NEW TECHNIQUES FOR HIGHER-ORDER CALCULATIONS IN LEPTON AND HADRON COLLISIONS

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Content

- Basic introduction and Loop-tree duality
- LTD/FDU approach
  - Location of IR singularities and renormalization
  - Toy-model examples; generalizations at NLO
- Physical example I: $A^* \to q\bar{q}(g)$@NLO
- Physical example II: Higgs @NLO
- Towards two-loops (**new!!**)
- Conclusions and perspectives

1. *Catani et al, JHEP 09 (2008) 065*
2. *Rodrigo et al, Nucl.Phys.Proc.Suppl. 183:262-267 (2008)*
3. *Buchta et al, JHEP 11 (2014) 014*

Rodrigo et al, JHEP 02 (2016) 044; JHEP 08 (2016) 160; JHEP 10 (2016) 162; arXiv:1702.07581 [hep-ph]

Basic introduction and LTD

Theoretical motivation

- When computing **IR-safe observables,** divergences cancel combining the real and virtual corrections (**KLN** theorem)
- For IR singularities, **phase-space integrals of real radiation** should originate the same structures that appear in **Feynman integrals for loop diagrams** → *Loop-tree theorems!*

Physical observable →

Virtual corrections (loop integrals)

$$\int \frac{d^D q}{(2\pi)^D}$$

\+ *Real corrections (PS integrals)*

$$\int \frac{d^{D-1}\vec{q}}{(2\pi)^{D-1}\,2q_0} = \int \frac{d^D q}{(2\pi)^D}(2\pi)\delta\left(q^2\right)\theta(q_0)$$

\+ *Renormalization counter-terms (ε poles times leading order)*

$$\frac{C_r}{\epsilon} \times d\sigma^{(0)}$$

Pole cancellation AFTER performing real-virtual integrals!!

WE WANT INTEGRAND LEVEL CANCELLATION!!!

Basic introduction and LTD

Dual representation of one-loop integrals

Loop Feynman integral

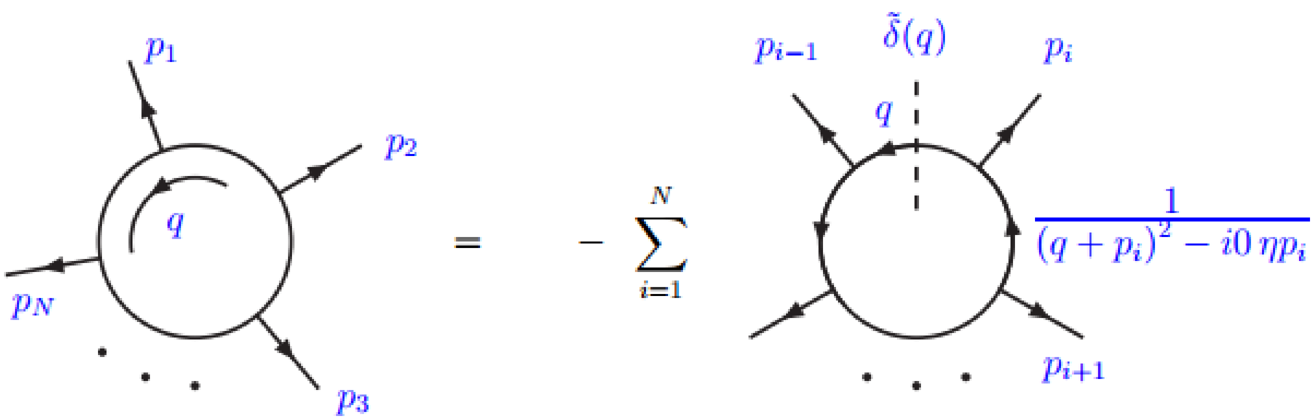
$$L^{(1)}(p_1,\ldots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\ldots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i; q_j)$$

Sum of phase-space integrals!

$$G_D(q_i, q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$$



Even at higher-orders, the number of cuts is equal the number of loops

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Basic introduction and LTD

Derivation (one-loop)

- **Idea:** *«Sum over all possible 1-cuts»* (but with a **modified prescription…**)
  - Apply Cauchy's residue theorem to the Feynman integral:

$$L^{(N)}(p_1, p_2, \dots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G(q_i) = \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G(q_i) = -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\, q_0<0\}} \left[\prod_{i=1}^{N} G(q_i) \right]$$

  - Compute the residue in the poles with negative imaginary part:

$$\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \left[\prod_{j=1}^{N} G(q_j) \right] = \left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}}\, G(q_i) \right] \left[\prod_{\substack{j=1 \\ j\neq i}}^{N} G(q_j) \right]_{\{i-\mathrm{th\ pole}\}}$$

$$\left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0\, \delta_+(q_i^2)$$

Put on-shell the particle crossed by the cut

$$\left[\prod_{j\neq i} G(q_j) \right]_{\{i-\mathrm{th\ pole}\}} = \prod_{j\neq i} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

Introduction of «dual propagators» (η prescription, a future- or light-like vector)

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Basic introduction and LTD

Derivation (general facts)

- *It is crucial to keep track of the prescription!* Duality relation involves the presence of dual propagators:

$$L^{(N)}(p_1, p_2, \ldots, p_N) = -\int_q \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

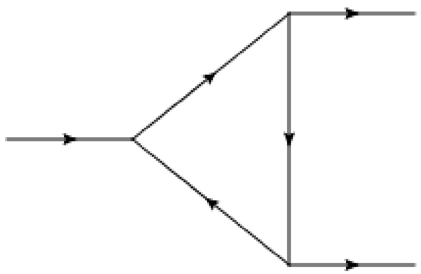
  - The prescription involves a future- or light-like vector (arbitrary) and could depend on the loop momenta (at 1-loop is always independent of *q*). It is related with the finite value of **i0** in intermediate steps
  - *Connection with Feynman Tree Theorem*: **dual prescription** encodes the information contained in **multiple cuts**
  - Implement a shift in each term of the sum to have the same measure: the loop integral becomes a phase-space integral!
  - The unification of coordinates allows a cancellation of singularities among dual components (*UV and soft/collinear divergences remaining*)

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

LTD/FDU approach

- Two different kinds of physical singularities: **UV and IR**
  - IR divergences: *massless triangle*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2 s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2} \right)^{-\epsilon}$$

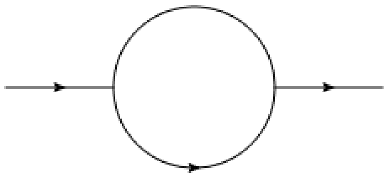


IR pole

IDEA: Define a proper MOMENTUM MAPPING to generate REAL EMISSION KINEMATICS, and use REAL TERMS as fully local IR counter-terms!

  - UV divergences: *bubble with massless propagators*

$$L^{(1)}(p, -p) = \int_l \prod_{i=1}^{2} G_F(q_i) = c_\Gamma \frac{\mu^{2\epsilon}}{\epsilon (1 - 2\epsilon)} (-p^2 - \imath 0)^{-\epsilon}$$



UV pole

IDEA: Define an INTEGRAND LEVEL REPRESENTATION of standard UV counter-terms, and combine it with the DUAL REPRESENTATION of virtual terms!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach

General strategy

- To find the dual representation of Feynman integrals, we follow some steps:
  - ✓ If there are only single poles, we replace standard propagators with dual ones. Otherwise, we compute the residue and remove the energy integral:

$$\mathrm{Res}\,(f, z_0) = \frac{1}{(n-1)!}\left[\frac{\partial^{n-1}}{\partial z^{n-1}}\left((z-z_0)^n f(z)\right)\right]_{z=z_0} \Rightarrow \int d\vec{q}_i \;\mathrm{Res}\left(\prod_j G_F(q_j), q_{i,0}^{(+)}\right)$$

  - ✓ Parametrize momenta; for instance, for 1->2 processes we used

$$\begin{aligned} p_1^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,1) \\ p_2^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,-1) \\ q_i^\mu &= \xi_{i,0}\frac{\sqrt{s_{12}}}{2}\left(1, \sqrt{1-y^2}\hat{e}_T^i, y\right) \end{aligned} \Rightarrow \begin{aligned} y &\in [-1,1] \\ \xi_{i,0} &\in [0,\infty) \\ y &= 1-2v_i \end{aligned}$$

Scalar variables

in the massless case (analogous expressions when massive particles are present)

