UNIVERSITY OF TRIESTE

DEPARTMENT OF PHYSICS

An introduction to the spontaneous wave function collapse models

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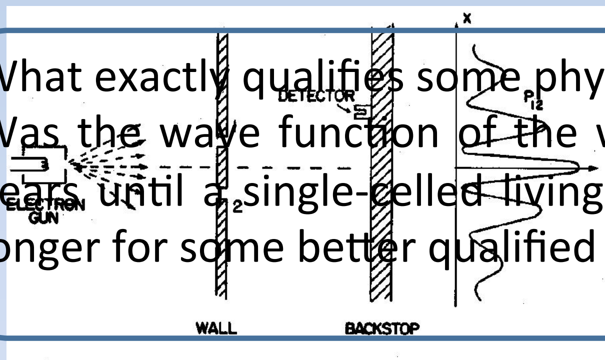
FRASCATI, 24/05/2017

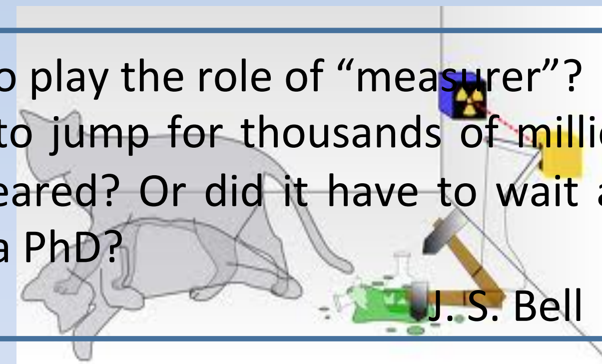
THE MEASUREMENT PROBLEM

The Schrödinger equation:

- Linear
- Deterministic

$$i\hbar\frac{d}{dt}\left|\Psi\left(t\right)\right\rangle = H\left|\Psi\left(t\right)\right\rangle$$





What exactly qualifies some physical systems to play the role of "measurer"? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some better qualified system...with a PhD?

J. S. Bell

OK

KO

The wave packet reduction postulate:

- Non Linear
- Stochastic

$$\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}} \xrightarrow{\text{measurement}} \begin{cases} \xrightarrow{\text{half of total cases}} |a_1\rangle \\ \xrightarrow{\text{half of total cases}} |a_2\rangle \end{cases}$$

There are two different laws for the evolution of the state vectors but it is not clear when to use which one.

COLLAPSE MODELS

IDEA: modify the Schrödinger dynamics with one that describes also the collapse.

The new dynamics must be:

1) *Non linear* ;
2) *Stochastic* ;
3) *Change the dynamics at the level of the ket states* .

$$|\psi\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \longrightarrow \rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

We want this state to evolve into:

$$\{50\% \ |+\rangle \ , \ 50\% \ |-\rangle\} \longrightarrow \rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Is the diagonalization of the density matrix a sufficient condition? NO.

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{array}{l} \nearrow \ \{50\% \ |+\rangle \ , \ 50\% \ |-\rangle\} \\ \searrow \ \left\{50\% \ \frac{|+\rangle + |-\rangle}{\sqrt{2}} \ , \ 50\% \ \frac{|+\rangle - |-\rangle}{\sqrt{2}}\right\} \end{array}$$

THE GRW MODEL

G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D **34**, 470 (1986).

1) Casual Localizations that follow a poissonian statistic with mean rate λ_i .

2) The localization around the point $\mathbf{a}$ is:

