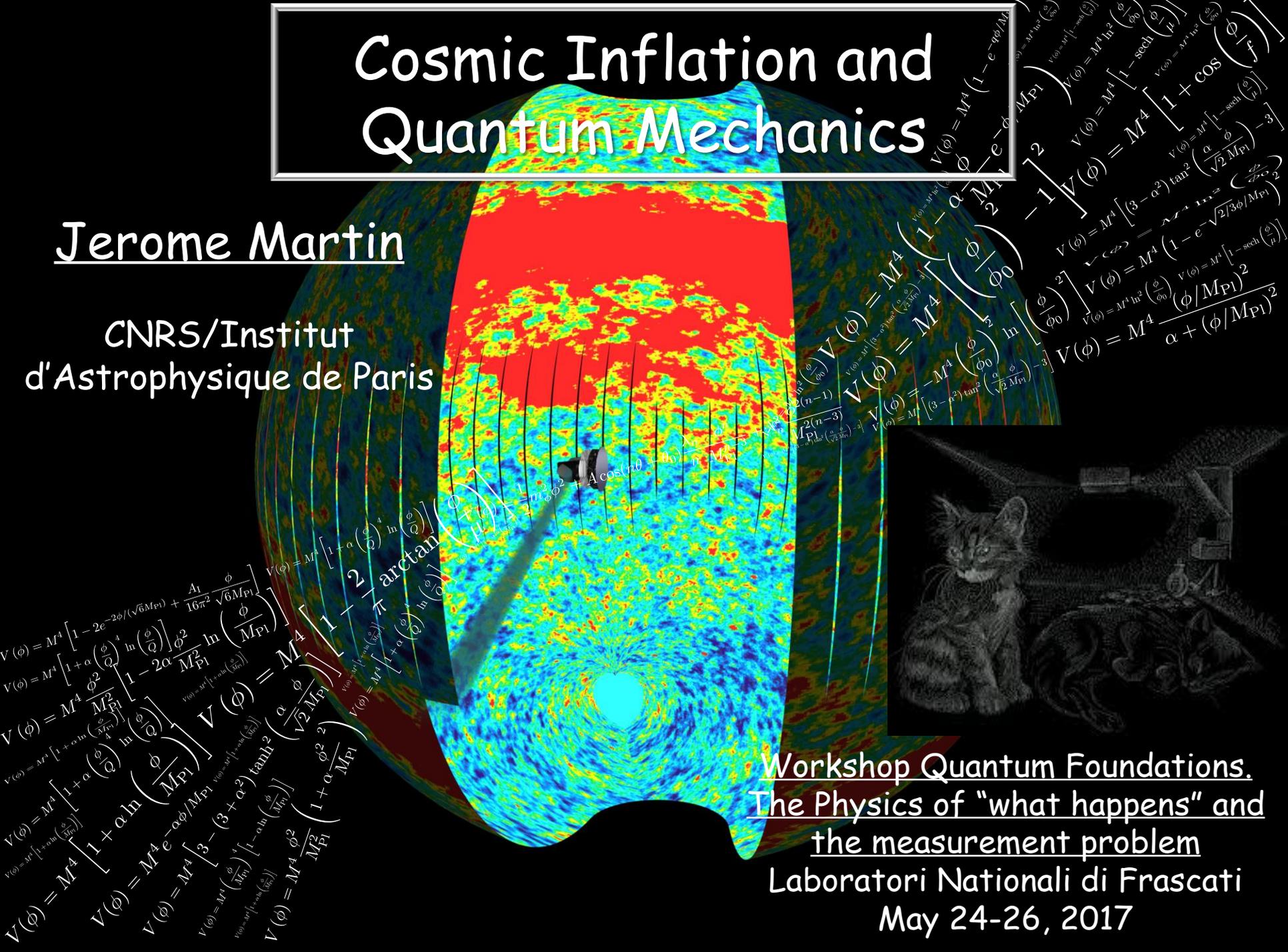
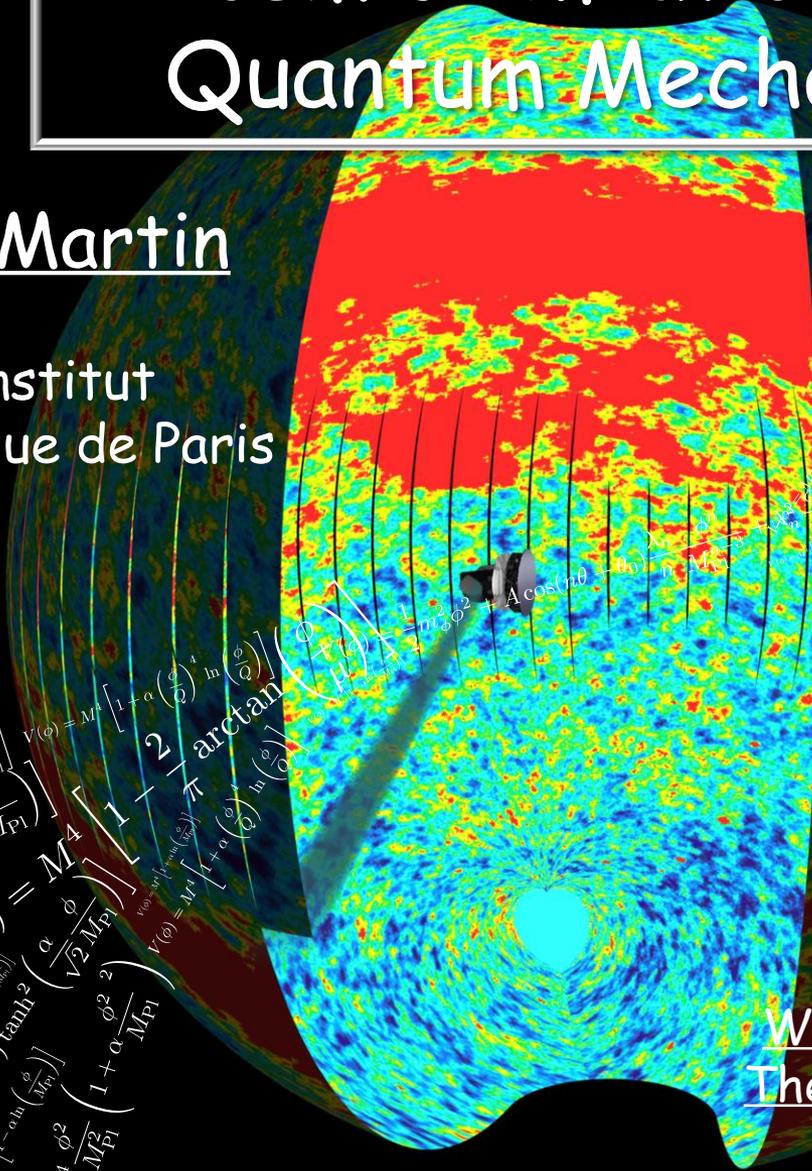


Cosmic Inflation and Quantum Mechanics

Jerome Martin

CNRS/Institut
d'Astrophysique de Paris



Workshop Quantum Foundations.
The Physics of "what happens" and
the measurement problem
Laboratori Nazionali di Frascati
May 24-26, 2017



Outline

- The theory of cosmic inflation in brief: basic principles & observational status

- The quantum origin of galaxies and CMB anisotropies in the theory of inflation

- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?