  - ✓ Factorize the measure in D-dimensions

$$d[\xi_{i,0}] = \frac{\mu^{2\epsilon}(4\pi)^{\epsilon-2}}{\Gamma(1-\epsilon)}\, s_{12}^{-2\epsilon}\, \xi_{i,0}^{-2\epsilon}\, d\xi_{i,0}$$

$$d[v_i] = (v_i(1-v_i))^{-\epsilon}\, dv_i$$

IMPORTANT: *We implement the method within DREG to establish a comparison with traditional results!*

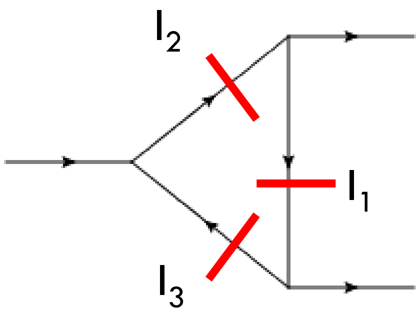
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach

- *Reference example: Massless scalar three-point function in the time-like region*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2}\left(-\frac{s_{12}}{\mu^2} - i0\right)^{-1-\epsilon} = \sum_{i=1}^{3} I_i$$

LTD



$$I_1 = \frac{1}{s_{12}} \int d[\xi_{1,0}]\, d[v_1]\, \xi_{1,0}^{-1}\, (v_1(1-v_1))^{-1}$$

$$I_2 = \frac{1}{s_{12}} \int d[\xi_{2,0}]\, d[v_2]\, \frac{(1-v_2)^{-1}}{1-\xi_{2,0}+i0}$$

$$I_3 = \frac{1}{s_{12}} \int d[\xi_{3,0}]\, d[v_3]\, \frac{v_3^{-1}}{1+\xi_{3,0}-i0}$$

To regularize threshold singularity

- This integral is UV-finite (power counting); there are only IR-singularities, associated to soft and collinear regions
- **OBJECTIVE:** Define a *IR-regularized* loop integral by adding real corrections at integrand level (i.e. no epsilon should appear, 4D representation)

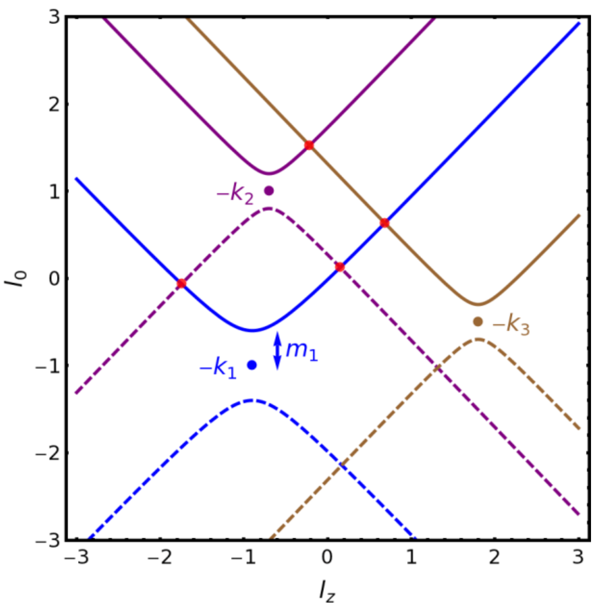
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach

Location of IR singularities in the dual-space

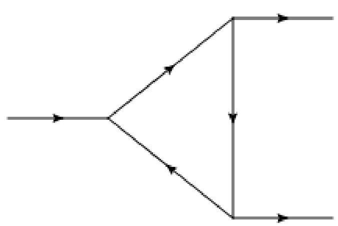
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition:

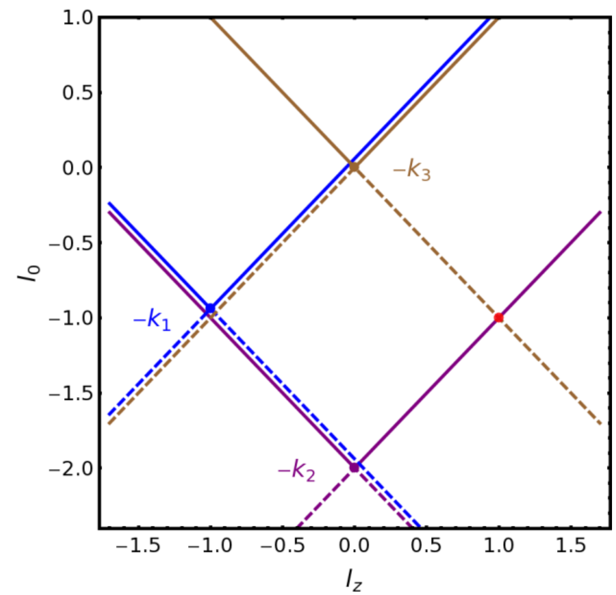
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy solutions.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)





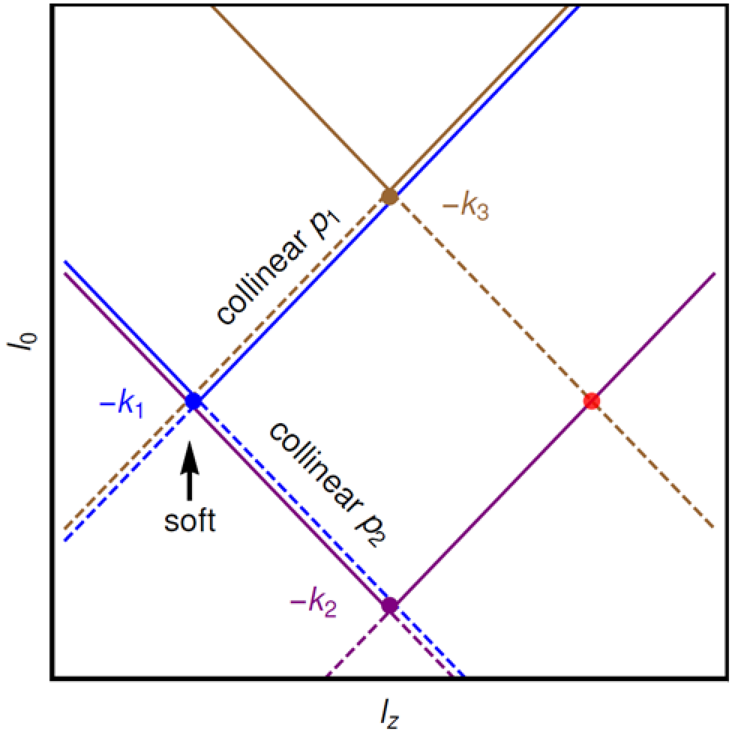
Massless case: light-cones

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

Location of IR singularities in the dual-space

- The application of LTD converts loop-integrals into PS ones: **integration over forward light-cones**.



- Only **forward-backward** interferences originate **threshold or IR poles** (*other propagators become singular in the integration domain*)
- **Forward-forward** singularities cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- **No threshold or IR singularity at large loop momentum**

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically unstable)

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

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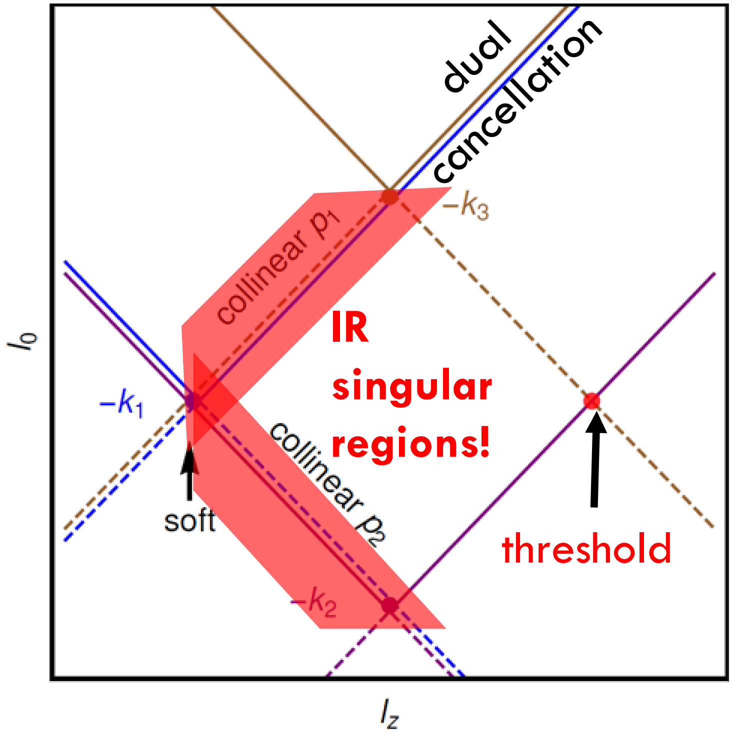

