Astroparticle physics Lectures at the PhD School of the University of Padua, 2017



Alessandro De Angelis

ASTROPARTICLE PHYSICS

by Alessandro De Angelis

Lectures for the PhD courses in Astronomy and Physics

Padova, 2017

Preface

One of the new frontiers of fundamental physics is the field called astroparticle physics, joining astrophysics and elementary particle physics. In particular the gamma-ray sky at high-energy (above the MeV) unveils the nature of fundamental astrophysical phenomena, and opens a window on new physics, possibly including the nature of dark matter.

This book collects the slides and some extra material presented in the course on astroparticle physics held for the PhD in Astronomy and for the PhD in Physics at the University of Padova in the Academical Year 2016/17. This course introduces such an interdisciplinary subject, providing students with the tools needed to understand current problems, read a modern article in the field, and analyze the data from the leading high-energy telescope Fermi/LAT - which are, as usual for NASA, public. Some basics links related to neutrino astrophysics and to the recent field of astrophysics with gravitational waves are also provided.

Students attending this course are expected to know already the basics of quantum mechanics and of special relativity; they should also know about basic physics processes and the expansion of the Universe. At the end of this course, the successful students are able to:

- 1. Understand the basic physical processes involving high-energy particles and originating the emission of high-energy messengers in particular, photons from astrophysical accelerators in high-density regions and from Dark Matter.
- 2. Know the methods and observing techniques to study high-energy emissions.
- 3. Describe the sky as seen with high-energy detectors.
- 4. Identify the kinds of astrophysical sources visible at high energies and relate them to relevant emission processes.
- 5. Have insight into current research in gamma and multimessenger astroparticle physics.
- 6. Read a scientific article related to gamma and multimessenger astroparticle physics.
- 7. Analyze the data from the Fermi/LAT gamma-ray satellite; extract a spectral energy distribution and a light curve for a generic source.

The lectures follow the textbook by De Angelis and Pimenta, "Introduction to particle and astroparticle physics", Springer 2015; some material has been updated, and this update is reflected in the slides presented in this book.

I thank Claudia Lazzaro and Francesco Longo, who wrote, respectively, the chapter on gravitational waves and the chapter on the analysis of Fermi-LAT data. M. Pimenta, S. Andringa, S. Ansoldi, U. Barres, D. Cannone, R. Conceiçao, R. Shellard, F. Simonetto, B. Tomé helped in the solution of the exercises. And mostly I thank the students, who motivated and inspired the course.

Alessandro De Angelis

Padova, May 2, 2017

Table of Contents

- 1. Introduction: high-energy phenomena in the Universe.
- 14. Detection of charged particles and neutrinos. Present detectors. (*)
- 58. The budget of energy and matter in the Universe: Dark Matter.
- 68. How to detect high-energy photons: (a) Techniques; (b) Present detectors.
- 115. How high-energy photons are produced and how they propagate. Acceleration sites. WIMPs and high-energy astrophysics.
- 159. Anomalous propagation of photons. Axion-Like Particles. Testing relativity and cosmology with photons. VHE neutrinos. (*)
- 173. Gravitational waves: physics, detectors, present and future, by Dr. C. Lazzaro. (*)
- 191. The future.
- 223. How to analyze Fermi data, by Dr. F. Longo.
- 265. Rules for the final exam.
- 267. Exercises (mostly solved).

(*) = Mandatory for physicists only.





and an and the second sec	Subject	Course material	Exercises	Notes	_
1) March 28, 201 (2h) Starting at 15 Room 207	7 Introduction; high- energy phenomena in the Universe. The budget of energy and matter in the Universe: Dark Matter.	14, 3.1, 8.1.4, 8.3.3, 8.4, 7.6 (from the textbook) <u>Slides</u>	1.6, 1.7, <u>8.11</u> , 8.12, 8.13		
2) March 29 (3) Starting at 10 Room 315	 Detection of charged particles, neutrinos, gravitational waves 		Service of	Mandatory physicists only	for
3-4) March 30 (2) Starting at 15 Room 315	 How to detect high- energy photons. 	4.1, 4.22, 4.23, 4.25, 4.3.2, 4.5 excluding 4.5.5 (from the textbook)	42, 44, 45, 46, 47, 48, 49, 4.12, 4.14, 10.4, 10.5, 10.8		
Starting at 15 Room 315	, ,	Sides			
5-6) April 5 (2) Starting at 10 Room 315 & April 6 (2) Starting at 15 Room 315	 How high-energy photons are produced and how they propagate. Acceleration sites. Dark matter and high-energy astrophysics. Multimessenger high- energy astrophysics. 	10.12, 10.2, 10.3 excluding 10.3.6, 10.4.1, 10.4.2, 10.4.3, 10.5.1, 10.5.2, 10.5.3 (from the textbook) <u>Slides</u>	103, 25, 10.12, 10.6, 10.14, 10.15, 10.13		
7) April 12 (3) Starting at 10 Room 315	 Results from the a, detection of charged particles, neutrinos, gravitational waves 			Mandatory physicists only	for
8) April 13 (2) Starting at 15 Room 315	 A look to the future. 	10.5.4, 10.5.5 (from the textbook); original part on future detectors.	No exercises ©		1
and the second second	and the second second	Slides		_	
9-10) April 18, h 1 (bh) & April 19, h 1 (bh) Aula <u>Informatica</u> Polo <u>Didattico</u> [Longo Menguzzato]	5 Tutorial: analysis of 0 real Fermi data. Students will learn how to access and analyze public data from the space-borne Fermi Large Area Telescope. They will extract the spectrum and light- curve from a real gamma-ray source, and discuss its properties in the theoretical learnin the theor	Slides			



About your lecturers

Alessandro De Angelis is a high-energy (astro)physicist. Professor in Udine and Lisbon, he is Director of Research at INFN Padova and PI of the e-ASTROGAM mission; he works for the MAGIC and CTA Collaborations. His main research interest is fundamental physics, especially astrophysics and elementary particle physics at accelerators. After graduating in Padova, Alessandro was employed at CERN in the 1990s, and he later was among the founders of the NASA Fermi gamma-ray telescope. He lectured electromagnetism and astroparticle physics in Italy and Portugal; has been visiting Professor at the ICRR Tokyo, MPI Munich, and Paris VI. Office phone: 049 967.7364; email <u>alessandro.deangelis@pd.infn.it</u>

Francesco Longo is a gamma-ray astrophysicist, staff at the University of Trieste and leader of the local Fermi/MAGIC/CTA group. He convenes the working group on Transients in MAGIC, and is involved in R&D of e-ASTROGAM. His main research topics are GRBs and fast transients. Graduated inTrieste, he made his PhD in Ferrara with Pierluigi Fortini in 2002. After this he worked at the University of Trieste with Guido Barbiellini and was involved in the development of the gamma-ray satellites AGILE and Fermi, particularly in their MonteCarlo simulations. Later he coordinated the Fermi working group on "Solar Flares. Currently he is teaching Particle Astrophysics at the University of Trieste. He was Visiting professor at the University of Ljubljana. Office phone: 0403756222; email francesco.longo@ts.infn.it





Messengers from the Universe - I

- Coulomb 1785: electroscopes can spontaneously discharge by the action of the air and not by defective insulation
- 1879: Crookes measures that the speed of discharge of an electroscope decreased when pressure was reduced (=> direct agent is the ionized air)
- Curie 1900: the discharge rate of an electroscope can be used to gauge the level of radioactivity
- Beginning of the XX century: is the radioactivity discharging electroscopes due to radioisotopes in the soil, or to radiation coming from the Cosmos?

Messengers from the Universe - II

- 1911/12: Domenico Pacini and Victor Hess perform two complementary experiments: Pacini discovers that ionizin gardiation decreases underwater, and Hess that it increases at high altitudes
 - 20% of the natural radiation at ground is due to cosmic radiation!!! Can we use this cosmic radiation for science?



















Conclusion: extremely difficult to use charged CR for astrophysics





31/03/17











(1) Interaction of accelerated charged particles with radiation and matter fields

- Gamma-ray production and absorption processes: several but well studied
- These phenomena generally proceed under extreme physical conditions in environments characterized by
 - huge gravitational, magnetic and electric fields,
 - very dense background radiation,
 - relativistic bulk motions (black-hole jets and pulsar winds)
 - shock waves, highly excited (turbulent) media, etc.
- They are related to, and their understanding requires knowledge of,
 - nuclear and particle physics,
 - quantum and classical electrodynamics,
 - special and general relativity,
 - plasma physics, (magneto) hydrodynamics, etc.
 - astronomy & astrophysics

Padova 2017

Alessandro De Angelis

20

















31/03/17



















Zwicky conjectures (1933)

- 1. Heavy enough stars collapse at the end of their lives into super-novae
- 2. Implosions produce explosions of cosmic rays
- 3. They leave behind neutron stars



Tycho's Supernova (SN 1572)



























PAMELA

- Obiettivi dell'esperimento:
 - Misurare lo spettro di antiprotoni, positroni e (anti)nuclei in un ampio intervallo di energie;
 - Ricerca di antimateria "primordiale"
 - Studio del flusso dei RC primari
- PAMELA è capace di misurare rigidità magnetiche (=impulso/ carica) sino a 700 GV/c.





Integrazione e posizionamento nel satellite
































31/03/17























LA FISICA DELLO SCIAME ESTESO

- La distanza media tra i contatori determina la *energia minima* dello sciame rivelabile.
- Il numero dei contatori, la precisione della misura
- L'area totale coperta, determina la *massima energia* misurabile.
- Ciascun contatore (*casetta*) misura in modo proporzionale la perdita di energia delle particelle che lo attraversa; da qui, si risale al numero di particelle incidenti
- Dalle misure della densità di particelle in ciascuna casetta dell'array, si risale alla distribuzione laterale *D*(*r*).
- Dalla misura di *D(r)* si risale all'energia del primario, *e* dalla frequenza del numero di conteggi si risale al flusso.
- La direzione dello sciame può essere determinata dalla misura dei tempi di ritardo temporale nell'arrivo dello sciame su diverse casette (le particelle dello sciame sono ⊥ al suo asse)





Composizione chimica dei RC nella regione degli EAS

- Il modello del *leaky box* prevede un arricchimento di elementi pesanti nei RC sino al ginocchio.
- Gli EAS possono misurare <A> con difficoltà.
- Le misure possono essere poi confrontate con modelli estremi (solo p o Fe) via MC



Altri metodi di Rivelazione

Le particelle cariche dello sciame EM che giungono al suolo possono essere rivelate da rivelatori di sciami estesi (§5.3)

- Gli sciami di particelle producono anche <u>luce nell'atmosfera per effetto Cherenkov</u> (gli elettroni con E>20+30 MeV).
- La luce Cerenkov può venire rivelata (telescopi Cherenkov) nelle notti senza luna da appositi rivelatori al suolo.
- Gli sciami EM inducono anche <u>l'eccitazione dell'azoto atmosferico</u>, che riemette irraggiando luce. Questa fluore-scenza può essere rivelata al suolo (<u>Rivelatori fluorescenza</u>).
- La componente di muoni può essere rivelata da rivelatori "underground".





























Main components of the observatory:

• Surface detector (SD): array of 1600 water-Cherenkov tanks, spaced by 1.6 km

• Fluorescence detector (FD): 24 telescopes in 4 buildings

Large detectors are needed!

Located in the southern hemisphere, it is the **world's largest experiment of UHECRs** with its extension of **3000 km**²

Powerful hybrid detection technique



AUGER: Un rivelatore ibrido

Rivelatore di sciami: 1600 taniche cilindriche (ciascuna di 10 m² ed alte 1.5 m) riempite di acqua, per rivelare gli sciami al suolo tramite la luce Cerenkov emessa dagli elettroni nell'acqua

Il rivelatore di sciami misura la distribuzione laterale e temporale dello sciame Distanza tra taniche: 1.5 km Area di forma esagonale, di 60×60 km2

Rivelatori di fluorescenza: 6 telescopi con ciascuno 4 "occhi" per determinare il profilo longitudinale dello sciame e l'altezza del suo massimo.





75





















UHECR study is an experimental challenge

Extremely low fluxes..

A direct detection can not be performed

We need very large surface detection areas

and we can measure the secondary fluxes of particles! Extensive air showers

NB: not only particles, also radiation, e.g. fluorescence light So, particle detectors and/or light detectors

and extremely high energies

Monte-Carlo simulations of EAS rely on models for the hadronic interactions, which are an extrapolation of LHC results to much higher energies and different kinematic regions















The Hubble constant and the energy density of the Universe determine the fate of the Universe

















Beyond the Minimal SM of Particle Physics The SM of PP has been incredibly successful. It looks however an ad-hoc model, and the SU(3) SU(2) U(1) looks like a lowenergy symmetry which is part of a bigger picture. - The SM looks a bit too complicated to be thought as the fundamental theory: There are many particles, suggesting some higher symmetries (between families, between quarks and leptons, between fermions and bosons) grouping them in supermultiplets - Compositeness? • There are many free parameters It does not describe gravity, which is the interaction driving the evolution of the Universe at large scale It does not include dark matter Interactions don't unify at high energy The fundamental constants have values consistent with conditions for life as we know; this requires a fine tuning. Is there any physics beyond the SM we would need anyway and can provide "for free" DM candidates?

va 201⁻












Multimessenger astroparticle physics

Alessandro De Angelis, INFN/INAF Padova and LIP/IST Lisboa

Lectures 4-5

How to detect high-energy photons (and, shortly, other kinds of cosmic rays).

Detecting particles

- Particle detectors measure physical quantities related to the outcome of a collision; they should ideally identify all the outcoming (and the incoming, if unknown) particles, and measure their kinematical characteristics (momentum, energy, velocity).
- In order to detect a particle, one must make use of its interaction with a sensitive material. The interaction should possibly not destroy the particle one wants to detect; however, for some particles this is the only way to obtain information about them.
- In order to study the properties of detectors, we shall first need to review the characteristics of the interaction of particles with matter.

Padova 2017

Some reminders of particle physics	
Cross-section = σ (normally given per particle, or per atom, in a reaction)	
Frequently used unit: 1 barn = 10^{-24} cm ² (surface of a large atom; π (0.5 fm) ² ~ few mb)	
Attenuation length or "mean free path" λ = 1/n σ , where n is the number density of atoms	
Attenuation of a beam I = $I_0 \exp(-x/\lambda)$	
For materials, we often use the attenuation coefficient, μ , which is the cross section per mass (cm ² /g) (this is what you usually find in the PDG)	
Then attenuation length λ = 1/n σ = 1/ $\mu\rho$, where ρ is density of the material	
Padova 2017 Alessandro De Angelis	3













Charged particles: "Collision" energy loss

This is one of the most important sources of energy loss by charged particles. The average value of the specific (i.e., calculated per unit length) energy loss due to ionization and excitation whenever a particle goes through a homogeneous material of density ρ are described by the so-called Bethe formula¹. This has an accuracy of a few % in the region $0.1 < \beta \gamma < 1000$ for materials with intermediate atomic number.

$$-\frac{dE}{dx} \simeq \rho D\left(\frac{Z}{A}\right) \frac{(z_p)^2}{\beta^2} \left[\frac{1}{2}\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta(\beta, \rho)}{2},\right]$$
(4.1)

where

- ρ is the material density, in g/cm³;
- Z and A are the atomic and mass number of the material, respectively;
- z_p is the charge of the incoming particle, in units of the electron charge;
- $D \simeq 0.307 \text{ MeV cm}^2/\text{g};$
- $m_e c^2$ is the energy corresponding to the electron mass;
- I is the mean excitation energy in the material; it can be approximated as $I \simeq 16 \text{eV} \times Z^{0.9}$ for Z > 1; Alessandro De Angelis

Padova 2017



Multiple scattering

When a charged particle passes near a nucleus it undergoes a deflection which, in most cases, is accompanied by a negligible (approximately zero) loss of energy. This phenomenon, called elastic scattering, is caused by the same electric interaction between the passing particle and the Coulomb field of the nucleus. The global effect is that the path of the particle becomes a random walk (Figure 1.5), and information on the original direction is partly lost – this fact can create problems for the reconstruction of direction in tracking detectors. For very-high energy hadrons, also hadronic cross section can contribute to the effect.

Summing up many relatively small random changes of the direction of flight for a thin layer of traversed material, the distribution of the projected scattering angle of a particle of unit charge can be approximated by a Gaussian distribution of standard deviation (projected on a plane: one has to multiply by $\sqrt{2}$ to determine the variance in space):

$$\theta_0 = \frac{13.6 \,\mathrm{MeV}}{\beta c p} z_p \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \frac{x}{X_0} \right] \,.$$

Padova 2017





Cherenkov radiation (ß>1/n) When $\beta > 1/n$ in a medium, light is emitted in a coherent cone at an angle such that $\cos\theta_c = \frac{1}{n\beta}$ from the direction of the emitting particle. The presence of a coherent wavefront can be easily derived by using the Huygens-Fresnel principle. The number of photons produced per unit path length and per unit energy interval of the photons by a particle with charge z_p at the maximum (limiting) angle is $\frac{d^2 N}{d\lambda dx} \simeq \frac{2\pi\alpha z_p^2}{\lambda^2} \sin^2\theta_c$ The total energy radiated is small, some 10⁻⁴ times the energy lost by ionization. In the visible range (300-700 nm), the total number of emitted photons is about 40/m in air, about 500/ cm in water. Due to the dependence on $\boldsymbol{\lambda}$, it is important that Cherenkov detectors be sensitive close to the ultraviolet region. However, both n and the absorption probability of light can

Hadronic interactions

- The nuclear or hadronic force is felt by hadrons, charged and neutral; at high energies (above a few GeV) the inelastic cross section for hadrons is dominated by nuclear interaction
 - Above some 100 EeV, the "hadronic" component of photons dominates their behavior, and this becomes also the most important interaction for photons
- High-energy nuclear interactions can be characterized by an inelastic interaction length $\lambda_{\rm H}$. Values for $\rho\lambda_{\rm H}$ are typically of the order of 100 g/cm²; a listing for some common materials is provided in the PDG where the inelastic length $\lambda_{\rm I}$ and the total length $\lambda_{\rm T}$ are separately listed, and the rule for the composition is

$$1/\lambda_{\rm T} = 1/\lambda_{\rm H} + 1/\lambda_{\rm I}$$
 .

 The final state products of inelastic high-energy hadronic collisions are mostly pions, since these are the lightest hadrons. The rate of positive, negative, and neutral pions is more or less equal—as we shall see, this fact is due to an important approximate symmetry of hadronic interactions, called the isospin symmetry.

Padova 2017



Bruno Rossi (founder of the Dipartimento)

- Expelled from Italy in 1938 with a bad treatment, moved to US
- Toward the end of the 1950s, as accelerator experiments came to dominate particle physics, Bruno Rossi turned to space research
- At MIT he initiated a program of detector development and rocket experiments aimed astrophysics (but the excuse was the control of nuclear explosions above the atmosphere)
- To implement his ideas about X-ray astronomy, Rossi addressed the young Giacconi (Giacconi & Rossi (1960): "A 'Telescope' for Soft X-Ray Astronomy") and they obtained support for rocket experiments from the Air Force. After two failures, the third satellite, launched in 1962, discovered a bright X-ray source.
- Giacconi won the Nobel prize in 2002 (Rossi Padodied in 1993). Alessandro De Angelis



Multiplicative showers (Rossi 1934)

- Cascades of particles produced as the result of a primary high-energy particle interacting with matter
 - The incoming particle interacts, producing multiple new particles with lesser energy; each of these interacts in turn, a process that continues until many particles are produced. These are then stopped in the matter and absorbed
- 2 basic types of showers:
 - electromagnetic showers are produced by a particle that interacts via the electromagnetic force, a photon or electron
- Hadronic showers are produced by hadrons, and proceed via the strong
 Padova 201 nuclear and the electromagnetic forces



Electromagnetic showers

- When a high-energy e or γ enters an absorber, it initiates an em cascade as pair production and bremsstrahlung generate more e and γ with lower energy
- The ionization loss becomes dominant < the critical energy E_c
 - $\rm E_{c}$ $^{\sim}$ 84 MeV in air, $^{\sim}73$ MeV in water; $^{\sim}$ (550/Z)MeV
 - Approximate scaling in y = E/E_c
 - The longitudinal development ~scales as the radiation length in the material: t = x/Xo
 - The transverse development scales approximately with the Moliere radius $R_M \sim (21 \text{ MeV/E}_c) \text{ Xo}$
 - In average, only 10% of energy outside a cylinder w/ radius $\rm R_{M}$
 - + In air, $\rm R_{M}$ ~ 80 m; in water $\rm R_{M}$ ~ 9 cm
- Electrons/positrons lose energy by ionization during the cascade process

Not a simple sequence: needs Monte Carlo calculations







Energy measurement

- The calorimetric approach: absorb the shower
 - As much as possible... But the logarithmic behavior helps
 - Typically (20-30) Xo give an almost full containment up to hundreds of GeV
 - But sometimes it is difficult (calorimeters in space)
 - Errors asymptotically dominated by statistical fluctuations:

$$\frac{\sigma_E}{E} \cong \frac{k_E}{\sqrt{E}} \oplus c$$

k can be a few per cent for a compact calorimeter



Hadronic showers and calorimeters

- Although hadronic showers are qualitatively similar to em, shower development is more complex because many different processes contribute
 - Larger fluctuations
- Some of the contributions to the total absorption may not give rise to an observable signal in the detector
 - Examples: nuclear excitation and leakage of secondary muons and neutrinos
- Depending on the proportion of π^0 s produced in the early stages of the cascade, the shower may develop predominantly as an electromagnetic one because of the decay $\pi^0 \rightarrow \gamma \gamma$
- The scale of the shower is determined by the nuclear absorption length λ_{H}
 - Since typically λ_{H} > Xo, hadron calorimeters are thicker than em ones
- The energy resolution of calorimeters is in general much worse for hadrons than for electrons and photons

- Energy resolution typically a factor of 5–10 poorer than in em calorimeters

Padova 2017













02/04/17













Tracking detectors (charged particles)

 A tracking detector reveals the path taken by a charged particle by measurements of sampled points (hits). Momentum measurements can be made by measuring the curvature of the track in a magnetic field, which causes the particle to curve into a spiral orbit with a radius proportional to the momentum of the particle. This requires the determination of the best fit to a helix of the hits (particle fit). For a particle of unit charge

$$p (GeV/c) \sim 0.3 B_{\perp}(T) R (m)$$

 A source of uncertainty for this determination is given by the errors in the measurement of the hits; another (intrinsic) noise is given by multiple scattering. In what follows we shall review some detectors used to determine the trajectory of charged tracks.

