Jets cross sections in NNLO QCD in lepton and hadron collisions

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LFC17: Old and New Strong Interactions from LHC to Future Colliders
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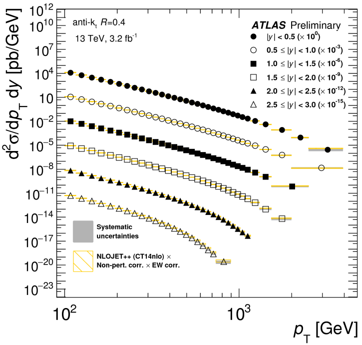
Introduction

Why jets?

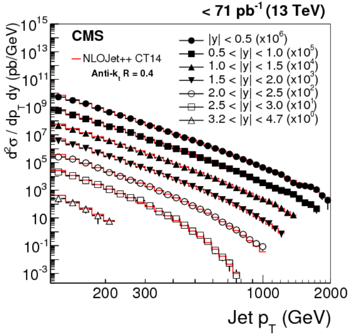
- Jet related studies important for understanding QCD
- Extraction of α_S and PDFs
- LHC is a jet factory: complex final states containing multiple hadronic jets copiously produced
- SM background to BSM searches
- Precisely measured over many orders of magnitude

Bottom line: jets are essential analysis tools, precise understanding needed

- Double-differential inclusive jet cross section at $\sqrt{s} = 13$ TeV
- Precise data over more than 10 orders of magnitude
- p_T range to (beyond) 2 TeV



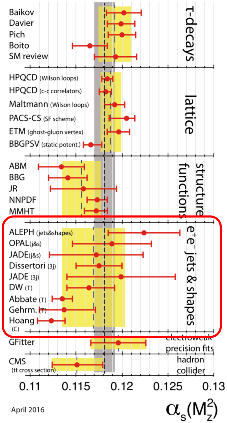
[ATLAS-CONF-2017-048]



[Eur. Phys. J. C76 (2016) no.8, 451]

[CMS-SMP-15-007, CERN-EP-2016-104]

[S. Bethke, Nucl. Part. Phys. Proc. 282-284 (2017) 149]

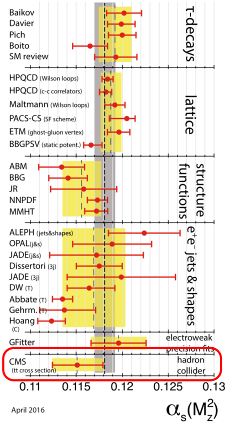


α_S at e^+e^- colliders

- Based on jet rates and event shapes (thrust, C-parameter, etc.)
- NNLO theory is used, with up to N^3LL resummation

α_S world average

[S. Bethke, Nucl. Part. Phys. Proc. 282-284 (2017) 149]



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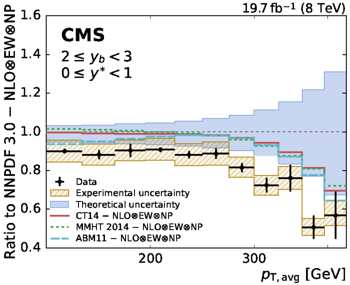
α_S at the LHC

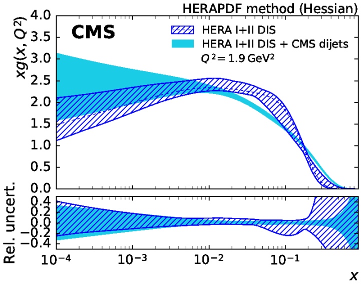
- Several determinations based on jet measurements at 7 and 8 TeV
- Typically NLO theory is used, except for the $t\bar{t}$ total cross section, which based on NNLO
- Uncertainties already dominated by theory, e.g., α_S from transverse energy-energy correlation at 8 TeV

$$\alpha_S(M_Z) = 0.1162 \pm 0.0011\ (\text{exp.})\ ^{+0.0076}_{-0.0061}\ (\text{scale}) \pm 0.0018\ (\text{PDF}) \pm 0.0003\ (\text{NP})$$

[ATLAS Coll., arXiv:1707.02562]

- Triple-differential dijet jet cross section at $\sqrt{s} = 13$ TeV
- Experimental uncertainties small enough to constrain PDFs
- Largest impact on the high-x region