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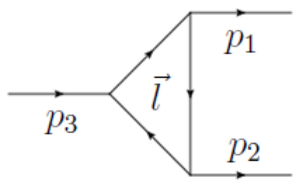
Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach: toy model

Real-virtual momentum mapping

- Suppose **one-loop** scalar scattering amplitude given by the triangle (scalar toy-model!):

Virtual

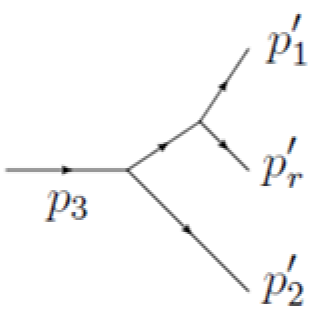


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \quad \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: toy model

Real-virtual momentum mapping

- **Mapping of momenta:** generate **1->3 real** emission kinematics (**3 external on-shell momenta**) starting from the variables available in the dual description of **1->2 virtual** contributions (**2 external on-shell momenta and 1 free three-momentum**)
  - ✓ Split the real phase space into two regions, i.e. $y'_{1r}<y'_{2r}$ and $y'_{2r}<y'_{1r}$, to separate the possible collinear singularities
  - ✓ Implement an optimized mapping in each region, to allow a fully local cancellation of IR singularities with those present in the dual terms

REGION 1:

$$p_r'^{\mu} = q_1^{\mu} \,, \qquad p_1'^{\mu} = p_1^{\mu} - q_1^{\mu} + \alpha_1 \, p_2^{\mu} \,,$$
$$p_2'^{\mu} = (1-\alpha_1)\, p_2^{\mu} \,, \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2} \,,$$

→

$$y'_{1r} = \frac{v_1 \, \xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}} \qquad y'_{12} = 1 - \xi_{1,0}$$
$$y'_{2r} = \frac{(1-v_1)(1-\xi_{1,0})\,\xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}}$$

REGION 2:

$$p_2'^{\mu} = q_2^{\mu} \,, \qquad p_r'^{\mu} = p_2^{\mu} - q_2^{\mu} + \alpha_2 \, p_1^{\mu} \,,$$
$$p_1'^{\mu} = (1-\alpha_2)\, p_1^{\mu} \,, \qquad \alpha_2 = \frac{q_1^2}{2 q_1 \cdot p_1} \,,$$

→

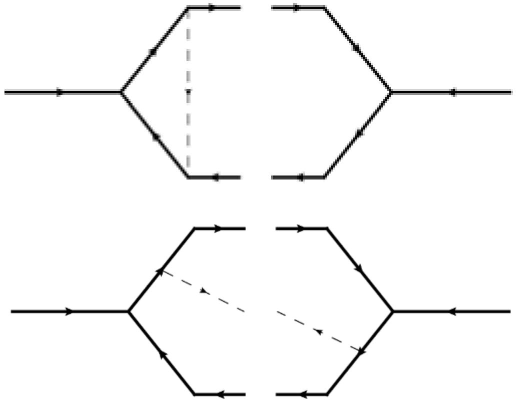
$$y'_{1r} = 1 - \xi_{2,0} \qquad y'_{2r} = \frac{(1-v_2)\,\xi_{2,0}}{1 - v_2\,\xi_{2,0}}$$
$$y'_{12} = \frac{v_2\,(1-\xi_{2,0})\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$

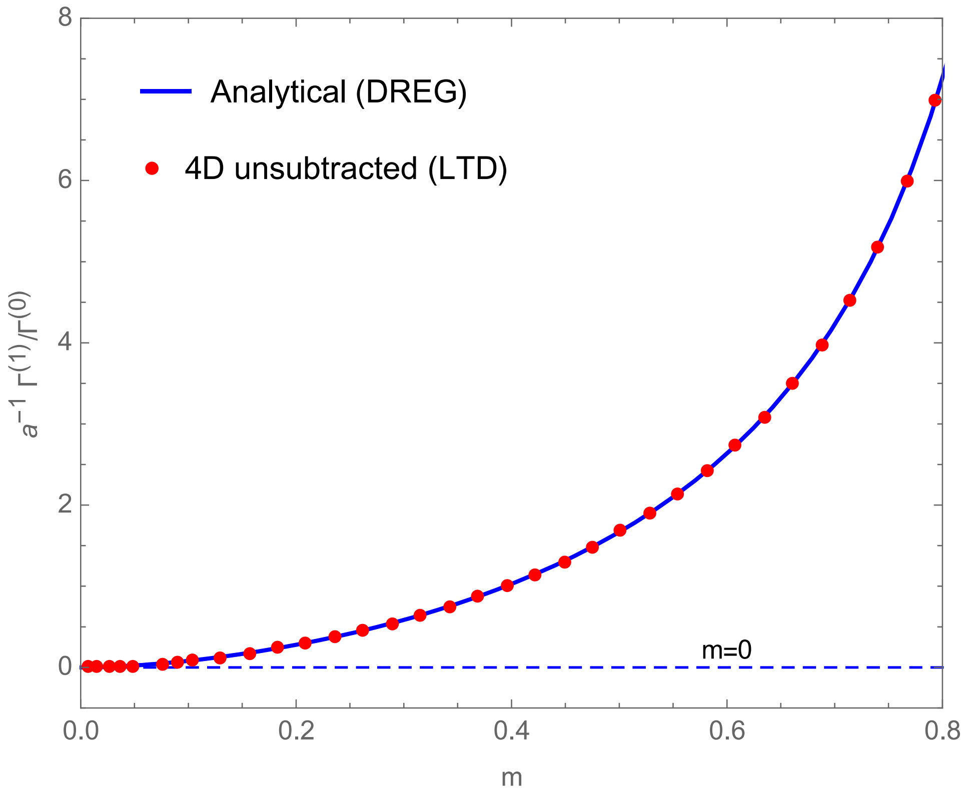
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: toy model

Example: massive scalar three-point function (DREG vs LTD)

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**



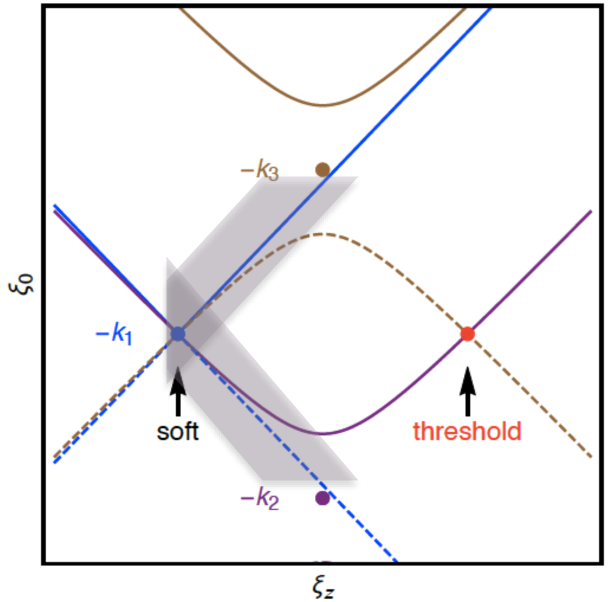


Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: toy model

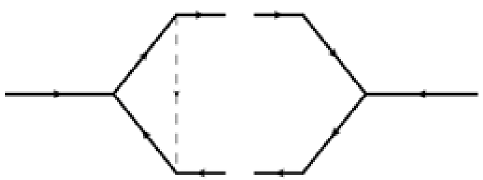
- *About the quasi-collinear configurations:* masses regulate IR singularities, but we need smooth transitions at **INTEGRAND** level to guarantee a smooth limit at **INTEGRAL** level.

