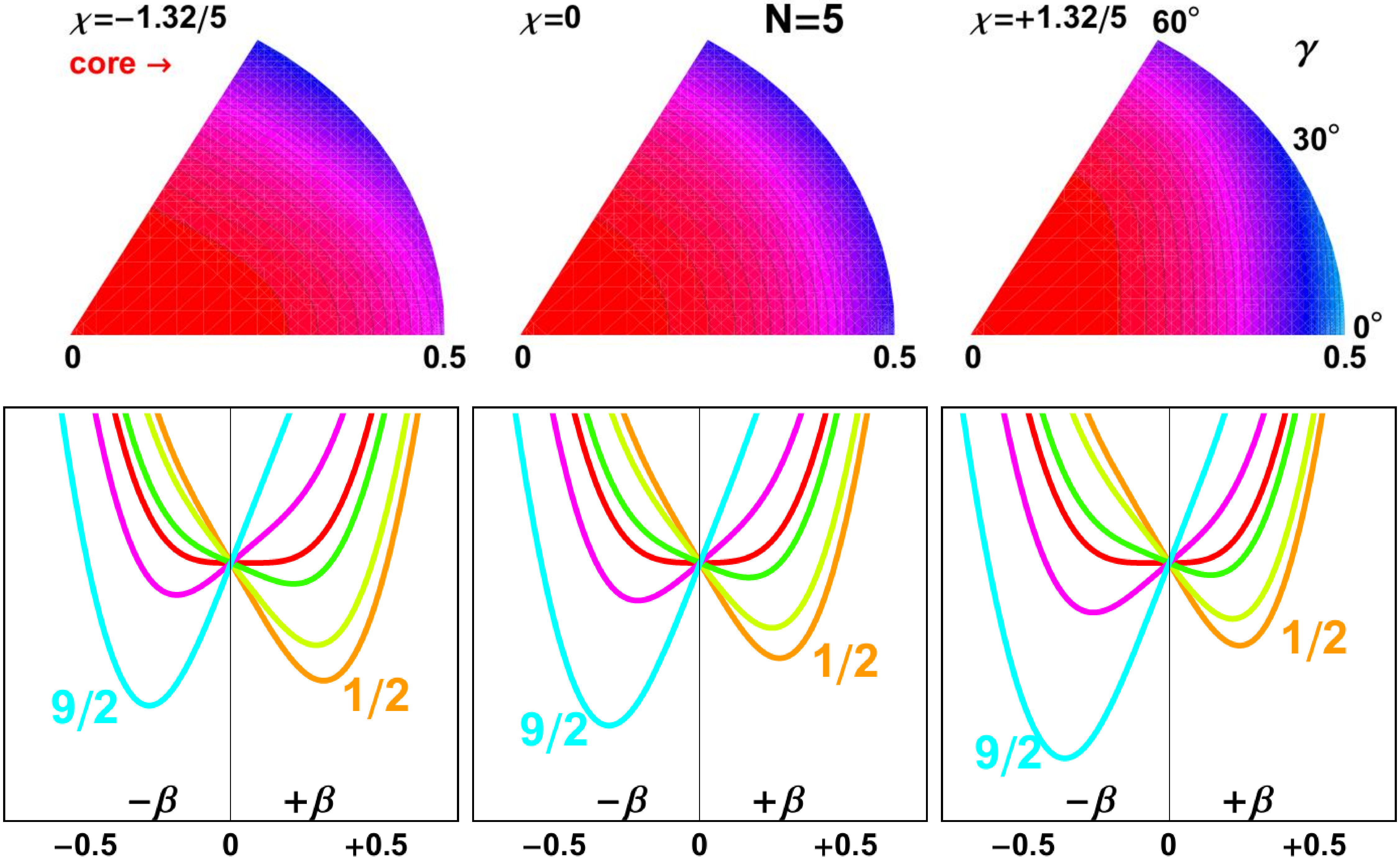
Spherical

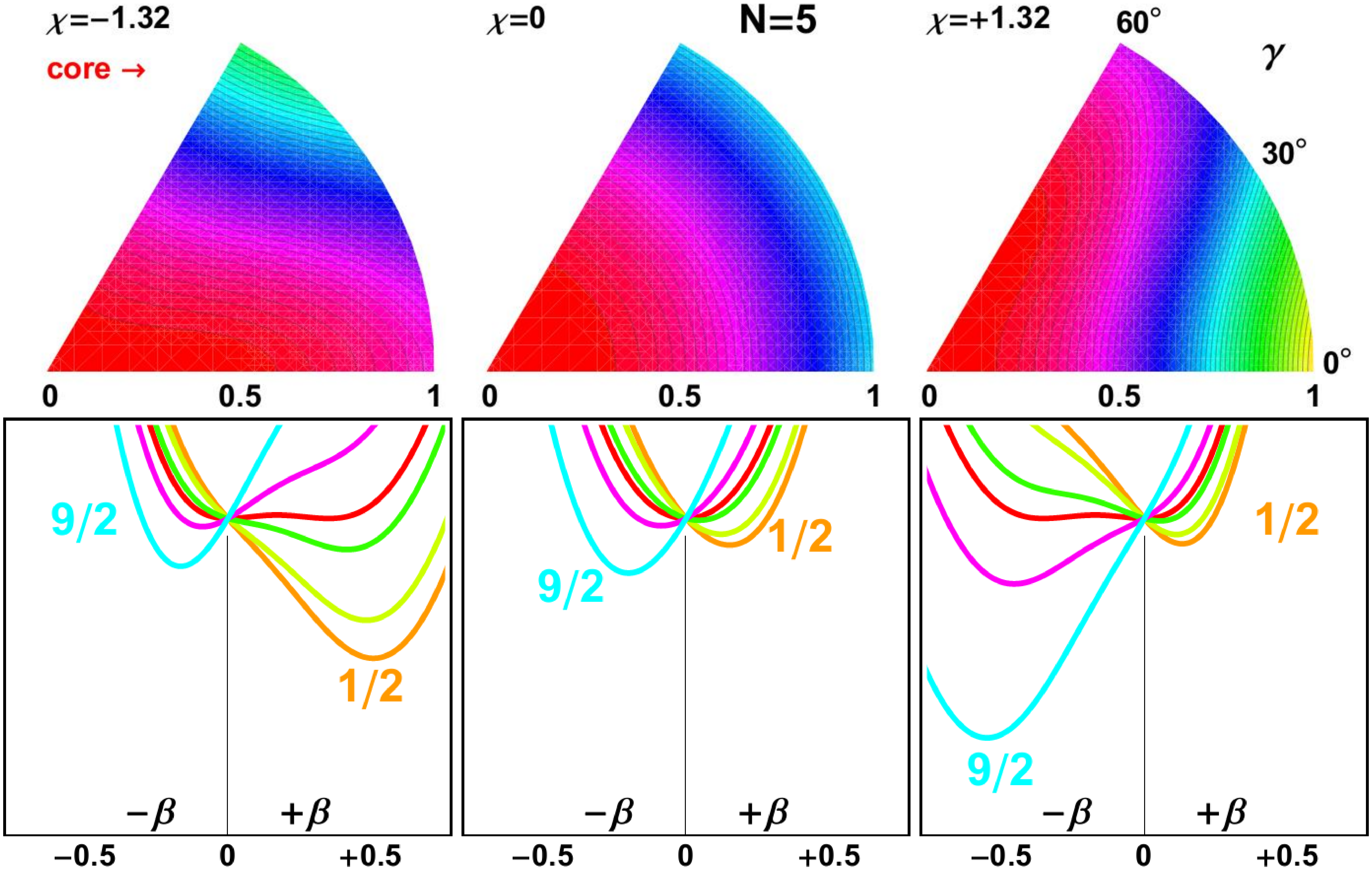
$$\boxed{H^B = (1-c)\hat{n}_d - \frac{c}{4N_B}\hat{Q}_B^{\chi}\cdot\hat{Q}_B^{\chi} \qquad -\frac{\sqrt{7}}{2} \le \chi \le \frac{\sqrt{7}}{2}}$$