- Conclusions



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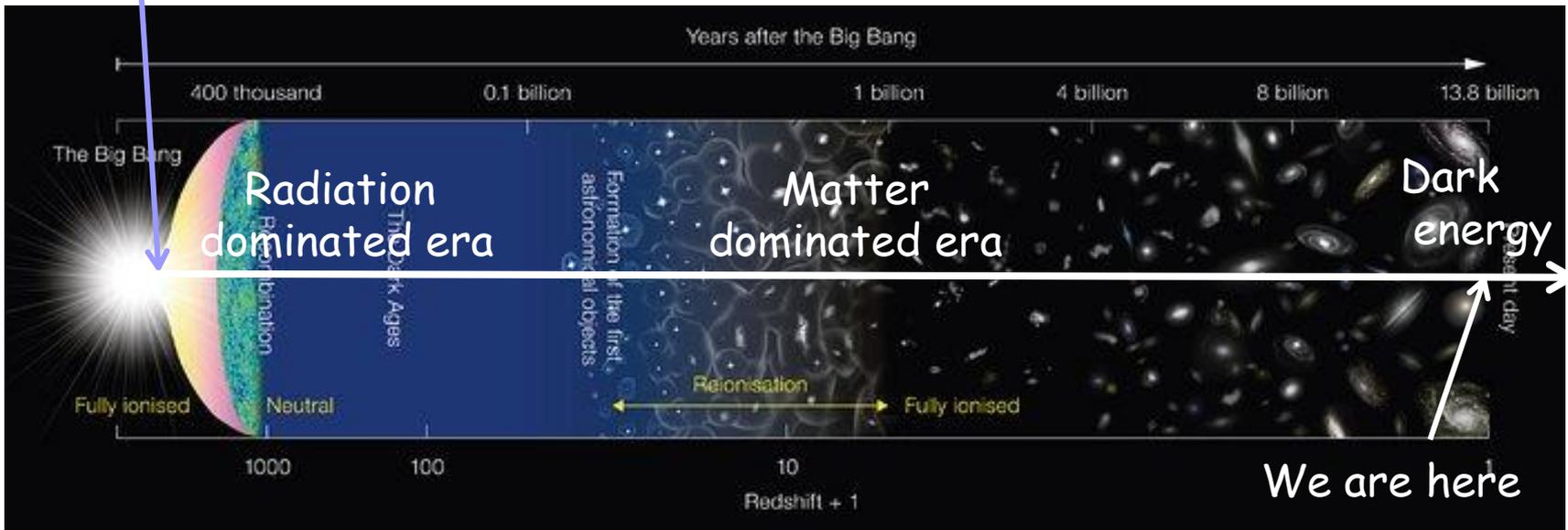
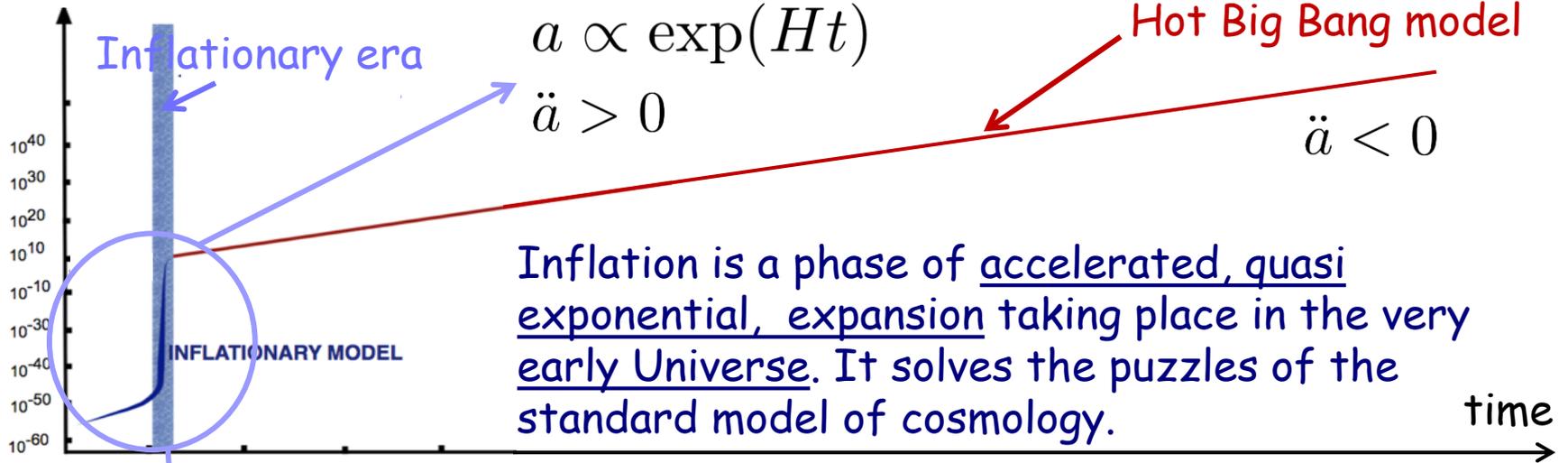
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Inflation in brief

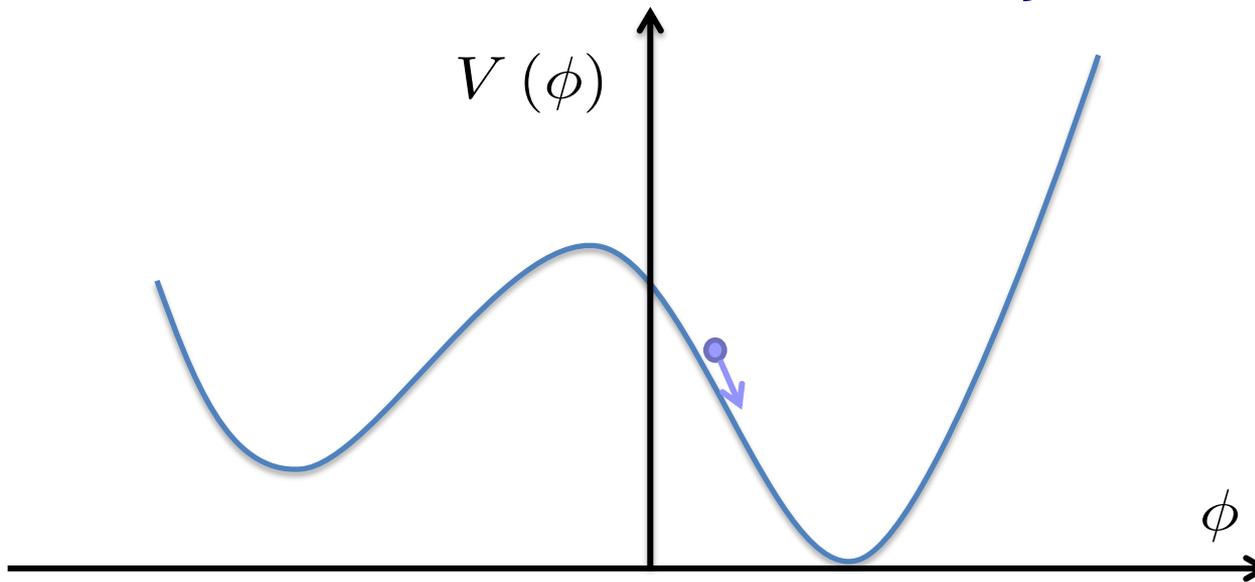
Scale factor $a(t)$





Inflation is (usually) realized with one (or many) scalar field(s)

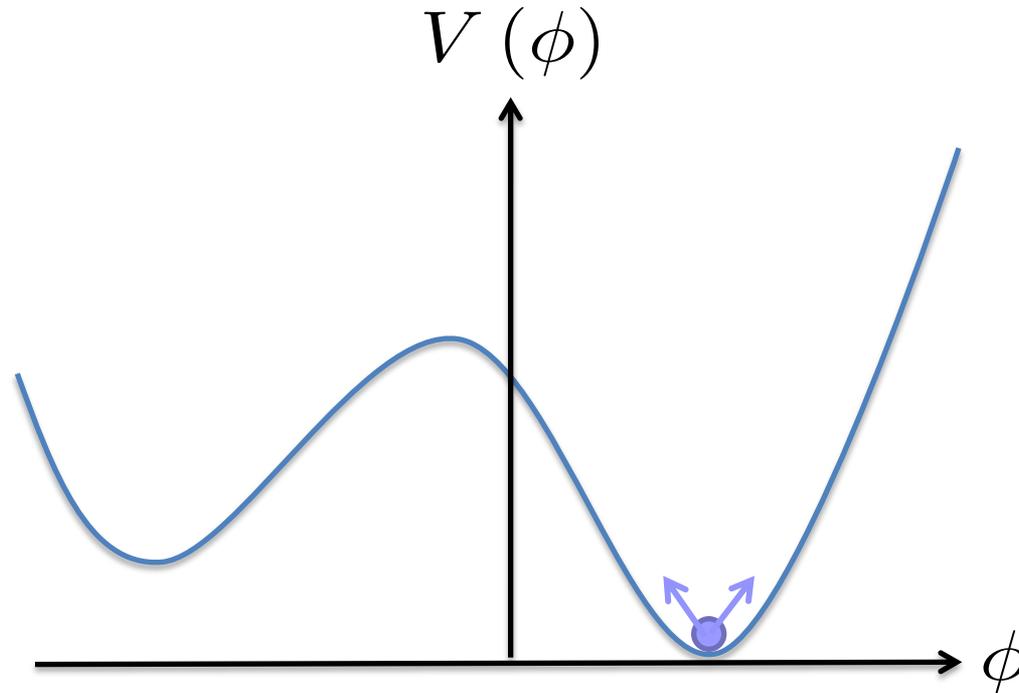
$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\text{Pl}}^2} (\rho + 3p) > 0 \quad \longleftrightarrow \quad \left. \begin{aligned} \rho &= \frac{\dot{\phi}^2}{2} + V(\phi) \\ p &= \frac{\dot{\phi}^2}{2} - V(\phi) \end{aligned} \right\} V(\phi) \gg \frac{\dot{\phi}^2}{2} \Rightarrow p < 0$$



If the scalar field moves slowly (the potential is flat), then pressure is negative which, in the context of GR, means accelerated expansion and, hence, inflation takes place.