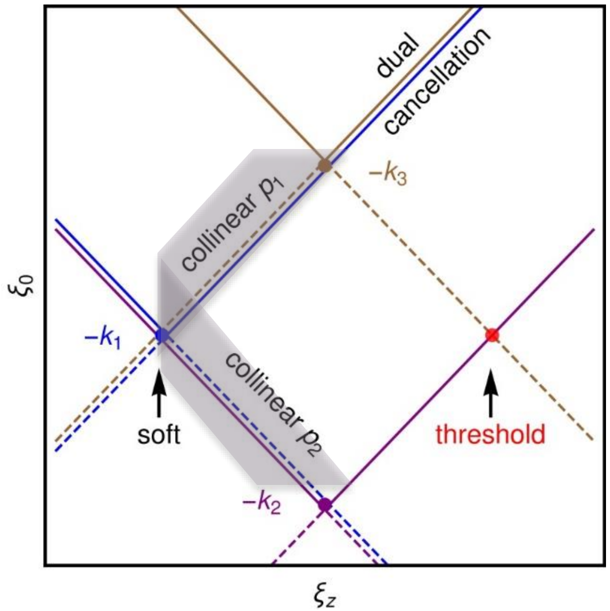
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: on-shell hyperboloids

- **Quasi-collinear** configurations lead to **Log(m²)**, which is singular in the massless limit
- We request a **smooth** behaviour in the massless limit





Massless case: light-cones

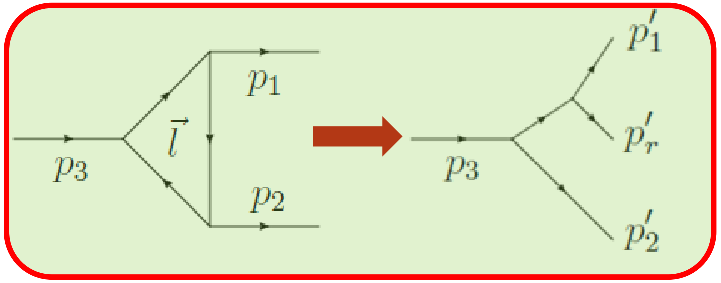
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: multileg

Real-virtual momentum mapping (GENERAL)

- **Real-virtual momentum mapping with massive particles:**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\begin{aligned}
p'^{\mu}_r &= q^{\mu}_1 \\
p'^{\mu}_1 &= (1-\alpha_1)\,\hat{p}^{\mu}_1 + (1-\gamma_1)\,\hat{p}^{\mu}_2 - q^{\mu}_1 \\
p'^{\mu}_2 &= \alpha_1\,\hat{p}^{\mu}_1 + \gamma_1\,\hat{p}^{\mu}_2
\end{aligned}
$$



Impose on-shell conditions to determine mapping parameters

with $\hat{p}_i$ massless four-vectors build using p_i (simplify the expressions)

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

Repeat in each region of the partition…

Rodrigo et al, JHEP10(2016)162

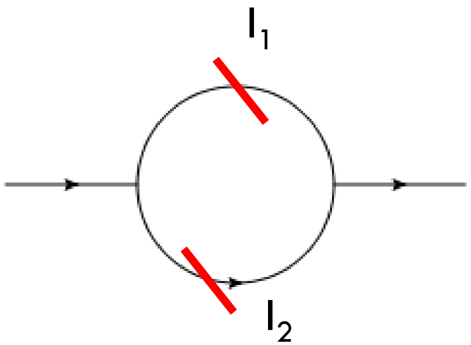
LTD/FDU approach: renormalization

UV singularities

- *Reference example: two-point function with massless propagators*

$$L^{(1)}(p,-p) = \int_\ell \prod_{i=1}^{2} G_F(q_i) = \frac{c_\Gamma}{\epsilon(1-2\epsilon)} \left(-\frac{p^2}{\mu^2} - i0 \right)^{-\epsilon} = \sum_{i=1}^{2} I_i$$

LTD



$$I_1 = -\int_\ell \frac{\tilde\delta(q_1)}{-2q_1\cdot p + p^2 + i0}$$

$$I_2 = -\int_\ell \frac{\tilde\delta(q_2)}{2q_2\cdot p + p^2 - i0}$$

To regularize threshold singularity

- In this case, the integration regions of dual integrals are two energy-displaced forward light-cones. This integral contains UV poles only!
- **OBJETIVE:** Define a *UV-regularized* loop integral by adding unintegrated UV counter-terms, and find a purely 4-dimensional representation of the loop integral

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: renormalization

Location of UV singularities and local counter-terms

- Divergences arise from the high-energy region (UV poles) and can be cancelled with a suitable renormalization counter-term. For the scalar case, we use

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

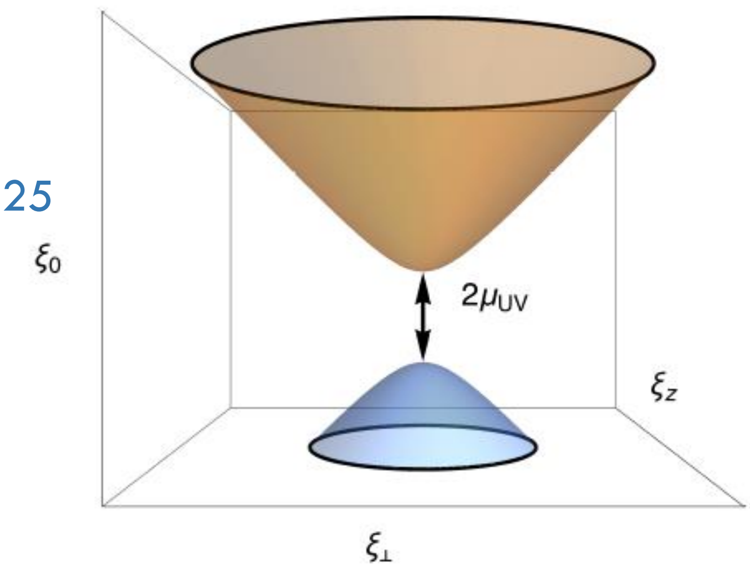
- Dual representation (**new: double poles in the loop energy**)

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

Bierenbaum *et al.* JHEP 03 (2013) 025

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

- Loop integration for loop energies larger than $\mu_{\rm UV}$



Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- LTD must be applied to deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2q_{\rm UV}\cdot k_{i,\rm UV} + k_{i,\rm UV}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2q_{\rm UV}\cdot k_{i,\rm UV})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2}\right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2}\right)$$

Becker, Reuschle, Weinzierl, JHEP12(2010)013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

LTD extended to deal with multiple poles

(use residue formula to obtain the dual representation)

- **3:** Take into account **wave-function and vertex renormalization** constants (not trivial in the massive case!)

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not{p}_1 = M) = 0 \qquad \left.\frac{d\Sigma_R(\not{p}_1)}{d\not{p}_1}\right|_{\not{p}_1=M} = 0$$

- ***Wave-function renormalization constant*** (**both IR and UV poles**):

$$\Delta Z_2(p_1) \;=\; -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2)\frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2}\right) G_F(q_3) \right)$$

- ***Vertex renormalization*** (**only UV**):