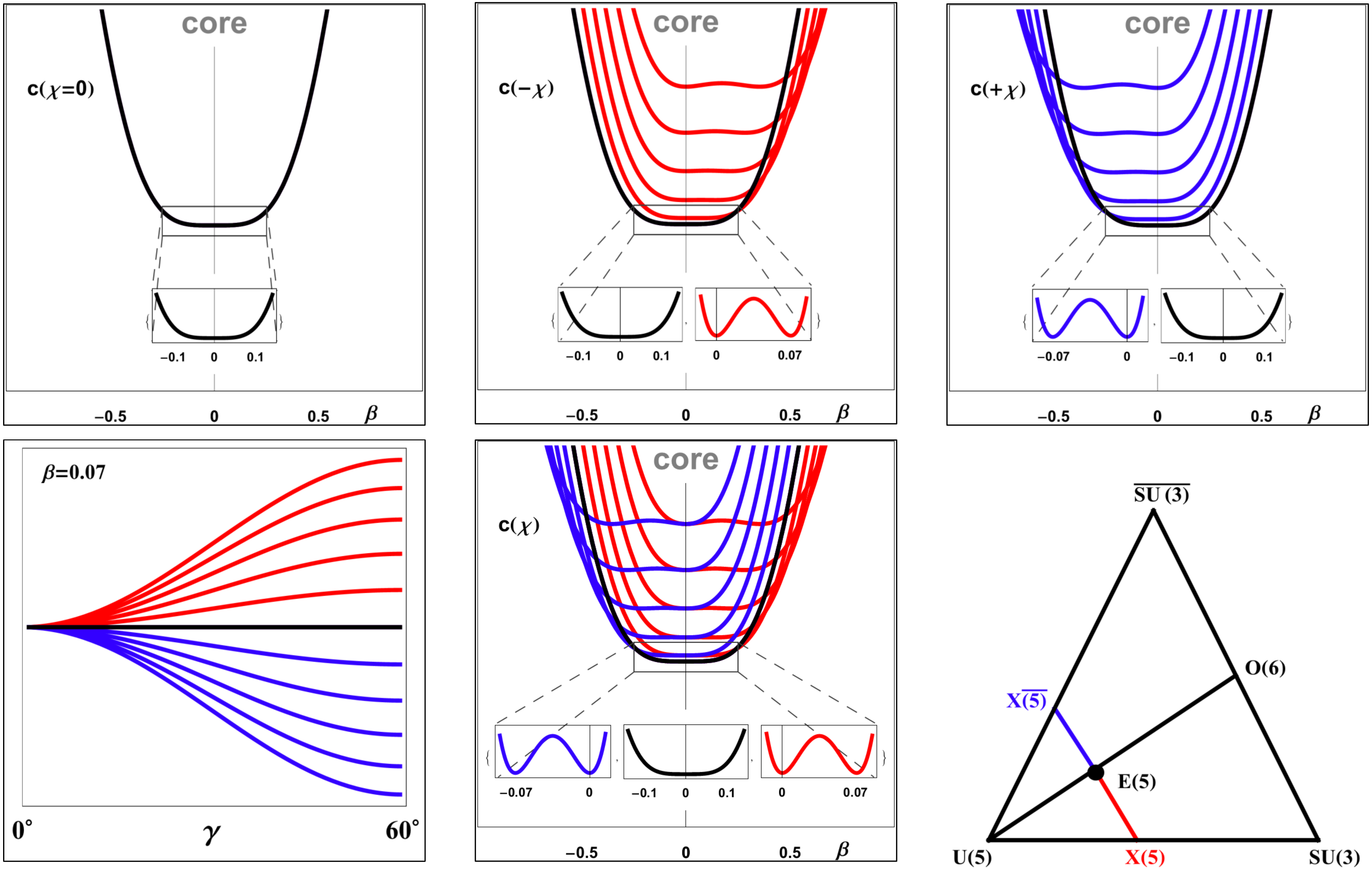
$$\boxed{c_{cr}(N,-\sqrt{7}/2 \le \chi \le +\sqrt{7}/2) = \frac{28N}{56(N-1)+\chi^2(5+2N)}}$$