Prototype: the ionization tube (Geiger-Muller, ...)

Padova 2017









	Exercises
1.	<i>Cherenkov radiation.</i> A proton with momentum 1.0 GeV/c passes through a gas at high pressure. The index of refraction of the gas can be changed by changing the pressure. Compute: (a) the minimum index of refraction at which the proton will emit Cherenkov radiation; (b) the Cherenkov radiation emission angle when the index of refraction of the gas is 1.6.
2.	<i>Photodetectors.</i> What gain would be required from a photomultiplier in order to resolve the signal produced by three photoelectrons from that due to two or four photoelectrons? Assume that the fluctuations in the signal are described by Poisson statistics, and consider that two peaks can be resolved when their centers are separated by more than the sum of their standard deviations.
3.	<i>Cherenkov counters.</i> Estimate the minimum length of a gas Cherenkov counter used in the threshold mode to be able to distinguish between pions and kaons with momentum 20GeV. Assume that 200 photons need to be radiated to ensure a high probability of detection and that radiation covers thewhole visible spectrum (neglect the variation with wavelength of the refractive index of the gas).
4.	Electromagnetic showers. If a shower is generated by a gamma ray of $E = 1 \text{ TeV}$ penetrating the atmosphere vertically, considering that the radiation length X0 of air is approximately 37 g/cm2 and its critical energy Ec is about 88 MeV, calculate the height hM of the maximum of the shower in the Heitler model and in the Rossi approximation B.
5.	<i>Electromagnetic calorimeters.</i> Electromagnetic calorimeters have usually 20 radiation lengths of material. Calculate the thickness (in cm) for a calorimeters made of of BGO, PbWO ₄ (as in the CMS experiment at LHC), uranium, iron, tungsten and lead. Take the radiation lengths from Appendix B or from the Particle Data Book.
6. Padova	Muon energy loss. A muon of 100 GeV crosses a layer of 1 m of iron. Determine the energy loss and the expected scattering angle.

















































Highlight in γ-ray astrophysics (mostly HESS, MAGIC, VERITAS)

- Thanks mostly to Cherenkov telescopes, imaging of VHE (> 30 GeV) galactic sources and discovery of many new galactic and extragalactic sources: ~ 200 (and >200 papers) in the last 9 years
 - And also a better knowledge of the diffuse gammas and electrons
- A comparable success in HE (the Fermi realm); a 10x increase in the number of sources
- A new tool for cosmic-ray physics Pado and fundamental physics












Instr.	Tels.	Tel. A	FoV	Tot A	Thresh.	PSF	Sens.
	#	(m^2)	(°)	(m^2)	(TeV)	(°)	(%Crab)
H.E.S.S.	4	107	5	✤ 428	0.1	0.06	0.7
MAGIC	2	236	3.5	472	0.05(0.03)	0.06	0.8
VERITAS	4	106	4	424	0.1	0.07	0.7
Plus teles operati	a 600 m2 cope (CT5 ng since 2	2 5) 2015					(0.03 for CT5)
the second				1	AA	*	-
Charles (12	-	W.		And the	and a second	
MAN PA	1	Ling			The second second	A CONTRACT	State Land
	All and	1	THE PARTY OF	-		Ast and	
		1~12	in A	zizona ol		200	
VERITAS: 4 U	elescop	les (°12r	nj m _i A	nzona oj	perational si	nce 200	
Padova 2017			Alessar	ndro De Angelis			75

































Detection of Cherenkov radiation vs. direct sampling

$$\theta_c = \frac{1}{n\beta}$$

from the direction of the emitting particle. The threshold velocity is thus $\beta = 1/n$, where n is the refractive index of the medium.

The number of photons produced per unit path length and per unit energy interval of the photons by a particle with charge $z_p e$ is

$$\frac{d^2 N}{dE dx} \simeq \frac{\alpha z_p^2}{\hbar c} \sin^2 \theta_c \simeq 370 \sin^2 \theta_c \,\mathrm{eV}^{-1} \mathrm{cm}^{-1} \tag{4.6}$$

or equivalently

$$\frac{d^2 N}{dEd\lambda} \simeq \frac{2\pi\alpha z_p^2}{\lambda^2} \sin^2 \theta_c \tag{4.7}$$

(the index of refraction n is in general a function of photon energy E).

For a particle unit charge: ~40 photons/m between 300nm and 700nm in air Total energy loss is about 10^{-4} the ionization loss

Attenuation length is ~ 3 km



Performance of different types of HE gamma detectors

arrays. Sen	sitivity computed over one yea	r for Fermi and the H	EAS, and over 50 h for the IACTs
Table 4.5	A comparison of the characteris	tics of Fermi, the IA	CTs and of the EAS particle detector

Quantity	Fermi	IACTs	EAS
Energy range	20 MeV-200 GeV	100GeV-50TeV	400 GeV-100 TeV
Energy res.	5-10%	15-20%	~ 50 %
Duty cycle	80%	15%	> 90 %
FoV	$4\pi/5$	$5 \text{ deg} \times 5 \text{ deg}$	$4\pi/6$
PSF (deg)	0.1	0.07	0.5
Sensitivity	1% Crab (1 GeV)	1% Crab (0.5 TeV)	0.5 Crab (5 TeV)
Sensitivity	1% Crab (1GeV)	1% Crab (0.5 leV)	0.5 Crab (5 leV)



Summary of lectures 2 and 3 Detectors for charged cosmic rays: (1) need large effective area for the UHE, (2) smart instruments on satellite for particle identification. For (1) we are close to the limit (Auger) unless we change technology, for (2) we are close to the limit Astrophysical neutrino detectors: we need several km3; we are close to the limit (Icecube) but still improving (Antares -> km3) Photons: In the MeV region, instruments did not reach the technological limit, yet In the GeV region, Fermi is close to the technological limit In the TeV region, the Cherenkov technique reigns. HESS, MAGIC and VERITAS have still potential, and there is room for improvement by "brute force"

 In the PeV region, only one detector presently active, and there is room for improvement by "brute force".

Padova 2017

Alessandro De Angelis

Exercises - II

- 1. Cherenkov telescopes. Suppose you have a Cherenkov telescope with 7m diameter, and your camera can detect a signal only when you collect 100 photons from a source. Assuming a global efficiency of 0.1 for the acquisition system (including reflectivity of the surface and quantum efficiency of the PMT), what is the minimum energy (neglecting the background) that such a system can detect at a height of 2 km a.s.l.?
- 2. Cherenkov telescopes. Show that the image of the Cherenkov emission from a muon in the focal plane of a parabolic IACT is a conical section (approximate the Cherenkov angle as a constant).
- 3. Energy loss. In the Pierre Auger Observatory the surface detectors are composed by water Cherenkov tanks 1.2m high, each containing 12 tons of water. These detectors are able to measure the light produced by charged particles crossing them. Consider one tank crossed by a single vertical muon with an energy of 5 GeV. The refraction index of water is n 1.33 and can be in good approximation considered constant for all the relevant photon wavelengths. Determine the energy lost by ionization and compare it with the energy lost by Cherenkov emission.

Padova 2017

Alessandro De Angelis



Tev Impact Highlights from HESS, MAGIC, VERITAS & MILAGRO • Microquasars: Science 309, 746 (2005), Science 312, 1771 (2006) • Pulsars: Science 322, 1221 (2008), Science 334, 69 (2011) • Pulsars: Science 322, 1221 (2008), Science 334, 69 (2011) • Supernova Remnants: Nature 432, 75 (2004) • The Galactic Centre: Nature 439, 695 (2006) • Surveys: Science 307, 1839 (2005), PRL 95, 251103 (2005) • Starbursts: Nature 462, 770 (2009), Science 326,1080 (2009) • AGN: Science 314,1424 (2006), Science 320, 752 (2008) • Dark Matter: PRL 96, 221102 (2006), PRL 106, 161301 (2011) • Lorentz Invariance: PRL 101, 170402 (2008) • Cosmic Ray Electrons: PRL 101, 261104 (2009) • Surveysing Participant (2005)















<u>imeractions</u>	s with matte	<u>r</u>
E-M:	VHE	
bremsstrahlung: $e N(e) \Rightarrow e' \gamma N(e)$	vie	$E\gamma \sim 1/2E_e$
pair production $\gamma N(e) \Rightarrow e^+e^- N(e)$	*	
$e+e-annihilation$ $e^+e^- \Rightarrow \gamma \gamma$ (511 keV line)	ne)	
Strong/week: pp (A) => π , K, A,	ste ste	Eγ ~ 1/10E
π, K, Λ => γ, ν, e, μ		
$\mu \Rightarrow \nu$		
also in the low energy region		
Nuclear: $p A \Rightarrow A^* \Rightarrow A' \gamma, n$		

intera	ctions with radiation	n and .	<u>B-fields</u>
Radiation field		VHE	
E-M:			
inverse Compton:	$e \gamma (B) => e' \gamma$	**	$E\gamma \sim \epsilon (Ee/mc^2)^2$ (T) to ~Ee (KN)
γγ pair production	γγ(B) => e+e-	**	
Strong/week	ργ=>π, Κ, Λ, …	*	
	$\pi, K, \Lambda \Rightarrow \gamma, v, e, \mu$		Eγ~ 1/10Ep
	μ => ν		
B-field	$A \gamma \Rightarrow A \Rightarrow A' \gamma$	*	Εγ~ 1/1000A Ep
synchrotron	$e(p) B \Rightarrow \gamma$	nje	
pair production	$\gamma B => e+e-$	*	$E\gamma \sim BE_e^2$; $hv_{max} \sim \alpha^{-1} mc^2$
•			













Description	Identified		Associated	
	Designator	Number	Designator	Number
Pulsar, identified by pulsations	PSR	143		
Pulsar, no pulsations seen in LAT yet			psr	24
Pulsar wind nebula	PWN	9	pwn	2
Supernova remnant	SNR	12	snr	11
Supernova remnant / Pulsar wind nebula			spp	49
Globular cluster	GLC	0	glc	15
High-mass binary	HMB	3	hmb	0
Binary	BIN	1	bin	0
Nova	NOV	1	nov	0
Star-forming region	SFR	1	sfr	0
Compact Steep Spectrum Quasar	CSS	0	CSS	1
BL Lac type of blazar	BLL	18	bll	642
FSRQ type of blazar	FSRQ	38	fsrq	446
Non-blazar active galaxy	AGN	0	agn	3
Radio galaxy	RDG	3	rdg	12
Seyfert galaxy	SEY	0	sey	1
Blazar candidate of uncertain type	BCU	5	bcu	568
Normal galaxy (or part)	GAL	2	gal	1
Starburst galaxy	SBG	0	sbg	4
Narrow line Seyfert 1	NLSY1	2	nlsy1	3
Soft spectrum radio quasar	SSRQ	0	ssrq	3
Total		238		1785
Unassociated				1010







Туре	Designator	Objects	Representatives
Pulsar wind nebula	PWN	31	Crab, Geminga, Vela X
Supernova remnant with shell	Shell	11	See Table 9.2
Supernova remnant with mol. clouds	SNR/Mol. Cloud	8	W28, W51
Binary systems	Binary	5	LS 5039, LSI +61 303
Massive star clusters, globular cl.	-	5	and the second sec
HBL Lac type of blazar	HBL	41	Mrk 421, Mrk 501
IBL Lac type of blazar	IBL	7	Bl Lac, W Comae
LBL Lac type of blazar	LBL	1	-
FSRQ type of blazar	FSRQ	3	3C 279
FRI type of blazar	FRI	3	Centaurus A, M87
Starburst galaxy	Starburst	2	
Unidentified	UNID	28	
Total		145	



Most are remnants of supernova (including PWN)

- Most of the VHE gamma emission in the Galaxy can be associated to "SNR".
- Shell Supernova Remnants (SNR). As the shockwave from a SN explosion plows through space, it produces a big shell of hot material in space. This process can continue up to 10⁴– 10⁵ years before the energy release becomes negligible. Magnetic field strengths are estimated to be B ~ 10 µG to 1 mG.
- Pulsar Wind Nebulae (PWN) are SNR with a young pulsar slowing down in rotation: the typical rate of decrease of kinetic energy lies in the range dE/dt ~ 10³²–10³⁹ erg/s. In most cases only a negligible fraction of this energy goes into the pulsed electromagnetic radiation observed from the pulsar, whereas most of it is deposited into an outflowing relativistic magnetized particle wind
 - Pulsars When a star collapses into a neutron star, its size shrinks to some 10–20 km, with a density of 5 10¹⁷ kg/m3. Since angular momentum is conserved, the rotation can become very fast, with periods of the order of a few ms up to 1 s. Neutron stars in young SNRs are typically pulsars
 - Can be derived from simple physics arguments, see the text
 - Pulsars have typical cutoffs at ~10 GeV, with two notable exceptions (Crab and Vela)

Padova 2017



An example of a different galactic emitter: microquasars and binaries























• Short GRBs are difficult to associate



Long and short GRBs

- Long and short duration GRBs are created by fundamentally different physical mechanisms
- Possible candidates for short GRBs are mergers of neutron star (NS) binaries or NS-BH binaries, which lose angular momentum and undergo a merger
 - Remember the recent gravitational wave event?
- Possible candidates for long GRBs: core collapse of a very massive star, a very energetic supernova (the "hypernova", 100 times the SN). Seen in association!
 - The explosion originates at the center of these massive stars. While a BH forms from the collapsing core, this explosion sends a blast wave moving through the star at speeds close to c. The gamma rays are created when the blast wave collides with stellar material still inside the star.
 - Erupting through the star surface, the blast wave of stellar material sweeps through space, colliding with intervening gas and dust, producing additional emission of photons. These emissions are believed responsible for the

Padova 2017 "afterglow" of progressively less energetic photons




































- The diffuse extragalactic background light (EBL) is all the accumulated radiation in the Universe due essentially to star formation processes
- This radiation covers a wavelength range between ~0.1 and 600 μm (consider the redshift and the reprocessing)
- After the CMB, the EBL is the second-most energetic diffuse background
- The understanding of the EBL is fundamental
 - To know the history of star formation
 - To model VHE photon propagation for extragalactic VHE astronomy. VHE photons coming from cosmological distances are attenuated by pair production with EBL photons. This interaction is dependent on the SED of the EBL.
- Therefore, it is necessary to know the SED of the EBL in order to study intrinsic properties of the emission in the VHE sources.

Padova 2017

















07/04/17









- Gamma Rays from annihilations in the galactic halo, near the galactic center, in dwarf galaxies, etc. Drawback: Unknown astrophysical background.
- Neutrinos from annihilations in the core of the Sun or in the sama sources as gamma rays (IceCube, Antares). Not the sensitivity, yet
- Positrons/Antiprotons from annihilations throughout the galactic halo. Drawback: Unknown astrophysical background.
 - Measured in space–based detectors: Fermi (gammas), PAMELA, AMS (antimatter) or in atmospheric Cherenkov telescopes: MAGIC















<image>

- Because of the large astrophysical foregrounds, we must first understand the γ-ray emission from the Galaxy and from known source classes
 - In the 1-100 GeV energy band these account for ~85% of the γ -rays in a 15°x15° box around the Galactic center







N-body simulations of Milky-Way like galaxies tend to show less DM signal in the inner few degrees than observations of the Galactic center (grey shaded region)









• Negligible astrophysical γ-ray production expected







Summary of Lecture 6

- During the recent years, we discovered thousands of astrophysical gammaray emitters in the HE region and >200 in the VHE region
 - New emitters and new classes of emitters
 - A diffuse background up to the TeV, maybe the sum of unresolved point-like emitters
- Both the leptonic and the hadronic gamma-ray mechanisms at work

 Identified mechanisms of emission explaining cosmic rays up to the PeV
- The SED of many emitters can be modeled in an effective way
- Although we are able to detect effectively gamma rays, the interaction with background photons in the Universe attenuates the flux of gamma rays

 The "enemies" of VHE photons are photons near the optical region (Extragalactic Background Light, EBL)

· Interesting perspects for fundamental physics from astroparticle physics

• Dark matter:

- A standard WIMP below 400 GeV is on reach for HE gamma detectors, if the particles was in thermal equilibrium and $\langle \sigma v \rangle$ is the same as at freeze-out
 - Dwarph spheroidals (no need for background models) and the GC region (a mess from the point of view of astronomy) are the favorite targets

Needs a laboratory experiment to confirm



















12/04/17



Is Lorentz invariance exact?

- For longtime violating Lorentz invariance/Lorentz transformations/Einstein relativity was a heresy
 - Is there an aether? (Dirac 1951)
 - Many preprints, often unpublished (=refused) in the '90s
- Then the discussion was open
 - Trans-GZK events? (AGASA collaboration 1997-8)
 - LIV => high energy threshold phenomena: photon decay, vacuum Cherenkov, GZK cutoff (Coleman & Glashow 1997-8)
 - GRB and photon dispersion (Amelino-Camelia et al. 1997)
 - Framework for the violation (Colladay & Kostelecky 1998)
 - LIV and gamma-ray horizon (Kifune 1999)

Padova 2017

LIV? New form of relativity?

- Von Ignatowsky 1911: {relativity, omogeneity/isotropy, linearity, reciprocity} => Lorentz transformations with "some" invariant c (Galilei relativity is the limit $c \rightarrow \infty$)
- CMB is kind of an aether: give away isotropy?
- QG motivation: give away linearity? (A new relativity with 2 invariants: "c" and E_{p})
- In any case, let's sketch an effective theory...
 - Let's take a purely phenomenological point of view and encode the general form of Lorentz invariance violation (LIV) as a perturbation of the Hamiltonian (Amelino-Camelia+)

A heuristic approach: modified dispersion relations (perturbation of the Hamiltonian)

We expect the Planck mass to be the scale of the effect

Padov

$$E_{p} = \sqrt{\frac{hc}{G}} \approx 1.2 \times 10^{19} \text{ GeV}$$

$$H^{2} = m^{2} + p^{2} \rightarrow H^{2} = m^{2} + p^{2} \left(1 + \xi \frac{E}{E_{p}} + ...\right)$$

$$H \xrightarrow{p \rightarrow p} p \left(1 + \frac{m^{2}}{2p^{2}} + \xi \frac{p}{2E_{p}} + ...\right)$$

$$v = \frac{\partial H}{\partial p} \approx 1 - \frac{m^{2}}{2p^{2}} + \xi \frac{p}{E_{p}} \Rightarrow v_{\gamma} \approx 1 + \xi \frac{E}{E_{p}}$$

$$\Rightarrow \text{ effect of dispersion relations at cosmological distances can be}$$

$$\Delta t_{\gamma} \approx T\Delta E \frac{\xi}{E_{p}}$$
Padova 2017































Padova 2017

Exercises

- 1. Neutrinos from SN1987A. Neutrinos from SN1987A, at an energy of about 50 MeV, arrived in a bunch lasting 13 s from a distance of 50 kpc, 3 h before the optical detection of the supernova. What can you say on the neutrino mass? What can you say about the neutrino speed (be careful...)?
- 2. Time lag in light propagation. Suppose that the speed c of light depends on its energy E in such a way that

$$c(E) \simeq c_0 \left(1 + \xi \frac{E^2}{E_P^2} \right)$$

where E_P is the Planck energy (second-order Lorentz Invariance Violation). Compute the time lag between two VHE photons as a function of the energy difference and of the redshift z.

Padova 2017
Gravitational waves searches

Claudia Lazzaro - Virgo group Seminar 12 April 2017 Experimental astroparticle physics

Gravitational waves and General Relativity

- **x** Gravity is a manifestation of curvature of space-time produced by matter-energy
- <code>x</code> Linearized Einstein equations admit wave solutions, as perturbations to a background geometry. $g_{
 u\mu}$ = $\eta_{
 u\mu}$ + $h_{
 u\mu}$ + $h_{
 u\mu}$ = $h_{
 u\mu}$
 $h_{
 u\mu}$
- **x** Any rapidly moving mass generates fluctuations in spacetime curvature which propagate at the speed of light.