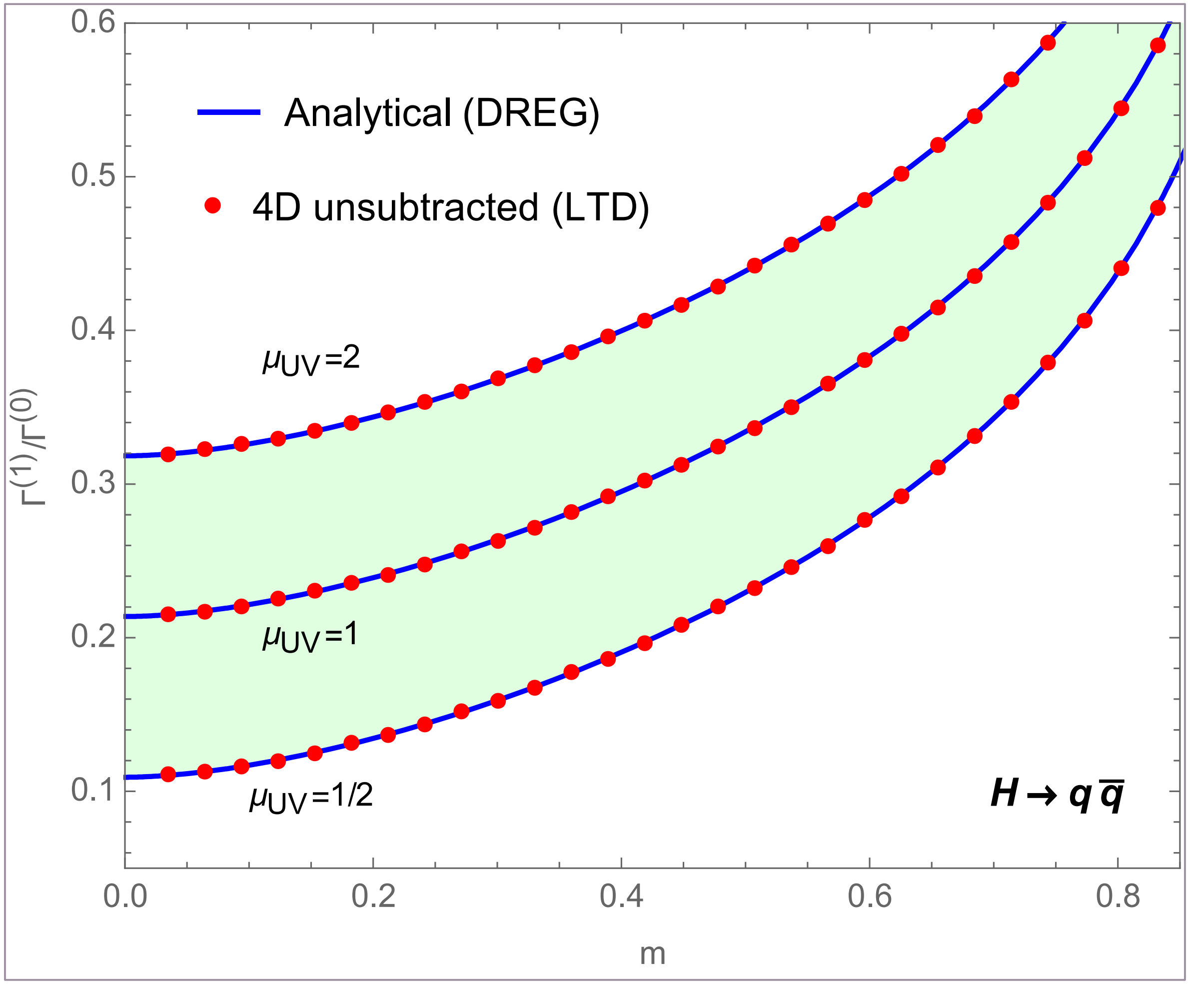
$$\mathbf{\Gamma}^{(1)}_{A,\mathrm{UV}} = g_S^2\, C_F \int_\ell (G_F(q_{\mathrm{UV}}))^3 \left[\gamma^\nu \not{q}_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A\, \not{q}_{\mathrm{UV}}\, \gamma_\nu - d_{A,\mathrm{UV}}\, \mu^2_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A\right]$$

- **Important features:**
  - Integrated results agrees with standard UV counter-terms!
  - **Smooth massless limit!**

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG

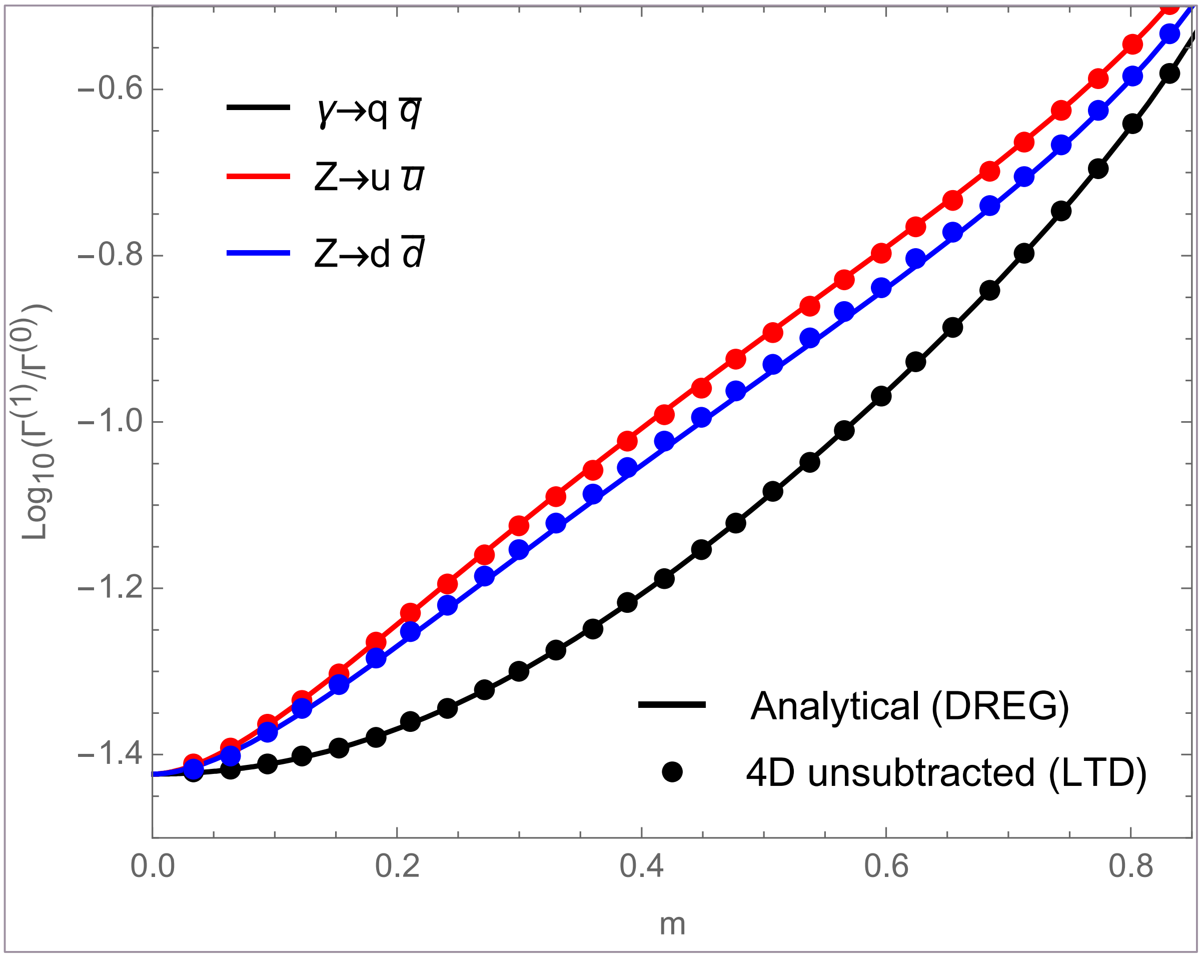


- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG



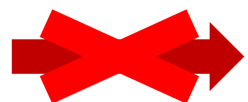
- *Total decay rate for a vector particle into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Efficient numerical implementation
  - Cancellation of UV log's (as in DREG…)

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Important remarks

- The total decay-rate can be expressed using purely **four-dimensional integrands** (which are **integrable** functions!!)
- We recover the total NLO correction**, avoiding to deal with** DREG (ONLY used for comparison with known results)
- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for **ε=0**)

    Finite integral for ε=0  Integrability with ε=0

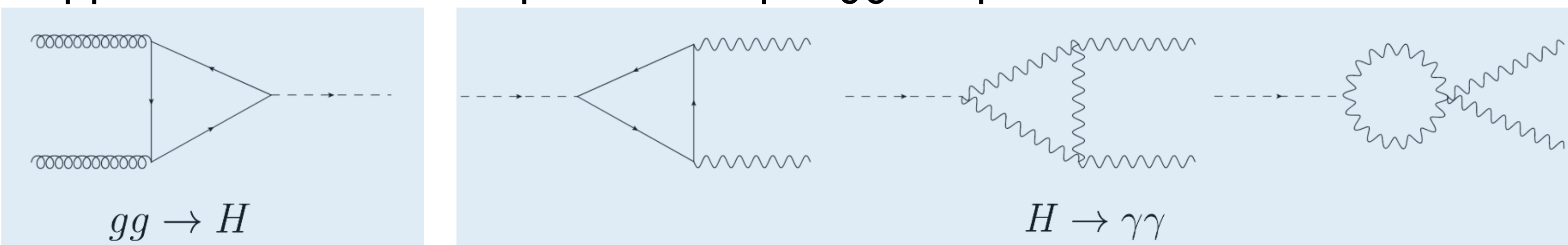
    With FDU is true!
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants,** which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)
  - ✓ **Mapped real-contribution used as a fully local IR counter-term for the dual contribution!**

Rodrigo et al, JHEP10(2016)162

Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:



- They are IR/UV finite BUT still not well-defined in 4D!!! Hidden cancellation of singularities leads to potentially undefined results (scheme dependence!!!)
- We start by defining a tensor basis and projecting (amplitude level!):

$$\mathcal{A}^{(1,f)}_{\mu\nu} = \sum_{i=1}^{5} \mathcal{A}^{(1,f)}_{i} T^{i}_{\mu\nu}$$

with

$$T^{\mu\nu}_i = \left\{ g^{\mu\nu} - \frac{2\, p_1^\nu\, p_2^\mu}{s_{12}}, g^{\mu\nu}, \frac{2\, p_1^\mu\, p_2^\nu}{s_{12}}, \frac{2\, p_1^\mu\, p_1^\nu}{s_{12}}, \frac{2\, p_2^\mu\, p_2^\nu}{s_{12}} \right\}$$

$$P_1^{\mu\nu} = \frac{1}{d-2}\left(g^{\mu\nu} - \frac{2\, p_1^\nu p_2^\mu}{s_{12}} - (d-1) P_2^{\mu\nu} \right)$$

$$P_2^{\mu\nu} = \frac{2\, p_1^\mu p_2^\nu}{s_{12}}$$

Projectors

- Then, scalar coefficients $P_i^{\mu\nu} \mathcal{A}^{(1,f)}_{\mu\nu} = \mathcal{A}^{(1,f)}_i$ are dualized.
- **IMPORTANT:** Take into account 1-2 exchange symmetry (different cuts and non-trivial cancellations!!!)