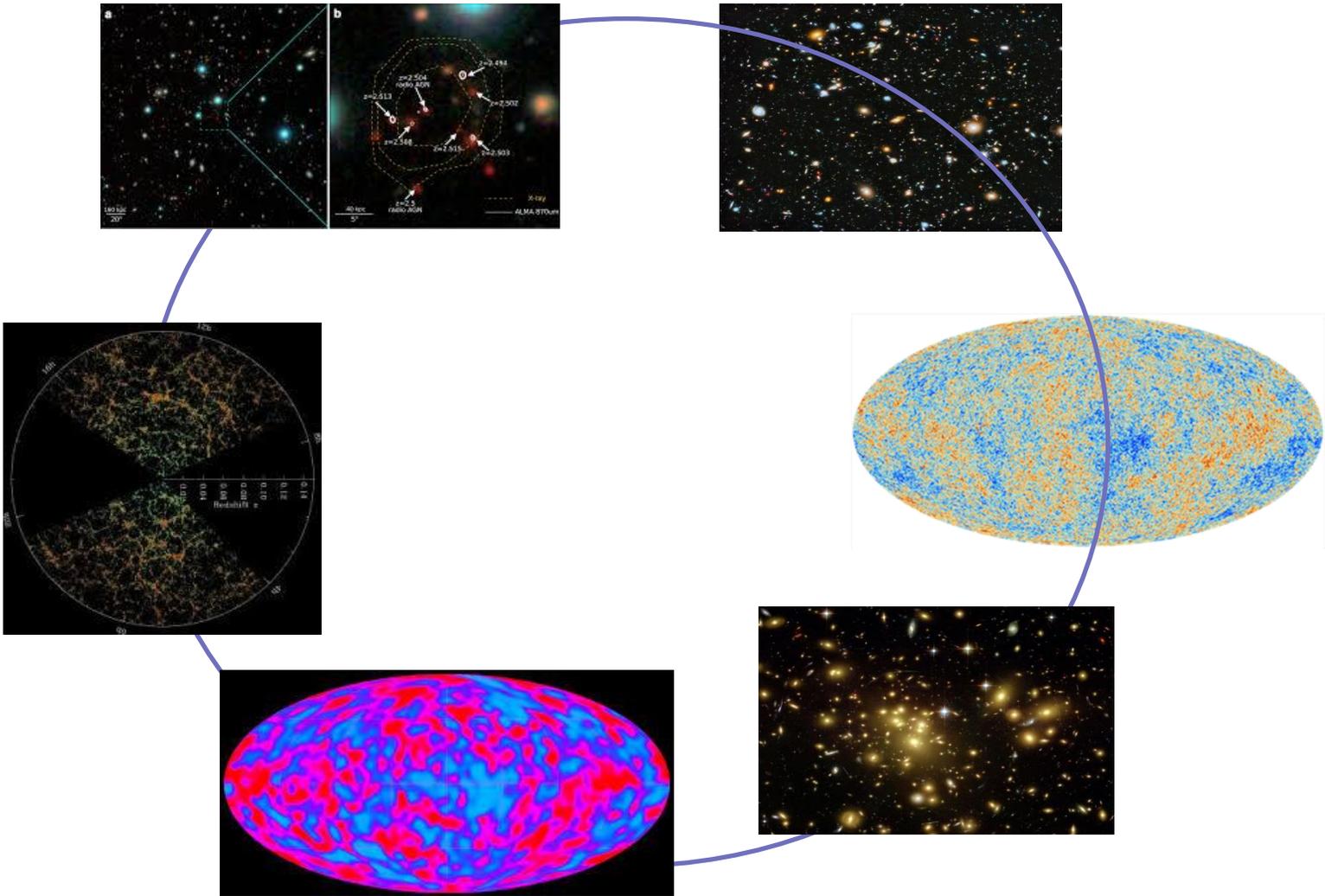


Inflation (usually) stops when the field reaches the bottom of the potential

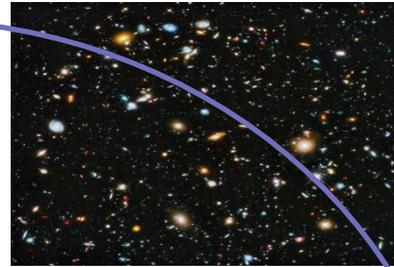
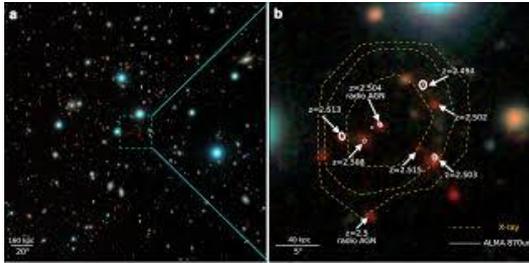


The field oscillates, decays and the decay products thermalize ... Then the radiation dominated era starts ...

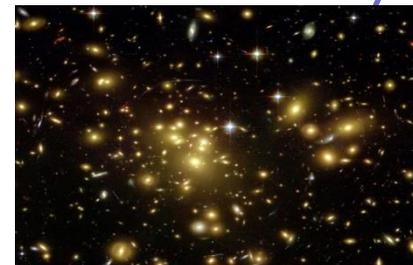
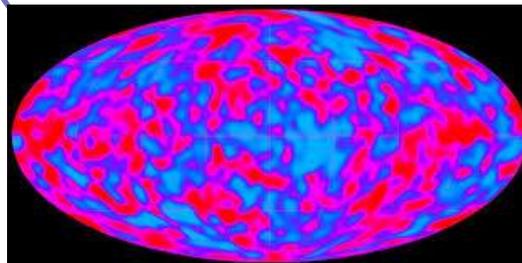
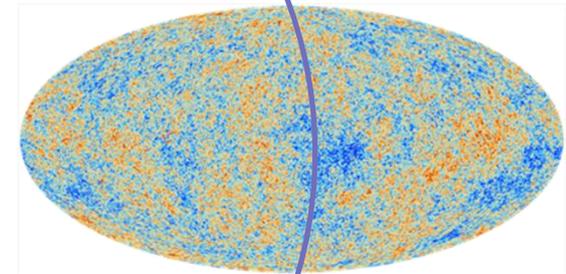
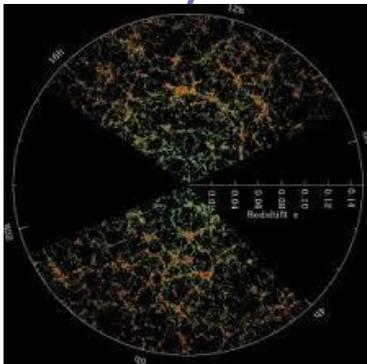
There are inhomogeneities (structures=galaxies, galaxy clusters, CMB anisotropies etc ...) in the Universe ...



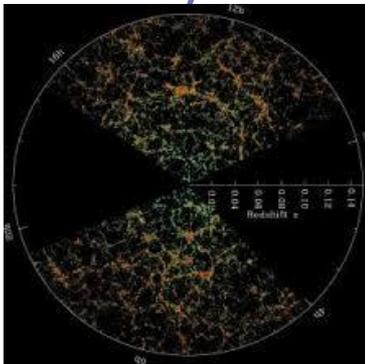
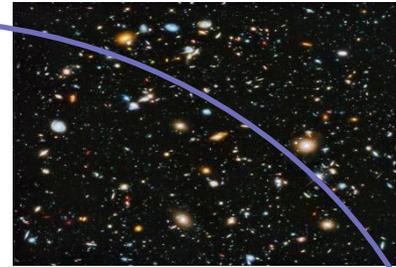
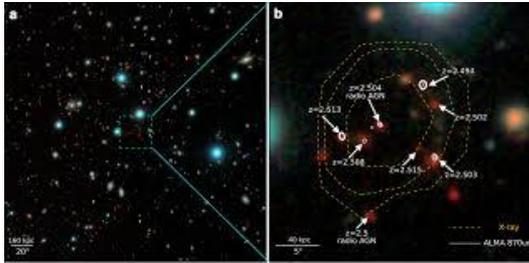
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What is their origin?

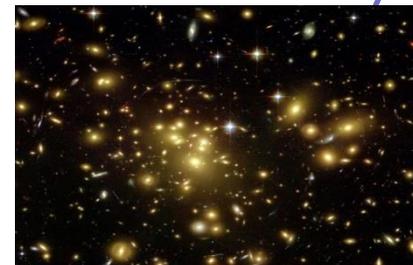
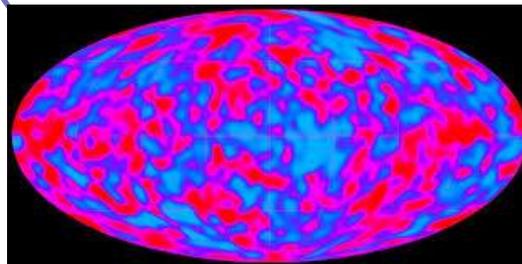
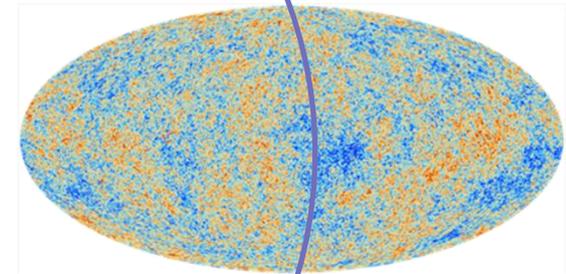


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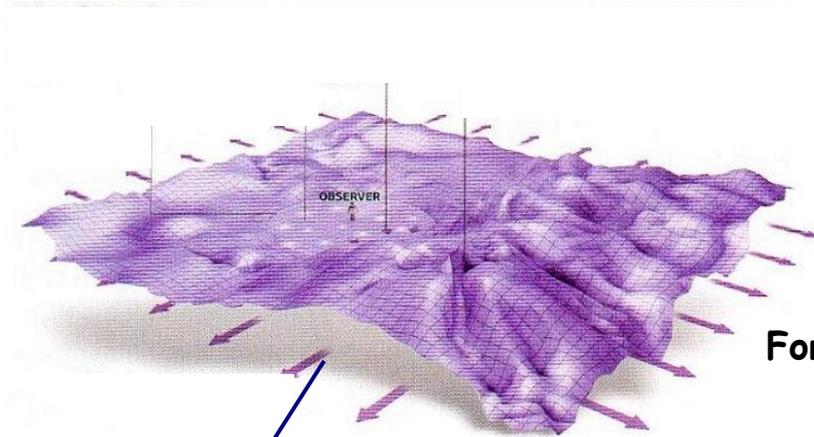


What is their origin?