Gravitational waves

- x Propagate with speed of light,
- **x** Transverse, traceless
- X Have two independent polarizations states "+" and "x"
- **x** Can be detected by their effect on the relative motion of test masses (stress and compress spacetime in two directions)



GW existence: Hulse and Taylor binary

GW emission has been indirectly demonstrated through the radiative energy loss of PSR191316

- Pulsar bound to a "dark companion", 7kpc from Earth. (vmax/c ~10--3)
- ✗ GR predicts such a system to lose energy via GW emission: orbital period decrease
- **x** Prediction of general relativity verified at 0.2% level

Symbol	Name	Value 1.441M _☉		
m_1	primary mass			
m_2	secondary mass	$1.387 M_{\odot}$		
P_{orb}	orbital period	7.751939106 hr		
a	semi-major axis	$1.9501\times 10^9~{\rm m}$		
e	eccentricity	0.617131		
D	distance	21,000 lyr		



Efficient sources of GW must be asymmetric,

GW detectors are sensitive to amplitude

Claudia Lazzaro

GW amplitude

 $Q_{\mu\nu}! = 0$

compact and fast

h: 1/r attenuation

Mass quadrupole

$$Q_{\mu\nu}$$

 $h_{\mu\nu} = \frac{2G}{c^4} \frac{1}{r} \ddot{Q}_{\mu\nu}$

Radiated power

 $P = \frac{G}{5c^2} Q^{\ddot{\mu}\nu} Q^{\ddot{\mu}\nu}$

Amplitude

Possible GW source:

Transient signal from core collapse supernovae:

$$h \sim 6 \times 10^{-21} \left(\frac{E}{10^{-7} \,\mathrm{M}_{\odot} c^2}\right)^{\frac{1}{2}} \left(\frac{1 \,\mathrm{ms}}{T}\right) \left(\frac{1 \,\mathrm{kHz}}{f}\right) \left(\frac{10 \,\mathrm{kpc}}{r}\right)$$

Continuous waves from non asymmetric spinning neutron stars

$$h_0 = \frac{4\pi^2 G I_{zz} f_{GW}^2}{c^4 r} \epsilon = (1.1 \times 10^{-24}) \left(\frac{I_{zz}}{I_0}\right) \left(\frac{f_{GW}}{1 \,\mathrm{kHz}}\right)^2 \left(\frac{1 \,\mathrm{kpc}}{r}\right) \left(\frac{\epsilon}{10^{-6}}\right) \quad \epsilon \equiv \frac{I_{xx} - I_{yy}}{I_{zz}}$$

Even in optimistic case: $h \! \leq \! 10^{-20}$

Need to measure: $\Delta L \sim 10^{-18}$ m

Michelson e Morley interferometers



Interferometers

Improving the sensitivity: Increase length of the interferometer arms, increase incident light power

Reaching h~10-22 would requires, kilometric arms scale and kilowatts of laser power



Interferometers: optical layout



Operation conditions (Locking conditions):

- Keep the FP Cavities In resonance (Maximize the phase response);
- keep the PR cavity in resonance (Minimize the shot noise);
- Keep the output on the "dark fringe" (Reduce the dependence on power fluctuations)
- -Keep the arm length constant within 10-15 m

Claudia Lazzaro

AdvVirgo interferometer

Arms: 3km length Vaccum volume 7000 m³ , 10⁻⁹ mbar



AdvVirgo interferometer

Arms: 3km length Vaccum volume 7000 m³ , 10⁻⁹ mbar Suspension Super-seismic isolation (from initial Virgo)

Reduction 10⁻¹² vibrations



Claudia Lazzaro

AdvVirgo interferometer

Arms:

3km length Vaccum volume 7000 m 3 , 10 $^{-9}$ mbar Suspension

Super-seismic isolation (from initial Virgo) Reduction 10⁻¹² vibrations

Mirrors

- **x** 42 kg, 35 cm diam., 20 cm thick
- **x** Flatness < 0.5 nm rms
- **x** Roughness < 0.1 nm rm
- **✗** Absorption < 0.5 ppm
- **x** Low mechanical losses, Low optical absorption



Ground based interferometers: main noise source

IFO sensitivity: power spectrum density (PSD, unit: $1/\sqrt{Hz}$)



Fundamental Noise Sources:

- **x** Seismic noise
- ★ Newtonian noise/Gravity gradients
- **x** Radiation pressure noise
- **x** Thermal noise
- **x** Mirror coatings
- **✗** Mirror substrates
- **X** Quantum shot noise
- * Seismic noise strongly suppressed by properly designed suspension systems).
- ✗ The two main thermal noise sources: wires suspending mirrors and mirrors themselves (mainly optical coatings on mirrors surface)
- Photon shot noise is due to the quantum nature of the light; it arise from statistical fluctuations in the number of detected photons. Claudia Lazzaro



Network of interferometers

Advantages of multiple interferometer:

- **x** Simultaneous detection, improve background rejection and detection confidence
- **x** Duty cycle
- **x** Enhanced network sky coverage, source parameter estimation(position and polarization)

Detector response to GW signal



Detector data:
$$x(t) = h(t) + n(t)$$

Interferometer response

to GW signal

Claudia Lazzaro



Astro-ph arXiv:1102.5421v2



$$h^2 = \int h_+^2 + h_x^2$$

Interferometers network response to GW signal

Multiple detector allows, improves:

- **x** network sky coverage
- **x** Source parameter estimation, polarization resolution

Network data allows coherent (not only coincident) analysis

Network response:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} F_1^+ & F_1^\times \\ F_2^+ & F_2^\times \\ \vdots & \vdots \\ F_N^+ & F_N^\times \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix}$$

Localization of the source position:

- **x** Triangulation: measure time of flight with 2 or more detector sites and reconstruct ToF rings
- **x** degeneracy along the rings can be reduce by using variability of antenna pattern



Interferometers network response to GW signal

How describe the coverage capability of the network:

- **X** Network sensitivity $S_{net} = \left(\sum_{k} S_{k}^{-1}\right)^{-1}$
- **x** Network acceptance $F = \sqrt{|F_{+}^{2}| + |F_{x}^{2}|} S_{net}$
- **x** Network alignment $A = |f_x|/f_+$
- multiple detectors improve coverage of the sky (acceptance & alignment) and strain sensitivity
- Even sensitive detectors, depending on source position, can act as spectator and not really participate in the measurement of event reconstruction
- Detectors with low sensitivity gives smaller contribution to the network SNR



NB: Hanford and Livigston are almost co-aligned Claudia Lazzaro

Interferometers network response to GW signal



Astrophysical sources



Astrophysical sources



CBC source



Till the ISCO (inner most circle orbital) characteristic frequency evolution in time: chirp signal

$$M_{chirp} = \frac{m_1 m_2^{3/5}}{m_1 + m_2^{1/5}} \qquad f_{GW}^{\cdot} = \frac{96}{5} \pi^{8/3} \left(\frac{GM_c}{c^3}\right)^{5/3} f_{GW}^{11/3}$$



The GW emission is not isotropic, it depends on line of sight and luminosity distance

$$h_{*} = \frac{1}{D_{L}} \left(\frac{5}{c(t_{c}-t)^{1/4}} \right) \frac{1+\cos i^{2}}{2} \cos[\Phi(t_{c}-t)] \qquad h_{x} = \frac{1}{D_{L}} \left(\frac{5}{c(t_{c}-t)^{1/4}} \right) \cos i \sin[\Phi(t_{c}-t)]$$

Inclination of the orbital plane $\mathbf{\hat{i}}$ & luminosity distance $D_{\rm I}$ Claudia Lazzaro

CBC (template) search

Need to build a bank of template that will cover the full parameter space:

- To accurately calculate inspiral, merger and ring-down stage, "hybrid" waveforms are built. Post Newtonian approximation (PN) used for inspiral phase, Numerical relativity and perturbative theory for merger and ringdown phase
- Source parameters are encoded in detected waveforms., up to 15 parameters to include/estimate (chirp mass, component masses, spins,



x template not fully available for complex systems such as eccentric compact binaries, spinning ...

Template analysis search, main step (generic):

- **x** matching filter for each template, find template that maximized matching template
- Same templates which are coincident in time (among the detector, including consistent time difference between sites) are combined in one event
- Coincident triggers are ranked according to a detection statistic (combined SNR, weighted likelihood, ...)
 Claudia Lazzaro

Possible sources

- **x** core-collapse supernovae
- CBC signal: the merger phase of binary compactobjects. In particular the burst search is crucial for eccentric binary black holes, and high precessing, spinning system; in this cases the template are not well known.
 Post merger phase of NS-NS phase (equation of state of NS)
- **x** neutron star instabilities, pulsar glitches
- **x** accretion disk instabilities
- **x** cosmic string cusps/kinks,
- \pmb{x} the un-expected

Waveform morphologies is poorly modeled or fully unknown. Short duration signal: hundreds milliseconds to a few seconds Long duration: signals lasting from few seconds up to hours

Claudia Lazzaro

Burst searches

Burst search is performed without assumption (or minimal) on the phase evolution of the signal

Goal: cover wide range of parameters space (can overlap the modeled searches)

Burst analysis search, main step (generic):

- x make time-frequency representation of the data, weight data by the noise at each frequency (whitening). Search for excess power in time-frequency domain, coincidently/coherently in different detectors data (considering time delay between detector sites and different antenna patterns of the detectors for any incoming direction).
- **x** Coherent analysis of the triggers and estimation of the signal parameters. Different algorithms
- **x** Ranked statistic

Joint GW and high-energy astrophysics

GWs and photons provide complementary information on the astrophysics source (and maybe on the environment)

Gravitational waves signal

Electromagnetic signal

Two scenario:

→GW triggered EM follow-up: low-latency GW data analysis pipelines promptly identify GW candidates and send GW alerts to trigger prompt EM observations



→EM triggered GW: an EM transient event is detected and GW triggered searches are are performed to look for possible associated GW events.

→even more: joint with neutrino, searches neutrino candidates with data of IceCube and ANTARES

Claudia Lazzaro

Joint GW and EM, astrophysics motivation

Coalescence of binary systems of NSs and/or BHs - Short GRBs: Prompt γ -ray emission (< 2 s) Multiwavelegth afterglow emission: X-ray, optical and radio (minutes, hours, days, months). -Kilonova: optical (days-weeks). Core collapse of massive stars: Supernovae: -X-rays, UV (minutes, days) -optical (week, months) -radio (years) Long GRBs (massive rapidly spinning star Collapse ??) Isolated neutron stars -soft γ -ray repeaters -radio/X-ray pulsar glitches



O1 data talking and sensitivity



O1 analysis: binary black holes



O1 analysis: first detection



- ✗ 3 minutes alert from the low latency searches pipeline coherent Wave Burst (cWB) (Florida, Hannover, Padova/Trento)
- ✗ Two independent sky maps available in short time (cWB 17 min, LIB 14 hours) within 600 deg² (90% c.l.)
- As the preliminary estimate of the event significance overcomes the planned threshold (1/month) within 48 hours, allert sent via GCN circular to 62 astronomer partners (including INAF PD).

Claudia Lazzaro

O1 analysis: first detection, confidence

The possibility that the transient GW150914 is due to environmental or instrumental excess noise is ruled out. **5.1 evidence of the astrophysical origin of the signal**

Two independent data analysis pipelines used to estimate the confidential level

Template search,

Generic (burst) search,

Estimated False Alarm rates: <1/203000 years Estimated False Alarm rates: <1/225000 years





analysis: first detection, binary parameters OI



- X The physical system is described by 8 parameters: $m_{1,2}$ and $S_{1,2}$
- **x** 9 additional parameters are required Luminosity distance, celestial coordinates, orbit inclination, time and phase of coalescence. Claudia Lazzaro

35 - 350 Hz band-passed strain time series. Coherent signals in both detectors

Waveform reconstruction, agreement between best fit theoretical waveforms, wavelet (unmodeled) and NR (Numerical Relativity)

Residual noise after waveform subtraction consistent with instrumental noise

Mass 1	$36.3^{+5.3}_{-4.5}M_{\odot}$
Mass 2	$28.6^{+4.4}_{-4.2}M_{\odot}$
Final mass	$62.0^{+4.4}_{-4.0}M_{\odot}$
Energy radiated in GW	$3.0^{+0.5}_{-0.5}M_{\odot}$
Spin magnitude $ a1 $	$0.32\substack{+0.45\\-0.28}$
Spin magnitude $ a2 $	$0.57\substack{+0.40 \\ -0.51}$
Final spin $ a_f $	$0.67\substack{+0.06\\-0.08}$
Luminosity distance	$410^{+160}_{-180}Mpc$

O1 analysis: first detection, follow up



Search for coincident neutrino candidates of IceCube and ANTARES (in 500s window): -ANTARSE: O neutrino candidates:

-IceCube: : 3neutrino candidates, consistent with the expected background (No one directionally coincident with GW150914)

Interferometers network, second generation



Claudia Lazzaro

Three detector localization



GW spectrum



Future

"Limit" to improvement of the second generation interferometer: length of the arms, seismic and newtonian noise, size of the beam (thermal noise), :

"3rd generation observatory":

- Possible new technology: squeezed light, alternative wavelengths + cryogenics, longer arms, go underground (access low frequencies)
- **x** Factor ~10 sensitivity increase over aLIGO (10 Hz few kHz); sensitivity x10 ⇒volume x10³
- **x** Low frequency sensitivity (down to ~ 5 Hz)

Einstein telescope design studies



- x 10 km arm length
- **x** Underground, cryogenics
- X New geometries or topologies, multiple interferometers Claudia Lazzaro

Space interferometers



- **✗** 3 Spacecraft 2.5 Million-km arm-lengths
- **X** Test masses in sub-femto-g free fall $(10^{-15} \text{m/s}^2/\sqrt{\text{Hz}})$
- **x** Laser interferometry between TMs

Successful LISA Pathfinder mission Mission proposal submitted (arXiv:1702:00786) Launch 2030-2034

Possible source: massive black hole binaries (MBHBs) detection, parameter estimation..and MBH formation. Extreme mass ratio inspirals (EMRIs) Cosmological background

Claudia Lazzaro

Back up





Summary of older lectures

- Detectors for charged cosmic rays: (1) need large effective area for the UHE, (2) smart instruments on satellite for particle identification. For (1) we are close to the limit (Auger) unless we change technology, for (2) we are close to the limit (AMS-02)
- Astrophysical neutrino detectors: we need several km³; we are close to the limit (IceCube) but still improving (Antares -> km³NeT)
- Photons:
 - In the MeV region, instruments did not reach the technological limit, yet (no new instrument since COMPTEL, 1991-2000)
 - In the GeV region, Fermi is close to the technological limit
 - In the TeV region, the Cherenkov technique reigns. HESS, MAGIC and VERITAS have still potential, and there is room for improvement by "brute force"
 - In the PeV region, only one detector presently active, and there is room for improvement by "brute force" – plus something



Alessandro De Angelis





The 20 GeV- 100 TeV region: how to do better with traditional IACT?

• More events

- More photons = better spectra, images, fainter sources
 - Larger collection area for gamma-rays

• Better events

- More precise measurements of atmospheric cascades and hence primary gammas
 - Improved angular resolution
 - Improved background
- rejection power

The CTA solution: More telescopes !

Simulation: Superimposed images from 8 cameras













Telescope	Telescope Specifications							
			SiPM Ca					
			3 SST types					
	LST "large"	MST "medium"	SCT "medium 2-M"	SST "small"				
Number	4 (S) 4 (N)	25 (S) 15 (N)	≤ 24 (S and N)	70 (S)				
Energy range	20 GeV to 1 TeV	200 GeV to 10 TeV	200 GeV to 10 TeV	> few TeV				
Effective mirror area	> 330 m ²	> 90 m ²	> 50 m ²	> 5 m²				
Field of view	> 4.4°	> 7º	> 7º	> 8º				
Pixel size ~PSF θ ₈₀	< 0.12°	< 0.18°	< 0.07°	< 0.25°				
Positioning time	50 s, 20 s goal	90 s, 60 s goal	90 s, 60 s goal	90 s, 60 s goal				
Target capital cost	7.4 M€	1.6 M€	< 2.0 M€	500 k€				
Padova 2017	Al	essandro De Angelis			13			















Alessandro De Angelis



19







SiPM: the technological challenge for small cameras

Cameras need high granularity, and typical PMT size of 5-6 mm

Difficult to do with standard PMT

New detectors (SiPM) under development



LST in the future: large surface SiPM?

Challenge: single sensor with large area (1 inch diameter)

Amplify-and-sum stage, one output per pixel

Prototype of analog sum scheme will be tested in MAGIC

Prototype cluster using Hamamatsu and developed by MPI mounted on MAGIC Jun 15

9 FBK 6x6 mm² sensors Sensor electronics by INFN Padova MAGIC cluster control electronics and

Signal: 2 mV per phe; noise: 0.5 mV rms Linearity: ok to >200 phe

Assembly and test now,; installed in MAGIC October 2015 for comparison with the standard PMT clusters (and with the similar Max Planck SiPM cluster, just installed) Padova 2017 Alessandro





















13/04/17






LHAASO

- Phase-0: Large Area Water Cherenkov Array (LAWCA)
 - >> YangBaJing, Tibet: around the ARGO detector
 - ► Completion end 2014
- Phase-1
 - Final site: Shangri-La
 4.3 km altitude
 - ► Sensitivity?
 - Will depend on background rejection power achieved in practice, but will be a very powerful instrument

































e-ASTROGAM scientific requirements

- 1. Achieve a sensitivity better than that of INTEGRAL/CGRO/COMPTEL by a factor of 20 50 100 in the range 0.2 30 MeV
- 2. Fully exploit gamma-ray polarization for both transient and steady sources
- 3. Improve significantly the angular resolution (to reach, e.g., ~ 10' at 1 GeV)
- 4. Achieve a very large field of view (~ 2.5 sr) \Rightarrow efficient monitoring of the γ -ray sky
- 5. Enable sub-millisecond trigger and alert capability for transients



















CONCLUSIONS

Gamma rays:

- A rich panorama of gamma experiments at VHE gamma proposed for the future. CTA will lead the field.
 - Besides CTA, new techniques. Exploration of the PeV region is fundamental and feasible. Northern projects approved, will produce nice science. Need to converge to a Southern 100 GeV-100 TeV EAS array.
- In the longer term, need taking care of multiwavelength aspects: priorities are
 - A MeV mission (room for smart improvement; 2 missions proposed)
 - A successor of Fermi
- Multimessenger astronomy gamma/neutrinos can help our understanding of cosmic accelerators, of physics under extreme environments and of fundamental particle physics
 - Neutrino detectors will grow (at high price), and we know what we can get for astronomy
 - In a few years we'll know the impact of GW (cfr the dedicated lecture)
 - Auger to be upgraded, but new technologies look far away in time



Gamma-ray Space Telescope	d Outline	
•	Overview of the Fermi Large AreaTelescope How it works LAT data LAT performance 	
•	Fermi Science Tools General Introduction	
•	Maximum Likelihood Overview Source modeling 	
•	One study case: • 3c454.3: analysis tutorial	
	2	





















































	odels		
Fermi Science Support C			
Home Observations	Data Propo	osals Library HE	ASARC Help Site Map
Data Policy Data Access LAT Data LAT Data LAT Casing LAT Casing LAT Casing LAT Case Cases LAT Case Cases LAT Case Cases LAT Avery Files Cold Data	LAT DECKGTOU Many analyses of LAT data regul offluse emission models, which refer to the binned or unknowel Ferri data analysis. Here is a lit fass for you to download. Howe SPERM_DIRKInetdata/termigadd For Pass 8, each event class an as examples.	re models of Galactic diffuse and lectropic e as available from this web page, have bee kelhood analysis subtraits for some exampl via: the files for the most recent data releas that directions. As a result, it is unknay that d event type combination has a dedicated IP	mission. Dataliet discussion of how the latest Gale in developed is available, Acero et. al. (2016). Pee es of how to incorporate these models into your o the variout data est. We have provided the mo- a are included in the source tools installation (in fu- you will need to constraid each the secondary data BF and isotropic model. Only a subset are shown he
Data Analysis Coveats	Galactic Interstellar emission model	Event Selection/ IRF Name	Isotropic spectral template
Newsletters FAQ	gl_iem_v06.fts (see below for usage notes)	Pass 8 Source (front+back, allPSF, allEDISP) P6R2_SOURCE_V6	te_PBR2_SOURCE_V6_v06.14
		Pass 8 Source (front only) P8R2_SOURCE_V6:FRONT	INO_PER2_SOURCE_VE_FRONT_V06.txt
		Pass 8 Source (back only) P6R2_SOURCE_V6:BACK	INO_PER2_SOURCE_VE_BACK_V06.04
		Pass 8 Clean (font+back, allPSF, alEDISP) P8R2_CLEAN_V8	mo_PBR2_CLEAN_V6_v06.txt
		Pass 8 Clean (PSF0) P8R2_CLEAN_V6:PSF0	ING_PER2_CLEAN_V6_PSF0_v06.txt
		Pass 8 Clean (PSF1) P8R2 CLEAN_V6:PSF1	IB0_PBR2_CLEAN_V6_PSF1_V0E.bt
		Pass 8 Clean (PSF2) P8R2_CLEAN_V6:PSF2	IND_PER2_CLEAN_V6_PEF2_v06.txt
		Pass 8 Clean (PSF3)	IND PERS CLEAN VE PSF3 VOEM



