WORKSHOP QUANTUM FOUNDATIONS
LNF Frascati 2017

CONSISTENCY OF VALUE ASSIGNMENT BY MEANS OF QUANTUM CORRELATIONS

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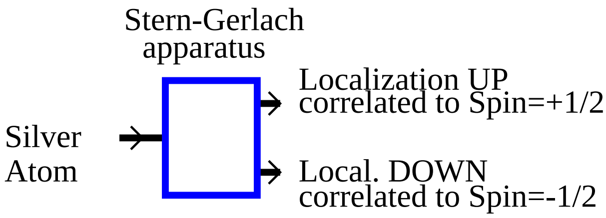
Indirect Measurement of Quantum Observables:

1. Spin of a Silver atom

PROBLEM: internal degree of freedom

Exploiting correlation with exit localization



CORRELATION:

Spin$= +\frac{1}{2}$ iff the atom outcomes from UP

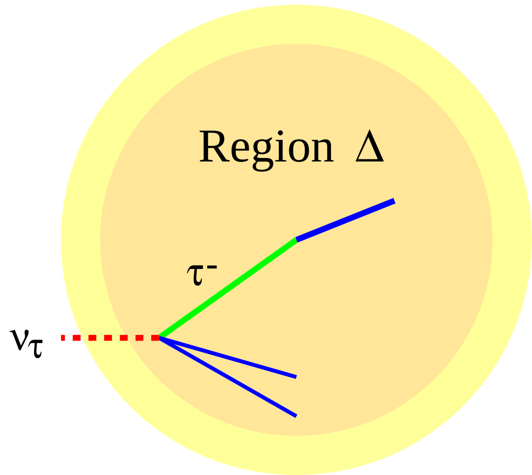
Spin$= -\frac{1}{2}$ iff the atom outcomes from DOWN

Indirect Measurement of Quantum Observables:

2. Localization of a *neutrino*

PROBLEM: too low interactivity

SOLUTION: To localize τ^-

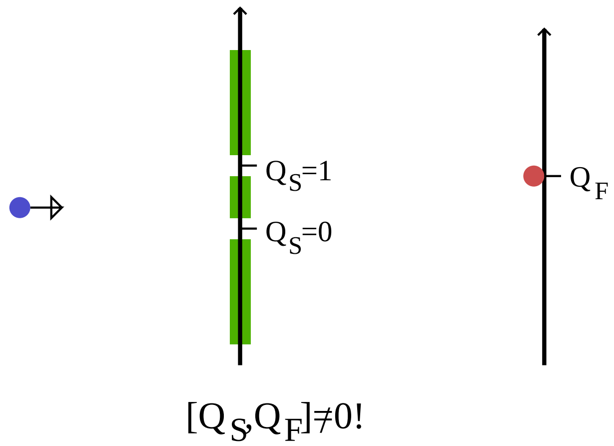


CORRELATION: ν_τ in Δ iff τ^- in Δ

Indirect Measurement of Quantum Observables:

3. Which Slit Q_S with the Final Position Q_F

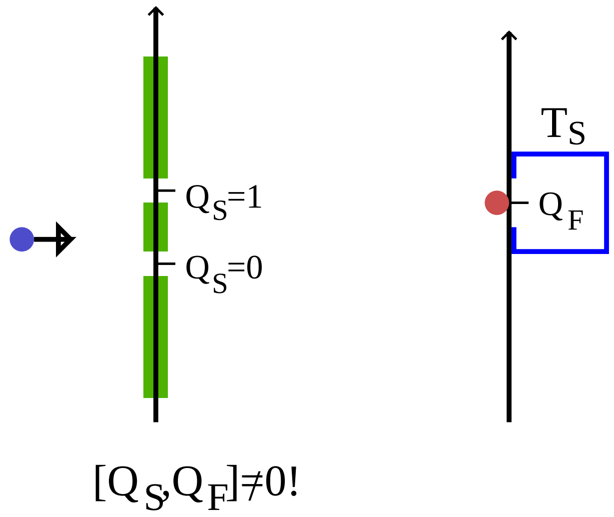
complementarity: $[\hat{Q}_S, \hat{Q}_F] = i\frac{t}{m}$