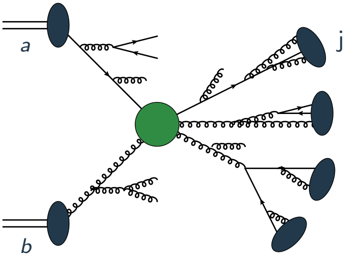


[CMS Coll., arXiv:1705.02628]

[CMS-SMP-16-011, CERN-EP-2017]

QCD at colliders

- To fully exploit the physics potential of colliders requires precision, QCD must be understood/modeled as best as feasible

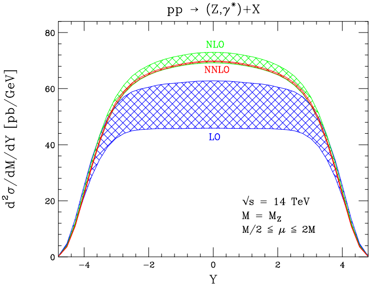


$$d\sigma = \sum_{a,b} \int dx_a \int dx_b \underbrace{f_a(x_a, \mu_F^2)\, f_b(x_b, \mu_F^2)}_{\text{non-pert. PDFs}} \times \underbrace{d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_S(\mu_R^2))}_{\text{pert. partonic x-sec}} + \mathcal{O}\left((\Lambda/Q)^m\right)$$

- One particular aspect of precision: calculation of exact higher order corrections to physical observables in perturbation theory

Why higher order corrections?

- NLO corrections are large, convergence is slow ($\alpha_S \sim 0.1$)
- Dependence on unphysical scales considerably reduced at higher orders
- Reliable estimate of theoretical uncertainties
- Benchmark processes measured with high experimental accuracy
- The lack of striking signals of new physics at LHC suggests that BSM effects will be accessible only through precision studies

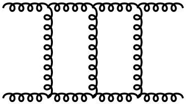


[Anastasiou, Dixon, Melnikov, Petriello, Phys. Rev. D69 (2004) 094008]

NNLO

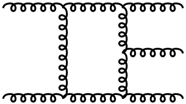
NNLO ingredients

- 2-loop (VV)



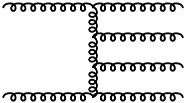
- Two-loop integrals $\Rightarrow$ explicit poles up to $1/\epsilon^4$
- $2 \to 2$ available (including VV production)
- Huge progress, but higher multiplicities a bottleneck?

- 1-loop (RV)



- One-loop integrals $\Rightarrow$ explicit poles up to $1/\epsilon^2$
- Real emission $\Rightarrow$ implicit poles up to $1/\epsilon^2$ from integration over unresolved phase space
- NLO complexity

- tree (RR)



- Tree level $\Rightarrow$ amplitudes trivial to compute
- Double real emission $\Rightarrow$ implicit poles up to $1/\epsilon^4$ from integration over unresolved phase space
- Higher multiplicities a bottleneck?

The problem

Assuming we know the relevant matrix elements, can we use those matrix elements to compute cross sections?

- Consider the NNLO correction to a generic m-jet observable

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

- All three terms are separately divergent in $d = 4$ dimensions

- Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (i.e. IR safe) observables (KLN theorem)

- How to make this cancellation explicit, so that the various contributions can be computed numerically?

Need a method to deal with implicit poles.

Phase space slicing: split phase space according to singular configurations

$$\int_0^1 |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V = \underbrace{\int_\delta^1 |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R}_{\text{regularized by cutoff}} + \underbrace{\int_0^\delta |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V}_{\text{can be obtained from resummation framework}}$$

Handling singularities: phase space slicing

Phase space slicing: split phase space according to singular configurations

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- Not used at NLO
- Generates large numerical cancellations on cutoff (must check independence)
- Can use existing NLO calculations as basis (X+jet)
- Local subtractions for NLO-like singularities
- Simpler to implement (resummation)

Handling singularities: phase space slicing

Two approaches based on different resummation frameworks

- q_T subtraction [Catani, Cieri, de Florian, Ferrera, Grazzini]
- N-jettiness subtraction [Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh]

q_T or jettiness used to disentangle “pure” NNLO regions

So far only for “simpler” configurations: one/zero colored particle in the final state