ů	Event types			
P8R2 Event Type Name	Event Type Partition	Event Type Value (evtype)		
FRONT	Conversion Type	1		
BACK	Conversion Type	2		
PSF0	PSF	4		
PSF1	PSF	8		
PSF2	PSF	16		
PSF3	PSF	32		
EDISP0	EDISP	64		
EDISP1	EDISP	128		
EDISP2	EDISP	256		
EDISP3	EDISP	512		

P8R2 IRF name	Event Class (evclass)	Class Hierarchy	Photon File	Extended File
P8R2_ULTRACLEANVETO_V6	1024	Standard	x	x
P8R2_ULTRACLEAN_V6	512	Standard	x	x
P8R2_CLEAN_V6	256	Standard	x	X
P8R2_SOURCE_V6	128	Standard	x	x
P8R2_TRANSIENT010_V6	64	Standard		X
P8R2_TRANSIENT020_V6	16	Standard		X
P8R2_TRANSIENT010E_V6	64	Extended		x
P8R2_TRANSIENT020E_V6	8	Extended		x
P8R2_TRANSIENT015S_V6	65536	No-ACD		x

na-ray Eelescope					
Event Selection I Analysis Type	Recommendat Minimum Energy (emin)	Maximum Energy (emax)	Max Zenith Angle (zmax)	Event Class (evclass)	IRF Name
Galactic Point Source Analysis	100 (MeV)	500000 (MeV)	90 (degrees)	128	P8R2_SOURCE_V6
Off-plane Point Source Analysis	100 (MeV)	500000 (MeV)	90 (degrees)	128	P8R2_SOURCE_V6
Burst and Transient Analysis (<200s)	100 (MeV)	500000 (MeV)	100 (degrees)	16	P8R2_TRANSIENT020_V6
Galactic Diffuse Analysis	100 (MeV)	500000 (MeV)	90 (degrees)	128	P8R2_SOURCE_V6
Extra-Galactic Diffuse Analysis	100 (MeV)	500000 (MeV)	90 (degrees)	1024	P8R2_ULTRACLEANVETO_V
Impulsive Solar Flare Analysis	100 (MeV)	500000 (MeV)	100 (degrees)	65536	P8R2_TRANSIENT015S_V6






































ce Telescope		follow the link		
Data Policy	Your search criteria were:			
Data Access + LAT Data	Equatorial coordinates (degrees)	(343,491,16,1482)		
+ LAT Catalog + LAT Data Oueries	Time range (MET)	(281318400,281923200)		
+ LAT Query Results	Time range (Gregorian)	(2009-12-01 00:00:00,2009-12-08 00:00:0	0)	
+ GBM Data	Energy range (MeV)	(100,300)		
Data Analysis	Search radius (degrees)	15		
Caveats Newsletters Eac	The state of your query is 2 (Query Server Pos	complete) ition in Queue Estimated Time F	temaining (sec	ì
T TAY	Photon Server Qu Spacecraft Server Qu	ery complete N/.	A	
	The Illenames of the result files cours from. The identifiers are of the fon generally be '00' unless the query of the database field are: • PH - Photon Database • SC - Spacecraft Pointing, Lh • EV - Extended Database In the event that you do not see an there may have been a problem.	nsist of the query ID string with an identifier a DDNN where DD indicates the databas resulted in a large data volume. In that case t vetime, and History Database ny files with the data type you requested liste	ppended to ind e and NN is th he data is broke d below, you sh	Icate which database the file ca e file number. The file number in up into multiple files. The valu
	Filename L14090420274034A4AC2B81_PH L14090420274034A4AC2B81_SC	Number of Entries 100.fits 3372 c00.fits 17120	Size (MB) 0.33 2.52	Status Available Available
	If you would like to download the fi	iles via waet, simply copy the following comp	nands and past	e them into a terminal window. T





Gamma-ray Space Telescope	gtbin (Counts Map)
[/home]	ն gtbin
Type of	output file (CCUBE CMAP LC PHA1 PHA2 HEALPIX) [CMAP]
Event da	ata file name[filtered_gti.fits]
Output f	ile name[CMAP.fits]
Spacecr	aft data file name[sc.fits]
Size of t	he X axis in pixels[120]
Size of t	he Y axis in pixels[120]
Image s	cale (in degrees/pixel)[0.25]
Coordina	ate system (CEL - celestial, GAL -galactic) (CEL GAL) [CEL]
First coc [343.494	ordinate of image center in degrees (RA or galactic I) 4812]
Second [16.1495	coordinate of image center in degrees (DEC or galactic b) 5]
Rotation	angle of image axis, in degrees[0]
Projectio [AIT]	on method e.g. AIT ARC CAR GLS MER NCP SIN STG TAN: 76





Use F	itsView to I	ook at	the liaht	curve:					
> fv L(C.fits &								
File Edit	Tools								Help
Index	Extension	Туре	Dimens	ion			View		
□ 0	Primary	Image	0 4 cols X 60 rows 2 cols X 1174 rows		Header Image			Table	
⊒ 1	RATE	Binary			Header	Hist	Plot	All	Select
_ 2	GTI	Binary			Header	Hist	Plot	All	Select
	00		X Select Pl	ot Colum	ns				
	Row Number	har	Click	on a colum	n name the	en selec	t the bar		
	TIME	uer	Axis Column name or expression to plot				t		
	TIMEDEL	TIMEDEL COUNTS ERROR		TIME					
	ERROR			Y COUNTS X Error			S		
				Y Error ERROR					
			Rows:		antantart				
			Use selected rows						





gtbin – II (Light Curve)

[/home]\$ gtexposure Light curve file[] lc.fits Spacecraft file[] sc.fits Response functions[CALDB] Source model XML file[none] Photon index for spectral weighting[-2.1]







sert	nev	N CC	olun	nn							
alcul	ate	rat	с 6 рг	rors							
licul	ale				8	TIM	EDEL	COUNTS	ERROR	EXPOSURE	RATE
	Sele	ct		D		6		3	-	E	1E Counts/cm**2s
		_	_		Tient	TV: S	arculate	л			Modify
										_	0.00000E+00
	sin	cos	tan	loge	exp			Columns		Calculate	0.00000000000
	asin	acos	atan	log10	sart	TIME				C1	0.000000E+00
						TIMEDEL	1			Close	0.000000E+00
	•			rgn	gu	THEFE	-		-	Help	0.0000002+00
	44	1	()	•	COUNTS	1				0.000000E+00
	PI	7	8	9	1	ERROR					0.0000002+00
		4	5	6	•	EXPOSURE					0.000000E+00
	EE	1	2	3		DATE	-			-	0.000000E+00
	CI	0	1000	#POW	1000	RAIE		-			0.000000E+00
	100	Hala	-	- function	-						0.000000E+00
	(204	neipi	or mor	e functi	, епк		Apply	only to selected	rows		0.000000E+00
						THE .				0	D.000000E+00

















Gamma-ray Space Telescope

./make3FGLxml.py gll_psc_v16.fit filtered_gti.fits -o 3c454.3.xml -G / home/grb/software/GlastExt/diffuseModels/v2r0/gll_iem_v06.fits -g gll_iem_v06 -I /home/grb/software/GlastExt/diffuseModels/v2r0/ iso_P8R2_SOURCE_V6_v06.txt -i iso_P8R2_SOURCE_V6_v06 -s 120 -p TRUE -v TRUE

Gamma-ray Space Telescope	Likelihood 3rd step: the XML model
 Backgrour 	nds
Diffuse Sour</td <th>></th>	>
<source galactic_background"="" name="</td><th>" type="DiffuseSource"/>	
<spectrum type="</td"><th>"PowerLaw"></th></spectrum>	"PowerLaw">
<parameter free<="" td=""><th>="1" max="10" min="0" name="Prefactor" scale="1" value="1"/></th></parameter>	="1" max="10" min="0" name="Prefactor" scale="1" value="1"/>
<parameter free<="" td=""><th>="0" max="1" min="-1" name="Index" scale="1.0" value="0"/></th></parameter>	="0" max="1" min="-1" name="Index" scale="1.0" value="0"/>
<parameter free<="" td=""><th>="0" max="2e2" min="5e1" name="Scale" scale="1.0" value="1e2"/></th></parameter>	="0" max="2e2" min="5e1" name="Scale" scale="1.0" value="1e2"/>
<spatialmodel fil<="" td=""><th>e="gll_iem_v06.fits" type="MapCubeFunction"></th></spatialmodel>	e="gll_iem_v06.fits" type="MapCubeFunction">
<parameter free<="" td=""><th>="0" max="1e3" min="1e-3" name="Normalization" scale="1.0" value="1.0"/></th></parameter>	="0" max="1e3" min="1e-3" name="Normalization" scale="1.0" value="1.0"/>
<source extragalactic_background"="" name="</td><th>" type="DiffuseSource"/>	
<spectrum file="</td><th><pre>iso_P8R2_SOURCE_V6_v06.txt" type="FileFunction"></spectrum>	
<parameter free<="" td=""><th>="1" max="10" min="1e-2" name="Normalization" scale="1" value="1"/></th></parameter>	="1" max="10" min="1e-2" name="Normalization" scale="1" value="1"/>
<spatialmodel td="" ty<=""><th>pe="ConstantValue"></th></spatialmodel>	pe="ConstantValue">
<parameter free<="" td=""><th>="0" max="10.0" min="0.0" name="Value" scale="1.0" value="1.0"/></th></parameter>	="0" max="10.0" min="0.0" name="Value" scale="1.0" value="1.0"/>
	94





Gamma-ray Space Telescope	Diffuse resp	ponse	
[/home/]\$gtdiffrs Spacecraft data Source model fil Response functi	pEvent data file[]filtered_gti.fits file[] sc.fits e[] 3c454.3.xml ons to use[] CALDB		
		97	



Ganma-ray Space Telescope	Likelihood output
<pre>{'3c454.3': {'Integral': '0. 'Index': '-2.29973 +/- 0.0 'LowerLimit': '100', 'UpperLimit': '300000', 'Npred': '4171.85', 'ROI distance': '0', 'TS value': '17548.4', 'Flux': '1.46192e-05 +/- 3 extragalactic_backgrourd 'Npred': '643.953', 'Flux': '0.000170707 +/- }, 'galactic_background': { 'Index': '0', 'Scale': '100', 'Npred': '357.929', 'Flux': '0.000215978 +/-</pre>	146106 +/- 0.00271733', 17189', gtlike creates two output files: 1) results.dat: fit results 2) counts_spectra.fits: the counts in a proper energy binning 2.7178e-07', nd': {'Normalization': '1.20197 +/- 0.23541', 3.34331e-05', 'Prefactor': '0.739969 +/- 0.251827', 7.35023e-05',
	99



Space Telescope	on of different models
Powerlaw * HE exp cut-off {'3c454.3': {'Prefactor': '0.39194 +/- 0.0	00793161',
'Index1': '-2.12802 +/- 0.03056', 'Cutoff': '5495.55 +/- 934.857 (MeV) 'Npred': '4157.04', 'ROI distance': '0', 'TS value': '17604.2', 'Flux': '1.41693e-05 +/- 2.72878e-07'	> Comparing TS values for different models! For this source, in this time interval, the model with the HE exponential cutoff is favoured with respect to the Simple Powerlaw
(pn cm-2 s-1) You can repeat analyses by yourself also follo site: - Standard Likelihood: http://fermi.gsfc.nasa.gov - PyLike: http://fermi.gsfc.nasa.gov/ssc/data/ar	wing instructive and complete Tutorials on the FSSC web- ov/ssc/data/analysis/scitools/likelihood_tutorial.html nalysis/scitools/python_tutorial.html 101

elescope	
Generate Spe	ectral Points
To generate spect use gtselect (filter Luckily, there's a s produced so you v If you've made it th play with the pyth saving everything	ral points to plot on top of the butterfly that we just produced, you need to go back to the data selection part and in python) to divide up your data set in energy bins and run the likelihood fit on each of these individual bins cript that does this for us that we'll employ to get a final result. This script also generates the butterfly plot we jus won't have to redo that again and it also does a lot of checks on the data to make sure that everything is going od his far, you might be a little curious as to why we didn't jump right into using this tool but now you're in a position to on tools and make them do what you want them to. The script is also much more careful in handling units and to files than we have been in this interactive session.
So download the information on the binned and unbinn	likeSED.py user contributed tool (it's in the SED_scripts package) and load it up into python. You can fine usage of this tool on the same page where you downloaded it. It can be used to generate an SED plot for both ed analyses but we're only going to work on a binned analysis here.
>>> from li This is like	<pre>keSED import * SED version 12.1, modified to handle Pass 7 selections.</pre>
Now you need to o as arguments. We have the module o double-wide bin at for the fits of the ir ahead and do tha other sources with	create a likelinput object that takes our unbinnedAnalysis object, the source name and the number of bins we wan re going to make 9 bins here like in the paper and we're also going to make custom bins and bin centers. You car hose bins and bin centers for you (via the getECent function) but we're going to do it this way so we can have the the end. We're also going to use the 'fit2' file (the result of our fit on the full dataset with the NewMinuit optimizer dividual bins but we need to doit it first to make everything fixed except for the integral value of PG 1553+113. Get to we're basically assuming that the spectral index is the same for all bins and that the contributions of the in the ROI are not changing with energy.
>>> inputs	= likeInput(like,'3FGL J1555.7+1111', nbins=9)

mi ay iscope	SED modeling
This last step will t requested (look in we request that it i be and then fit eac	take a while (approximately 30 minutes) because it's creating an expmap and event file for each of the bins that we your working directory for these files). Once it is done we'll tell it to do the fit of the full energy band and make sure keep the covariance matrix. Then we'll create a likeSED object from our inputs, tell it where we want our centers to sh of the bands. After this is done, we can plot it all with the Plot function.
<pre>>>> inputs. Full energy 3FGL 11555.7 Spectrum: Po 32 Prefactor 33 Index: 1. 44 Scale: 1. Flux 0.1-300 Test Statist >>> sed.get >>> sed.fit -Running Lik -Running -Running -Runnin</pre>	fullFit(CoVar=True) range model for 3FGL J1555.7+1111: +1111 werLaw : 5.928e-01 2.586e-02 1.000e-04 1.000e+04 (1.000e-11) 621e+00 2.674e-02 0.000e+00 1.000e+00 (-1.000e+00) 648e+03 0.000e+00 3.000e+01 5.000e+00 fixed .00 GeV 7.2e-08 +/- 4.2e-09 cm^-2 s^-1 :c 3141.46 ikeSED(inputs) ECent() Bands() Helihood for banda- telihood for banda-



Examination of the course "Astroparticle Physics" PhD School in Astronomy and in Physics 2016/17

Choose one of the following (a, b or c):

a) Give a seminar of 25' (+ ~15' questions) on an article, scientific or technical.

Some **scientific** articles you might choose for the final exam (of course you can propose your own, and I'll answer you if it's OK for me)

- 1. Acceleration of petaelectronvolt protons in the Galactic Centre. By HESS Collaboration (F. Aharonian et al.). Nature 531 (2016) 476.
- Search for Spectral Irregularities due to Photon–AxionLike-Particle Oscillations with the Fermi Large Area Telescope. By Fermi-LAT Collaboration (M. Ajello et al.). Phys. Rev. Lett. 116 (2016) no.16, 161101.
- 3. Detection of the Characteristic Pion-Decay Signature in Supernova Remnants. By Fermi-LAT Collaboration (M. Ackermann et al.). Science 339 (2013) 807.
- 4. Searches for Dark Matter annihilation signatures in the Segue 1 satellite galaxy with the MAGIC telescope. By MAGIC Collaboration (J. Aleksic et al.). JCAP 1106 (2011) 035.
- 5. Search for a Dark Matter annihilation signal from the Galactic Center halo with H.E.S.S. By HESS Collaboration (A. Abramowski et al.). Phys. Rev. Lett. 106 (2011) 161301.
- 6. Very-High-Energy Gamma Rays from a Distant Quasar: How Transparent Is the Universe? By MAGIC Collaboration (E. Aliu et al.). Science 320 (2008) 1752.
- Evidence for a new light spin-zero boson from cosmological gamma-ray propagation? By Alessandro De Angelis, Marco Roncadelli, Oriana Mansutti. Phys. Rev. D76 (2007) 121301.
- 8. The energy spectrum of cosmic-ray electrons at TeV energies. By HESS Collaboration (F. Aharonian et al.). Phys. Rev. Lett. 101 (2008) 261104.
- 9. High Statistics Measurement of the Positron Fraction in Primary Cosmic Rays of 0.5-500 GeV with the Alpha Magnetic Spectrometer on the International Space Station. By AMS Collaboration (L. Accardo et al.). Phys. Rev. Lett. 113 (2014) 121101.
- 10. Probing Quantum Gravity using Photons from a flare of the active galactic nucleus Markarian 501 Observed by the MAGIC telescope. By MAGIC and Other Contributors (J. Albert et al.). Phys. Lett. B668 (2008) 253.
- 11. Observation of Gravitational Waves from a Binary Black Hole Merger. By LIGO and Virgo Collaborations (B. Abbott et al.). Phys. Rev. Lett. 116 (2016) 061102. [Physicists only]

Some **technical** papers/subjects you might choose

(of course you can propose your own, and I'll answer you if it's OK for me)

- 1. The e-ASTROGAM mission (A. De Angelis, V. Tatischeff et al.), 2017. https://arxiv.org/abs/1611.02232 . Accepted for publication in Experimental Astronomy. Take only Sections 1, 3, 4, 5, 6.
- 2. Describe the principle of operation of the AMS-02 detector.
- 3. Describe the operation principle of a system of Imaging Air Cherenkov Telescopes.
- 4. Describe the operation of a Silicon photomultiplier. Compare it to a CCD.
- b) Analyze the data on a Fermi source that you judge interesting. Write a short report (you can copy the general structure from a Fermi paper; you'll not be accused of plagiarism). Give a seminar of 15' (+ ~15' questions) on your result.
- c) Propose an original scientific article on a subject covered in the course, write it and submit it to a journal (all the classroom will help you, and it will be a "social" article. Independent of the fact that the journal will accept it or not, the professor will offer a good luck party).

Exercises

Chapter 1

1. Number of stars in the Milky Way. Our galaxy consists of a disk of a radius $r_d \simeq 15$ kpc about $h_d \simeq 300$ pc thick, and a spherical bulge at its center roughly 3 kpc in diameter. The distance between our Sun and our nearest neighboring stars, the Alpha Centauri system, is about 1.3 pc. Estimate the number of stars in the our galaxy.

If we approximate the shape of the disk as a cylinder, the volume of the disk is

$$V_d \simeq 4\pi r_d^2 h = 12.56 \times (15 \,\mathrm{kpc})^2 \times (0.3 \,\mathrm{kpc}) = 848 \,\mathrm{kpc}^3$$
.

We also know that the closest star to the Sun is about 1.3 pc away, and that we are in an "ordinary" part of the galactic disk. Assuming that the average distance between stars throughout the galaxy is 1.3 pc, there are a total of

 $N_d \simeq (848 \,\mathrm{kpc}^3) / (9.18 \times 10^{-9} \mathrm{kpc}^3) \simeq 9.2 \times 10^{10} stars$.

The volume of the bulge is 100 times smaller than the disk:

$$V_b \simeq \frac{4}{3}\pi r_b^3 \simeq 4.18 \times (1.5 \text{kpc})^3 = 14.1 \text{kpc}^3$$
,

so it would contain about 1.5×10^9 stars if the density of stars in the bulge is the same as the disk - indeed we observe the density to be larger. In any case, since the volume is so smaller, we can approximate in a robust way

$$N_s \simeq 10^{11} stars$$

in the full Galaxy.

2. Number of nucleons in the Universe. Estimate the number of nucleons in the Universe.

If dark matter is, as indicated by observations, not made of nucleons, then we can approximate the total mass of the observable universe by the total mass of stars (planets and clouds have negligible mass compared to stars). There are about 10^{11} galaxies in the observable universe, and about 10^{11} stars per galaxy. We assume that a typical star weights like our Sun, which has a mass $M_{\odot} \simeq 2 \times 10^{30}$ kg. Thus, the mass of all the stars in the observable universe is

$$M_U \simeq 10^{11} \times 10^{11} \times (2 \times 10^{30}) \simeq 2 \times 10^{52} \text{kg},$$

dominated by nucleons (electrons have negligible mass). Being the nucleon mass $m \simeq m_p \simeq 1.67 \times 10^{-27}$ kg,

$$N_{nucleons} \simeq \frac{M_U}{m_p} \simeq 1.2 \times 10^{79}$$

We expect most of them to be protons, due to the relatively short neutron lifetime.

3. *Galactic and extragalactic emitters of gamma rays.* In Figure 1.10, more than half of the emitters of high-energy photons lie in the Galactic plane (the equatorial line). Guess why.

Two effects can play a role (indeed they both do, in general): (a) present detectors do not have a resolution large enough to observe the structure of galaxies out of the Milky Way, and (b) the signal from distant galaxies is attenuated – in first approximation, as $1/d^2$, where *d* is the distance. Notice that the size of the Milky Way is about 100 kly, while the distance of the nearest large galaxy, Andromeda, is about 2.5 Mly.