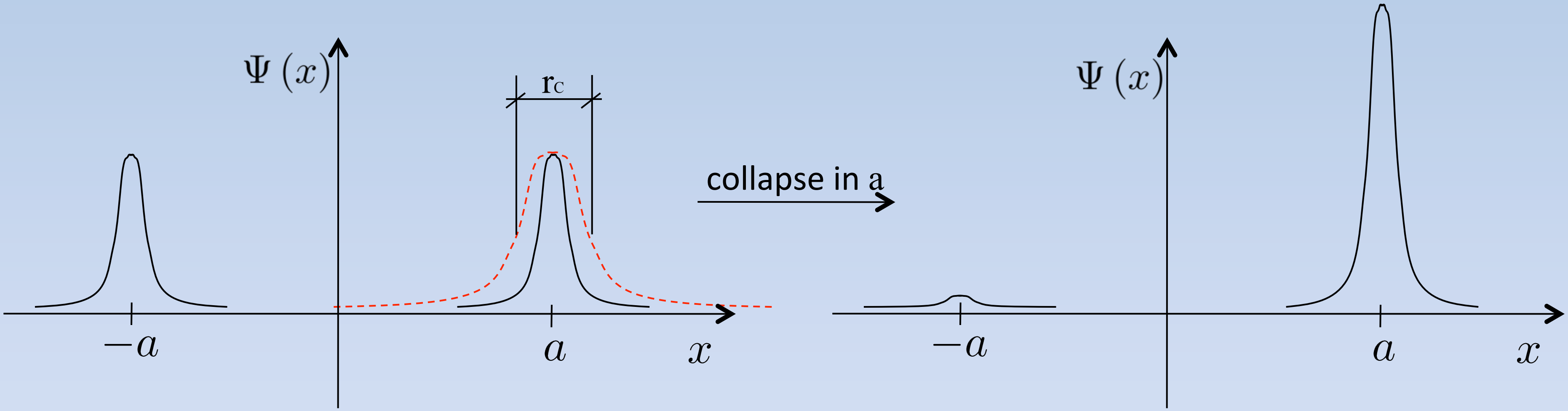
$$|\Psi\rangle \longrightarrow \frac{L_{\mathbf{a}}^{i}|\Psi\rangle}{\left\|L_{\mathbf{a}}^{i}|\Psi\rangle\right\|} \qquad \text{where} \qquad L_{\mathbf{a}}^{i} = \left(\pi r_c^2\right)^{-3/4} e^{-\frac{(\mathbf{q_i}-\mathbf{a})^2}{2r_c^2}}$$

3) The probability of localization around $\mathbf{a}$ is given by: $P^{i}(\mathbf{a}) = \left\|L_{\mathbf{a}}^{i}|\Psi\rangle\right\|^2$

4) Between two localizations the system's state evolves following the Schrödinger equation:

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

LOCALIZATION MECHANISM

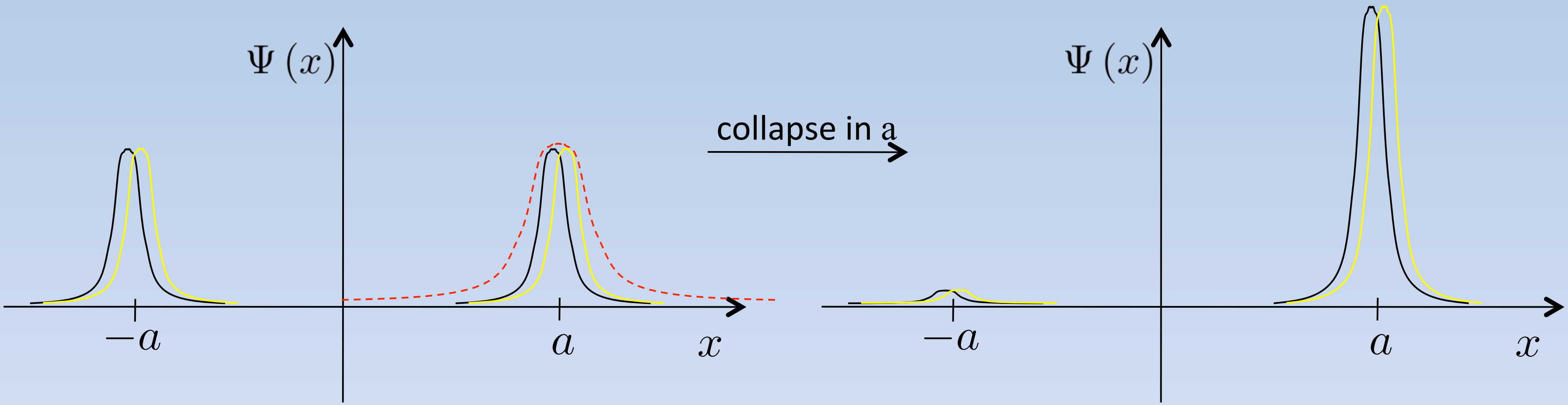


$$\Psi(x) = \frac{\psi_{-a}(x)+\psi_a(x)}{\sqrt{2}} \longrightarrow \Psi(x) = \psi_a(x)$$

$$P(\mathbf{a}) = \| L_{\mathbf{a}} |\Psi\rangle \|^2 \simeq \frac{1}{2} = P(-\mathbf{a}) \qquad P(\mathbf{0}) \simeq 0$$