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Physical example: Higgs@NLO

 Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12}\, M_f^2}{(2\,\ell \cdot p_1)(2\,\ell \cdot p_2)}\, c_1^{(f)} + c_2^{(f)} \right) + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2}\, c_3^{(f)} \right]$$

$$\mathcal{A}_2^{(1,f)} = g_f \, \frac{c_3^{(f)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right)$$

with $q_1 = \ell + p_1,\ q_2 = \ell + p_{12}$ and $q_{1,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_1)^2 + M_f^2}\,, \quad q_{4,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_2)^2 + M_f^2}\,,$

$q_3 = \ell,\ q_4 = \ell + p_2$ $\quad \ell_0^{(+)} = q_{2,0}^{(+)} = q_{3,0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + M_f^2}\,.$

- **Comments:**
  - Generic result valid for $gg \to H$ and $H \to \gamma\gamma$!!
  - Process dependence codified in the coefficients. **Valid for scalar, fermion and vector massive particles inside the loop!!!**

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Combine expressions (use "zero integrals" in DREG associated with Ward identities):

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \frac{s_{12}\, M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)}\, c_1^{(f)} + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_{23}^{(f)} \right]$$

Well defined in 4-d!!

UV divergent

$\mathcal{O}(\epsilon)$

Non-commutativity of limit and integration!!!!

- Use local renormalization (equivalent to Dyson's prescription…)

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}\Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,f)} \right)_{d=4}$$

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,f)} = -g_f \int_\ell \frac{\tilde{\delta}(\ell)\, \ell_0^{(+)}\, s_{12}}{2(q_{\mathrm{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\mathrm{UV},0}^{(+)})^2} \frac{3\,\mu_{\mathrm{UV}}^2}{d-4} \right) c_{23}^{(f)}$$

  - Counter-term mimics UV behaviour at integrand level.
  - Term proportional to μ_{UV}^2 used to fix DREG scheme (vanishing counter-term in d-dim!!)
  - Valid also for W amplitudes in unitary-gauge (naive Dyson's prescription fails to subtract subleading terms due to enhanced UV divergences)

$$-i \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \frac{1}{q_i^2 - M_W^2 + \imath 0}$$

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Physical example: Higgs@NLO

- *Infinite-mass limit used to define effective vertices.* Equivalent to explore asymptotic expansions!
- Expansions at **integrand level** are non-trivial in **Minkowski** space (i.e. within Feynman integrals) and additional factors are neccesary
- **Dual amplitudes** are expressed as **phase-space** integrals ➡ **Euclidean** space!!

$$\tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{s_{12}+2q_3\cdot p_{12}-\imath 0} \xrightarrow{M_f^2 \gg s_{12}} \tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3\cdot p_{12}}\right)^n$$

Expansion of the dual propagator (q_3 on-shell)

- *Example*: Higgs amplitudes with heavy-particles within the loop

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}(s_{12}<4M_f^2)\Big|_{d=4} = \frac{M_f^2}{\langle v\rangle} \int_\ell \tilde{\delta}(\ell) \left[\frac{3\,\mu_{\mathrm{UV}}^2\,\ell_0^{(+)}}{(q_{\mathrm{UV},0}^{(+)})^5}\,\hat{c}_{23}^{(f)} + \frac{M_f^2}{(\ell_0^{(+)})^4}\left(\sum_{n=0}^{\infty} Q_n(z)\left(\frac{s_{12}}{(2\ell_0^{(+)})^2}\right)^n\right) c_1^{(f)}\right]$$

$$z = (2\boldsymbol{\ell}\cdot\mathbf{p}_1)/(\ell_0^{(+)}\sqrt{s_{12}}) \quad \text{and} \quad Q_n(z) = \frac{1}{1-z^2}\left(P_{2n}(z)-1\right)$$

Reproduces all the known-results!!

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Towards two loops…

Introducing the notation

- Dual amplitudes can be defined at **higher-orders** (even with multiple poles)

  Bierenbaum, Catani, Draggiotis, Rodrigo; JHEP 10 (2010) 073

- *Standard example:* two-loop N-point scalar amplitude

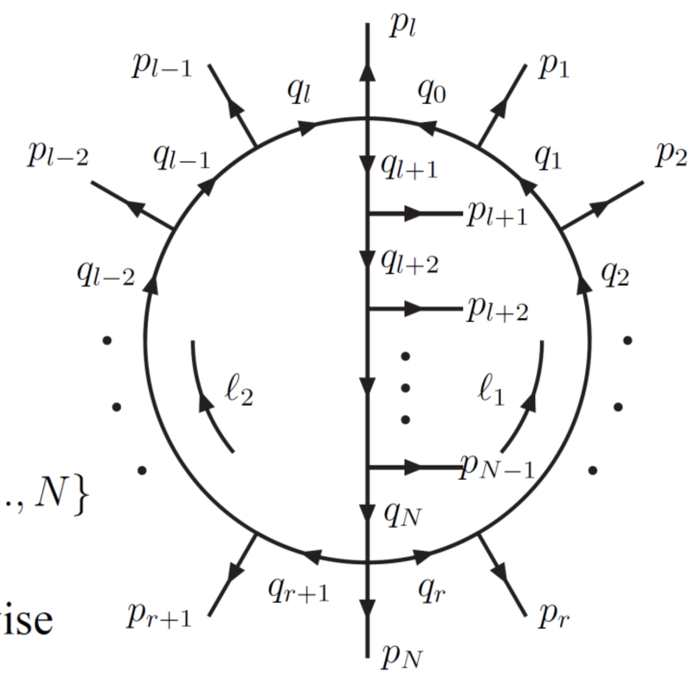
$$L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3)$$

$$\int_{\ell_i} \bullet = -\imath \int \frac{d^d \ell_i}{(2\pi)^d} \bullet \quad , \quad G_F(\alpha_k) = \prod_{i \in \alpha_k} G_F(q_i)$$

- Three possible sets of momenta, according to their dependence on l_1, l_2 or l_1+l_2 (*integration variables*)

$$\alpha_1 \equiv \{0, 1, ..., r\} \, , \alpha_2 \equiv \{r+1, r+2, ..., l\} \, , \alpha_3 \equiv \{l+1, l+2, ..., N\}$$

$$q_i = \begin{cases} \ell_1 + p_{1,i} & , i \in \alpha_1 \\ \ell_2 + p_{i,l-1} & , i \in \alpha_2 \\ \ell_1 + \ell_2 + p_{i,l-1} & , i \in \alpha_3 \end{cases} \quad \text{with} \quad \begin{array}{ll} \ell_1 & \text{anti–clockwise} \\ \ell_2 & \text{clockwise} \end{array}$$