Chapter 2

- 1. *GZK threshold*. The Cosmic Microwave Background fills the Universe with photons with a peak energy of 0.37 meV and a density of $\rho \sim 400/\text{cm}^3$. Determine:
 - (a) The minimal energy (known as the GZK threshold) that a proton should have in order that the reaction $p\gamma \rightarrow \Delta$ may occur.
 - (b) The interaction length of such protons in the Universe considering a mean cross-section above the threshold of 0.6 mb.
 - (a) In order that the reaction $p \gamma \to \Delta^+$ may occur the center-of-mass energy should be greater than the mass of the Δ particle:

$$egin{aligned} & \left(p_p + p_{\gamma CMB}
ight)^2 \geq m_\Delta^2, \ & E_p \ \geq \ rac{m_\Delta^2 - m_p^2}{2 \ E_{\gamma CMB} \ (1 - eta \ cos heta)} \end{aligned}$$

Assuming head-on collisions,

$$E_p \gtrsim \frac{m_{\Delta}^2 - m_p^2}{4 E_{\gamma CMB}} \simeq 3.7 \ 10^{20} \text{eV} \,.$$

In fact the GZK is not a sharp cut-off. Taking into account the full distributions of the energy of the CMB photons (for instance the mean value of this distribution is 0.635 meV) and of the $p \gamma \rightarrow \pi^0 p$ and $p \gamma \rightarrow \pi^+ n$ cross sections, the effect of the GZK suppression should start to be effective for proton energies of the order of $E_p \sim 6 \times 10^{19}$ eV.

(b) The interaction length L_{int} is given by

$$L_{int} = \frac{1}{\sigma n}$$

where σ is the total cross section and *n* the number density of targets (in this case the CMB photons). Replacing the given values one obtains

$$L_{int} \simeq 4 \ 10^{24} \text{ cm} \simeq 1.3 \text{ Mpc}$$

In fact 0.6 mb is the peak photon-pion cross section at the Δ resonance. Taking into account all the relevant GZK energy range, a mean photon-pion cross section of around 0.1 mb should be instead considered and a more realistic value for L_{int} would be around 6 Mpc.

- 2. *Photon conversion*. Consider the conversion of one photon in one electron-positron pair. Determine the minimal energy that the photon must have in order that this conversion would be possible if the photon is in presence of:
 - (a) one proton;
 - (b) one electron;
 - (c) when no charged particle is around.

In the presence of a charged particle of mass M, considered to be initially at rest, the minimal energy that the photon must have is the energy required to produce a final state where all particles are at rest in the center-of-mass frame. The total 4-momenta in the initial and final state, p_{μ}^{i} and p_{μ}^{f} , expressed respectively in the laboratory and in the center-of-mass frames are:

$$p^{i,Lab}_{\mu} = (E_{\gamma} + M, \vec{P}_{\gamma}) \tag{2.1}$$

$$p_{\mu}^{f,CM} = (M + 2m_e, \vec{0}).$$
(2.2)

Since $p_{\mu}^{i} p^{i,\mu}$ and $p_{\mu}^{f} p^{f,\mu}$ are Lorentz invariants and 4-momentum is conserved, one has:

$$\left(p^{i,\text{Lab}}\right)^2 = \left(p^{i,\text{CM}}\right)^2 = \left(p^{f,\text{CM}}\right)^2 \tag{2.3}$$

yielding the relation:

$$(E_{\gamma} + M)^2 - P_{\gamma}^2 = (M + 2m_e)^2$$
(2.4)

which leads to

$$E_{\gamma}^{2} + M^{2} + 2E_{\gamma}M - P_{\gamma}^{2} = M^{2} + 4m_{e}^{2} + 4Mm_{e}$$
(2.5)

and finally:

$$E_{\gamma} = 2m_e \left(1 + \frac{m_e}{M}\right). \tag{2.6}$$

(a) For a spectator particle with mass $M >> m_e$ one has:

$$E_{\gamma} \simeq 2m_e \,. \tag{2.7}$$

In particular, for the case of the conversion in presence of a proton, the minimal energy that the photon must have is just a fraction of about 5×10^{-4} above the mass of the electron-positron pair. In fact, for a fixed momentum transferred to the spectator particle, its gain in kinetic energy decreases as the mass increases. In the limit of a very large mass, where one can assume that the velocity is small:

$$T \simeq \frac{P^2}{2M},\tag{2.8}$$

the spectator recoils in order to conserve the momentum, but carries only a small fraction of the energy of the converted photon and most of the available energy is converted into the masses of the electron and of the positron. Hence, as M increases the minimal energy that the photon must have approaches the limit $2m_e$.

(b) In the case of the photon conversion in the presence of an electron, $M = m_e$, and one has:

$$E_{\gamma} = 4 m_e \,. \tag{2.9}$$

- (c) Considering now the photon conversion in vacuum, the minimum energy of the photon is obtained by letting M → 0. In this case, according to equation 2.6 we have E_γ → ∞. A photon cannot convert in an electron-positron pair in the absence of a spectator charged particle, otherwise the total momentum would not be conserved. This can be visualised by placing ourselves in the center-of-mass frame of the electron-positron. Here we would see an initial photon converting into a final state with total momentum M^P = 0, which would violate momentum conservation.
- 3. π^- decay. Consider the decay of a flying π^- into $\mu^- \bar{\nu_{\mu}}$ and suppose that the μ^- was emitted along the flight line of flight of the π^- . Determine:
 - (a) The energy and momentum of the μ^- and of the $\bar{v_{\mu}}$ in the π^- frame.
 - (b) The energy and momentum of the μ^- and of the $\bar{v_{\mu}}$ in the laboratory frame, if the momentum $P_{\pi}^- = 100$ GeV/c).

- (c) Same as the previous question but considering now that was the v_{μ} that was emitted along the flight line of the π^- .
- (a) Energy-momentum conservation in the π^- frame leads to:

$$m_{\pi} = E_{\mu} + E_{\nu} \tag{2.10}$$

$$\vec{0} = \vec{P}_{\mu} + \vec{P}_{\nu} \,. \tag{2.11}$$

Using the first equality and the relation between energy and momentum gives:

$$E_{\mu}^{2} = (m_{\pi} - E_{\nu})^{2} = m_{\pi}^{2} + E_{\nu}^{2} - 2m_{\pi}E_{\nu}$$
(2.12)

$$E_{\mu}^{2} = P_{\mu}^{2} + m_{\mu}^{2}$$
(2.13)

and, since

$$E_{\nu} = \left| \vec{P}_{\nu} \right| = \left| \vec{P}_{\mu} \right| \tag{2.14}$$

we get:

$$E_{\nu} = \frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}}.$$
 (2.15)

Inserting E_v in Eq. 2.10 we obtain:

$$E_{\mu} = \frac{m_{\pi}^2 + m_{\mu}^2}{2m_{\pi}}.$$
(2.16)

Taking the particle masses $m_{\pi} = 139.58 \,\text{MeV}$ and $m_{\mu} = 105.66 \,\text{MeV}$, gives:

$$E_{\mu} \simeq 109.78 \,\mathrm{MeV}$$
; $E_{\nu} \simeq 29.80 \,\mathrm{MeV}$. (2.17)

Finally, the momenta are computed through Eq. 2.14.

(b) Defining the z axis along the direction of flight of the π^- , the momentum vectors of the final state particles in the π^- frame are:

$$\vec{P}_{\mu} \equiv (0, 0, P_{\mu}) \tag{2.18}$$

$$\vec{P}_{\nu} = -\vec{P}_{\mu} \tag{2.19}$$

and the Lorentz transformation to the laboratory frame gives

$$E_{\mu}^{lab} = \gamma \left(E_{\mu} + \beta P_{\mu} \right) \quad ; \quad E_{\nu}^{lab} = \gamma \left(E_{\nu} - \beta P_{\nu} \right) \tag{2.20}$$

$$P_{\mu}^{lab} = \gamma \left(\beta E_{\mu} + P_{\mu}\right) \quad ; \quad P_{\nu}^{lab} = \gamma \left(\beta E_{\nu} - P_{\nu}\right) \tag{2.21}$$

where γ and β are the Lorentz boost and the velocity of the π^- :

$$\gamma = \frac{E_{\pi}}{m_{\pi}} \quad ; \quad \beta = \frac{P_{\pi}}{E_{\pi}} \tag{2.22}$$

or

$$\beta \gamma = \frac{P_{\pi}}{m_{\pi}} = 719 \quad ; \quad \gamma = \sqrt{(\beta \gamma)^2 + 1} \simeq \beta \gamma.$$
 (2.23)

Note that from the above relations one gets $\beta \sim 1$ ($\beta \simeq 0.99999902$). Using the results for the energy and momentum of the μ^- and of the $\bar{\nu}_{\mu}$ in the π^- frame, Eqs. 2.14, 2.15 and 2.16, their energy and momentum in the laboratory frame can be written as:

$$E_{\mu}^{lab} = \frac{E_{\pi}}{2} \left[(1+\beta) + \frac{m_{\mu}^2}{m_{\pi}^2} (1-\beta) \right] ; \ E_{\nu}^{lab} = \gamma E_{\nu} (1-\beta)$$
(2.24)

$$P_{\mu}^{lab} = \frac{E_{\pi}}{2} \left[(1+\beta) + \frac{m_{\mu}^2}{m_{\pi}^2} (\beta - 1) \right] ; P_{\nu}^{lab} = -E_{\nu}^{lab} .$$
 (2.25)

For $\beta \to 1$ the energy of the neutrino goes to zero and the muon energy approaches E_{π} . In particular this is the case for $P_{\pi}^{-} = 100 \text{ GeV}/c$, see Eq. 2.23.

(c) In this case we have

$$\vec{P}_{\mu} \equiv (0, 0, -P_{\mu})$$
 (2.26)

$$\vec{P}_{\nu} = -\vec{P}_{\mu} \tag{2.27}$$

and the Lorentz transformation to the laboratory frame yields,

$$E_{\mu}^{lab} = \gamma \left(E_{\mu} - \beta P_{\mu} \right) \quad ; \quad E_{\nu}^{lab} = \gamma \left(E_{\nu} + \beta P_{\nu} \right) \tag{2.28}$$

$$P_{\mu}^{lab} = \gamma \left(\beta E_{\mu} - P_{\mu}\right) \quad ; \quad P_{\nu}^{lab} = \gamma \left(\beta E_{\nu} + P_{\nu}\right) \tag{2.29}$$

or, using again the results from Eqs. 2.14, 2.15 and 2.16,

$$E_{\mu}^{lab} = \frac{E_{\pi}}{2} \left[(1-\beta) + \frac{m_{\mu}^2}{m_{\pi}^2} (1+\beta) \right] ; \ E_{\nu}^{lab} = \gamma E_{\nu} (1+\beta)$$
(2.30)

$$P_{\mu}^{lab} = \frac{E_{\pi}}{2} \left[(\beta - 1) + \frac{m_{\mu}^2}{m_{\pi}^2} (1 + \beta) \right] ; P_{\nu}^{lab} = E_{\nu}^{lab} .$$
(2.31)

Now for $\beta \rightarrow 1$ we have

$$E_{\mu}^{lab} \simeq E_{\pi} \frac{m_{\mu}^2}{m_{\pi}^2} \quad ; \quad E_{\nu}^{lab} \simeq 2\,\gamma E_{\nu} \tag{2.32}$$

$$P_{\mu}^{lab} \simeq E_{\mu}^{lab} \; ; \; P_{\nu}^{lab} = E_{\nu}^{lab} \; .$$
 (2.33)

It should be noted that the boost due to the large momentum of the π^- results in that the μ^- reversed its direction of motion (in the π^- frame it was emitted opposite to the flight line of the π^-). Also, in this case $P_{\mu}^{lab} \sim E_{\mu}^{lab}$, i.e. the muon mass is negligible in comparison with its energy.

- 4. π^0 decay. Consider the decay of a π^0 into $\gamma\gamma$ (with pion momentum of 100 GeV/c). Determine:
 - (a) The minimal and the maximal angles that the two photons may have in the laboratory frame.
 - (b) The probability of having one of the photon with an energy smaller than an arbitrary value E_0 in the laboratory frame.
 - (c) Same as (a) but considering now that the decay of the π^0 is into e^+e^- .
 - (d) The maximum momentum that the π^0 may have in order that the maximal angle in its decay into $\gamma\gamma$ and in e^+e^- would be the same.
 - (a) Let us start by evaluating the decay in the CM reference frame. In this reference frame the π^0 is at rest and the two photons, that arise from the π^0 decay, have opposite directions such that the relation between their momentum is $\vec{p}_1 = -\vec{p}_2$. Since for photons, in natural units, the momentum is equal to its energy, then

$$E_{\gamma 1}^* = E_{\gamma 2}^* = \frac{M_{\pi^0}}{2} \,. \tag{2.34}$$

In order to obtain the energy of the photons one needs to apply the Lorentz transformations

$$\begin{pmatrix} E \\ P \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ P^* \end{pmatrix}$$
(2.35)

where (E, P) are the energy and the momentum of the particle in the laboratory reference frame while (E^*, P^*) are the same quantities in the CM frame.

Using Eq. 2.35 one can now transform the energy of the photons from CM \rightarrow LAB,

$$E_{\gamma 1} = \gamma (E_{\gamma 1}^* + \beta E_{\gamma 1}^* \cos \theta^*) \tag{2.36}$$

$$E_{\gamma 2} = \gamma (E_{\gamma 2}^* - \beta E_{\gamma 2}^* \cos \theta^*).$$
(2.37)

The γ and β that relate reference frames can be obtained from

$$\gamma = \frac{E_{\pi}}{M_{\pi}} \quad ; \quad \beta = \frac{P_{\pi}}{E_{\pi}} \tag{2.38}$$

leading to

$$E_{\gamma 1} = \frac{E_{\pi}}{2} \left(1 + \frac{P_{\pi}}{E_{\pi}} \cos \theta^* \right)$$
(2.39)

$$E_{\gamma 2} = \frac{E_{\pi}}{2} \left(1 - \frac{P_{\pi}}{E_{\pi}} \cos \theta^* \right).$$
(2.40)

In oder to determine the minimum and maximum angle between the photons in the laboratory frame one can start by identifying two limiting situations: the photons directions are aligned with the boost direction; both photons are emitted transversely to the boost direction.

• Maximum angle: in the CM reference frame the angle between the photons is always $\alpha^* = 180^\circ$. As the boost cannot reverse the momentum of the photon travelling against the boost direction, the angle that the photons will do in the LAB frame will continue to be $\alpha = 180^\circ$. Furthermore, for $\theta^* = 0^\circ$,

$$E_{\gamma 1} = \frac{E_{\pi}}{2} + \frac{P_{\pi}}{2} \tag{2.41}$$

$$E_{\gamma 2} = \frac{E_{\pi}}{2} - \frac{P_{\pi}}{2} \,. \tag{2.42}$$

• Minimum angle: this occurs when both photons are emitted transversely to the boost direction, i.e., $\cos \theta^* = 0$. To evaluate the minimum angle let us start by noting that the Lorentz transformation only alter physical quantities that have a non-zero projection along the boost axis. Therefore, the transverse momentum of γ_1 in the LAB is equal to its transverse momentum in the CM frame, leading to $P_{\gamma 1}^T = E_{\gamma 1}^*$. The longitudinal momentum (in the boost direction) can be obtained through the Lorentz transformations (Eq. 2.35),

$$P_{\gamma 1}^{\parallel} = \gamma \beta E_{\gamma 1}^{*} + \underbrace{\gamma P_{\gamma 1}^{*} \cos \theta^{*}}_{=0}$$
(2.43)

Therefore, the minimum angle between the two photons is given by

$$\alpha = 2 \arctan\left(\frac{E_{\gamma 1}}{\gamma \beta E_{\gamma 1}}\right) \simeq 2 \frac{M_{\pi}}{P_{\pi}}$$
(2.44)

where the approximation for small angles $\tan \theta \sim \theta$ was used; the factor of 2 comes from the symmetry of the system, i.e., both photons will have the same angle with the boost direction.

(b) As seen in the previous problem

$$E_{\gamma l} \in \left[\frac{E_{\pi}}{2}; \frac{E_{\pi}}{2} + \frac{P_{\pi}}{2}\right] \tag{2.45}$$

$$E_{\gamma 2} \in \left[\frac{E_{\pi}}{2} - \frac{P_{\pi}}{2}; \frac{E_{\pi}}{2}\right]. \tag{2.46}$$

Therefore, the photon that fulfils the problem condition is the one moving in the opposite direction to the boost, i.e., photon 2.

The photon energy in the LAB has to be smaller than E_0 , and so

$$E_{\gamma 2} = \frac{E_{\pi}}{2} \left(1 - \frac{P_{\pi}}{E_{\pi}} \cos \theta^* \right) \le E_0 \tag{2.47}$$

leading to the condition

$$\delta \equiv \cos \theta^* \ge \frac{E_\pi}{P_\pi} - \frac{2E_0}{P_\pi}.$$
(2.48)

The emission of the photons in the CM frame is isotropic. Thus, the probability of finding a photon with a given solid angle is uniform. As a consequence, we can write

$$\operatorname{Prob} = \int_{\delta}^{1} d\cos\theta^* = 1 - \delta.$$
(2.49)

Notice that the cosine of an angle is bounded between -1 and +1, which justifies the upper bound of the integral.

Substituting Eq. 2.48 in Eq. 2.49 and noticing that $E_{\pi}/P_{\pi} \simeq 1$ we finally obtain

$$\operatorname{Prob} = \frac{2E_0}{P_{\pi}}.$$
(2.50)

(c) In this problem we are in a situation similar to π^0 decay but now photons 1 and 2 are substituted by the e^- and the e^+ , respectively (this is an arbitrary choice). Like before we have two limiting situations when $\cos \theta^* = \pm 1$ and $\cos \theta^* = 0$. However, as now the particles produced during the decay are massive, their momentum might be reversed by the boost, even if they are produced backwards.

The condition for the particle to be emitted in the LAB frame, always in the direction of the boost, even if the particle is emitted in the opposite direction of the boost ($\cos \theta^* = 180^\circ$), can be easily obtained from the Lorentz transformation

$$P_e = \gamma \beta E_e^* - \gamma P_e^* > 0 \tag{2.51}$$

This means that if the particle is always emitted forward in the LAB frame then the following condition has to be fulfilled:

$$\frac{P_e^*}{E_e^*} = \beta_e < \beta_\pi = \frac{P_\pi}{E_\pi} \,. \tag{2.52}$$

As the electron and the positron have the same mass, then in the CM reference frame, they will share the same amount of energy coming from the decay of the pion and so

$$E_e^* \equiv E_{e^-}^* = E_{e^+}^* = \frac{M_\pi}{2}.$$
(2.53)

Hence, using the relation $P = \sqrt{E^2 - m^2}$, it is possible evaluate numerically Eq. 2.52,

$$0.999973 < 0.99999(9). \tag{2.54}$$

The above result confirms that, in the LAB frame, all particles will be produced in the direction of the boost, independent of their production angle. Thus, in the LAB, the **minimum angle** is zero and will occur when $\cos \theta^* = \pm 1$.

Naturally, the **maximum angle** between the electron and the positron, α , will be in the opposite limiting situation, i.e. $\cos \theta^* = 0$. The procedure to compute this angle is the same used to evaluate the minimum angle in (a),

$$\alpha = 2 \arctan\left(\frac{P_e^T}{P_e^{\parallel}}\right) \simeq 2 \frac{\sqrt{(E_e^*)^2 - m_e^2}}{\gamma \beta E_e^*}$$
(2.55)

where the approximation $\tan \theta \sim \theta$ was used for small angles.

By neglecting the electron mass with respect to the pion mass, one gets finally for the minimum angle

$$\alpha \simeq \frac{1}{\beta \gamma} \tag{2.56}$$

with $\beta = P_{\pi}/E_{\pi}$ and $\gamma = E_{\pi}/m_{\pi}$.

(d) The maximum angle between the two photons is $\alpha = 180^{\circ}$, independent of the pion momentum. For the electron and positron to make an angle of 180° in the LAB frame, the particle must be aligned with the boost direction and the condition in Eq. 2.52 must not be fulfilled (otherwise the momentum of the particle emitted backwards would be reversed). Therefore, the condition is

$$\beta_e^* > \beta_{CM} \tag{2.57}$$

and thus

$$P_{\pi} < \sqrt{\frac{m_{\pi}^2 \left(\frac{P_{\pi}^2}{P_{\pi}^2 + m_{\pi}^2}\right)^2}{1 - \left(\frac{P_{\pi}^2}{P_{\pi}^2 + m_{\pi}^2}\right)^2}} \simeq 19 \,\text{GeV}/c\,.$$
(2.58)

Chapter 3

1. *The measurement by Hess.* Discuss why radioactivity decreases with elevation up to some 1000 m, and then increases. Can you make a model? This was the subject of the thesis by Schrödinger in Wien in the beginning of XX century.

Ionizing radiation must carry more than 10 eV in energy in order to be able to ionize atoms and molecules. The level of radioactivity decreases with elevation up to about 1000 m because in these altitudes the sources of ionising radiation are mostly on ground. This can be modelled as $e^{-h/\lambda}$. There is an inflection on the profile density of ionising particles in the atmosphere, when radiation from the soil attains a minimum. After this height, radiation begins to increase with altitude because of the contribution of cosmic rays to ionisation. Therefore, the particle flux can be written as

$$F = Ae^{-h/\lambda} + Bh + c$$

where *h* is the height measured from ground level, λ a parameter related to atmospheric attenuation, the other terms being model constants.

2. *Antimatter.* The total number of nucleons minus the total number of antinucleons is believed to be constant in a reaction – you can create nucleon-antinucleon pairs. What is the minimum energy of a proton hitting a proton at rest to generate an antiproton?