Localization/correlation length : $r_C = 10^{-7}\,\mathrm{m}$

AMPLIFICATION MECHANISM



$$\Psi(x,y)=\frac{\psi_{-a}(x)\phi_{-a}(y)+\psi_{a}(x)\phi_{a}(y)}{\sqrt{2}} \xrightarrow[\Lambda=\lambda_1+\lambda_2]{} \psi_a(x)\phi_a(y)$$

For a system with N particles: $\Lambda = N\lambda$

$\lambda = 10^{-17}\ \mathrm{s}^{-1}$ ⟶ for few particles low probability of collapse

$\lambda_{macro} = N\lambda \simeq 10^{6}\ \mathrm{s}^{-1}$ ⟶ for macrosystems the collapse is instantaneous

THE CSL MODEL

(CONTINUOUS SPONTANEOUS LOCALIZATIONS MODEL)

G.C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A **42**, 78 (1990).

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}H dt + \frac{\sqrt{\gamma}}{m_0}\int d\mathbf{x}\,\left(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_{\psi_t}\right) dW_t(\mathbf{x}) \right.$$
$$\left. -\frac{\gamma}{2m_0}\int d\mathbf{x}\,d\mathbf{y}\,g(\mathbf{x}-\mathbf{y})\left(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_{\psi_t}\right)\left(N(\mathbf{y}) - \langle N(\mathbf{y})\rangle_{\psi_t}\right) dt\right]|\psi_t\rangle$$

Schrödinger

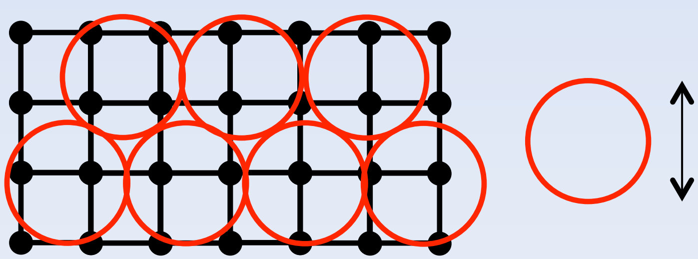
Stochasticity

Non linearity

$$\gamma = \lambda 8\pi^{3/2} r_C^3$$
$$N(\mathbf{x}) = m\psi^\dagger(\mathbf{x})\psi(\mathbf{x})$$

$$\mathbb{E}\left[dW_t(\mathbf{x})dW_t(\mathbf{y})\right] = g(\mathbf{x}-\mathbf{y})dt \qquad g(\mathbf{x}-\mathbf{y}) = \frac{e^{-(\mathbf{x}-\mathbf{y})^2/4r_C^2}}{(4\pi r_C^2)^{3/2}}$$

- First model valid also for identical particles;
- Localization in space;
- Amplification mechanism.



$\Lambda = N_c n^2 \lambda$

$N_c =$ number of red circles.

$n =$ number of particles inside each red circle;

PROBLEM IN GRW AND CSL MODELS

A. Bassi, et al., Rev. Mod. Phys. **85**, 471-527 (2013).

ENERGY INCREASE

$$\langle E_t \rangle = \langle E_0 \rangle + \frac{\lambda \hbar^2}{4 m r_c^2} t$$

Steady increase of energy!

For an electron: 10^{14} years to increase the energy 1 eV

For a monoatomic gas: increase of temperature in one year is: $\triangle T \simeq 10^{-10}\,\mathrm{K}$

WHITE NOISE

Problem: Simple for modelling but not realistic.

Motivation: 1) How predictions depend on the noise?

2) Connection with realistic noise fields of nature (maybe cosmological?)

DIOSI-PENROSE (DP) MODEL

L. Diósi, Phys. Rev. A **40**, 1165–1174 (1989)
R. Penrose, Gen. Relativ. Gravit. **28**, 581–599 (1996)

IDEA: Is wave function collapse induced by gravity?

$$d|\phi_t\rangle = \left[-\frac{i}{\hbar} H dt + \sqrt{\frac{G}{\hbar}} \int d\mathbf{x} \, (f(\mathbf{x}) - \langle f(\mathbf{x}) \rangle) \, dW_t(\mathbf{x}) - \frac{G}{2\hbar} \int d\mathbf{x}_1 \int d\mathbf{x}_2 \, \frac{(f(\mathbf{x}_1) - \langle f(\mathbf{x}_1) \rangle) \, (f(\mathbf{x}_2) - \langle f(\mathbf{x}_2) \rangle)}{x_{12}} dt \right] |\phi_t\rangle$$

$$\mathrm{E}\left[dW_t(\mathbf{x})\right] = 0, \quad \mathrm{E}\left[dW_t(\mathbf{x}_1) dW_t(\mathbf{x}_2)\right] = \frac{dt}{x_{12}}, \qquad x_{12} = |\mathbf{x}_1 - \mathbf{x}_2|\,, \quad f(\mathbf{x}) = \frac{M}{V} \theta \left(R - |\mathbf{q} - \mathbf{x}| \right)$$

- Introduction of finite volume for particles necessary to avoid divergences.