Handling singularities: subtraction method

Subtraction method: use local counterterm to rearrange singularities

$$\int_0^1 |\mathcal{M}_R|^2 \, \mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \, \mathrm{d}\phi_V = \underbrace{\int_0^1 (|\mathcal{M}_R|^2 - \mathcal{D}) \, \mathrm{d}\phi_R}_{\text{integrable}} + \underbrace{\int_0^1 \mathcal{D} \, \mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \, \mathrm{d}\phi_V}_{\text{poles cancel analytically}}$$

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- Method of choice at NLO
- Subtractions can be completely local (good convergence)
- At NNLO lots of singular configurations with overlaps
- Integration of subtraction term quite complicated (can be numerical)

Handling singularities: subtraction method

Definition of the subtraction term is not unique, several approaches

- Sector decomposition [Anastasiou, Melnikov, Petriello; Binoth, Heinrich]
- Antenna subtraction [Gehrmann, Gehrmann-de Ridder, Glover]
- Sector-improved residue subtraction (STRIPPER) [Czakon; Boughezal, Melnikov, Petriello]
- Projection-to-born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi]
- CoLoRFulNNLO subtraction [Del Duca, GS, Trócsányi]

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Personal opinion: general solution not yet available

Several approaches – why this one?

- general and explicit expressions, including color and flavor
  (automation, color space notation is used)
- fully local counterterms, taking account of all color and spin correlations
  (mathematical rigor, efficiency)
- analytic cancellation of explicit ϵ poles in loop amplitudes
  (mathematical rigor)
- option to constrain subtractions to near singular regions (α_{max})
  (efficiency, important check)
- very algorithmic construction
  (valid at any order in perturbation theory)

The recipe

Use the same framework that was successful at NLO: local subtractions

The NLO correction to some m-jet observable J

$$\sigma^{\mathrm{NLO}}[J] = \int_{m+1} \left[\mathrm{d}\sigma^{\mathrm{R}}_{m+1} J_{m+1} - \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} J_m \right]_{d=4} + \int_m \left[\mathrm{d}\sigma^{\mathrm{V}}_m + \int_1 \mathrm{d}\sigma^{\mathrm{R,A_1}}_{m+1} \right]_{d=4} J_m$$

The NNLO correction is the sum of three pieces

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_m \mathrm{d}\sigma^{\mathrm{VV}}_m J_m$$

The three contributions are separately IR divergent in $d = 4$

- RR: double and single unresolved real emission
- RV: single unresolved real emission $\oplus$ ϵ-poles from $m+1$ parton one-loop
- VV: ϵ poles from m parton two-loop

For the RR contribution subtractions are needed to regularize one- and two-parton emissions

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right] \right\}_{d=4}$$

- A_1 and A_2 have overlapping singularities $\Rightarrow$ A_{12} is needed to cancel

For the RV contribution emissions are like at NLO but for one-loop-tree interference

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}_{d=4}$$

- Notice the integrated A_1 from RR which is still singular $\Rightarrow$ subtraction is needed (last term)

The m-parton contribution contains the double virtual and integrated subtractions

$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\}_{d=4} J_m$$

Use known ingredients

Collinear and soft factorization of QCD matrix elements at NNLO known

- Tree level 3-parton splitting functions and double soft gg and $q\bar{q}$ currents

[Campbell, Glover 1997; Catani, Grazzini 1998; Del Duca, Frizzo, Maltoni 1999; Kosower 2002]

- One-loop 2-parton splitting functions and soft gluon current

[Bern, Dixon, Dunbar, Kosower 1994; Bern, Del Duca, Kilgore, Schmidt 1998-9; Kosower, Uwer 1999; Catani, Grazzini 2000; Kosower 2003]

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But note

- Unresolved regions in phase space overlap
- Quantities appearing in factorization formulae are only well-defined in the strict limit

Defining the subtraction scheme

The following three problems must be addressed

1. Matching of limits to avoid multiple subtraction in overlapping singular regions of PS. Easy at NLO: collinear limit + soft limit - collinear limit of soft limit.

$$\mathbf{A}_1 |\mathcal{M}^{(0)}_{m+1}|^2 = \sum_i \left[\sum_{i \neq r} \frac{1}{2} \mathbf{C}_{ir} + \mathbf{S}_r - \sum_{i \neq r} \mathbf{C}_{ir} \mathbf{S}_r \right] |\mathcal{M}^{(0)}_{m+1}|^2$$

2. Extension of IR factorization formulae over full PS using momentum mappings that respect factorization and delicate structure of cancellations in all limits.