The reaction involving the minimal number of new particles while assuring charge and baryon number conservation is:

$$pp \rightarrow ppp\bar{p}$$
.

The minimum energy is obtained when the antiproton and the three protons are produced at rest in the center mass frame, i.e.:

$$\sqrt{s} = 4m_p, \ s = 16m_p^2$$

(proton and antiproton have the same mass). In the laboratory frame, the value of s before the reaction is

$$s = (p_1 + p_2)^2 = 2m_p^2 + 2E_pm_p$$

As s is conserved in the reaction and it is also Lorentz-invariant, we can equate the two expressions, obtaining:

$$16m_p^2 = 2m_p^2 + 2E_pm_p;$$

 $E_p = 7m_p \simeq 6.57 \,\text{GeV}.$

3. *Fermi maximum accelerator.* According to Enrico Fermi, the ultimate human accelerator, the "Globatron", would be built around 1994 encircling the entire Earth and attaining energy of around 5000 TeV (with an estimated cost of 170 million US1954 dollars...). Discuss the parameters of such an accelerator.

For the sake of reference, the circumference of the LHC is 27 km, using 8.3 T magnets. The corresponding proton-proton center of mass energy achieved is 14 TeV.

The CM energy that a proton-proton collider can achieve is related to its perimeter and the magnetic field strength of its bending magnets. Fermi assumed a magnetic field of 2 T, which would allow obtaining a maximum energy

of 5 PeV, comparable to that of CR at the "knee" of the spectrum. This "Globatron", going around Earth's equator would be some 40.000 km long. Using the same magnetic field as in the LHC, the Globatron would achieve energies of about 20 PeV.

4. *Cosmic pions and muons*. Pions and muons are produced in the high atmosphere, at a height of some 10 km above sea level, as a result of hadronic interactions from the collisions of cosmic rays with atmospheric nuclei. Compute the energy at which charged pions and muons respectively must be produced to reach in average the Earth's surface.

You can find the masses of the lifetimes of pions and muons in your Particle Data Booklet.

An unstable particle produced in the high atmosphere can only reach the Earth's surface if the time of flight, t, is smaller than the average lifetime in the Earth reference frame, τ . This condition imposes that

$$\frac{L}{c} = \beta \tau = \beta \gamma \tau_0 \tag{3.1}$$

with τ_0 the average lifetime in the rest frame. Given that

$$\beta \gamma = \sqrt{\gamma^2 - 1}, \qquad (3.2)$$

one has

$$\gamma = \sqrt{\left(\frac{L}{c\,\tau_0}\right)^2 + 1}\,.\tag{3.3}$$

Taking

$$m_{\pi} = 139.57 \,\mathrm{MeV}/c^2 \qquad c \,\tau_{\pi} = 7.8 \,\mathrm{m}$$
 (3.4)

$$m_{\mu} = 105.66 \,\mathrm{MeV}/c^2 \quad c \,\tau_{\mu} = 658.6 \,\mathrm{m}$$
 (3.5)

the minimum energy at which charged pions and muons must be produced at a height of 10 km above sea level to reach in average the Earth's surface is:

$$E_{\pi} \simeq 180 \,\text{GeV} \tag{3.6}$$

$$E_{\mu} \simeq 1.6 \,\text{GeV}\,. \tag{3.7}$$

Given that the pion and muon masses are of the same order, they have identical boost for the same production energy. Hence, in this case it is the difference in the lifetimes that dictates the distance they can travel before decaying. In fact the minimum energies computed above differ by a factor of ~ 100 , which is approximately the factor between the pion and muon lifetimes.

5. *Very-high-energy cosmic rays.* Justify the sentence "About once per minute, a single subatomic particle enters the Earth's atmosphere with an energy larger than 10 J" in Chapter 1.

The differential CR spectrum can be written as $dF/dE = AE^{-\alpha}$, so that the flux above a certain energy E_1 is given by:

$$F(E > E_1) = \int_{E_1}^{E_{\text{inf}}} (dF/dE) dE = 1/2A/E_1^2, \qquad (3.8)$$

for $\alpha = -3$. Here we use $A = 3.375 \times 10^{22} \text{ eV}^2 \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}$, derived from the fact that, at $E \simeq 1.5 \times 10^9 \text{ eV}$, the differential flux is of $\simeq 1 \times 10^{-5} \text{ eV}^2 \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}$. So, $F(E > E_1) = 4.33 \times 10^{-18} \text{ m}^{-2} \text{s}^{-1} \text{sr}^{-1}$, and finally, the number of particles reaching the Earth per minute is, $N = 2\pi AF(E > E_1)$, where *A* is the surface area of the Earth and 2π [sr] counts the entire solid angle of arrival of particles. From which we obtain the value of one sub atomic particle per minute.

6. *Very-high-energy neutrinos*. The IceCube experiment in the South pole can detect neutrinos crossing the Earth from the North pole. If the cross section for neutrino interaction on a nucleon is $(6.7 \times 10^{-39} E)$ cm² with E

expressed in GeV (note the linear increase with the neutrino energy E), what is the energy at which half of the neutrinos interact before reaching the detector? Comment the result.

The mean free path of the neutrino can be calculated from the cross section:

$$\lambda \simeq \frac{1}{\mathcal{N}\rho\sigma}$$

where ρ is the density of the material, and \mathcal{N} is Avogadro's number. One has thus the equation

$$e^{-2R/\lambda} = \frac{1}{2}\,,$$

where *R* is the radius of the Earth.

Solving for σ one has

$$2R\mathcal{N}\rho\sigma = \ln 2 \Longrightarrow \sigma = \frac{\ln 2}{2R\mathcal{N}\rho}$$

and assuming an average density $\rho \simeq 5.51$ g/cm³, a diameter $2R \simeq 12.7 \times 10^8$ cm, and the Avogadro's number 6.02×10^{23} , one has

$$\sigma \simeq 1.64 \times 10^{-34} \,\mathrm{cm}^2$$

In the approximation of the problem, one has thus

$$E \sim 25 \,\mathrm{TeV}$$
.

- 7. If a π^0 from a cosmic shower has an energy of 2 GeV:
 - (a) Assuming the two γ -rays coming from its decay are emitted in the direction of the pion's velocity, how much energy does each have?
 - (b) What are their wavelengths and frequencies?
 - (c) How far will the average neutral pion travel, in the laboratory frame, from its creation to its decay? Comment on the difficulty to measure the pion lifetime.

Chapter 4

1. Compton scattering. A photon of wavelength λ is scattered off a free electron initially at rest. Let λ' be the wavelength of the photon scattered in the direction θ . Compute: (a) λ' as a function of λ , θ and universal parameters; (b) the kinetic energy of the recoiling electron.

We will apply to this scattering problem the conservation of four-momentum in the frame in which the electron is initially at rest: we will choose the *x*-asis along the line containing the photon and the electron before the scattering. As in the text, we will indicate with primes the quantities after the scattering, e.g. λ' , and without primes those before the scattering, e.g. λ . We will call the four momenta (before and after the scattering) with q_{μ} and q'_{μ} for the photon, and with p_{μ} and p'_{μ} for the electron. In the chosen frame we have

$$q_{\mu} = (h\nu/c, h\nu/c, 0, 0) \qquad q'_{\mu} = (h\nu'/c, h\nu'\cos\theta/c, h\nu'\sin\theta/c, 0),$$
$$p_{\mu} = (m_{e}c, \vec{0}) \qquad p'_{\mu} = (\gamma'm_{e}c, \vec{p}'),$$

where $\gamma' = (1 - v'^2/c^2)^{-1/2}$. From the time component of energy momentum conservation,

$$q_\mu + p_\mu = q'_\mu + p'_\mu,$$

we obtain

$$\frac{h\nu}{c} + m_{\rm e}c = \frac{h\nu'}{c} + m_{\rm e}\gamma'c \quad \Rightarrow \quad h\nu - h\nu' + m_{\rm e}c^2 = m_{\rm e}\gamma'c. \tag{4.1}$$

Conservation of three-momentum can be expressed using the angle θ between the incident and scattered photon. Since from the spatial part of momentum conservation the total three momentum before the scattering (and only related to the photon in the chosen frame) must equal the sum of the three momenta after scattering, the three momenta \vec{q} , \vec{q}' and \vec{p}' form a triangle, with an angle θ at the vertex formed by \vec{q} and \vec{q}' . Using Carnot rule on this triangle, we get

$$m_{\rm e}^2 \gamma'^2 v'^2 = \frac{h^2 v'^2}{c^2} + \frac{h^2 v'^2}{c^2} - 2 \frac{h^2 v v'}{c^2} \cos \theta.$$
(4.2)

Equation (4.1) can be first rewritten as

$$(h\nu - h\nu' + m_{\rm e}c^2)^2 = m_{\rm e}^2\gamma'^2c^2, \qquad (4.3)$$

and, solving for v'^2 by remembering that $\gamma'^2 c^2 = c^2 - v'^2$, results in

$$v'^2 = c^2 - \frac{m_e^2 c^6}{(hv - hv' + m_e c^2)^2}$$

From the above and (4.3), with a little algebra, we then obtain

$$m_{\rm e}^2 \gamma'^2 v'^2 = \frac{(h\nu - h\nu' + m_{\rm e}c^2)^2}{c^2} - m_{\rm e}^2 c^2,$$

and, by comparison with (4.2),

$$\frac{(h\nu - h\nu' + m_{\rm e}c^2)^2}{c^2} - m_{\rm e}^2 c^2 = \frac{h^2\nu^2}{c^2} + \frac{h^2\nu'^2}{c^2} - 2\frac{h^2\nu\nu'}{c^2}\cos\theta.$$

The above result can be now simplified into

$$-\frac{2h^2\mathbf{v}\mathbf{v}'}{c^2} + 2hm_{\rm e}(\mathbf{v} - \mathbf{v}') = -\frac{2h^2\mathbf{v}\mathbf{v}'}{c^2}\cos\theta \quad \Rightarrow \quad \frac{h}{m_{\rm e}c^2}(1 - \cos\theta) = \frac{1}{\mathbf{v}'} - \frac{1}{\mathbf{v}}$$

The sane expression in terms of the photon wavelength before and after the scattering, $\lambda = c/v$ and $\lambda' = c/v'$, respectively, is

$$\lambda' - \lambda = \frac{h}{m_{\rm e}c}(1 - \cos\theta),$$

and gives us the relationship between λ , λ' , the photon scattering angle θ , and the universal parameters c, h, and m_e . The kinetic energy of the scattered electron, T'_e , can then be calculated as

$$T'_{e} = m_{e}\gamma'c^{2} - m_{e}c^{2}$$

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$= \frac{hc}{\lambda} \left(1 - \frac{1}{1 + h(1 - \cos\theta)/(\lambda m_{e}c)}\right)$$

$$= \frac{hc}{\lambda} \left(\frac{(1 - \cos\theta)}{\lambda m_{e}c/h + (1 - \cos\theta)}\right), \qquad (4.4)$$

again in terms of c, h, m_e, λ , and the scattering angle of the photon θ .

- Cherenkov radiation. A proton with momentum 1.0 GeV/c passes through a gas at high pressure. The index of refraction of the gas can be changed by changing the pressure. Compute: (a) the minimum index of refraction at which the proton will emit Cherenkov radiation; (b) the Cherenkov radiation emission angle when the index of refraction of the gas is 1.6.
 - (a) The boost can be obtained using

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2 + m^2}}{m} = 1.46 \tag{4.5}$$

where *m* is the mass of the proton and *p* its momentum. The velocity, β , can obtained inverting the boost formula:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}.\tag{4.6}$$

The condition for Cherenkov light emission is

$$\cos\theta = \frac{1}{n\beta} \le 1 \tag{4.7}$$

where n is the medium refraction index.

Hence, using Eq. 4.7 and Eq. 4.6, one finds that the refraction index necessary to produce Cherenkov light is

$$n \ge \frac{1}{\sqrt{1 - \frac{1}{\gamma^2}}} \simeq 1.37.$$
 (4.8)

(b) Again, using Eq. 4.7, and taking now n = 1.6, and β from Eq. 4.6, one gets

$$\cos\theta = \frac{1}{n\beta} = \frac{1}{1.6 \times 0.729} \simeq 0.86$$
 (4.9)

which means that $\theta \simeq 31^{\circ}$.

3. *Nuclear reactions*. The mean free path of fast neutrons in lead is of the order of 5 cm. What is the total fast neutron cross section in lead?

$$\sigma_t = \frac{\omega_A}{LN_{\text{int}}} = 6 \times 10^{-24} \text{ cm}^2.$$
 (4.10)

Using equation 2.25 in the textbook, we have:

4. *Photodetectors.* What gain would be required from a photomultiplier in order to resolve the signal produced by 3 photoelectrons from that due to 2 or 4 photoelectrons? Assume the fluctuations in the signal are described by Poisson statistics, and consider that two peaks can be resolved when their centers are separated by more than the sum of their standard deviations.

The number of electrons, N, produced by $N^{(pe)}$ photoelectrons is

$$N = GN^{(pe)} \tag{4.11}$$

where G is the PMT gain.

From the problem one has that the condition to have resolved peaks is

$$\mu_1 - \mu_2 \ge \sigma_1 + \sigma_2 \tag{4.12}$$

where μ is the distribution mean and σ its standard deviation. If the distributions follow Poisson statistics then

$$\mu_i = N_i \text{ and } \sigma_i = \sqrt{N_i}.$$
 (4.13)

Therefore, Eq. 4.12 becomes

$$N_{i+1} - N_i \ge \sqrt{N_{i+1}} + \sqrt{N_i} \,. \tag{4.14}$$

Using Eq. 4.11 the above equation provides

$$G(N_{i+1}^{(pe)} - N_i^{(pe)}) \ge \sqrt{G} \left(\sqrt{N_{i+1}^{(pe)}} + \sqrt{N_i^{(pe)}} \right)$$
(4.15)

and so the gain must be

$$G \ge \left(\frac{\sqrt{N_{i+1}^{(pe)}} + \sqrt{N_i^{(pe)}}}{N_{i+1}^{(pe)} + N_i^{(pe)}}\right)^2 \tag{4.16}$$

From the above condition one has that for $N_i^{(pe)} = 2$ and $N_{i+1}^{(pe)} = 3$

$$G \ge 9.9 \simeq 10 \tag{4.17}$$

and for $N_i^{(pe)} = 3$ and $N_{i+1}^{(pe)} = 4$,

$$G \ge 13.93 \simeq 14$$
. (4.18)

5. *Cherenkov counters*. Estimate the minimum length of a gas Cherenkov counter used in the threshold mode to be able to distinguish between pions and kaons with momentum 20 GeV. Assume that 200 photons need to be radiated to ensure a high probability of detection and that radiation covers the whole visible spectrum (neglect the variation with wavelength of the refractive index of the gas).

The number of Cherenkov photons emitted per unit of track length and per unit of the photon energy interval is:

$$\frac{d^2 N_{\gamma}}{dE \, dx} \simeq 370 \sin^2 \theta_c \,\mathrm{eV}^{-1} \,\mathrm{cm}^{-1} \,. \tag{4.19}$$

In the whole visible spectrum ($\lambda \in [300, 700]$ nm) the Cherenkov light yield in a length L (in cm) is:

$$N_{\gamma} = \int_{0}^{L} \int_{E(700\,\text{nm})}^{E(300\,\text{nm})} \frac{d^{2}N_{\gamma}}{dEdx} dE \, dx \simeq 875 \sin^{2}\theta_{c} L \tag{4.20}$$

where

$$\sin^2 \theta_c = 1 - \cos^2 \theta_c = 1 - \frac{1}{n^2 \beta^2}$$
(4.21)

and the variation with wavelength of the refractive index was neglected.

From these relations it is clear that for a fixed number of radiated photons, the length of the Cherenkov counter can be minimised by increasing the refractive index of the gas. However, in order to be able to distinguish between pions and kaons, by operating the detector in threshold mode, the maximum *n* that can be used is the value matching the kaon threshold velocity, $\beta = 1/n$. Writing the kaon threshold velocity in terms of its momentum, p_K , gives for the refractive index,

$$n^{2} = \frac{1}{\beta^{2}} = \frac{m_{K}^{2} + p_{K}^{2}}{p_{K}^{2}} \simeq 1 + 6 \times 10^{-4}$$
(4.22)

and, inserting this relation in Eq. 4.21, the angle of the Cherenkov photons emitted by the pion is then

$$\sin^2 \theta_c = 1 - \frac{1}{n^2 \beta_\pi^2} = 1 - \frac{m_\pi^2 + p_\pi^2}{m_K^2 + p_K^2} = \frac{m_K^2 - m_\pi^2}{m_K^2 + p_K^2}$$
(4.23)

where use was made of the assumption $p_K = p_{\pi}$. From Eqs. 4.20 and 4.23, the length of the Cherenkov counter is then

$$L = \frac{N_{\gamma}}{875} \frac{m_K^2 + p_K^2}{m_K^2 - m_{\pi}^2} \,\mathrm{cm} = \frac{N_{\gamma}}{875} \frac{1 + \left(\frac{p_K}{m_K}\right)^2}{1 - \left(\frac{m_{\pi}}{m_K}\right)^2} \,\mathrm{cm} \simeq \frac{N_{\gamma}}{875} \left(\frac{p_K}{m_K}\right)^2 \,\mathrm{cm},\tag{4.24}$$

using the approximations $p_K^2 >> m_K^2$ and $m_\pi^2 << m_K^2$.

Taking $N_{\gamma} = 200$, $m_{\pi} = 139.58$ MeV and $m_K = 493.68$ MeV the minimum length of the gas Cherenkov counter is

$$L \simeq 3.6 \,\mathrm{m}$$
. (4.25)

6. *Electromagnetic calorimeters*. Electromagnetic calorimeters have usually 20 radiation lengths of material. Calculate the thickness (in cm) for a calorimeters made of of BGO, PbWO₄ (as in the CMS experiment at LHC), uranium, iron, tungsten and lead. Take the radiation lengths from Appendix B or from the Particle Data Book.

Let us compute the thickness, and compare it to the weight per unit area per radiation length:

$$\begin{split} L_{BGO} &= 22.3\,\mathrm{cm}; \quad \rho X_0 = 7.97\,\mathrm{g/cm^2} \\ L_{PBWO_4} &= 17.8\,\mathrm{cm}; \quad \rho X_0 = 7.39\,\mathrm{g/cm^2} \\ L_U &= 6.4\,\mathrm{cm}; \quad \rho X_0 = 6.00\,\mathrm{g/cm^2} \\ L_{Fe} &= 35.2\,\mathrm{cm}; \quad \rho X_0 = 13.84\,\mathrm{g/cm^2} \\ L_W &= 7.0\,\mathrm{cm}; \quad \rho X_0 = 6.76\,\mathrm{g/cm^2} \\ L_{Pb} &= 11.2\,\mathrm{cm}; \quad \rho X_0 = 6.37\,\mathrm{g/cm^2}. \end{split}$$

Besides uranium, a material quite expensive and complicated to treat, if your main problem is space a good material to build a converter for an electromagnetic calorimeter is lead, or, even better, tungsten – but tungsten will be a bit heavier and quite more expensive. Iron is cheap and performs reasonably well. BGO and lead tungstate (PbWO₄) occupy a space a larger than Pb or W, but they are active materials, and thus you can use them for a very performant homogeneous calorimeter (you do not need to add a sensitive detector).

7. *Cherenkov telescopes.* Suppose you have a Cherenkov telescope with 7 m diameter, and your camera can detect a signal only when you collect 100 photons from a source. Assuming a global efficiency of 0.1 for the acquisition system (including reflectivity of the surface and quantum efficiency of the PMT), what is the minimum energy (neglecting the background) that such a system can detect at a height of 2 km a.s.l.?

Considering a global efficiency of 0.1, the minimum number of Cherenkov photons that must be collected by the telescope in order to detect a signal is $N_{\gamma} = 1000$. Since the area of the telescope is 38.5 m^2 , this corresponds to a minimum density of about 26 photons/m² in the Cherenkov "light pool" of the shower. The mean density of Cherenkov photons at a height of 2 km a.s.l. is about 10 photons/m², for a primary of 100 GeV, and about 150 photons/m² for a primary of 1 TeV (see Chapter 4 of the textbook). Taking the conservative number of 100 photons/m² per 1 TeV of primary energy, the minimum energy of the primary yielding the required density of photons is thus of about 260 GeV.
8. *Cherenkov telescopes.* If a shower is generated by a gamma-ray of E=1 TeV penetrating the atmosphere vertically, considering that the radiation length X_0 of air is approximately 37 g/cm² and its critical energy E_c is about 88 MeV, calculate the height h_M of the maximum of the shower in the Heitler model and in the Rossi approximation B.

See the answer to question a. in Problem 10.8.

- 9. *Cherenkov telescopes*. Show that the image of the Cherenkov emission from a muon in the focal plane of a parabolic IACT is a conical section (approximate the Cherenkov angle as a constant).
- 10. Energy loss. In the Pierre Auger Observatory the surface detectors are composed by water Cherenkov tanks 1.2 m high, each containing 12 tons of water. These detectors are able to measure the light produced by charged particles crossing them. Consider one tank crossed by a single vertical muon with an energy of 5 GeV. The refraction index of water is $n \simeq 1.33$ and can be in good approximation considered constant for all the relevant photons wavelengths. Determine the energy lost by ionization and compare it with the energy lost by Cherenkov emission. Consider that the mean energy loss rate for water is somewhere between the helium gas and carbon.