How do they grow?

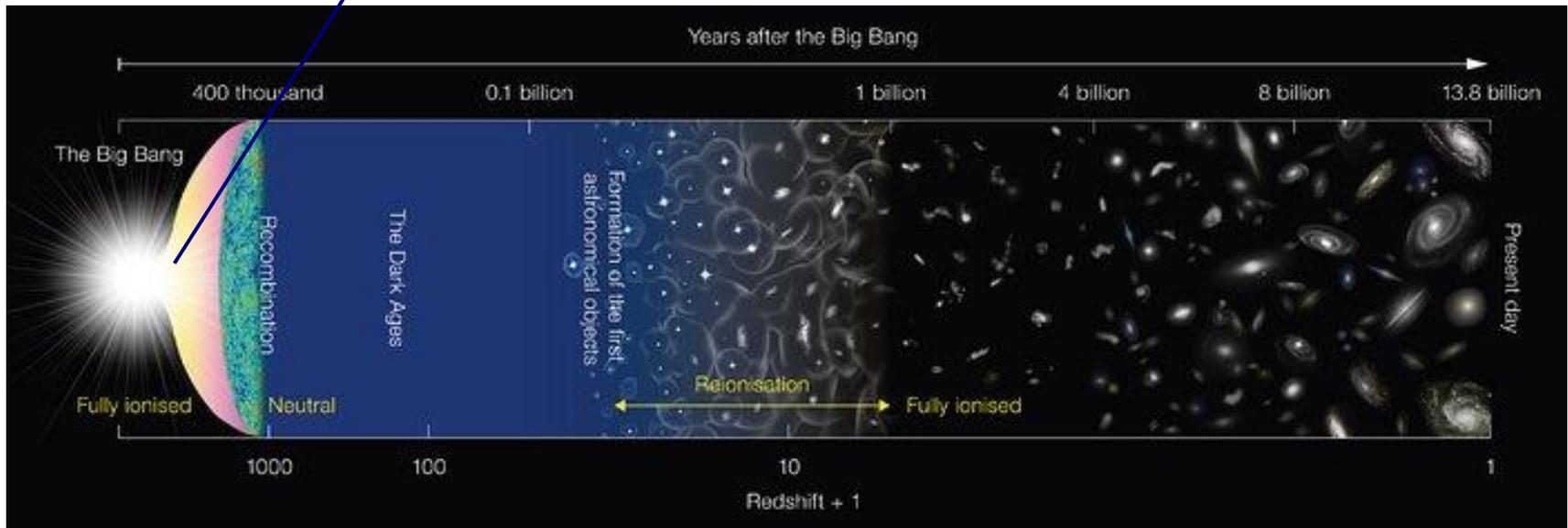


What is their origin?



Quantum fluctuations during inflation

For a review, see J. Martin, Lect. Notes Phys. 669 (2005), 199, hep-th/040611





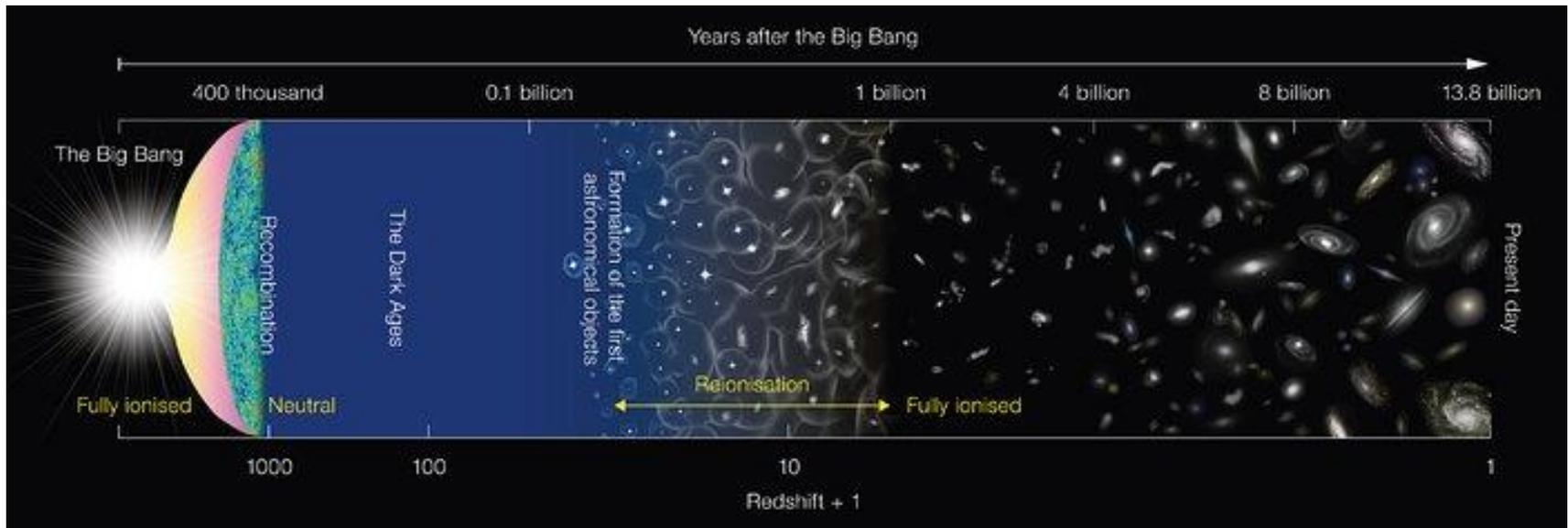
How do they grow?