DOUBLE SLIT EXPERIMENT SOLUTION

Correlation: $\exists T_S$, $[\hat{T}_S, \hat{Q}_S] = [\hat{T}_S, \hat{Q}_F] = \mathbf{0}$

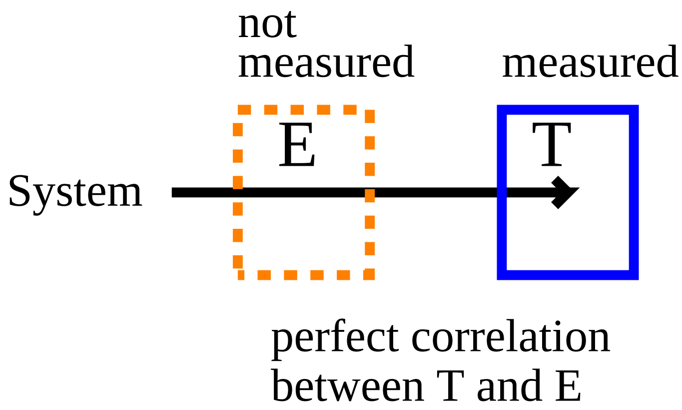
$T_S = 1$ iff $Q_S = 1$, $T_S = 0$ iff $Q_S = 0$



Q_S is not measured! Its value is inferred!

GENERALIZATION

EVALUATION OF E BY MEASURING T:



Condition: T perfectly correlated with E

To assign E the actual outcome of T

FIRST QUESTION: **Evaluations of E by T, are they Measurements of E?**

[Spin $\leftrightarrow$ exit localization: practice answers YES]

Tasks for a satisfactory scientific answer

i. to formally establish the different concepts of Measurements and Evaluations

ii. to find out the formal statement for the Identification Measurement$\equiv$Evaluation

iii. to check whether it holds in the theory

I. Quantum Formalism of Measurements

Support $\mathcal{S}(\rho)$ of ρ (density operator):

any concrete non-empty set of specimens whose quantum state is ρ

$\mathcal{E}$ set of *elementary* (1-0) observables

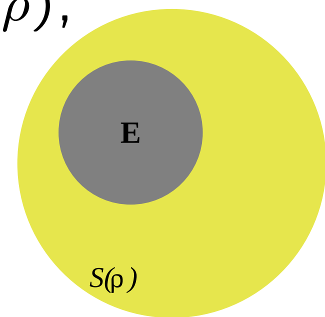
$E \in \mathcal{E}$ is represented by projection $\widehat{E}$

Given $E \in \mathcal{E}$ and support $\mathcal{S}(\rho)$,

specimen $x \in \mathbf{E}$ means

$x \in \mathcal{S}(\rho)$ and

E actually measured on x.



I. Quantum Formalism of Measurements

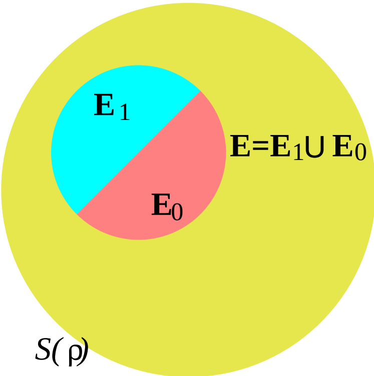
For each $E \in \mathcal{E}$,

$\mathbf{E}_1$: specimens in $\mathbf{E}$ with outcome 1

$\mathbf{E}_0$: specimens in $\mathbf{E}$ with outcome 0

$$\mathbf{E}_1 \cap \mathbf{E}_0 = \emptyset$$

$$\mathbf{E}_1 \cup \mathbf{E}_0 = \mathbf{E}$$



I. Quantum Formalism of Measurements

E, F are measurable together (*comeasurable*) iff for every ρ a support $\mathcal{S}(\rho)$ exists such that $\mathbf{E} \cap \mathbf{F} \neq \emptyset$.