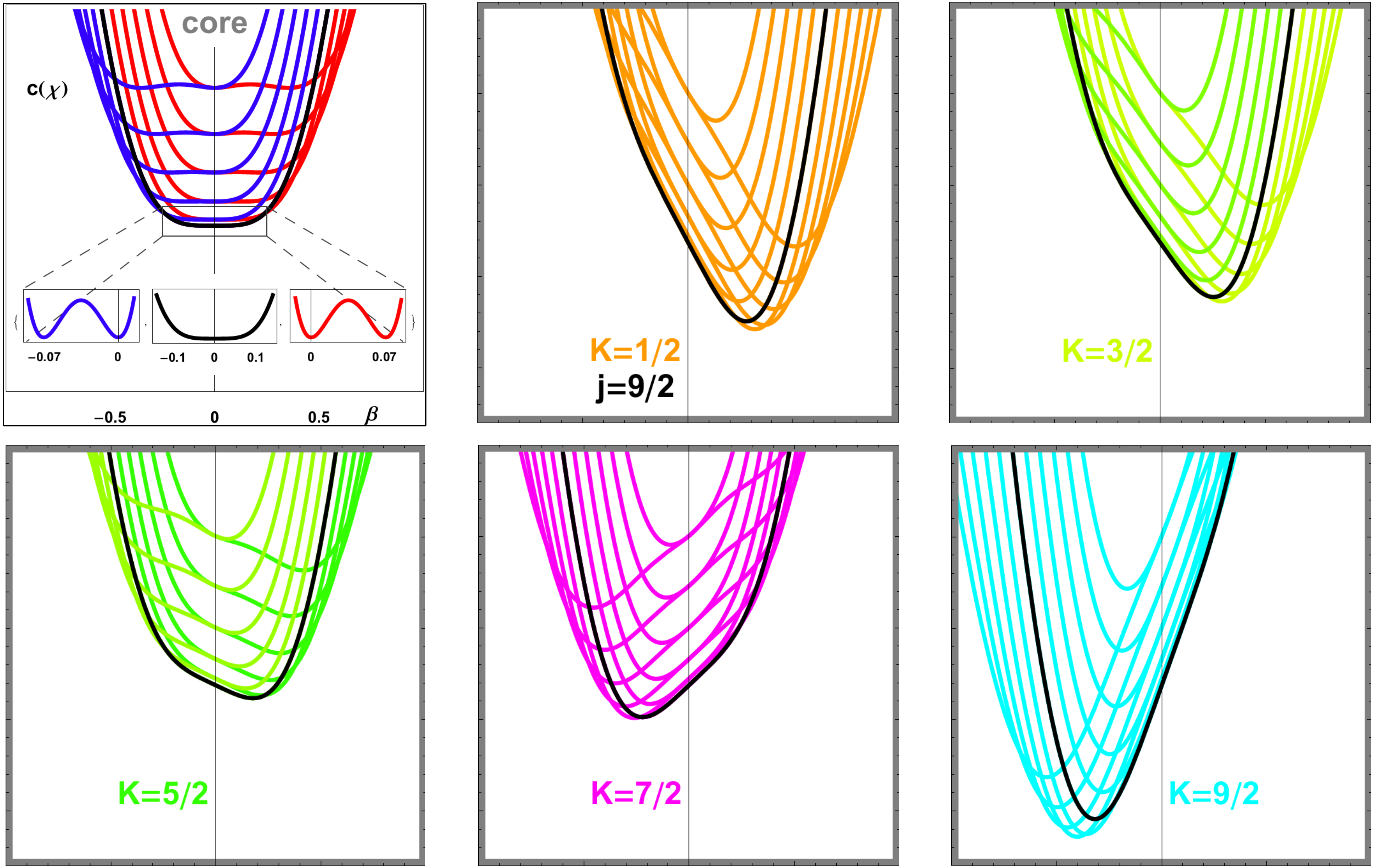
$$\boxed{c_{cr}(N=5,\chi) = \frac{140}{224+15\chi^2}}$$