- For $R = 10^{-15}$ m $\longrightarrow$ unacceptable heating: for a gas of proton $\Delta T = 10^{-4}$ K/s

- For $R = 10^{-7}$ m $\longrightarrow$ $\Delta T = 10^{-28}$ K/s ok but arbitrary value.

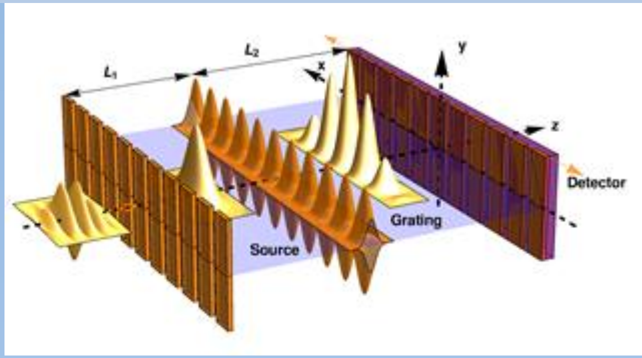
- With dissipation one can take $R = 10^{-15}$ m but the model is valid only for systems with mass larger than $m = 10^{11}$ amu

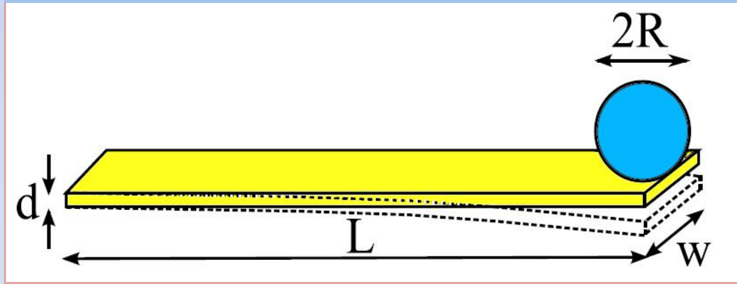
RELEVANT EXPERIMENTS

Matter-wave interferometry

S. Eibenberger, S. Gerlich, M. Arndt, M. Mayor, and J. Tuxen. Phys. Chem. **15,** 14696 (2013).

M. Toros and A. Bassi. Link to Arxiv: http://arxiv.org/abs/1601.03672, http://arxiv.org/abs/1601.02931.

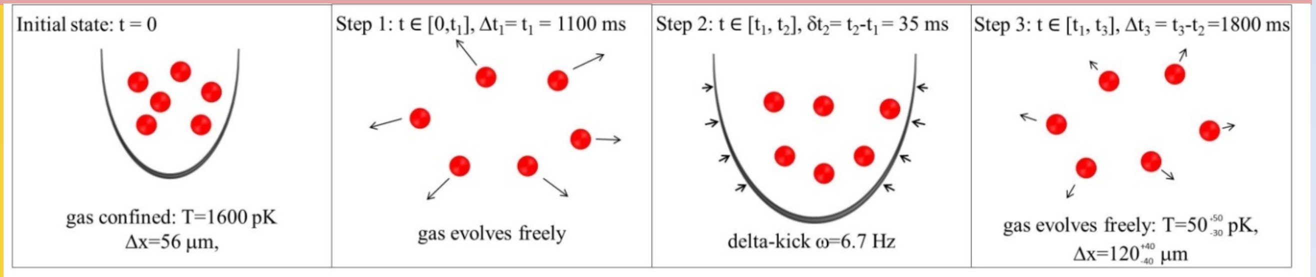




Cantilever

A. Vinante, M. Bahrami, A. Bassi, O. Usenko, G. Wijts, T.H. Oosterkamp, Phys. Rev. Lett. **116**, 090402 (2016).