According to Quantum Theory

($\mathbf{q.1}$) If $[\hat{E}, \hat{F}] = \mathbf{0}$ then $\forall\rho$, $\mathcal{S}(\rho)$ exists
such that $\mathbf{E} \cap \mathbf{F} \neq \emptyset$ (E, F comeasurable);

I. Quantum Formalism of Evaluations

$T \in \mathcal{E}$ Evaluates $E \in \mathcal{E}$ in state ρ, *conceptually,*

if whenever they are measured together, then their outcomes coincide, i.e. if

the restrictions of $\mathbf{T}_1$ and $\mathbf{E_1}$ to $\mathbf{E} \cap \mathbf{T}$ coincide, and those of $\mathbf{T}_0$ and $\mathbf{E_0}$ coincide too.

Def. T evaluates E in ρ, written $E \prec \rho \succ T$, if

(D.1) $\exists \mathcal{S}(\rho)$ such that $\mathbf{E} \cap \mathbf{T} \neq \emptyset$

(D.2) $\forall x \in \mathbf{E} \cap \mathbf{T}$ ("if measured together")

$$x \in \mathbf{T}_1 \text{ iff } x \in \mathbf{E}_1, \qquad x \in \mathbf{T}_0 \text{ iff } x \in \mathbf{E}_0$$

II. Identification Measurement$\equiv$Evaluation:

To identify evaluations of E by T

with authentic measurements of E

means "If $x \in \mathbf{T}$ then $x \in \mathbf{E}$".

Evaluations are identifiable with measurements

if and only if

$T \prec \rho \succ E$ implies $\mathbf{T}_1 = \mathbf{E}_1$, $\mathbf{T_0} = \mathbf{E}_0$ (Id)

(*tout court*, not for the restricions to $\mathbf{T} \cap \mathbf{E}$)

III. (Id) Contradicts Quantum Physics!

$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \otimes \mathcal{H}_4$, and $\mathcal{H}_k = \mathbb{C}^2$, $\forall k$.

Seven observables with spectrum $\{1, -1\}$:

$$\widehat{A}^\alpha = \widehat{\sigma}_x \otimes \mathbf{1}_2 \otimes \mathbf{1}_3 \otimes \mathbf{1}_4, \quad \widehat{A}^\beta = \widehat{\sigma}_y \otimes \mathbf{1}_2 \otimes \mathbf{1}_3 \otimes \mathbf{1}_4$$

$$\widehat{B} = \mathbf{1}_1 \otimes \widehat{\sigma}_x \otimes \mathbf{1}_3 \otimes \mathbf{1}_4$$

$$\widehat{C}^\alpha = \mathbf{1}_1 \otimes \mathbf{1}_2 \otimes \widehat{\sigma}_x \otimes \mathbf{1}_4, \quad \widehat{C}^\beta = \mathbf{1}_1 \otimes \mathbf{1}_2 \otimes \tfrac{1}{2}\widehat{\sigma}_y \otimes \mathbf{1}_4$$

$$\widehat{D}^\alpha = \mathbf{1}_1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_3 \otimes \widehat{\sigma}_x, \quad \widehat{D}^\beta = \mathbf{1}_1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_3 \otimes \tfrac{1}{2}\widehat{\sigma}_y.$$

A quantum state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ where ψ_0 is

$$\frac{1}{\sqrt{2}}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}_2 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}_3 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}_4 - \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2 \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}_3 \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}_4 \right\}$$