Let us start by evaluating the energy lost by ionisation. Using the dE/dX plot in the Particle Data Group booklet (PDG) one gets that a 5 GeV muon in a material with characteristic between carbon and helium gas loses about

$$\frac{dE}{dX} \simeq 2.1 \,\mathrm{MeV} \,\mathrm{cm}^2 \,\mathrm{g}^{-1} \tag{4.26}$$

The amount of matter traversed by the muon is

$$X = l\rho_{water} \simeq 120 \,\mathrm{g\,cm}^{-2} \tag{4.27}$$

where l = 120 cm is the height of the tank and ρ_{water} the water density.

Thus, the muon energy lost by ionisation is

$$E_{loss}^{ionisation} = X \frac{dE}{dX} \simeq 252 \,\mathrm{MeV}\,.$$
 (4.28)

Let us now compute the energy lost due to Cherenkov radiation. Assuming that in average each photon carried about 3.5 eV, then

$$E_{loss}^{Cherenkov} = \langle E \rangle_{\gamma} N_{\gamma} \tag{4.29}$$

with N_{γ} being the number of produced Cherenkov photons. This last quantity can be obtained through

$$\frac{d^2N}{dE\,dx} = 370\sin^2(\theta_c)\,\mathrm{photons}\,\mathrm{eV}^{-1}\,\mathrm{cm}^{-1}\,.\tag{4.30}$$

Knowing that $E_{\mu} = 5 \text{ GeV}$ and that the boost is $\gamma = E_{\mu}/m_{\mu}$ one gets that the velocity of the particle is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \left(\frac{m_{\mu}}{E_{\mu}}\right)^2} \simeq 0.9998$$
(4.31)

and the Cherenkov emission angle is

$$\theta_c = \arccos\left(\frac{1}{\beta n}\right) = 41.25^\circ.$$
(4.32)

We can now obtain the number of produced Cherenkov photons by integrating equation 4.30 in energy and path:

$$N_{\gamma} \simeq 370 \sin^2(\theta_c) \Delta E \Delta X = 19302 \text{ photons}$$
(4.33)

where it was assumed that n(E) is constant.

Finally, the ratio between the energy lost by ionisation and the one lost by Cherenkov emission is

$$\frac{E_{loss}^{ionisation}}{E_{loss}^{Cherenkov}} \simeq 3730 \tag{4.34}$$

which means that the energy lost by Cherenkov emission can be neglected for practical purposes.

11. *Bremsstrahlung radiation.* Consider a circular synchrotron of radius R_0 which is capable of accelerating charged particles up to an energy of E_0 . Compare the Bremsstrahlung radiation emitted by a proton and an electron and discuss the difficulties to accelerate these particles with this technology.

12. *Muon energy loss*. A muon of 100 GeV crosses a layer of 1 m of iron. Determine the energy loss and the expected scattering angle.

The electrons lose energy due to Bremsstrahlung at a rate $\sim 10^{13}$ times higher than the protons, the reason being that the radiative power scales with $1/m^4$, where *m* is the mass of the charged particle. Furthermore, radiation losses in the presence of magnetic field also scale with B^2 . Therefore, a circular collider, which needs to apply all the way intense magnetic fields to bend and confine the particle's trajectories, cannot be used to accelerate electrons efficiently at very-high energies due to prohibitive, intense radiative losses, a factor which is less critical for protons.

1. *Grey disk model in proton-proton interactions.* Determine, in the framework of the grey disk model, the mean radius and the opacity of the proton as a function of the center-of-mass energy (you can use Figure 6.70 to extract the total and the elastic proton-proton cross-sections).

1. *Cosmological principle and Hubble law.* Show that the Hubble law does not contradict the Cosmological principle (all points in space and time are equivalent).

The cosmological principle principle expresses the fact that, on large enough scales, the matter distribution in the universe is homogeneous and isotropic. The fact that the Hubble law does not contradict this statement can be shown by proving that, given a homogeneous and isotropic distribution of matter, the Hubble law results as a consequence.

If we consider a spacetime with homogeneous and isotropic spatial sections, in comoving coordinates the metric has the form

$$ds^2 = dt^2 - a^2(t) \left\{ d\chi^2 + \Sigma^2(\chi) (d\theta^2 + \sin^2\theta d\phi^2) \right\},$$

where $\Sigma^2(\chi) = \sin^2 \chi$ (resp. χ^2 , $\sinh^2 \chi$), depending on the closed (respectively flat, open) character of the spatial sections. If we call *D* the distance between an observer in the origin of the above reference system and another point, we measure D = D(t) along a surface of constant time *t*; by also choosing to consider the radial distance (i.e., constant θ and ϕ), we end up with the three conditions $dt = d\theta = d\phi = 0$, so that $D(t) = a(t)\chi$. For the observed velocity V(t) of a body at a constant value of the coordinate χ we then obtain

$$V(t) = \frac{dD(t)}{dt} = \dot{a}(t)\chi = \frac{\dot{a}(t)}{a(t)}D(t).$$

In the above relation $\dot{a}(t)/a(t) = H(t)$ is the Hubble constant, so that the above relation gives the Hubble law V(t) = H(t)D(t). This shows that the Hubble law is consistent with a homogeneous and isotropic cosmological model that respects the cosmological principle.

2. Asymptotically matter-dominated Universe. Consider a Universe composed only by matter and radiation. Show that whatever would have been the initial proportion between the matter and the radiation energy densities this Universe will be asymptotically matter dominated.

$$\rho_{\rm r} \propto a^{-4}$$
 and $\rho_{\rm r} \propto a^{-3}$, (8.1)

where a is the scale factor of the Friedmann metric. In the two component universe the Friedmann equation reads

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_{\rm r} + \rho_{\rm m})$$

Let us now consider a universe that is initially radiation dominated. In this case we can neglect ρ_m with respect to ρ_r , and we can write with good approximation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{\rm r} \propto a^{-4}$$

for the Friedmann equation above. Then, $a \propto t^{1/2}$, and, using (8.1) this shows that, in this epoch, the radiation density behaves as $\rho_r \propto t^{-2}$, while the matter density evolves as $\rho_m \propto t^{-3/2}$. Thus the radiation content dilutes

Let us imagine a two components universe, filled by matter and radiation. Let us assume that these two components are not interacting. This last hypothesis allows us to write two independent energy conservation laws for matter and radiation. If ρ_r is the radiation density, and ρ_m is the matter density, the corresponding conservation laws imply

faster as the universe expands in the radiation dominated epoch. Eventually the radiation will dilute so much that the radiation domination condition will not be satisfied anymore. After enough time, the matter content will become dominant. In this other epoch, the universe evolution will be governed by the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{\rm m} \propto a^{-3},$$

in which we are now neglecting ρ_r with respect to ρ_m . Accordingly, the scale factor now evolves as $a \propto t^{2/3}$, and the radiation and matter densities scale as $\rho_r \propto t^{-2-2/3}$ and $\rho_m \propto t^{-2}$, respectively.

As we could anticipate from the conservation laws, radiation will continue to become less and less important for the evolution of the universe, so that the universe is eventually matter dominated. Of course, the same conclusion would have been reached if we would have started with a matter dominated universe, as we would have been from the beginning in the second regime. In presence of matter and radiation, the universe will then become, eventually, matter dominated.

3. WIMP "miracle". Show that a possible Weak Interacting Massive Particle (WIMP) with a mass of the order of m_{χ} 100 GeV would have the relic density needed to be the cosmic dark matter (this is the so called WIMP "miracle").

As we are talking about cold Dark Matter, the appropriate asymptotic form for the equilibrium number density of particles χ with mass $m_{\chi} \simeq 100 \,\text{GeV}$ is the nonrelativistic limit:

$$n \sim (m_{\chi}T)^{3/2} \exp\left(-m_{\chi}/T\right)$$
 (8.2)

Decoupling happens when $\Gamma \sim n\sigma \sim H$, where *H* is the Hubble constant, which, using the Friedmann equations, can be rewritten as $H \sim T^2/M_{\rm pl}$ with $M_{\rm pl} = (8\pi G)^{-1/2}$. Using this information we can recast the freeze-out condition as:

$$x^{-1/2}e^{-x} \sim \frac{1}{M_{\rm pl}\,\sigma\,m_{\chi}}\,,$$
(8.3)

where $x = m_{\chi}/T$. Numerical values can be now inserted noting that for a weak interacting particle $\sigma \simeq G_F^2 m_{\chi}^2 \simeq 10^{-8} \text{ GeV}^{-2}$. Solving numerically the previous equation in the range $10^{-10} - 10^{-20}$, one finds x in the range 20 - 50. Now, we know that:

$$\Omega_{\chi} = \frac{\rho_{\chi}}{\rho_c} = \frac{m_{\chi}n}{\rho_c} = \frac{m_{\chi}}{\rho_c} \frac{n}{T_0^3} T_0^3 , \qquad (8.4)$$

with $T_0 = 2.75 \text{ K} \simeq 10^{-4} \text{ eV}$. After decoupling annihilations cease and the density of dark matter particles will just decrease with a^{-3} , exactly like temperature does, so the ratio n/T^3 remains constant and we can take its value at freeze-out. In particular, using $n\sigma \sim H$, we can write $n \simeq T^2/M_{\text{pl}}\sigma$ and finally:

$$\Omega_{\chi} = \left(\frac{T_0^3}{\rho_c M_{\rm pl}}\right) \frac{x}{\sigma} \simeq \frac{x}{40} \left(\frac{10^{-8} \,\mathrm{GeV}^{-2}}{\sigma}\right) \,, \tag{8.5}$$

which gives the right relic density for dark matter $\Omega_{\chi} \simeq 0.2$, given that $\sigma \simeq 10^{-8} \text{ GeV}^{-2}$ and $x \simeq 20$.

4. Classical black hole. Compute the relation between the radius and the mass of a black hole in classical physics.

$$< E_p >= \frac{GMm}{r} = \frac{(6.67 \times 10^{-11}) \times (4 \times 10^{44}) \times (2 \times 10^{42})}{1 \text{Mpc} \times (3.1 \times 10^{22} \text{m/kpc})}$$

^{5.} *Virial theorem.* A cluster of galaxies, called Abell 2715 (at a redshift $\simeq 0.114$), contains about 200 galaxies, each the mass of the Milky Way. The average distance of the galaxies from the center of the cluster is 1 Mpc. If Abell 2715 is a virialized system, what is the approximate average velocity of the galaxies with respect to the center? The Milky Way galaxy has a mass of about 2.0×10^{42} kg.

If Abell 2715 is a virialized system, then its potential energy E_p will be twice its entire kinetic energy E_k . If it has 200 galaxies, each of mass 2×10^{42} kg, then the mass of the entire cluster is $200 \times 2 \times 10^{42} = 4 \times 10^{44}$ kg. So on average, each galaxy will have potential and kinetic energies

$$< E_k > = \frac{mv^2}{2} = \frac{2 \times 10^{42}}{2}v^2$$

where v is the average velocity of the galaxies in this cluster.

For the entire cluster, we can use the virial theorem to solve for v.

$$[200 < E_p >] = 2 \times [200 \times < E_k >$$

$$200 \times 1.7 \times 10^{52} = 2 \times 200 \times (10^{42} \times v^2)$$

$$v = \sqrt{0.85 \times 10^{10}} = 9.2 \times 10^5 \text{m/s} = 920 \text{ km/s}.$$

- 6. *M/L*. At $r = 10^5$ light-years from the center of a galaxy the measurement yields $v_{meas} = 225$ km/s while the expected velocity calculated from the luminous mass is of $v_{calc} = 15$ km/s. Calculate the visible and the true galaxy mass, and the ratio M/L between the total and the luminous masses. How high is the average dark matter mass density?
- 7. The WIMP annihilation prefers the production of heavy fermions. Demonstrate that, in the reaction $\chi \chi \rightarrow f \bar{f}$,

$$\left(1 - \frac{|p|}{E + m_f}\right) = \frac{m_f}{2m_\chi + m_f},$$

where χ is a generic WIMP and *E*, *p* are respectively the energy and the momentum of the fermion produced in the decay. Use the above relation to compute, for a WIMP of mass $m_{\chi} = 30 GeV$, the ratios of the branching fractions into $\tau^+\tau^-$ and into $b\bar{b}$, in the hypothesis that

- (a) the WIMP has spin 0;
- (b) the WIMP has spin 1/2 (note: in this case, use the Clebsch-Gordan coefficients to compute the relative probabilities of the singlet and of the triplet state).

1. *Neutrino interaction cross section*. Explain the peak in the cross section in the (anti)neutrino interaction cross section for an (anti)neutrino energy around 10¹⁶ eV (Fig. 9.1).

The *s*-channel excitation of a W^- boson on-shell, commonly known as the Glashow resonance, can be initiated by the electron antineutrino hitting an electron at a laboratory energy around 6.3 PeV.

If we call *x* the direction of motion of the (anti)neutrino, the energy-momentum 4-vector of the system constituted by the (anti)neutrino and by an electron at rest can be written, neglecting the neutrino mass, as

$$(E_v^2 + m_e, E_v^2, 0, 0)$$

(we omitted the bar over the antineutrino symbol). The square of the invariant mass of the system is thus

$$m_{inv}^2 = (E_v^2 + m_e)^2 - E_v^2 \simeq 2E_v m_e$$
.

The energy at which one can produce a W boson on shell is

$$E_{\rm v}\simeq rac{m_W^2}{2m_e}\simeq 6.3\,{
m PeV}\,,$$

and this corresponds to a peak on the cross section.

- 2. Neutrino from Supernova 1987A. In 1987, a supernova explosion was observed in the Large Magellanic Cloud, and neutrinos were measured in three different detectors. The neutrinos, with energies between 10 and 50 MeV, arrived with a time span of 10 s, after a travel distance of about $c(5 \times 10^{12} \text{s})$, and three hours before photons at any wavelength.
 - (a) Can this information be used to determine a neutrino mass? Discuss the quantitative mass limits that could be derived from the SN1987A.
 - (b) This was the only SN observed in neutrinos, up to now, but the same reasoning can be used in pulsed accelerator beams. Derive the needed time and position precision to measure masses $\sim 1 \text{ eV}$, given a beam energy $E \sim 1 \text{ GeV}$ and a distance L.
 - (a) It is noted that the neutrinos arrive before light; if we would assume that they leave the source at the same time, this would mean that neutrinos are faster than light,

$$\beta = \frac{v}{c} = \frac{p}{E} > 1 \,,$$

which implies $E^2 - m^2 > E^2$ and $m^2 < 0$. In fact, this is not the correct interpretation, but that the light itself is delayed by interaction on the source (more than neutrinos).

Likewise, some neutrinos can be emitted earlier than others, and that explains the time span. However, if they are emitted at the same time but have a large mass, then the most energetic should arrive earlier, which is not reported by the experiments. From the observed time and energy span, an upper limit can still be derived for the mass.

For both calculations we use

$$\Delta\beta = \Delta(\frac{v}{c}) = \frac{\Delta T}{L} \,.$$

 $\Delta\beta \simeq -2 \times 10^{-8}$, between the neutrinos and the light, $\Delta\beta \simeq 2 \times 10^{-12}$, between the neutrinos themselves. On the other hand,

$$\begin{split} \beta &= \frac{p}{E} = \sqrt{1 - \frac{m^2}{E^2}} \simeq -\frac{1}{2} \frac{m^2}{E^2} \\ \Delta\beta &\simeq \frac{m^2}{2} \frac{E_1^2 - E_2^2}{E_1^2 E_2^2} \\ m &= \sqrt{2\Delta\beta \frac{E_1^2 E_2^2}{E_1^2 - E_2^2}} \,. \end{split}$$

The comparison between neutrinos and gammas would give an imaginary mass, m = 10 i keV, while the comparison between neutrinos would give an upper limit of 20 eV.

(b) Accelerator beams are in general pulsed, and thus we have the time of emission of the neutrino (known within this pulse), and the length until they are detected, as well as their energy. To measure a mass $m \sim 1$ eV with $E \sim 1$ GeV, we need a precision $m/E \sim 10^{-9}$. The error on $(\Delta\beta/\beta)$ is about $\sqrt{(\Delta L/L)^2 + (\Delta T/T)^2}$, since the relative uncertainties are summed in quadrature, and thus both the distance and timing precision must be of this order of magnitude or better. Present experiments have accuracies of $L \sim 1$ cm and $T \sim 1$ ns.

 Neutrinos from SN1987A. Neutrinos from SN1987A, at an energy of about 50 MeV, arrived in a bunch lasting 13 s from a distance of 50 kpc, 3 hours before the optical detection of the supernova. What can you say on the neutrino mass? What can you say about the neutrino speed (be careful...)?

This exercise is very similar to 9.2, and we refer to that for the first question. About the second question, no relevant conclusions can really be drawn unless a hypothesis is done on the relative time of emission of neutrinos compared to the time of emission of photons. If you would assume that the delay between the photon bunch and the neutrino bunch is due to a superluminal speed c_v of neutrinos, you would obtain

$$\frac{c_v}{c} \simeq 1 + 2 \times 10^{-9} \dots$$

Notice they the OPERA experiment had claimed that neutrinos were traveling from CERN to Gran Sasso (a distance of 730 lm) 60 ns before photons; this would imply $c_v/c \simeq 1 + 2 \times 10^{-5}$.

2. Time lag in light propagation. Suppose that the speed c of light depends on its energy E in such a way that

$$c(E) \simeq c_0 \left(1 + \xi \frac{E^2}{E_P^2}\right),$$

where E_P is the Planck energy (second-order Lorentz Invariance Violation). Compute the time lag between two VHE photons as a function of the energy difference and of the redshift *z*.

3. *Photon spectrum in hadronic cascades.* Demonstrate that in a decay $\pi^0 \rightarrow \gamma\gamma$, once boosted for the energy of the emitting π^0 , the probability to emit a photon of energy E_{γ} is constant over the range of kinematically allowed energies.

- (a) Write the analytical expressions for the number of particles and for the energy of each particle at depth *X* as a function of *d*, *n* and E_0 .
- (b) The multiplication of the cascade stops when the particles reach a critical energy, E_c (when the decay probability surpasses the interaction probability). Using the expressions obtained in the previous question, write as a function of E_0 , E_c and $\lambda = d/\ln(2)$, the expressions, at the shower maximum, for:
 - i. the average energy of the particles,
 - ii. the number of particles, N_{max},
 - iii. the atmospheric depth X_{max} .

^{4.} *Extensive electromagnetic air showers*. The main characteristic of an electromagnetic shower (say, initiated by a photon) can be obtained using a simple Heitler model. Let E_0 be the energy of the primary particle and consider that the electrons, positrons and photons in the cascade always interact after travelling a certain atmospheric depth $d = X_0$, and that the energy is always equally shared between the two particles. With this assumptions, we can schematically represent the cascade as in Figure 4.10.

(a) At the *n*-th generation,

$$X = n \times d \tag{10.1}$$

and the number of produced particles is simply

$$N = 2^n. (10.2)$$

As the energies of the particles are the same at the end of each generation, the energy of each particle is equal is the primary energy divided by the number of particles at this level, i.e.,

 $E = E_c$

$$E_i = \frac{E_0}{2^n}.$$
 (10.3)

(b) i. By construction:

ii.

(10.4)

$$N_{max} = \frac{E_0}{E_c} \,. \tag{10.5}$$

iii. Using Eq. (10.2),

$$N_{max} = 2^{n_{max}} \Leftrightarrow n_{max} = \frac{\ln(N_{max})}{\ln(2)}$$
(10.6)

where n_{max} is the maximum number of levels.

Since $d = \lambda \ln(2)$ the maximum atmospheric depth can be written as

$$X_{max} = n_{max} \times d = \frac{\ln(N_{max})}{\ln(2)} d = \lambda \ln\left(\frac{E_0}{E_c}\right)$$
(10.7)

where Eq. (10.6) and Eq. (10.5) were used to evaluate n_{max} and N_{max} , respectively.

- 5. *Extensive hadronic air showers*. Consider a shower initiated by a proton of energy E_0 . We will describe it with a simple Heitler-like model: after each depth *d* an equal number of pions, n_{π} , and each of the 3 types is produced: π^0, π^+, π^- . Neutral pions decay through $\pi^0 \rightarrow \gamma\gamma$ and their energy is transferred to the electromagnetic cascade. Only the charged pions will feed the hadronic cascade. We consider that the cascade ends when these particles decay as they reach a given decay energy E_{dec} , after *n* interactions, originating a muon (plus an undetected neutrino).
 - (a) How many generations are needed to have more that 90% of the primary energy, E_0 in the electromagnetic component?
 - (b) Assuming the validity of the superposition principle, according to which a nucleus of mass number A and energy E_0 behaves like A nucleons of energy E_0/A , derive expressions for:
 - i. the depth where this maximum is reached, X_{max} ,
 - ii. the number of particles at the shower maximum,
 - iii. the number of muons produced in the shower, N_{μ} .

In this case:

$$N_{tot} = n_{\pi}^{n}; N_{ch} = \left(\frac{2}{3}n_{\pi}\right)^{n}; E_{i} = \frac{E_{0}}{n_{\pi}^{n}}$$
 (10.8)

where N_{ch} is the number of charged particles at the level *n* and n_{π} the number of pions produced at each interaction.