$$\{p\}_{m+1} \xrightarrow{r} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+1}(\{p\}_{m+1}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{1,m}]$$
$$\{p\}_{m+2} \xrightarrow{r,s} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+2}(\{p\}_{m+2}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{2,m}]$$

3. Integration of the counterterms over the phase space of the unresolved parton(s).

Defining the subtraction scheme

Specific issues at NNLO

1. Matching is cumbersome if done in a brute force way. However, an efficient solution that works at any order in PT is known.
2. Extension is delicate. E.g., counterterms for single unresolved real emission (unintegrated and integrated) must have universal IR limits. This is not guaranteed by QCD factorization.
3. Choosing the counterterms such that integration over the unresolved phase space is (relatively) straightforward generally conflicts with the delicate cancellation of IR singularities.

Computing the integrated subtractions

Strategy for computing the phase space integrals: direct integration

1. Write phase space in terms of angles and energies
2. Angular integrals in terms of Mellin-Barnes representations
3. Resolve the ϵ poles by analytic continuation
4. MB integrals to Euler-type integrals, pole coefficients are finite parametric integrals

1. Choose explicit parametrization of phase space
2. Write the parametric integral representation in chosen variables
3. Resolve the ϵ poles by sector decomposition
4. Pole coefficients are finite parametric integrals

5. Evaluate the parametric integrals in terms of multiple polylogs
6. Simplify result (optional)

General features of CoLoRFulNNLO

CoLoRFulNNLO: Completely Local subtRactions for Fully differential NNLO

Subtractions built using universal IR limit formulae and exact PS factorization

- Altarelli-Parisi splitting functions, soft currents
- PS factorizations based on momentum mappings that can be generalized to any number of unresolved partons

Completely local in color $\otimes$ spin space, fully differential in phase space

- No need to consider the color decomposition of real emission ME's
- Azimuthal correlations correctly taken into account in gluon splitting
- Can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit

Poles of integrated subtraction terms computed analytically

- Can check pole cancellation in (double) virtual contribution explicitly

Explicit formulae for processes with colorless initial state

- Automation is possible

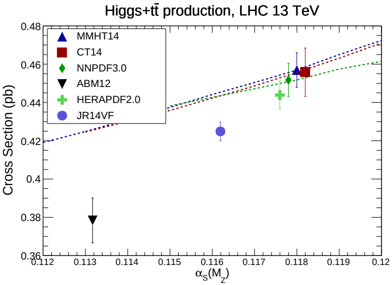
MCCSM is a Monte Carlo for the CoLoRFulNNLO Subtraction Method

- Completely general and fully automatic
- Highly flexible and tunable
- Phase space is recursively constructed, MINT is used for MC integration
- Histogram output in YODA format through an interface to YODA
- Written in standard fortran90 (by Á. Kardos)
- User must provide only the squared MEs, including color- and spin-correlated (since subtraction terms are local)

Jet production at lepton colliders

Why $e^+e^- \to$ jets?

- Relevant for extracting α_S from data – the value of the strong coupling matters
- Three-jet event shapes and jet rates are sensitive to α_S and have been extensively measured
- Can compute new observables which may be better suited extraction of the strong coupling
- Good testing ground for higher order technology



[LHC Higgs Cross Section Working Group]

$e^+e^- \to$ jets: status

NNLO corrections to event shapes and jet rates in $e^+e^- \to 2, 3$ jets known

- Antenna subtraction: `EERAD3` now superseded by `NNLOJET`

  [Gehrmann, Gehrmann-de Ridder, Glover, Heinrich 2007; Gehrmann, Glover, Huss, Niehues, Zhang 2017]

- Another implementation of the same scheme is available

  [Weinzierl 2009]

- CoLoRFulNNLO subtraction: `MCCSM`

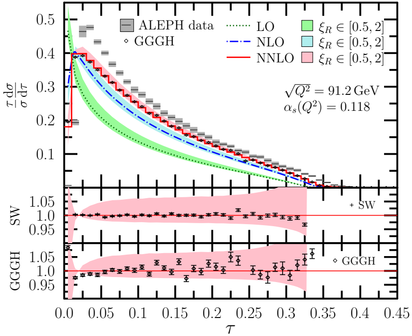
  [Del Duca, Duhr, Kardos, GS, Szőr, Trócsányi, Tulipánt 2016]