$$\frac{\delta\rho}{\rho} \simeq 10^{-5}$$

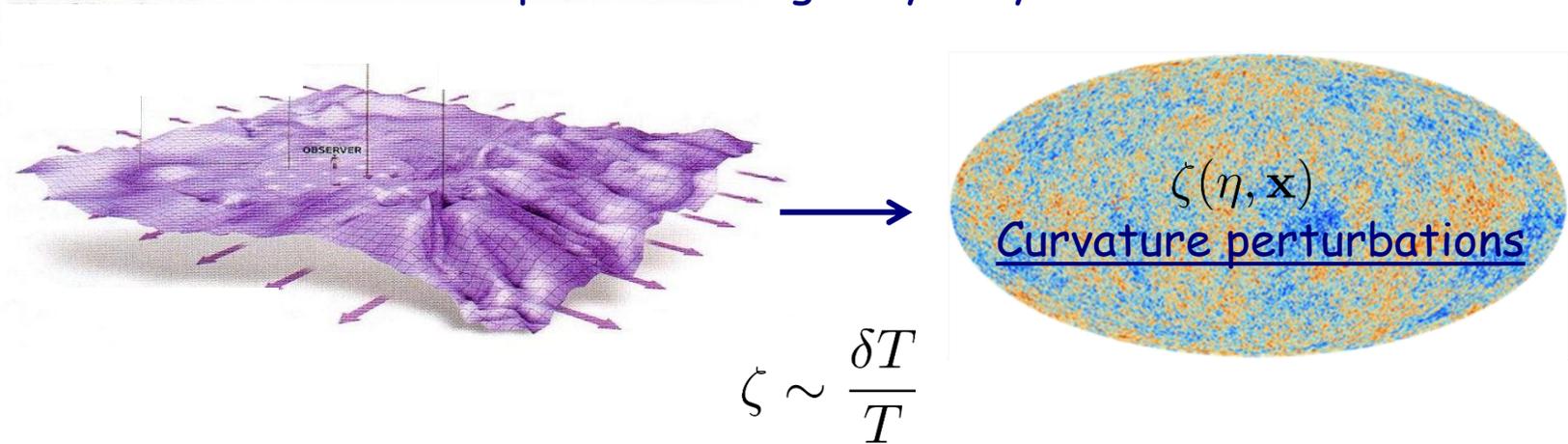
Gravitational instability



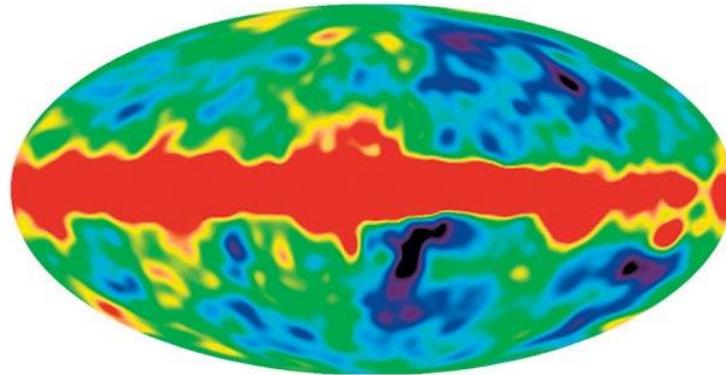
$$\frac{\delta\rho}{\rho} \simeq 1$$



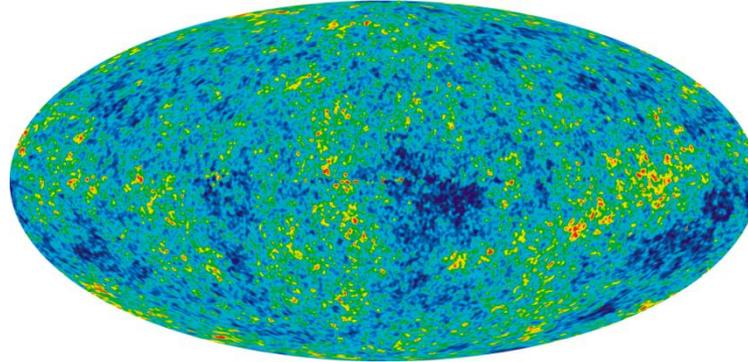
Inflation is phenomenologically very successful



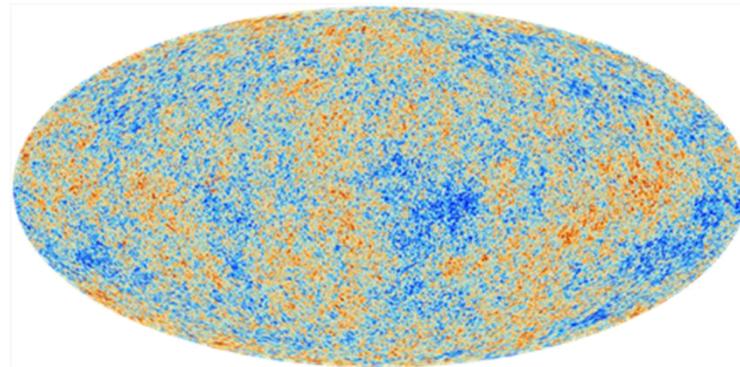
- The quantum fluctuations amplified during inflation are responsible for the formation of galaxies and for the CMB anisotropies
- Therefore, by accurately measuring the CMB anisotropies, one can not only check that the data are consistent with the predictions of inflation, but we can also use the data to learn about inflation



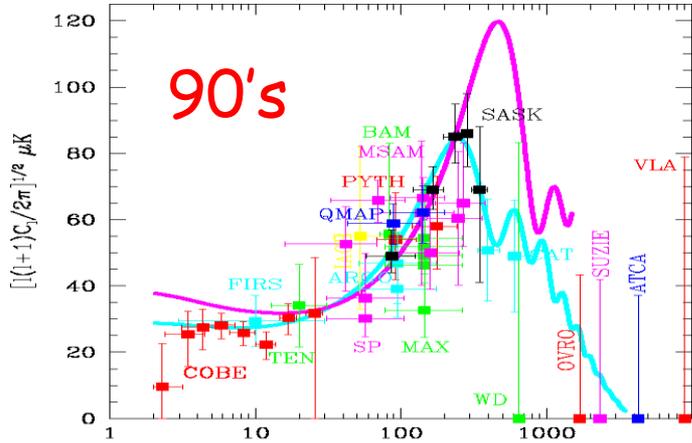
COBE (1992)



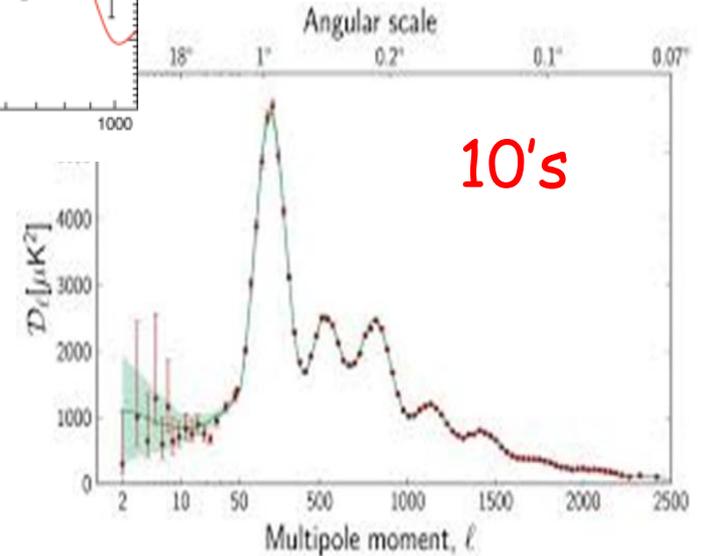
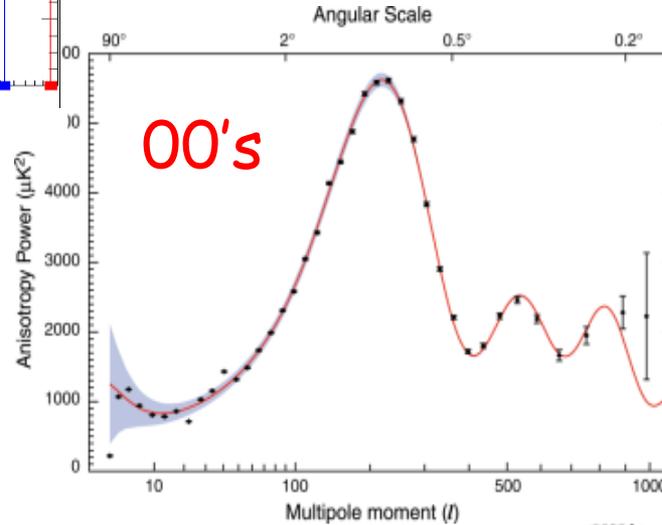
WMAP (2003)

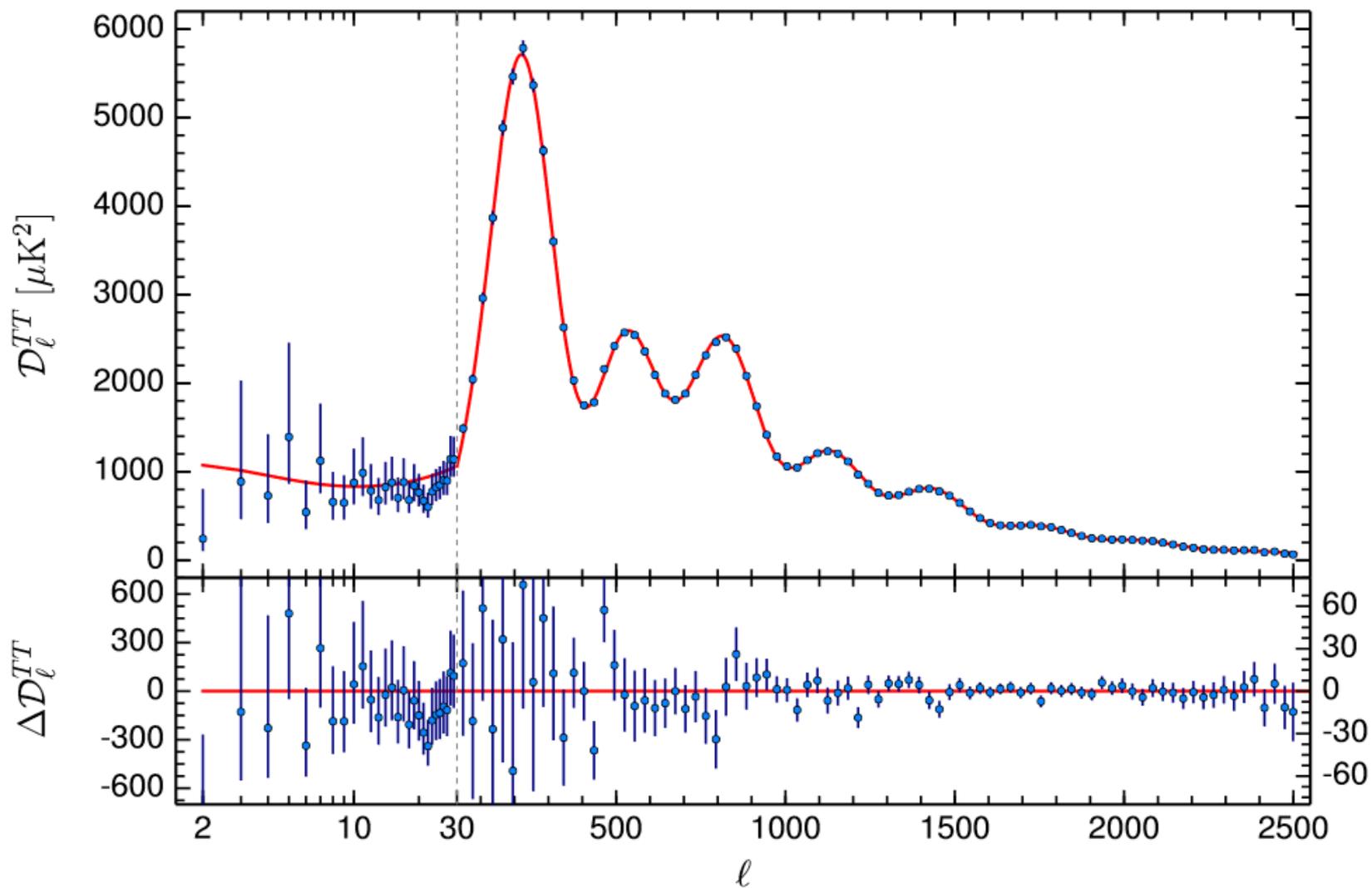


Planck (2013 & 2015)