(a) At each interaction 1/3 of the energy goes into the electromagnetic channel through the π^0 decay. Therefore the energy that remains for the charged particles is

$$E_{ch} = \left(\frac{2}{3}\right)^n E_0. \tag{10.9}$$

Thus the fraction of electromagnetic energy rises as $E_{em} = E_0 - E_{ch}$. Hence

$$\frac{E_{em}}{E_0} = 1 - \left(\frac{2}{3}\right)^n.$$
 (10.10)

Taking $E_{em}/E_0 = 0.9$ and inverting Eq. (10.10) to obtain the number of generations (levels), one gets

$$n = \frac{\ln(0.1)}{\ln(2/3)} \simeq 5.7 \text{ generations.}$$
(10.11)

(b) i. Let us start by evaluating X_{max} for protons. In this case from Eq. (10.8), and recalling that the shower development stops when the energy of the particles reaches E_{dec} , one obtains

$$X_{max} = d \times n_{dec}. \tag{10.12}$$

The maximum number of generations is $E_{max} = E_0/n_{\pi}^{n_{dec}}$. Inverting this last expression one gets

$$n_{dec} = \frac{\ln(E_0/E_{dec})}{\ln(n_{\pi})},$$
(10.13)

which leads, using Eq. (10.12), to

$$X_{max} = d \frac{\ln(E_0/E_{dec})}{\ln(n_{\pi})}.$$
 (10.14)

For iron we have 56 nucleons (i.e. the atomic number A = 26 protons + 30 neutrons). Using the superposition principle each nucleon carries $E_0/56$ of the primary energy. Substituting in Eq. (10.14),

$$X_{max} = d \frac{\ln\left(\frac{E_0}{AE_{dec}}\right)}{\ln(n_{\pi})} = \frac{d}{\ln(n_{\pi})} \left(\ln\left(\frac{E_0}{E_{dec}}\right) - \ln(A)\right).$$
(10.15)

Notice that the X_{max} evolution with energy is the same for proton and iron and the curves are separated by a constant term: $\ln(A)$.

ii. Again starting with protons we have for the number of particles at the shower maximum

$$N_{max} = n_{\pi}^{n_{dec}} = \frac{E_0}{E_{dec}} \,. \tag{10.16}$$

For iron primaries,

$$N_{max} = A n_{\pi}^{n_{dec}} \,. \tag{10.17}$$

Using the superposition principle and the result of Eq. (10.13) it is easy to see that

$$N_{max} = A n_{\pi}^{\frac{\ln\left(\frac{E_0}{AE_{dec}}\right)}{\ln(n_{\pi})}} = A \frac{E_0}{AE_{dec}} = \frac{E_0}{E_{dec}}$$
(10.18)

which means that the number of particles at the shower maximum does not depend on the primary mass composition.

iii. The number of muons in the shower, for this simplified model, is given by

$$N_{\mu} = N_{ch}|_{X = X_{max}} = \left(\frac{2}{3}n_{\pi}\right)^{n_{dec}}.$$
(10.19)

Therefore for proton primaries, using Eq. (10.13),

$$N_{\mu} = \left(\frac{2}{3}n_{\pi}\right)^{\frac{\ln(E_{0}/E_{dec})}{\ln(n_{\pi})}} = \left[\left(\frac{2}{3}n_{\pi}\right)^{\log_{\frac{2}{3}n_{\pi}}(E_{0}/E_{dec})}\right]^{\frac{\ln\left(\frac{2}{3}n_{\pi}\right)}{\ln(n_{\pi})}} = \left(\frac{E_{0}}{E_{dec}}\right)^{\frac{\ln\left(\frac{2}{3}n_{\pi}\right)}{\ln(n_{\pi})}} = \left(\frac{E_{0}}{E_{dec}}\right)^{\beta} \quad (10.20)$$

where β is a parameter related with the multi-particle production in the hadronic interactions, in particular, the ratio between the hadronic and the electromagnetic component of the interaction. For iron, using again the superposition principle and the final result of Eq. (10.20), one gets

$$N_{\mu} = A \left(\frac{E_0/A}{E_{dec}}\right)^{\beta} = A^{1-\beta} \left(\frac{E_0}{E_{dec}}\right)^{\beta}.$$
 (10.21)

- 6. Propagation. The transparency of the Universe to a given particle depends critically on its nature and energy. In fact, whenever it is possible to open an inelastic channel of the interaction between the *travelling* particle and the Cosmic Microwave Background, its mean free path diminishes drastically. Assuming that the only relevant phenomena that rules the mean free path of the *travelling* particle is the CMB (CvB), estimate the order of magnitude energies at which the transparency of the Universe changes significantly, for:
 - (a) Photons;
 - (b) Protons;
 - (c) Neutrinos.

Assume $\langle E_{\gamma_{CMB}} \rangle \simeq 0.24$ meV; $\langle E_{\nu_{CVB}} \rangle \simeq 0.17$ meV.

(a) For photons the dominant process is the interaction with the photons of the cosmic microwave background, γ_{CMB} , through pair creation:

$$\gamma + \gamma_{CMB} \to e^+ + e^- \,. \tag{10.22}$$

To determine the minimum energy at which this process can occur it is useful to compute the inner product of the four-momentum vector. This quantity is a Lorentz invariant and thus we can easily relate quantities in the laboratory frame with the ones in the center-of-mass frame. For convenience, the calculations for the photons (before the interaction) will be considered in the laboratory while the products of such interaction will be considered in the center-of-mass. Therefore,

$$P_{\mu}^{LAB} = (E_{\gamma} + E_b, \vec{P}_b + \vec{P}_{\gamma}) \tag{10.23}$$

$$P_{\mu}^{CM} = (2m_e, \vec{0}). \tag{10.24}$$

where it was assumed that $E_b \equiv E_{\gamma_{CMB}}$. Using

$$s = (P_{\mu}P^{\mu})_{LAB} = (P_{\mu}P^{\mu})_{CM} \tag{10.25}$$

$$P_{\mu}P^{\mu} = E^2 - \vec{P} \cdot \vec{P}$$
 (10.26)

one obtains the following equation:

$$4m_e^2 = (E_b + E_\gamma)^2 - (P_b^2 + P_\gamma^2 + 2\vec{P}_b \cdot \vec{P}_\gamma).$$
(10.27)

Taking into account that for photons $E = |\vec{P}|$ and that $\vec{P_1} \cdot \vec{P_2} = |P_1||P_2|\cos\theta$, one gets that the energy of the incoming photon is given by

$$E_{\gamma} = \frac{2m_e^2}{E_b(1 - \cos\theta)}.$$
(10.28)

Notice that we are looking for the minimal energy that allows for this process to happens, so $\cos \theta = -1$. Inputing the values given in the problem one gets

$$E_{\gamma} = \frac{m_e^2}{E_b} \simeq 10^{14} \,\mathrm{eV}\,.$$
 (10.29)

(b) For protons the dominant inelastic channel is via

$$p + \gamma_{CMB} \to (\Delta^+) \to p + \pi^0$$

$$p + \gamma_{CMB} \to (\Delta^+) \to n + \pi^+.$$
(10.30)

As in the previous problem, we want to find the minimum energy of the proton for which the process is possible. Moreover, we will use the Lorentz invariant *s* and consider once again that the proton and the gamma are in the Lab frame and the products of the interaction in the center-of-mass frame. Let us then start by defining our kinematics:

$$P_{\mu}^{LAB} = (E_p + E_b, \vec{P}_p + \vec{P}_b) \tag{10.31}$$

$$P_{\mu}^{CM} = (m_p + m_{\pi}, \vec{0}). \tag{10.32}$$

Therefore, using Eq. (10.26), one can write:

$$(E_b + E_p)^2 - P_b^2 - P_p^2 - 2\vec{P}_b \cdot \vec{P}_p = (m_p + m_\pi)^2.$$
(10.33)

Solving the above equation for E_p and recalling that $P_p = \sqrt{E_p^2 + m_p^2}$ one gets:

$$2E_b E_p - 2E_b \sqrt{E_p^2 + m_p^2} \cos(\theta) = m_\pi^2 + 2m_p m_\pi.$$
(10.34)

Since $E_p \gg m_p$ the proton momentum can be approximated by its energy $(P_p \simeq E_p)$. Therefore,

$$E_p = \frac{m_\pi^2 + 2m_p m_\pi}{4E_b} \simeq 6 \times 10^{19} \,\text{eV}\,. \tag{10.35}$$

where $\cos \theta$ was taken to be -1, i.e., the proton and the photon have opposite directions.

(c) The Universe becomes opaque to the neutrinos when they the following inelastic interaction channel opens:

$$\nu + \nu_{C\nu B} \to Z \,. \tag{10.36}$$

Again considering that the interaction occurs in the laboratory frame and noticing that the interaction occurs if there is enough energy to produce a Z at rest, one can write

$$P_{\mu}^{LAB} = (E_{\nu} + E_b, \vec{P}_{\nu} + \vec{P}_b) \tag{10.37}$$

$$P_{\mu}^{CM} = (m_Z, \vec{0}). \tag{10.38}$$

and similarly as before one can write the following expression:

$$(E_b + E_v)^2 - P_b^2 - P_v - 2\vec{P}_b \cdot \vec{P}_v = m_Z^2$$
(10.39)

$$E_{v} = \frac{m_{Z}^{2}}{2E_{b}(1 - \cos\theta))}.$$
 (10.40)

Therefore, the minimum energy of the neutrino that allows the process in Eq. (10.36) is

$$E_{\rm v} = \frac{m_Z^2}{4E_b} \simeq 10^{24} \,{\rm eV}\,.$$
 (10.41)

7. *Fermi acceleration mechanisms*. Calculate, for first and second order Fermi acceleration mechanism, how many times has the particle to cross the cloud (shock) to gain a factor 10 on its initial energy. Assume $\beta = 10^{-4}$ for the magnetic cloud and $\beta = 10^{-2}$ for the shock wave. Repeat the previous exercise assuming $\beta = 10^{-4}$ for both acceleration mechanisms.

$$E = E_0 (1 + \varepsilon)^n. \tag{10.42}$$

where ε is the gain and it is proportional to β for the acceleration in a shock wave (Fermi first order acceleration mechanism) and proportional to β^2 for the *collision* with the magnetic cloud (Fermi second order acceleration mechanism). β is the velocity of the astrophysical object (shock wave or cloud). Inverting Eq. (10.42) one can obtain the number of times that a particle should cross in order to increase its energy from E_0 to E,

$$n = \frac{\ln\left(\frac{E}{E_0}\right)}{\ln(1+\varepsilon)}.$$
(10.43)

In this problem we want to know how many times a particle should cross a cloud or a shock wave to increase its energy by a factor of 10, so $E/E_0 = 10$. Finally, using Eq. (10.43), we have

$$n(\varepsilon \propto \beta; \beta = 10^{-2}) \simeq 2.3 \times 10^2 \text{ cycles}$$
 (10.44)

$$n(\varepsilon \propto \beta^2; \beta = 10^{-4}) \simeq 2.3 \times 10^8 \text{ cycles}.$$
 (10.45)

Therefore, in realistic astrophysical conditions the particle needs to cross only 230 times the shock wave to gain a factor of 10 on its energy while it should cross a cloud 2.3×10^8 times to gain the same energy.

⁽a) At each passage through the cloud or shock wave the particle gains some energy that is proportional to its energy. Therefore for *n* crossings the energy of the particle relatively to its initial energy, E_0 is given by

(b) Here we assume that both the shock wave and the magnetic cloud have the same velocity $\beta = 10^{-4}$.

$$n(\varepsilon \propto \beta; \beta = 10^{-4}) \simeq 2.3 \times 10^4 \text{ cycles}$$
 (10.46)

 $n(\varepsilon \propto \beta^2; \beta = 10^{-4}) \simeq 2.3 \times 10^8 \text{ cycles}.$ (10.47)

Even considering the same velocity for the two astrophysical phenomena the Fermi first order acceleration needs 10000 times less cycles than the Fermi second order mechanism.

8. *Imaging Array Cherenkov Telescopes*. In the isothermal approximation, the depth *x* of the atmosphere at a height *h* (i.e., the amount of atmosphere above *h*) can be approximated as

$$x \simeq X e^{-h/7 \,\mathrm{km}}$$
,

with $X \simeq 1030$ g/cm². If a shower is generated by a gamma ray of E=1 TeV penetrating the atmosphere vertically, considering that the radiation length X_0 of air is approximately 36.6 g/cm² (440 m) and its critical energy E_c is about 88 MeV and using Rossi approximation B:

	Incident electron	Incident photon
Peak of shower t_{max}	$1.0 \times (\ln y - 1)$	$1.0\times(\ln y - 0.5)$
center of gravity t_{med}	$t_{max} + 1.4$	$t_{max} + 1.7$
Number of e^+ and e^- at peak	$0.3y/\sqrt{\ln y - 0.37}$	$0.3y/\sqrt{\ln y - 0.31}$
Total track length	У	У

- (a) Calculate the height h_M of the maximum of the shower in the Heitler model and in the Rossi approximation B.
- (b) If 2000 useful Cherenkov photons per radiation length are emitted by charged particles in the visible and near UV, compute the total number N_γ of Cherenkov photons generated by the shower (note: the critical energy is larger than the Cherenkov threshold).
- (c) Supposing that the Cherenkov photons are all emitted at the center of gravity of the shower that in the Heitler approximation is just the maximum of the shower minus one radiation length, compute how many photons per square meter arrive to a detector at a height h_d of 2000 m, supposing that the average attenuation length of photons in air is 3 km, and that the light pool can be derived by a opening of $\sim 1.3^{\circ}$ from the shower maximum (1.3° is the Cherenkov angle and 0.5° comes from the intrinsic shower spread). Comment on the size of a Cherenkov telescope, considering an average reflectivity of the mirrors (including absorption in transmission) of 70%, and a photodetection efficiency (including all the chains of acquisition) of 20%.
- (d) Re-do the calculations for E = 50 GeV, and comment.
- (a) Let us start by computing the depth of the shower maximum, X_{max} , for the two models and afterwards convert it into an altitude using the atmosphere model provided in the problem

$$X_{max} \simeq X e^{-\frac{h}{7\mathrm{km}}} \tag{10.48}$$

In the Heitler model,

$$X_{max} = X_0 \left(1 + \frac{\ln(E/E_c)}{\ln(2)} \right) \simeq 530 \,\mathrm{g \, cm^{-2}}$$
(10.49)

where E = 1 TeV, $E_c = 88$ MeV and $X_0 = 36.6 \text{ g cm}^{-2}$. In the Rossi approximation B model,

$$X_{max} = X_0 \left[\ln \left(\frac{E}{E_c} \right) - 0.5 \right] \simeq 325 \,\mathrm{g \, cm^{-2}} \,. \tag{10.50}$$

Inverting Eq. (10.48) to obtain the height of X_{max} one gets

$$h_M = -7\mathrm{km}\ln\left(\frac{X_{max}}{X}\right). \tag{10.51}$$

Hence,

- Heitler: $h_M \simeq 4.9 \,\mathrm{km}$
- Rossi: $h_M \simeq 8.6 \,\mathrm{km}$
- MC: $h_M \simeq 9.5 \,\mathrm{km}$

where MC is the value for a full Monte Carlo EAS simulation.

(b) The total number of Cherenkov photons generated by the shower is

$$N_{\gamma}^{total} = \left(\frac{E}{E_c}\right) N_{\gamma}^{Ch} \simeq 2.27 \times 10^7 \,\text{photons} \tag{10.52}$$

where N_{γ}^{Ch} is the number of Cherenkov photons per radiation length and (E/E_c) the total track length in units of radiation length.

(c) In this problem we will use the Rossi approximation B model to evaluate the shower main characteristics for a 1 TeV photon induced shower. It shall be assumed that all the photons are coming from the center of gravity of the shower, t_{med} . This quantity can be computed using

$$t_{med} = t_{max} + 1.7 = \ln\left(\frac{E}{E_c}\right) - 0.5 + 1.7 \simeq 10.5$$
 (10.53)

or, in traversed matter units,

$$X_{med} = t_{med} X_0 \simeq 385.7 \,\mathrm{g} \,\mathrm{cm}^2 \,. \tag{10.54}$$

Using the *isothermal* approximation as atmosphere model one obtains for the altitude

$$h_{med} = -7\ln\left(\frac{X_{med}}{X}\right) \simeq 6.88\,\mathrm{km}\,.\tag{10.55}$$

Using this altitude as emission point and knowing that the light pool can be derived from an opening angle of $\simeq 1.3^{\circ}$, one gets for r_p

$$r_p = (h_{med} - h_d) \tan \theta \simeq 110.6 \,\mathrm{m.}$$
 (10.56)

Therefore the number of photons at ground is

$$n_{\gamma} = \frac{N_{\gamma}|_{ground}}{A} = \frac{N_{\gamma} e^{-(h_{med} - h_d)/3}}{\pi r_p^2} = 116 \,\text{photons m}^{-2} \,. \tag{10.57}$$

where *A* is the area of the light pool at ground and $N_{\gamma}|_{ground}$ is the number of Cherenkov photons that reach the ground. N_{γ} is the total number of photons produced by the shower, calculated in the previous problem, while the exponential term represents the attenuation of these photons while travelling through the atmosphere.

IACTs telescopes of VERITAS and HESS have areas, A_{det} of $\simeq 100 \text{ m}^2$. Thus, the number of detected photons is

$$n_{\gamma}^{det} = n_{\gamma} A_{det} \, \varepsilon_{ref} \, \varepsilon_{acq} \simeq 1629 \, \text{photons}$$
 (10.58)

where ε_{ref} and ε_{act} are the reflective and acquisition efficiencies, respectively.

(d) This problem is solved in the same way by taking into account that the primary energy is now of 50 GeV. We summarize the results in the following table.

E	t _{med}	X_{med}	h _{med}	r_p	N_{γ}	n_{γ}	N_{γ}^{det}
[GeV]		(g/cm^2)	[km]	[m]		(ph/m^2)	-
50	7.5	276.0	9.22	163.7	1.1×10^{6}	1	17
1000	10.5	385.7	6.88	110.6	2.2×10^{7}	116	1629

For showers induced by photons with E = 50 GeV the number of detected Cherenkov photons is extremely low (of the order of the background fluctuations). Therefore these kind of IACT are not suited to measure gamma-ray induced showers below 50 GeV.

9. Flux of cosmic rays. Translate Eq. 10.1 in the textbook into TeV.

10. *Flux of photons from Crab.* Consider Eq. 10.1 in the text and let us assume that the flux of cosmic rays between 0.05 TeV and 2 PeV follows this expression.

The flux from the most luminous steady (or almost steady) source of gamma rays, the Crab Nebula, follows, according to the measurements from MAGIC, a law

$$N_{\gamma}(E) \simeq 3.23 \times 10^{-11} \left(\frac{E}{\text{TeV}}\right)^{-2.47 - 0.24 \left(\frac{E}{\text{TeV}}\right)} \text{TeV}^{-1} \text{s}^{-1} \text{m}^{-2} \,.$$
(10.59)

Compute the number of photons from Crab hitting every second a surface of $10\,000 \text{ m}^2$ above a threshold of 50 GeV, 100 GeV, 200 GeV, 1 TeV, up to 500 TeV. Compare this number to the background from the flux of cosmic rays in a cone of 1 degree of radius.

- 11. If the average magnetic field in the Milky Way is 1 μ G, what is the minimum energy of a proton coming from Crab Nebula (at a distance of 2 kpc from the Earth) we can detect as "pointing" to the source?
- 12. $\gamma\gamma \rightarrow e^+e^-$. Compute the energy threshold for the process as a function of the energy of the target photon, and compare it to the energy for which the absorption of extragalactic gamma-rays is maximal.
- 13. *Mixing photons with paraphotons.* The existence of a neutral particle of tiny mass μ , the paraphoton, coupled to the photon, has been suggested to explain possible anomalies in the CMB spectrum and in photon propagation (the mechanism is similar to the one discussed to the photon-axion mixing, but there are no complications related to spin here). Calling ϕ the mixing angle between the photon and the paraphoton, express the probability of oscillation of a photon to a paraphoton as a function of time (note: the formalism is the same as for neutrino oscillations). Supposing that the paraphoton is sterile, compute a reasonable range of values for ϕ and μ that could explain an enhancement by a factor of 2 for the signal detected at 500 GeV from the AGN 3C279 at $z \simeq 0.54$.
- 14. *Standard model of particle physics cannot provide dark matter.* Name all particles which are described by the SM and write down through which force(s) they can interact. Why can we rule out that a dark matter particle does interact through the electromagnetic force? Why can we rule out that a dark matter particle does interact through the strong force? Now mark all particles which pass the above requirements and could account for dark matter, and comment.

Compare this value to the results m < 0.2 eV, from cosmology. What do you conclude?

^{15.} *Tremaine-Gunn bound*. Assume that neutrinos have a mass, large enough that they are non-relativistic today. This neutrino gas would not be homogeneous, but clustered around galaxies. Assume that they dominate the mass of these galaxies (ignore other matter). We know the mass M(r) within a given radius r in a galaxy from the velocity v(r) of stars rotating around it. The mass could be due to a few species of heavy neutrinos or more species of lighter neutrinos. But the available phase space limits the number of neutrinos with velocities below the escape velocity from the galaxy. This gives a lower limit for the mass of neutrinos. Assume for simplicity that all neutrinos have the same mass. Find a rough estimate for the minimum mass required for neutrinos to dominate the mass of a galaxy. Assume spherical symmetry and that the escape velocity within radius r is the same as at radius r.