III. (Id) Contradicts Quantum Physics!

PROP.3. If (**Id**) holds, then x_0 exists and

i) $x_0 \in \mathbf{A}^\alpha \cap \mathbf{A}^\beta \cap \mathbf{B} \cap \mathbf{C}^\alpha \cap \mathbf{C}^\beta \cap \mathbf{D}^\alpha \cap \mathbf{D}^\beta$

ii) the values $a^\alpha, a^\beta, b, c^\alpha, c^\beta, d^\alpha, d^\beta$ measured on x_0 must satisfy the relations

$$a^\alpha b = -c^\alpha d^\alpha, \quad a^\beta b = -c^\beta d^\alpha$$
$$a^\beta b = -c^\alpha d^\beta, \quad a^\alpha b = c^\beta d^\beta$$

These relations are contradictory, because:

(ii) and $a^\alpha, a^\beta, b, c^\alpha, c^\beta, d^\alpha, d^\beta \in \{-1, +1\}$ imply

$$c^\alpha c^\beta = -c^\alpha c^\beta \quad !$$

CONCLUSION:

Identification Evaluation$\equiv$Measurement

Is Impossible in Quantum Physics!

QUESTION:

What's the Physical Meaning of Evaluations related to the evaluated Observable?

What are we doing when we Evaluate an Observable instead of measuring it?

A Quick (limited) answer is obtained

By Theoretically Comparing

Physical Consequences of Occurrences of E

with

Physical Consequences of Occurrences of T

RESULT OF THE COMPARISON:

these consequences are the same:

Evaluations are perfect simulations of measurements

Algebraic characterization of $E \prec \rho \succ T$

$E \prec \rho \succ T$ implies E, F comeasurable.
According to Quantum Theory $[\hat{T}, \hat{E}] = \mathbf{0}$.

PROP.1. $E \prec \rho \succ T$ if and only if
$[\hat{T}, \hat{E}] = \mathbf{0}$ and $\hat{E}\rho = \hat{T}\rho$.

This point should be treated with more care.

I do so to expedite the presentation

Consequences of a Measurements of E:

Correlations Between

occurrences of actual outcomes of E and

occurrences of actual outcomes of F

Expressed by Quantum conditional probability:

$$P(F \mid E) = \frac{Tr(\rho \widehat{F} \widehat{E})}{Tr(\rho \widehat{E})} \quad \text{Condition: } [\widehat{F}, \widehat{E}] = \mathbf{0}$$

Consequences of a Measurements of T:

Quantum conditional probability:

$$P(F \mid T) = \frac{Tr(\rho \widehat{F} \widehat{T})}{Tr(\rho \widehat{T})} \quad \text{Condition: } [\widehat{F}, \widehat{T}] = \mathbf{0}$$

Comparing consequences of E and T

PROP. 4. If $E \prec \rho \succ T$ (T evaluates E) then

$P(F \mid T) = P(F \mid E) \quad$ for all $F \in \mathcal{F}_T(E)$

where $\mathcal{F}_T(E) = \{F \in \mathcal{E} \mid [\hat{F}, \hat{T}] = [\hat{F}, \hat{E}] = \mathbf{0}\}$.

Thus, if T evaluates E, then

Measurable consequences of outcomes of T are indistinguishable from those of E

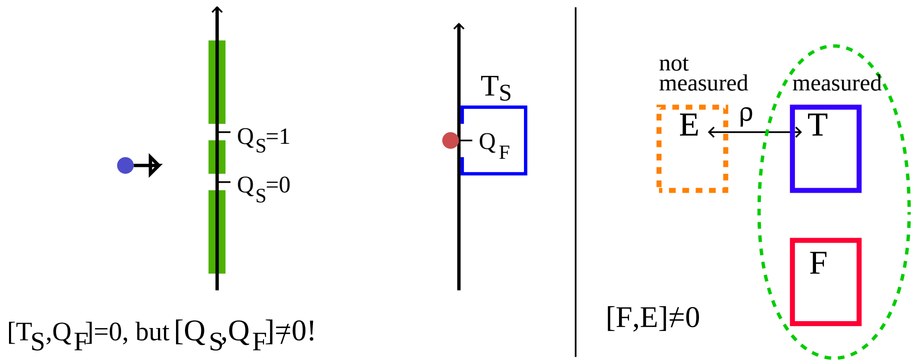
Evaluations of E by T perfectly simulate measurements of E.