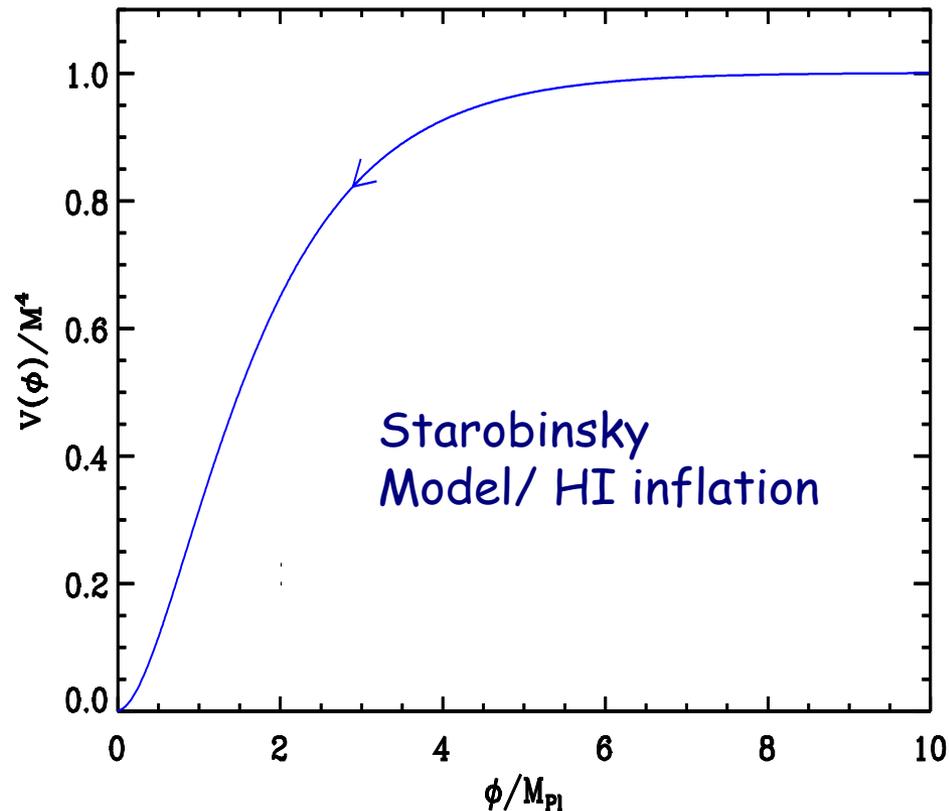
From COBE to Planck ...





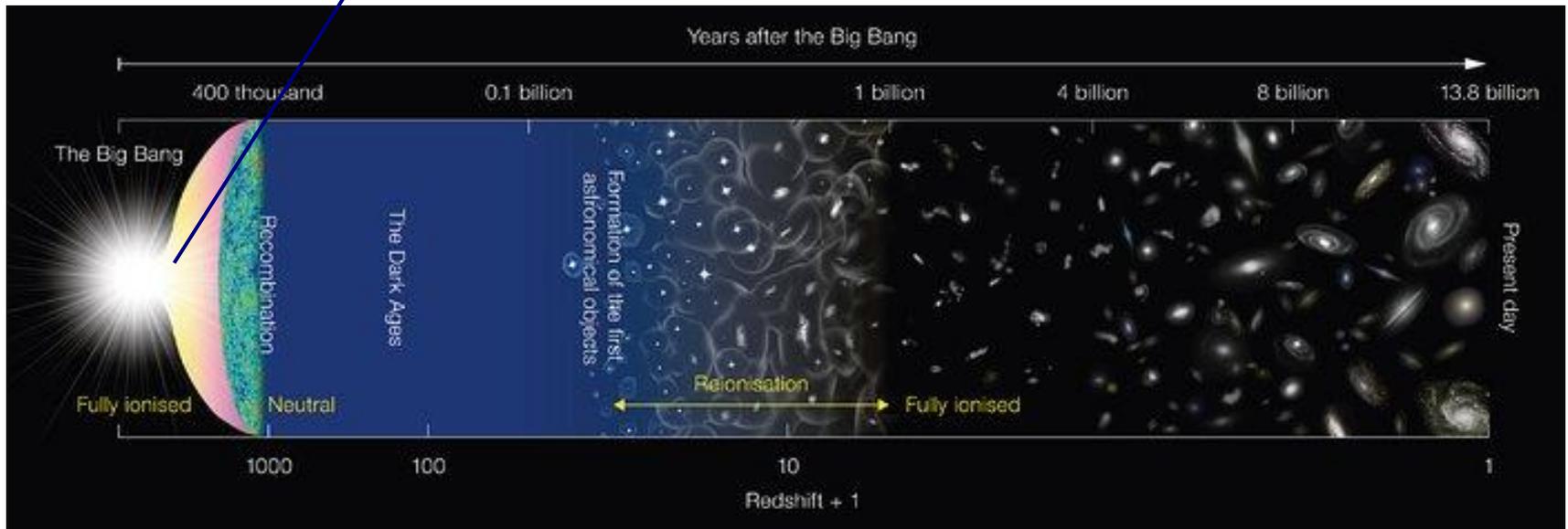
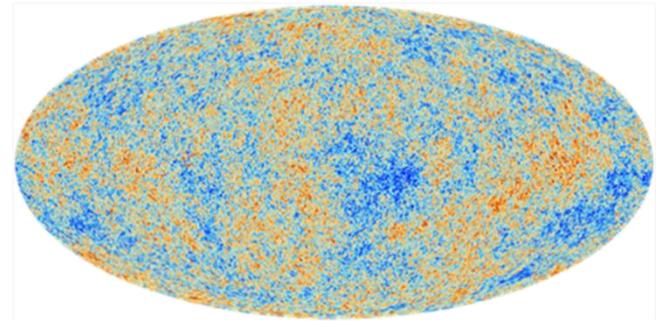
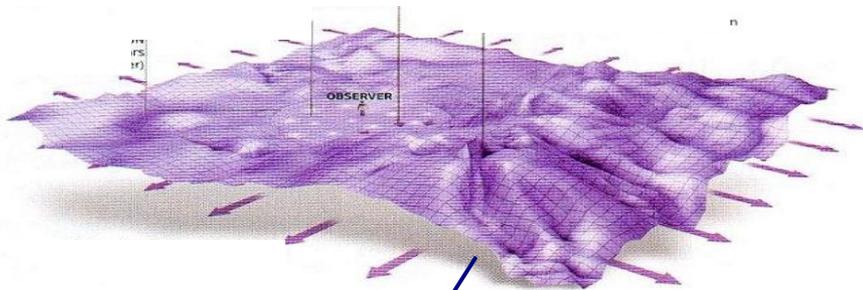
Plateau inflationary models are the winners!

J. Martin, C. Ringeval R. Trotta & V. Vennin, JCAP1403 (2014), 039, arXiv:1312.3529



$$V(\phi) = M^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}} \right)^2$$

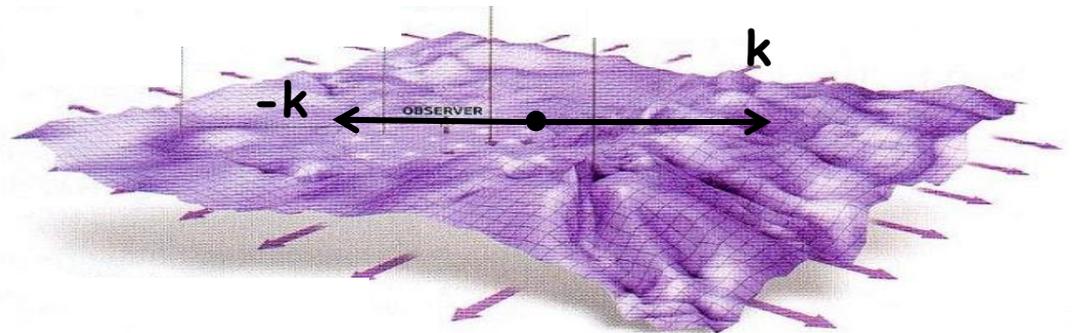
Quantum origin of the fluctuations

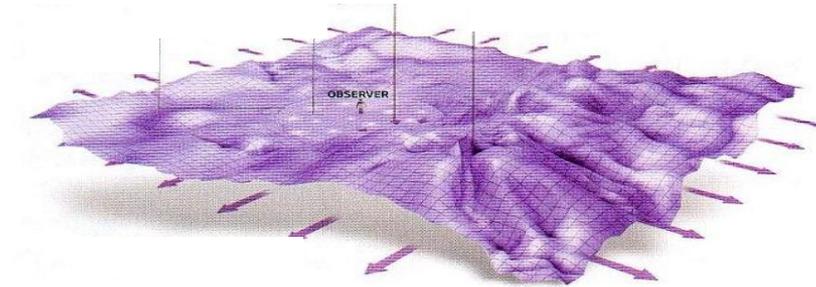


In the Heisenberg picture, this corresponds to creation of particle out of the vacuum (with opposite momenta), thanks to the dynamical background