- Used to extract α_S from e^+e^- data, in conjunction with resummation

  [Dissertori et al. 2009, Abbate et al. 2011, Gehrmann et al. 2013, Hoang et al. 2015]

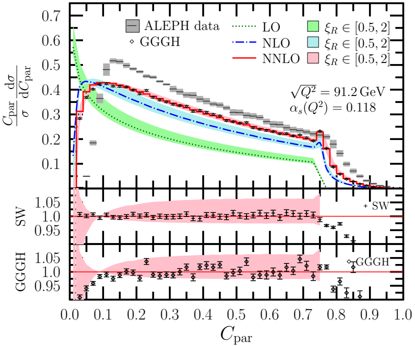
Event shapes at NNLO

- Thrust



$$\tau = 1 - \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$$

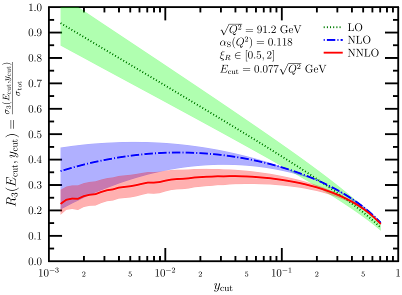
- C-parameter



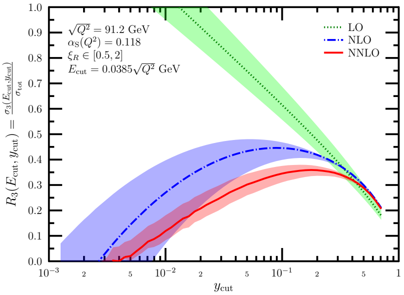
$$C_{\mathrm{par}} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

Jet rates at NNLO

- Three-jet rate, anti-$k_\perp$ jets ($E_{\rm cut} = 0.077\sqrt{Q^2}$)



- Three-jet rate, anti-$k_\perp$ jets ($E_{\rm cut} = 0.0385\sqrt{Q^2}$)



$$R_3(y_{\rm cut}, E_{\rm cut}) = \frac{\sigma_{\text{3-jet}}(y_{\rm cut}, E_{\rm cut})}{\sigma_{\rm tot}}\,, \qquad y_{ij} = \frac{Q^2}{8}\min\left\{E_i^{-2}, E_j^{-2}\right\}(1 - \cos\theta_{ij})$$

PRELIMINARY

$e^+e^- \to$ jets: outlook

More jets?

- Necessary two-loop amplitudes not yet ready, but on the way
- Antenna: general form of approximate cross sections for $e^+e^- \to n$ jets not recorded in literature, in particular subleading color is complicated
- STRIPPER: can handle $e^+e^- \to n$ jets in principle, but cancellation of poles is numeric
- N-jettiness: some pieces of the resummation framework still missing, numerics could be a major challenge
- CoLoRFulNNLO: approximate cross sections for the general case known, some integrated counterterms for $n > 3$ missing
- Numerics for double real radiation will be challenging for any method

Jet production at hadron colliders

Jets at LHC: status

NNLO corrections known to

- Single jet inclusive production using antenna subtraction: `NNLOJET` (leading color, all partonic channels)

  [Currie, Glover, Pires 2016]

- Dijet production using antenna subtraction: `NNLOJET` (leading color, all partonic channels)

  [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires 2017]

- First qualitative comparisons to data

  [ATLAS-CONF-2017-048]

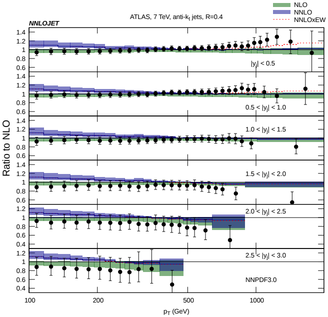
- Will not discuss $H/W/Z$ + jet and VBF, apologies

  [Boughezal, Focke, Giele, Liu, Petriello 2015; Boughezal, Caola, Melnikov, Petriello, Schulze 2015, Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier 2016; Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan 2016; Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello 2016; Boughezal, Liu, Petriello 2016; Cacciari, Dreyer, Karlberg, Salam, Zanderighi 2015]