Cold atoms



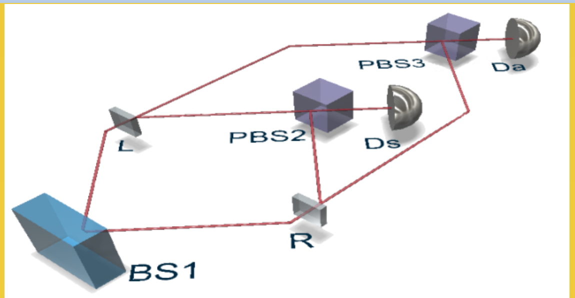
T. Kovachy, et al., Phys. Rev. Lett. **114**, 143004 (2015)
M. Bilardello, S. Donadi, A. Vinante and A. Bassi, Physica A **462**, 764-782 (2016).
F. Laloe, Franck, W. J. Mullin and P. Pearle, Phys. Rev. A **90** (5), 052119 (2014).

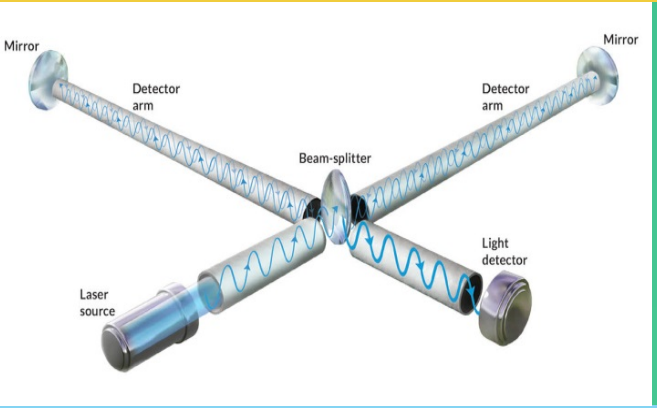
RELEVANT EXPERIMENTS

Entangling macroscopic diamonds

K. C. Lee, et al., Science **334**, 1253 (2011).

S. Belli, et al., Phys. Rev. A **94.1**, 012108 (2016).





Gravitational waves detection

B. P. Abbott, et al., Phys. Rev. Lett. **116.6**, 061102 (2016).

B. Helou, B. Slagmolen, D. E. McClelland, and Y. Chen, arXiv 1606.03637(2016).

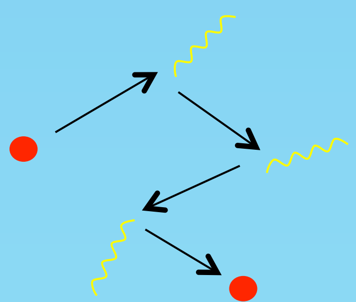
M. Carlesso, A. Bassi, P. Falferi, A.Vinante, Phys. Rev. D **94.12**, 124036 (2016).