$$\hat{H} = \int d^3\mathbf{k} \left[\frac{k}{2} \left(\hat{c}_{\mathbf{k}}\hat{c}_{\mathbf{k}}^\dagger + \hat{c}_{-\mathbf{k}}\hat{c}_{-\mathbf{k}}^\dagger \right) - \frac{i}{2} \frac{(a\sqrt{\epsilon_1})'}{a\sqrt{\epsilon_1}} \left(\hat{c}_{\mathbf{k}}\hat{c}_{-\mathbf{k}} - c_{-\mathbf{k}}^\dagger c_{\mathbf{k}}^\dagger \right) \right]$$

The pump source vanishes if space-time is not dynamical, $a'=0$





The system starts in the vacuum and evolves into a two-mode squeezed state

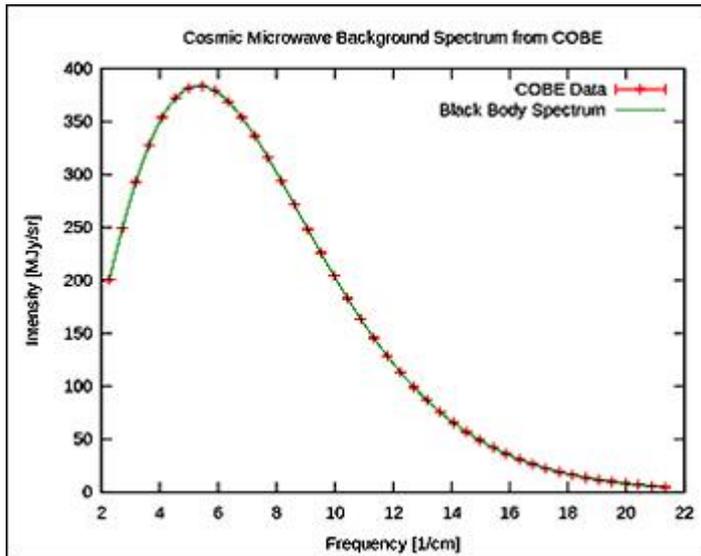
$$|\Psi_{\mathbf{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{+\infty} e^{2in\varphi_k} (-1)^n \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

- r and φ are the squeezing parameters
- It is an entangled state
- There should be quantum correlation in the sky
- The sky can be viewed as a bipartite system $\mathcal{E} = \mathcal{E}_{\mathbf{k}} \otimes \mathcal{E}_{-\mathbf{k}}$

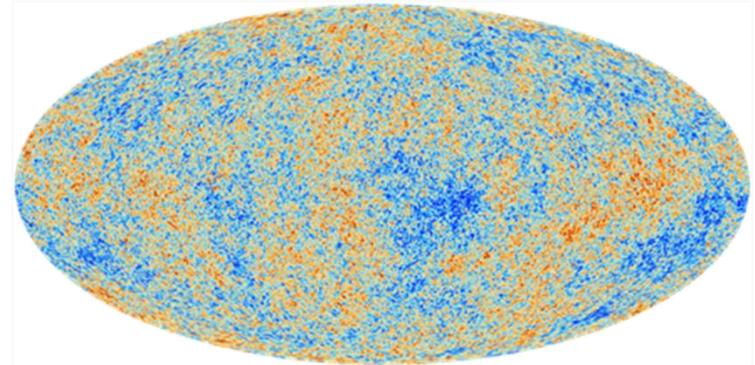
The cosmological two-mode squeezed state is (very!) strongly squeezed

CMB is the most accurate
black body ever produced in
Nature

CMB anisotropy is the strongest
squeezed state ever produced in
Nature



$$r_k = \mathcal{O}(10^2)$$





Cosmic inflation is the only situation in Physics where

- We need GR and QM to derive the predictions of the theory
- And, at the same time, we have high accuracy data to test this theory

Therefore, for somebody interested in the foundations of QM, this is a perfect playground!



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A non-discordant state has necessarily the following form

$$\hat{\rho}_{\text{cl}} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

- There a choice of p_{nm} 's that leads to the same correlations functions

$$\hat{\rho}_{\text{cl}} = (1 - e^{-\beta_k}) \sum_{n=0}^{+\infty} e^{-\beta_k n} |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle \langle n_{\mathbf{k}}, n_{-\mathbf{k}}|$$

↳ $\langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle_{\text{cl}} = \langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle$ One reproduces exactly the power spectrum of curvature perturbations

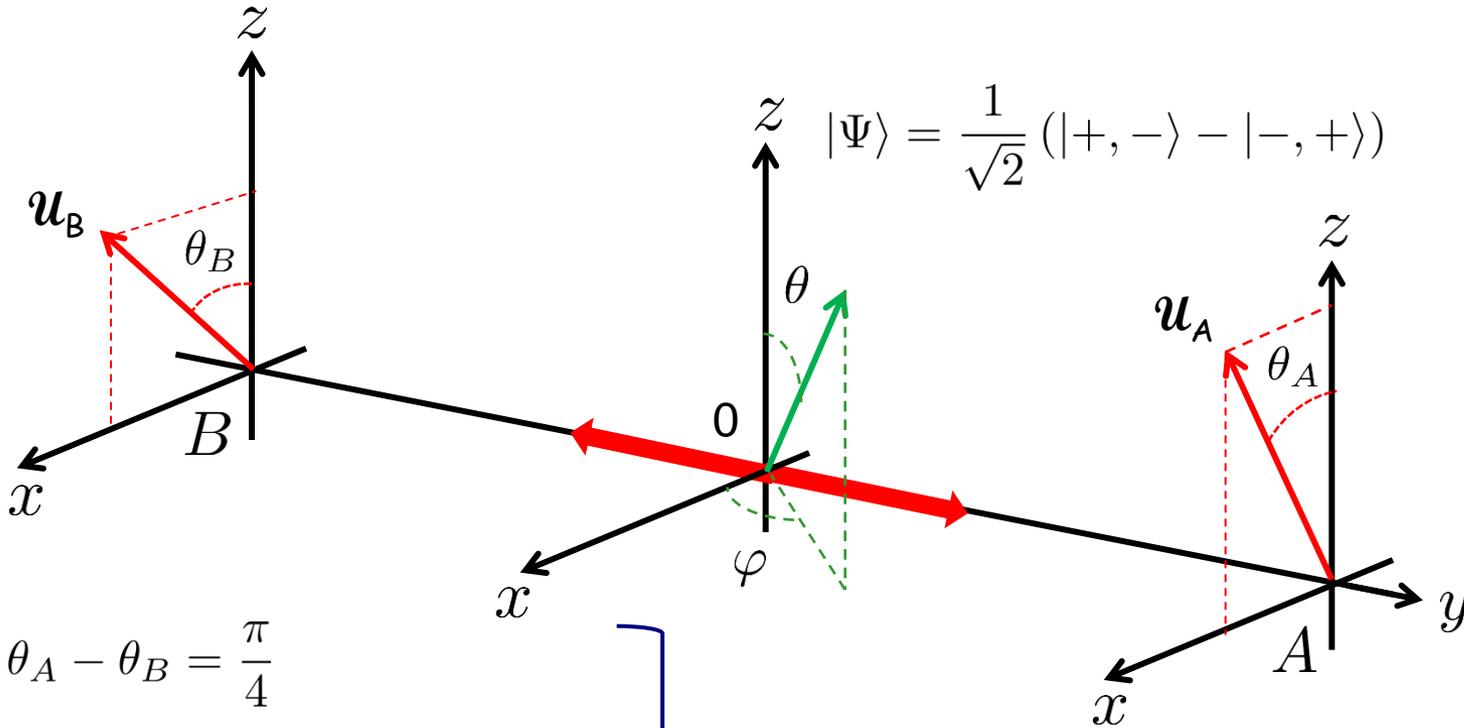
- The other correlation functions are not correctly predicted ... but are unobservable

$$\left. \begin{aligned} \langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle_{\text{cl}} &= e^{-4N_*} \\ \langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle &= e^{-2N_*} \end{aligned} \right\} \frac{\langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle}{\langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle_{\text{cl}}} = e^{2N_*} \gg 1$$



Usual Bell setup (CHSH version)

$$\hat{B}_{\text{CHSH}} = \mathbf{u}_A \cdot \hat{\mathbf{S}}_A \otimes \mathbf{u}_B \cdot \hat{\mathbf{S}}_B + \mathbf{u}_A \cdot \hat{\mathbf{S}}_A \otimes \mathbf{u}'_B \cdot \hat{\mathbf{S}}_B + \mathbf{u}'_A \cdot \hat{\mathbf{S}}_A \otimes \mathbf{u}_B \cdot \hat{\mathbf{S}}_B - \mathbf{u}'_A \cdot \hat{\mathbf{S}}_A \otimes \mathbf{u}'_B \cdot \hat{\mathbf{S}}_B$$



$$\theta_A - \theta_B = \frac{\pi}{4}$$

$$\theta_A - \theta'_B = \theta'_A - \theta_B = -\frac{\pi}{4}$$

$$\theta'_A - \theta'_B = -3\frac{\pi}{4}$$

$$\langle \hat{B}_{\text{CHSH}} \rangle = -2\sqrt{2}$$



How to construct a cosmic Bell experiment?