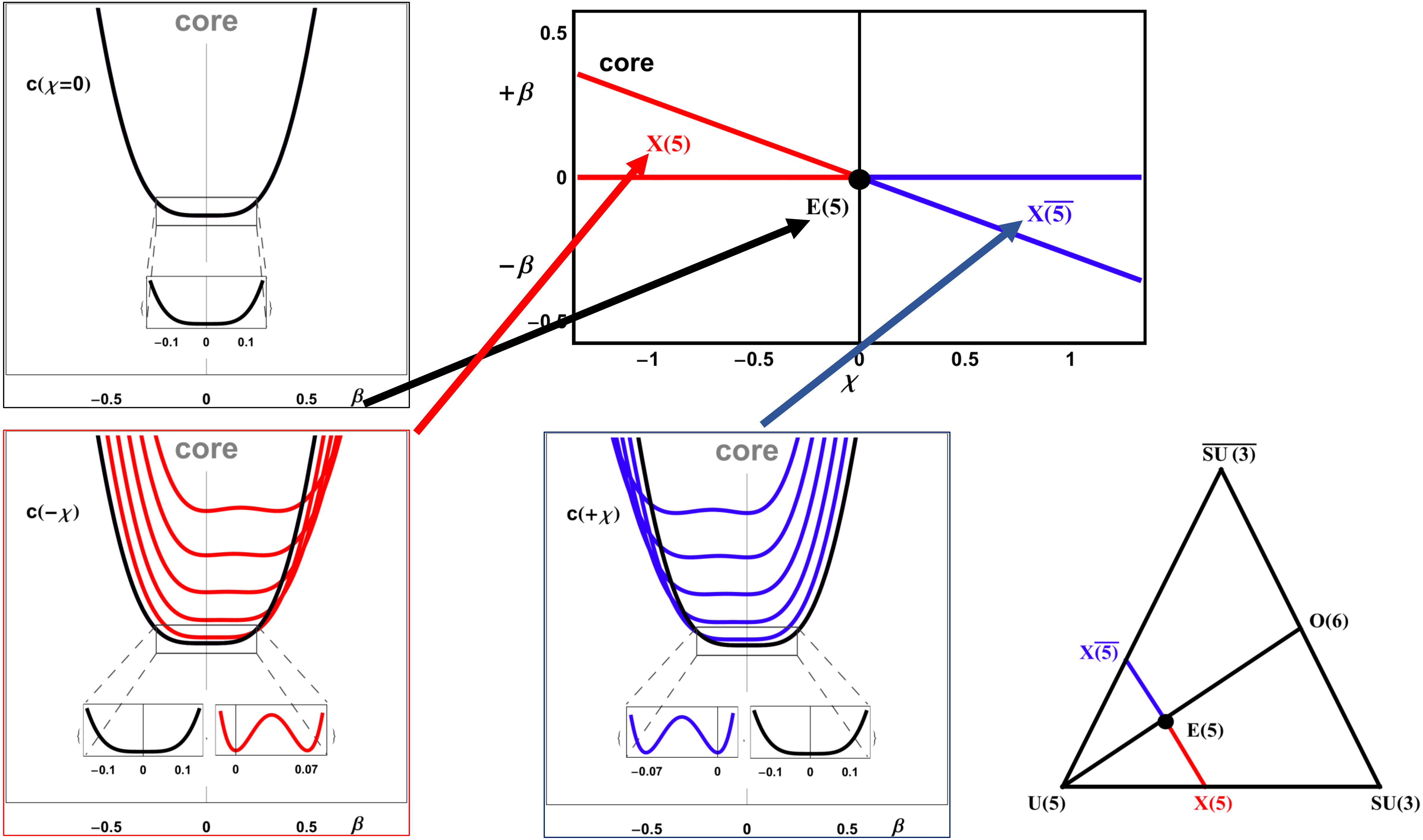


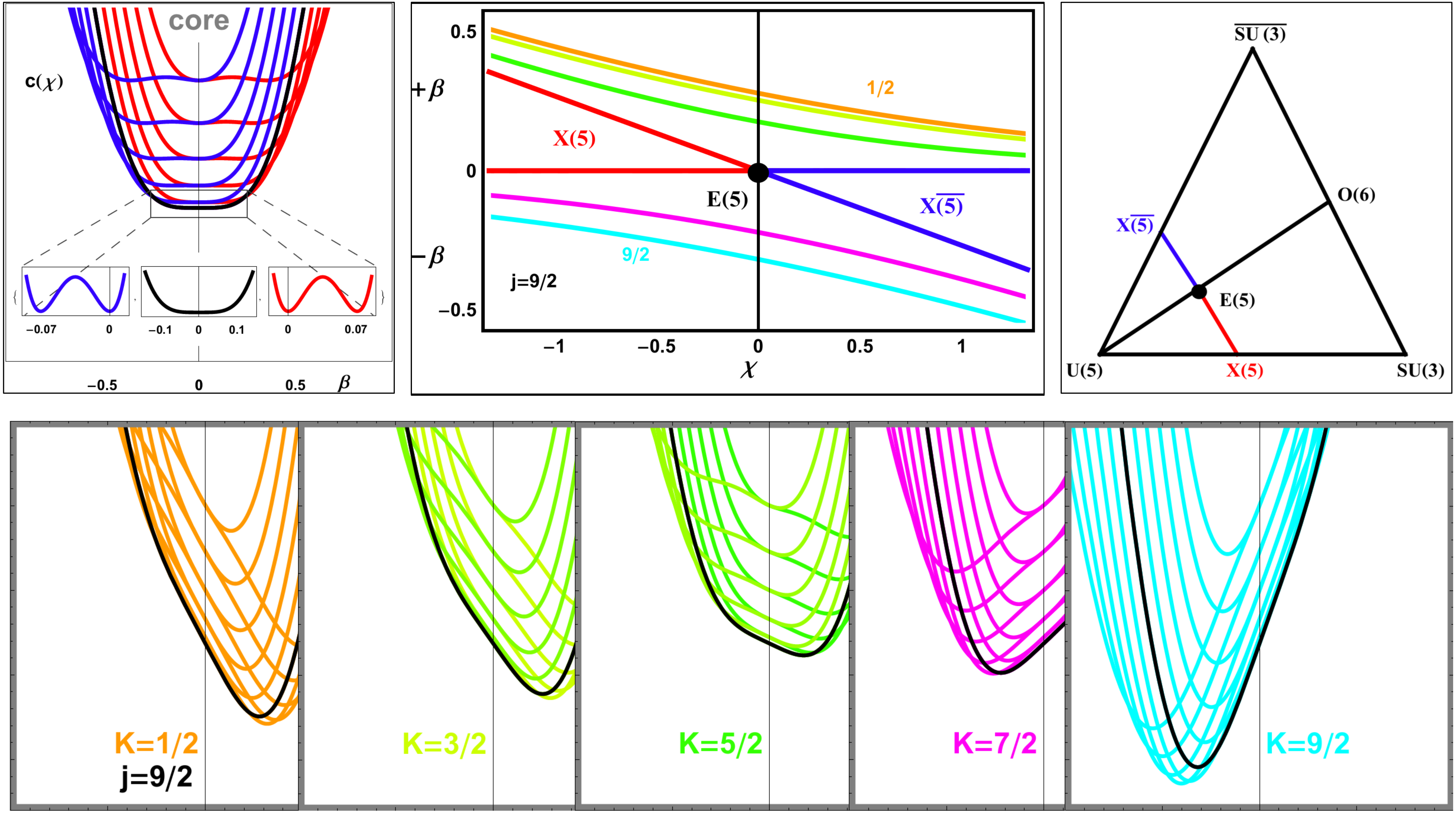

χ=−1.32/5
core →
χ=0
N=5
χ=+1.32/5
60°
γ
30°
0°
0
0.5
0
0.5
0
0.5
9/2
1/2
$-\beta$
$+\beta$
9/2
1/2
$-\beta$
$+\beta$
9/2
1/2
$-\beta$
$+\beta$
−0.5
0
+0.5
−0.5
0
+0.5
−0.5
0
+0.5