Radiation emission

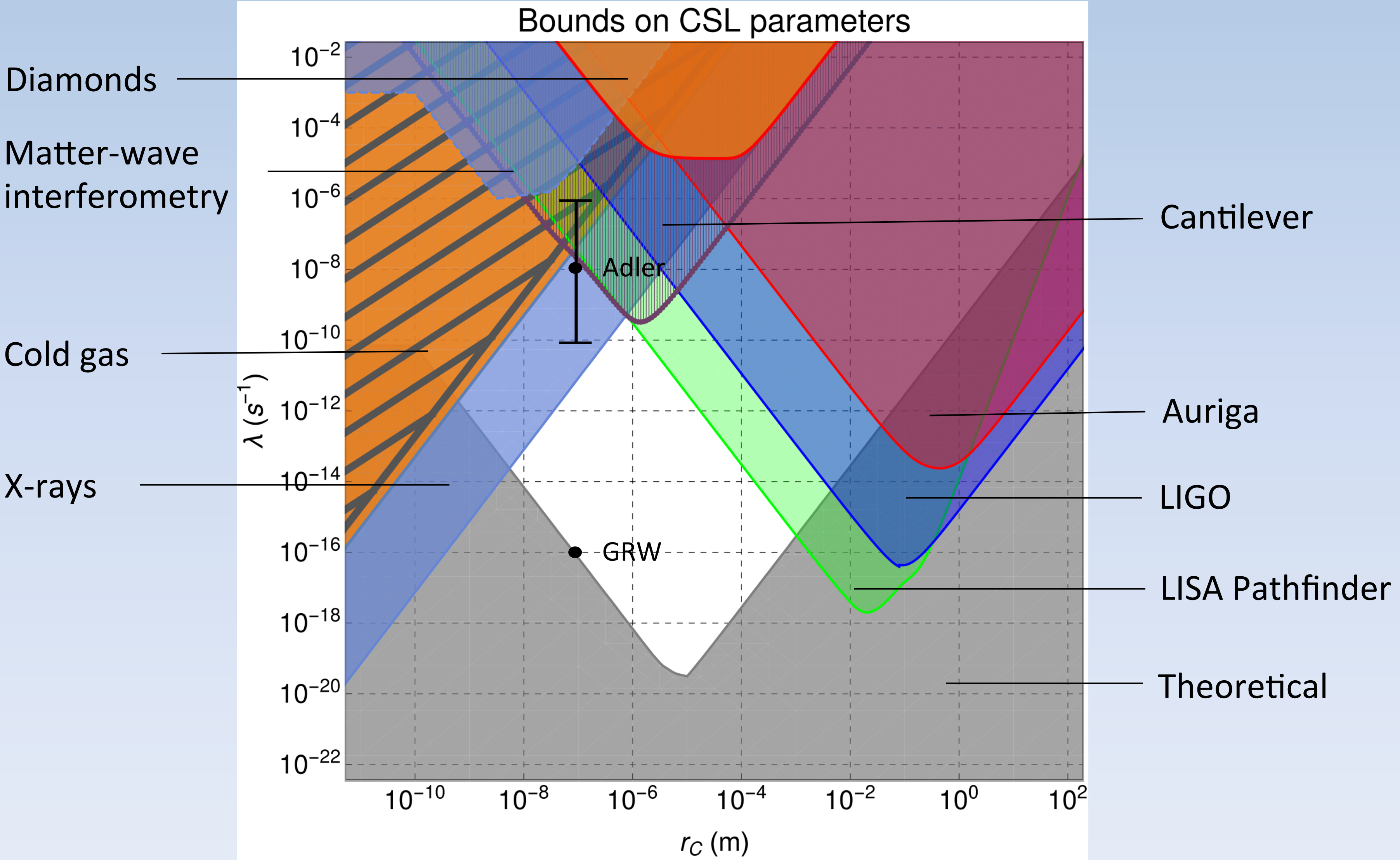
Q. Fu, Phys. Rev. A **56**, 1806 (1997).

S.L. Adler, A. Bassi and S. Donadi, J. Phys. A **46**, 245304 (2013).

C. Curceanu, B. C. Hiesmayr, K. Piscicchia. J. Adv. Phys. **4(3)**, 263–266 (2015).



EXCLUSION PLOT



WHY GOING TO SPACE?

Maqro2015

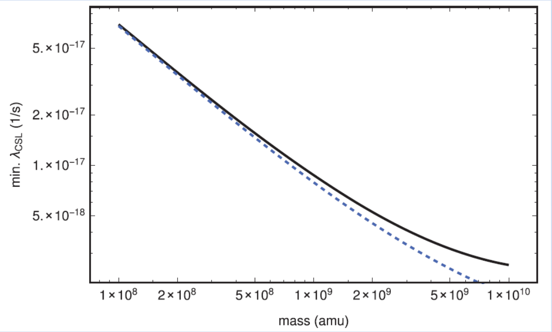
Interferometry: Masses in the range 10^8 - 10^{11} amu. Advantages:

1) More time for spreading: 10^2 s vs 10^{-3} s; ($\sigma_x(t) = f(t/m)$)

2) De Broglie wavelenght does not decrease too much; ($\lambda_{\mathrm{db}} = h/mv$)

3) Masses increased by 4-7 orders of magnitude!

Wave function spread: Experiments are expected to test GRW values!
Relevant also for SN equation!



Cold atoms: - Time of experiment was only 2.8 sec because of gravity.
- Short time of experiment was limiting the cooling
- t=100s implies improving the bounds of 3 orders of magnitude.

$$\frac{\lambda}{r_C^2} \leq \frac{4\epsilon\sqrt{\langle \hat{\mathbf{v}}^2 \rangle_{t_0}} m_0^2}{\sqrt{N-1}\hbar^2 t^2}$$

Rotation of a rod or a plane (?) : no friction and less decoherece effects

CONCLUSIONS

1) Collapse models: dynamical description of the wave function collapse.
   - modifications very little for microsystems;
   - but relevant for macrosystems.

2) Collapse models make different predictions compared to standard quantum theory.
   - many experiments set bounds on the parameters of the models;
   - still a large region of the exclusion plot is not excluded.

3) Micro gravity environment: essentialy the fact that systems can evolve freely for large times (order of 100 s) allows to improve current tests of several orders of magnitudes.

THANKS

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