1- We need a bipartite system, ie Alice and Bob



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How to construct a cosmic Bell experiment?

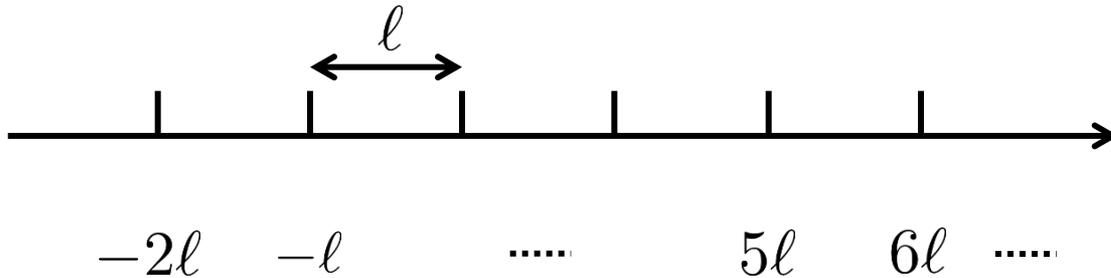
- 1- We need a bipartite system, ie Alice and Bob: in cosmology, we have the modes k and $-k$
- 2- We need a spin variable with eigenvalues ± 1



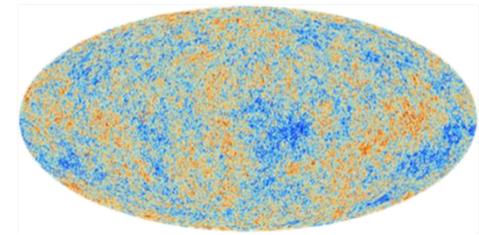
How to construct a cosmic Bell experiment?

1- We need a bipartite system, ie Alice and Bob: in cosmology, we have the modes k and $-k$

2- We need a spin variable with eigenvalues ± 1 . In cosmology, we deal with a continuous variable system (the continuous variable is the amplitude of the fluctuations). But we can introduce pseudo spin operators:



Continuous variable Q



$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} dQ |Q\rangle \langle Q|$$

$$\hat{S}_z^2(\ell) = 1 \quad \text{It is a spin!}$$



How to construct a cosmic Bell experiment?

1- We need a bipartite system, ie Alice and Bob: in cosmology, we have the modes k and $-k$

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3- We need a quantum state corresponding to $|\Psi\rangle = \frac{1}{\sqrt{2}} (|+, -\rangle - |-, +\rangle)$ in the standard case



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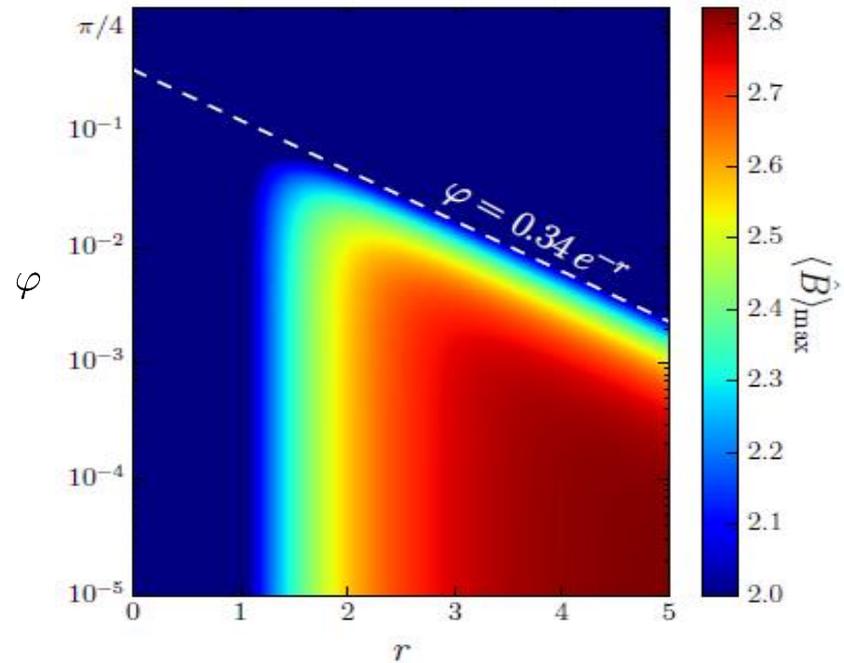
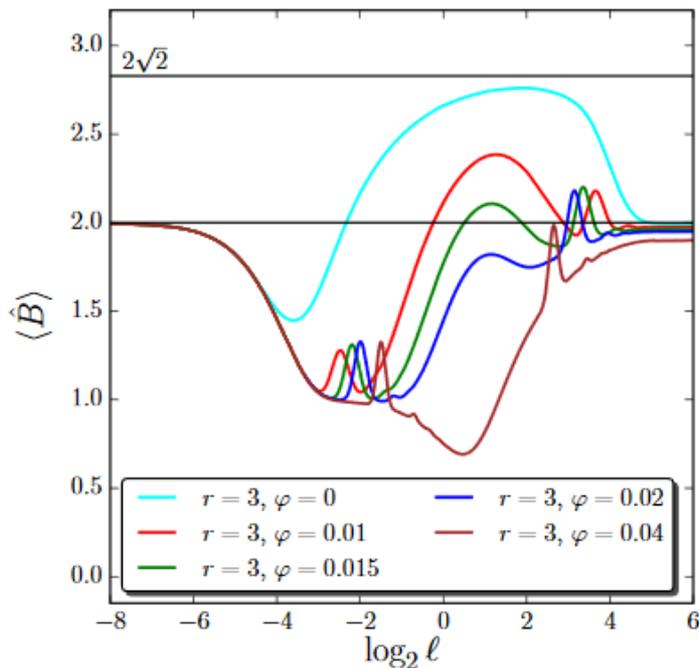
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Bell's operator for two-mode squeezed state (1 is k and 2 is $-k$)

$$\hat{B}(\ell) = \left[\mathbf{n} \cdot \hat{\mathbf{S}}^{(1)}(\ell) \right] \otimes \left[\mathbf{m} \cdot \hat{\mathbf{S}}^{(2)}(\ell) \right] + \left[\mathbf{n} \cdot \hat{\mathbf{S}}^{(1)}(\ell) \right] \otimes \left[\mathbf{m}' \cdot \hat{\mathbf{S}}^{(2)}(\ell) \right] \\ + \left[\mathbf{n}' \cdot \hat{\mathbf{S}}^{(1)}(\ell) \right] \otimes \left[\mathbf{m} \cdot \hat{\mathbf{S}}^{(2)}(\ell) \right] - \left[\mathbf{n}' \cdot \hat{\mathbf{S}}^{(1)}(\ell) \right] \otimes \left[\mathbf{m}' \cdot \hat{\mathbf{S}}^{(2)}(\ell) \right]$$



J. Martin & V. Vennin, PRA93 (2016), 062117,
arXiv:1605.02944



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- 1- We need a bipartite system, ie Alice and Bob: in cosmology, we have the modes k and $-k$
- 2- We need a spin variable with eigenvalues ± 1 . In cosmology, we deal with A continuous variable system (the continuous variable is the amplitude of the fluctuations). But we can introduce pseudo spin operators:
- 3- We need a quantum state corresponding to $|\Psi\rangle = \frac{1}{\sqrt{2}} (|+, -\rangle - |-, +\rangle)$ in the standard case. In cosmology, one takes the two-mode squeezed state
- 4- In cosmology the problem is that, in order to "measure" the pseudo spin, we need to measure the decaying mode ...



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Recap

- ❑ Inflation is a very successful model for the early Universe. It is now part of the standard model of cosmology
- ❑ According to cosmic inflation, galaxies are nothing but vacuum quantum fluctuations stretched over cosmological distances and amplified by gravitational instability
- ❑ According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which is a discordant and entangled state. There should be quantum correlations in the sky.
- ❑ It is the only situation in Physics where an effect based on GR and QM can be experimentally tested with high accuracy.
- ❑ So far, however, quantum effects in the sky are hidden by a peculiar feature of inflation, the momentum of the fluctuations turns out to be tiny and hence unobservable.
- ❑ Take away message: inflation is not only a successful scenario of the early Universe, it is also a very interesting playground for foundational issues of quantum mechanics