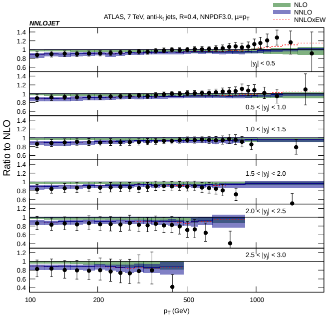
Single jet inclusive production at the LHC

Leading color, all partonic channels using antenna subtraction: `NNLOJET`

$\mu = p_{T_1}$ leading jet



$\mu = p_T$ individual jet

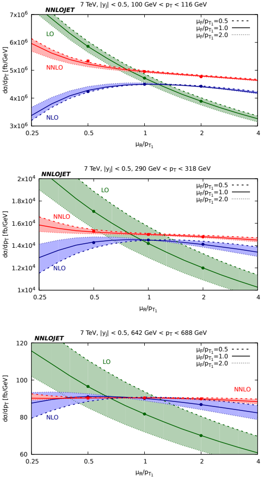


- Moderate NNLO corrections
- Two different scale choices: leading jet vs. individual jet transverse momentum
  - Equivalent at large transverse momentum
  - Differences outside scale band at low transverse momentum
  - $\mu = p_T$ provides better description of data
  - Requires further studies

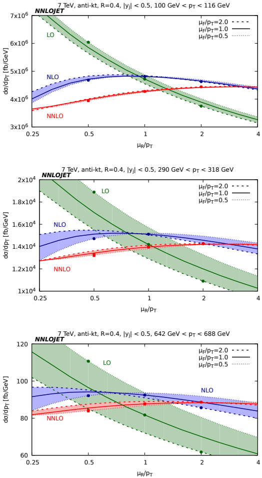
Single jet inclusive production at the LHC

Single inclusive cross section in three p_T bins

$\mu = p_{T_1}$ leading jet

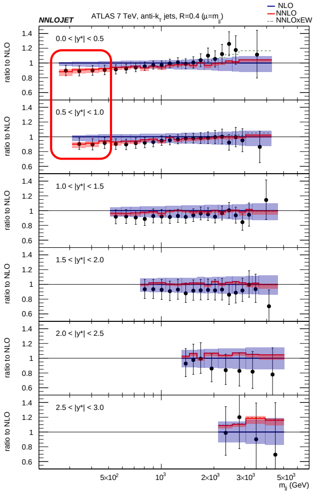


$\mu = p_T$ individual jet



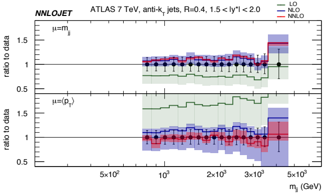
Dijet production at the LHC

Leading color, all partonic channels using antenna subtraction: `NNLOJET`



- Moderate NNLO corrections ($\sim 10\%$)
- Better description of data at low m_{jj}, y^*
- NLO underestimates uncertainty at low m_{jj}, y^*
- NNLO scale dependence smaller than experimental uncertainty
- Natural scale choice is invariant mass (better convergence)

$$\mu = m_{jj} \quad \text{vs.} \quad \mu = (p_{T_1} + p_{T_2})/2$$



$pp \to$ jets: outlook

More jets?

- Available computations for one and two jets still only leading color
- Necessary two-loop amplitudes not yet ready, but on the way
- Antenna: general form of subtraction terms not recorded in literature, in particular subleading color is complicated
- STRIPPER: can handle $pp \to n$ jets in principle, but cancellation of poles is numeric
- N-jettiness: some pieces of the resummation framework still missing, numerics could be a major challenge
- CoLoRFulNNLO: work on extending to initial state radiation ongoing
- Numerics for double real radiation will be challenging for any method

Conclusions

Conclusions

Jets are essential analysis tools: precise understanding mandatory

Amazing progress in fixed order calculations in the past decade

- Automation of NLO
- NNLO for 3 jets at lepton colliders
- NNLO for several $2 \to 2$ processes, including dijet, at hadron colliders
- Even N^3LO for simplest LHC kinematics, first set of splitting functions

NNLO results are being used for analyses

- Extraction of α_S, constraining PDFs, searches, ...
- First comparison of LHC jet data with NNLO

But reaching new bottlenecks, in particular NNLO still very challenging beyond $2 \to 2$

- Two-loop (massive) amplitudes
- Real radiation not trivial

Will need significant developments: new understanding, new ideas, new tools

The future is challenging but exciting