Generic two-loop diagram

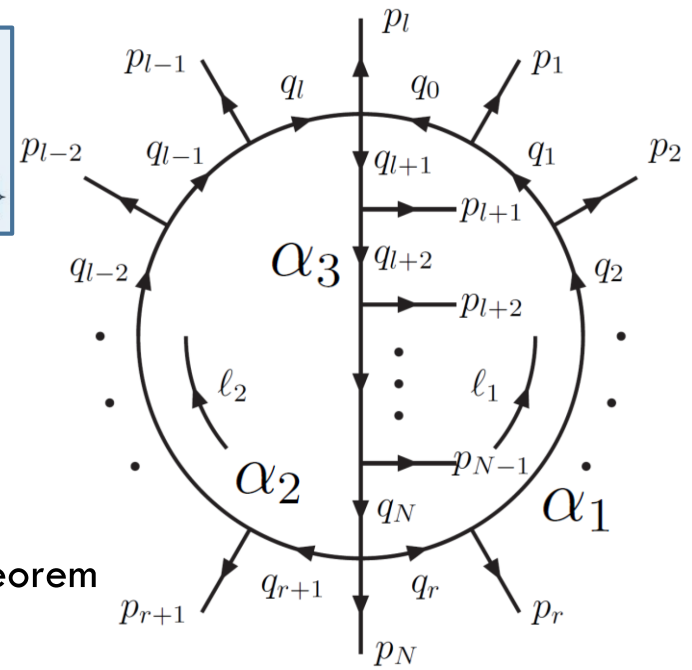
Driencourt-Mangin, Rodrigo and G.S., in progress

Towards two loops…

Cuts and LTD formula

- **"The number of cuts equals the number of loops"**
- *Derivation:* "Iterate" the one-loop formula and use propagator properties
- *Standard example:* two-loop N-point scalar amplitude

$$L^{(2)}(p_1, p_2, \dots, p_N) = \int_{\ell_1} \int_{\ell_2} \{G_D(\alpha_2)\, G_D(\alpha_1 \cup \alpha_3) + G_D(-\alpha_2 \cup \alpha_1)\, G_D(\alpha_3) - G_F(\alpha_1)\, G_D(\alpha_2)\, G_D(\alpha_3)\}$$



where we used $G_D(\alpha_k) = \sum_{i \in \alpha_k} \tilde{\delta}(q_i) \prod_{\substack{j \in \alpha_k \\ j \neq i}} G_D(q_i; q_j)$

- **Remarks and subtleties:**
  - Modified prescription depends on loop momenta.
  - Not a "trivial" iteration: connection with Cauchy's theorem and multivariable residues.
  - *Thesis:* "Virtual-real amplitudes mapped with one-loop formulae" (*partial cancellations*), but a new mapping required for double-real emission.

Driencourt-Mangin, Rodrigo and G.S., in progress

Conclusions and perspectives

- ✓ Loop-tree duality allows to treat **virtual and real** contributions **in the same way** (implementation simplified)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals (light-cone diagrams)
- ✓ **Combined virtual-real terms are integrable in 4D!!**
- ✓ **Realistic physical implementation!!!**
- ✓ **Universal & compact expressions for Higgs amplitudes**

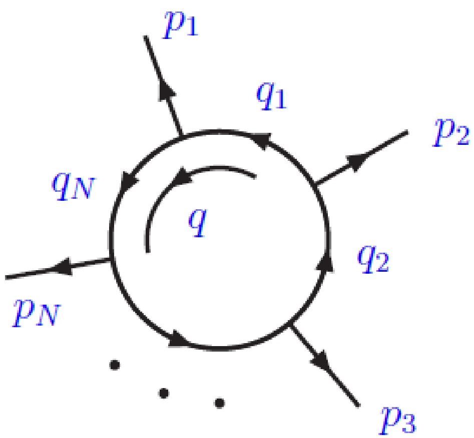
- **Perspectives:**
  - Automation of multileg processes at NLO (ongoing) and beyond (…)
  - Generalization of universality relations and simplified asymptotic expansions
  - *Carefull comparison with other schemes* →

"Workstop-Thinkstart meeting"
UZH, Zurich, Sep. 2016
Eur.Phys.J. C77 (2017) no.7, 471
arXiv:1705.01827 [hep-ph]

Thanks!!!!

Extra material

Cauchy's theorem and prescriptions



$$L_R^{(N)}(p_1, p_2, \cdots, p_N) = -\imath \int \frac{d^d \ell}{(2\pi)^d} \prod_{i=1}^{N} G_R(q_i)$$

Generic one-loop Feynman integral

$$q_i = \ell + \sum_{k=1}^{i} p_k$$

Momenta definition

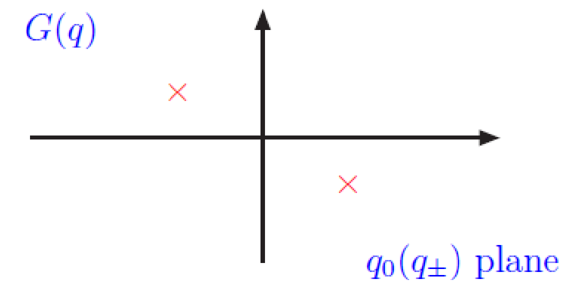
Prescriptions are useful to avoid poles. Different prescriptions are possible; connection between FTT and LTD theorems!

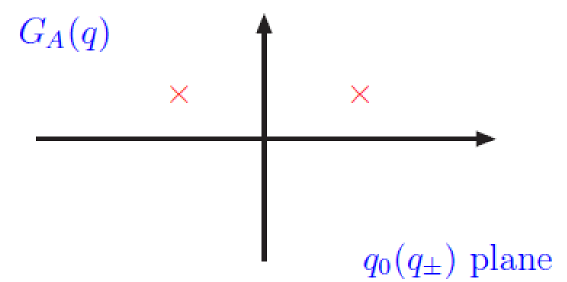
Feynman propagator

$$G(q) \equiv \frac{1}{q^2 + \imath 0}$$

Advanced propagator

$$G_A(q) \equiv \frac{1}{q^2 - \imath 0\, q_0}$$

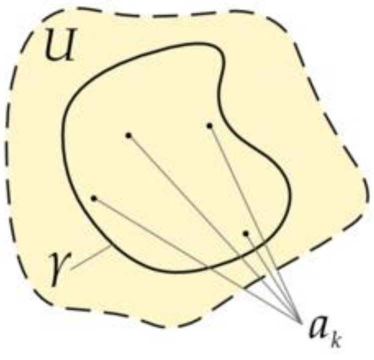




Cauchy's theorem and prescriptions

Residue theorem (*from Wikipedia*)

$$\oint_\gamma f(z)\,dz = 2\pi i \sum \mathrm{Res}(f, a_k)$$

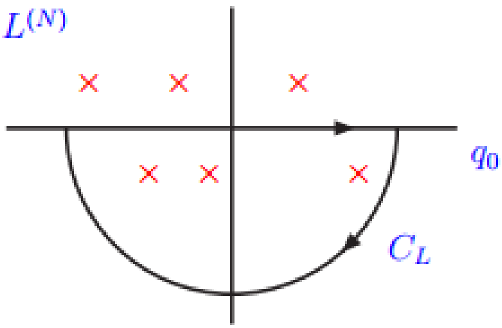


«If f is a holomorphic function in U/{a_i}, and γ a simple positively oriented curve, then the integral is given by the sum of the residues at each singular point a_i»

Residue theorem can be used to compute integrals involving propagators: the prescription and the contour that we choose determine the result!

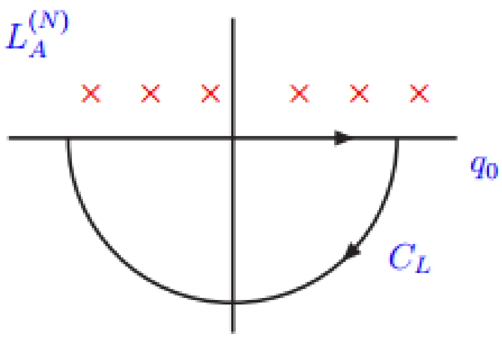
Feynman propagator

$$[G(q)]^{-1} = 0 \implies q_0 = \pm\sqrt{\mathbf{q}^2 - i0}$$



Advanced propagator

$$[G_A(q)]^{-1} = 0 \implies q_0 \simeq \pm\sqrt{\mathbf{q}^2} + i0$$



NO POLES CLOSED BY C_L!