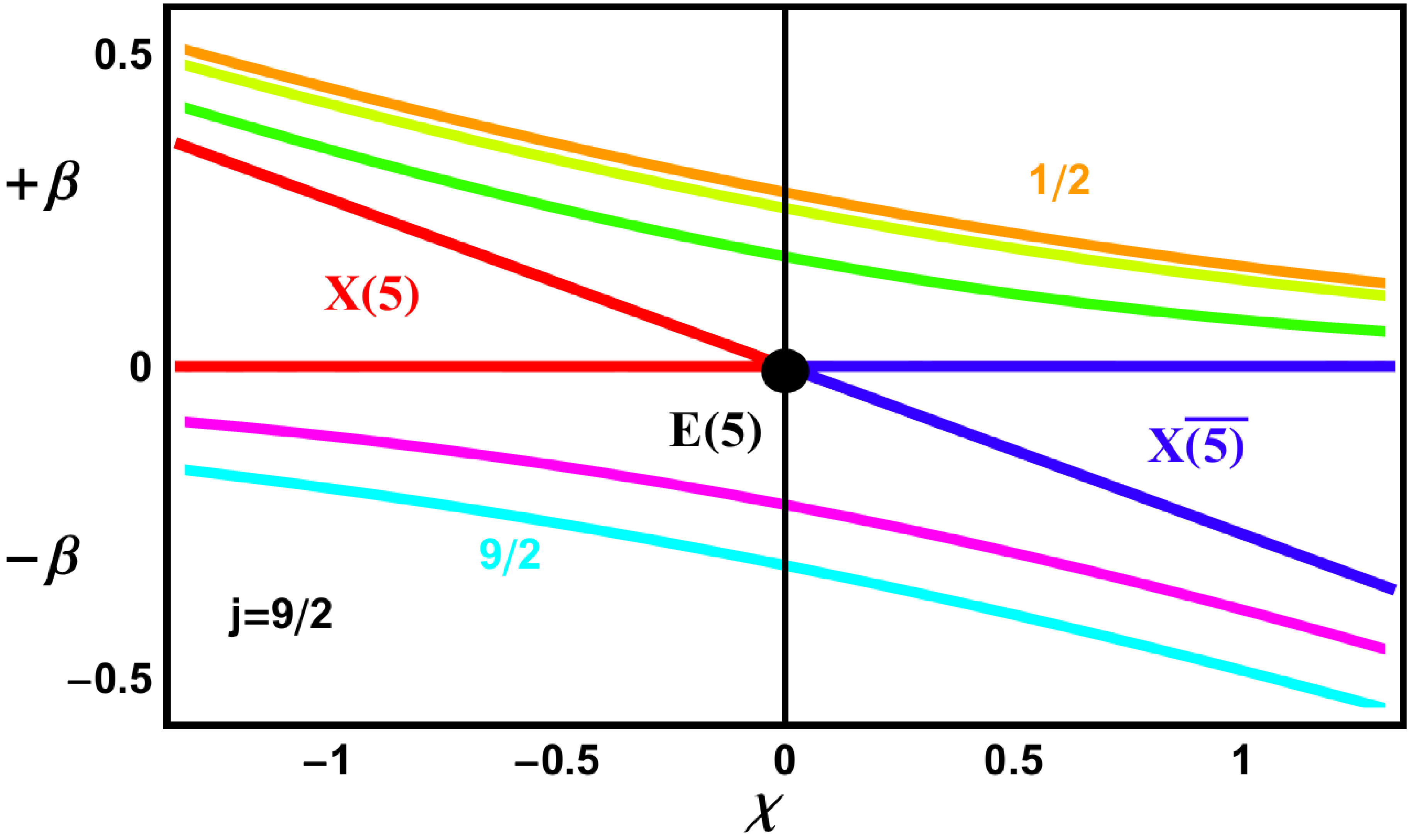

$\chi=-1.32$
core →
$\chi=0$
N=5
$\chi=+1.32$
60°
γ
30°
0°
0
0.5
1
0
0.5
1
0
0.5
1
9/2
1/2
$-\beta$
$+\beta$
−0.5
0
+0.5
1/2
9/2
$-\beta$
$+\beta$
−0.5
0
+0.5
1/2
9/2
$-\beta$
$+\beta$
−0.5
0
+0.5


core
c(χ=0)
−0.1
0
0.1
−0.5
0
0.5
β
core
c(−χ)
−0.1
0
0.1
0
0.07
−0.5
0
0.5
β
core
c(+χ)
−0.07
0
−0.1
0
0.1
−0.5
0
0.5
β
β=0.07
0°
γ
60°
core
c(χ)
−0.07
0
−0.1
0
0.1
0
0.07
−0.5
0
0.5
β
$\overline{\text{SU (3)}}$
O(6)
$\text{X}\overline{(5)}$
E(5)
U(5)
X(5)
SU(3)


core
c(χ)
{
−0.07
0
,
−0.1
0
0.1
,
0
0.07
}
−0.5
0
0.5
β
K=1/2
j=9/2
K=3/2
K=5/2
K=7/2
K=9/2


core
c(χ=0)
−0.1
0
0.1
−0.5
0
0.5
β
core
+β
0.5
X(5)
0
E(5)
X$\overline{(5)}$
−β
−0.5
−1
−0.5
0
0.5
1
χ
core
c(−χ)
−0.1
0
0.1
0
0.07
−0.5
0
0.5
β
core
c(+χ)
−0.07
0
−0.1
0
0.1
−0.5
0
0.5
β
$\overline{\text{SU (3)}}$
O(6)
X$\overline{(5)}$
E(5)
U(5)
X(5)
SU(3)


core
c(χ)
−0.07
0
−0.1
0
0.1
0
0.07
−0.5
0
0.5
β
0.5
+β
0
−β
−0.5
X(5)
E(5)
$X\overline{(5)}$
1/2
9/2
j=9/2
−1
−0.5
0
0.5
1
χ
$\overline{SU(3)}$
O(6)
$X\overline{(5)}$
E(5)
U(5)
X(5)
SU(3)
K=1/2
j=9/2
K=3/2
K=5/2
K=7/2
K=9/2


0.5
+β
0
−β
−0.5
−1
−0.5
0
0.5
1
χ
1/2
X(5)
E(5)
$X\overline{(5)}$
9/2
j=9/2

Q P T n 9

Padova (Italy)
22-25 May
2018

THANK YOU VERY MUCH

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