Evaluations of E perfectly simulate measurement of E if $[\hat{F}, \hat{E}] = \mathbf{0}$.

$E \prec \rho \succ T$ and $[\hat{T}, \hat{F}] = \mathbf{0}$

T can evaluate E in measuring F

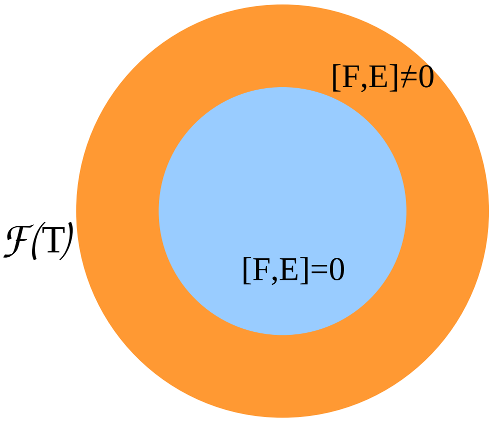
But if $[\hat{F}, \hat{E}] \neq \mathbf{0}$, there is nothing to simulate!



Given E and ρ, $E \prec \rho \succ T$ and $[\hat{T}, \hat{F}] = \mathbf{0}$,
$p_Q(E\&F)$ quantum probability of joint event

"outcome of E is 1" & "otucome of F is 1"

is defined on $\mathcal{F}(E) = \{F \in \mathcal{E} \mid [\hat{F}, \hat{E}] = \mathbf{0}\}$.



Theorem 1. Put $E' = \mathbf{1} - E$. The mappings

$p_\rho(E\& \cdot\,) : \mathcal{F}(T) \to [0,1]$, $p_\rho(E\&F) = Tr(\rho\widehat{E}\widehat{F}\widehat{E})$

$p_\rho(E'\& \cdot\,) : \mathcal{F}(T) \to [0,1]$, $p_\rho(E'\&F) = Tr(\rho\widehat{E}'\widehat{F}\widehat{E}')$

are the unique functionals such that

C.1. If $F \in \mathcal{F}(T)$ and $[\widehat{F}, \widehat{E}] = \mathbf{0}$, then

$p_\rho(E\&F) = Tr(\rho\widehat{E}\widehat{F})$ and

$p_\rho(E'\&F) = Tr(\rho\widehat{E}'\widehat{F})$;

C.2. if $\{F_j\}_{j\in J} \subseteq \mathcal{F}(T)$ and $\widehat{F}_j \perp \widehat{F}_k$, then

$p_\rho(E\& \sum_j \widehat{F}_j) = \sum_{j\in J} p_\rho(E\&F_j)$ and

$p_\rho(E'\& \sum_j \widehat{F}_j) = \sum_{j\in J} p_\rho(E'\&F_j)$.

Meaning of Theorem 1.

There is a unique possibility for a probability ruling over values assignments to E consistent with measurements and Quantum theoretical predictions about all observables in $\mathcal{F}(T)$,

$$\mathcal{F}(T) = \{F \in \mathcal{E} \mid [\widehat{F}, \widehat{T}] = \mathbf{0}\}.$$

Theorem 2. Such a unique probability

is empirically realized by assigning

- E the measured value of T
- F the value actually measured.

CONCLUSION

I. To interpret an evaluation of E by T
as a measurement of E
is consistent with all performable
measurements in the domain $\mathcal{F}(T) \subseteq \mathcal{E}$.

II. The consistency is guaranteed
if T *is actually measured*,
not if T is only evaluated.

III. Given E, its evaluation can be performed
by different T with different $\mathcal{F}(T)$.

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