Feynman tree theorem

Derivation

- **Idea:** *«Sum over all possible m-cuts»*

$$L_A^{(N)}(p_1, p_2, \dots, p_N) = \int_q \prod_{i=1}^{N} G_A(q_i) = 0$$

Residue theorem (using a proper integration path)

$$G_A(q) = G(q) + \tilde{\delta}(q)$$

Using PV prescription

$$L_A^{(N)}(p_1, p_2, \dots, p_N) = \int_q \prod_{i=1}^{N} \left[G(q_i) + \tilde{\delta}(q_i) \right]$$

$$= L^{(N)}(p_1, p_2, \dots, p_N) + L^{(N)}_{1-\mathrm{cut}}(p_1, p_2, \dots, p_N) + \dots + L^{(N)}_{\mathrm{N-cut}}(p_1, p_2, \dots, p_N)$$

m-cut definition:

$$L^{(N)}_{\mathrm{m-cut}}(p_1, p_2, \dots, p_N) = \int_q \left\{ \tilde{\delta}(q_1) \dots \tilde{\delta}(q_m) \, G(q_{m+1}) \dots G(q_N) + \text{uneq. perms.} \right\}$$

$$L^{(N)}(p_1, p_2, \dots, p_N) = - \left[L^{(N)}_{1-\mathrm{cut}}(p_1, p_2, \dots, p_N) + \dots + L^{(N)}_{\mathrm{N-cut}}(p_1, p_2, \dots, p_N) \right]$$

Feynman tree theorem

Derivation

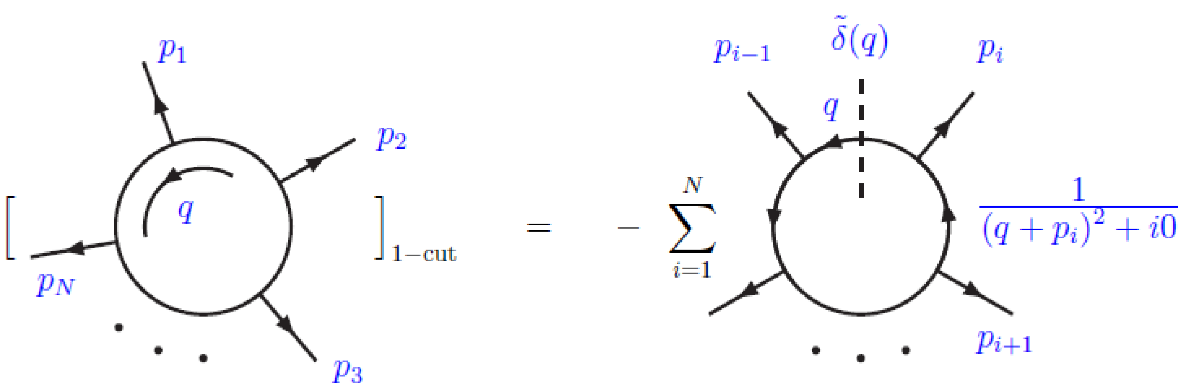
- Some remarks:
  - Making m-cuts decomposes the original **one-loop** diagram into **m-tree level terms,** all of them using the **same prescription**
  - *1-cut = sum over «tree level» terms*

$$L^{(N)}_{1-\mathrm{cut}}(p_1, p_2, \ldots, p_N) = \int_q \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G(q_j)$$

$$I^{(n)}_{1-\mathrm{cut}}(k_1, k_2, \ldots, k_n) = \int_q \tilde{\delta}(q) \prod_{j=1}^{n} G(q+k_j) = \int_q \tilde{\delta}(q) \prod_{j=1}^{n} \frac{1}{2qk_j + k_j^2 + i0}$$

Basic 1-cut integral (shift in loop momentum)

Loop



Sum of multiple cuts

IR singularities within LTD

Compactness of IR singular regions (massless triangle)

- From the previous plots, we define three contributions:

IR-divergent contributions ($x_0<1+w$)

- *Originated in a **finite region** of the loop three-momentum*
- *All the IR singularities of the original loop integral*

$$I^{\rm IR} = I_1^{(\rm s)} + I_1^{(\rm c)} + I_2^{(\rm c)} = \frac{c_\Gamma}{s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2}\right)^{-\epsilon} \times \left[\frac{1}{\epsilon^2} + \left(\ln(2)\ln(w) - \frac{\pi^2}{3} - 2{\rm Li}_2\left(-\frac{1}{w}\right) + \imath\pi\ln(2)\right)\right] + \mathcal{O}(\epsilon)$$

Forward integrals ($v<1/2, x_0>1$)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\rm f)} = \sum_{i=1}^{3} I_i^{(\rm f)} = c_\Gamma \frac{1}{s_{12}} \left[\frac{\pi^2}{3} - \imath\pi \, \log(2)\right] + \mathcal{O}(\epsilon)$$

Backward integrals ($v>1/2, x_0>1+w$)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\rm b)} = c_\Gamma \frac{1}{s_{12}} \left[2{\rm Li}_2\left(-\frac{1}{w}\right) - \ln(2)\ln(w)\right] + \mathcal{O}(\epsilon)$$

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

IR singularities within LTD

Technical details

- Let's stop and make some remarks about the structure of these expressions:
  - Introduction of an **arbitrary cut** w to **include threshold regions.**
  - Forward and backward integrals can be performed in 4D because the sum does not contain poles.
  - Presence of extra Log's in (F) and (B) integrals. They are originated from the expansion of the measure in DREG, i.e.

$$\xi_r^{-1-2\epsilon} = -\frac{Q_S^{-2\epsilon}}{2\epsilon}\delta(\xi_r) + \left(\frac{1}{\xi_r}\right)_C - 2\epsilon\left(\frac{\ln(\xi_r)}{\xi_r}\right)_C + \mathcal{O}(\epsilon^2)$$

for both v and ξ (keep finite terms only). **Unify coordinate system to avoid them!**

  - IR-poles isolated in I^{IR}! → **IR divergences originated in compact region of the three-loop momentum!!!**

$$L^{(1)}(p_1, p_2, -p_3) = \underbrace{I^{\mathrm{IR}}}_{\text{Explicit poles still present...}} + \underbrace{I^{(b)} + I^{(f)}}_{\text{Can be done in 4D!}}$$

Explicit poles still present…

Can be done in 4D!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

UV regularized bubble with LTD

Cancellation of UV singularities

- Using the standard parametrization we define

Regularized two-point function

$$L^{(1)}(p,-p) - I_{\rm UV}^{\rm cnt} = c_\Gamma \left[-\log\left(-\frac{p^2}{\mu_{\rm UV}^2} - i0 \right) + 2 \right] + \mathcal{O}(\epsilon)$$

- Since it is finite, we can express the regularized two-point function in terms of 4-dimensional quantities (i.e. no epsilon required!!)

- ***Physical interpretation of renormalization scale*:** Separation between on-shell hyperboloids in UV-counterterm is $2\mu_{UV}$. To avoid intersections with forward light-cones associated with l_1 and l_2, the renormalization scale has to be larger or of the order of the hard scale. So, the minimal choice that fulfills this agrees with the standard choice (i.e. ½ of *the hard scale*).

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

γ>qqbar@NLO: 4D formulae

- Integration regions: $$\mathcal{R}_1(\xi_0, v) = \theta(1-2v_1)\,\theta\left(\frac{1-2v_1}{1-v_1} - \xi_{1,0}\right)\bigg|_{\{\xi_{1,0},v_1\}\to\{\xi_{3,0},v_3\}=\{\xi_0,v\}}$$ $$\mathcal{R}_2(\xi_0, v) = \theta\left(\frac{1}{1+\sqrt{1-v}} - \xi_0\right)$$
- Four-dimensional cross-sections:

$$\widetilde{\sigma}_1^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{1,0} \int_0^{1/2} dv_1\, 4\,\mathcal{R}_1(\xi_{1,0}, v_1) \left[2\left(\xi_{1,0} - (1-v_1)^{-1}\right) - \frac{\xi_{1,0}(1-\xi_{1,0})}{(1-(1-v_1)\,\xi_{1,0})^2}\right]$$

$$\widetilde{\sigma}_2^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{2,0} \int_0^{1} dv_2\, 2\,\mathcal{R}_2(\xi_{2,0}, v_2)\,(1-v_2)^{-1} \left[\frac{2\,v_2\,\xi_{2,0}\left(\xi_{2,0}(1-v_2)-1\right)}{1-\xi_{2,0}}\right.$$

$$\begin{aligned}
\overline{\sigma}_V^{(1)} = {} & \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^\infty d\xi \int_0^1 dv \left\{ -2\,(1-\mathcal{R}_1(\xi,v))\,v^{-1}(1-v)^{-1}\,\frac{\xi^2(1-2v)^2+1}{\sqrt{(1+\xi)^2-4v\,\xi}} \right. \\
& + 2\,(1-\mathcal{R}_2(\xi,v))\,(1-v)^{-1}\left[2\,v\,\xi\,\left(\xi(1-v)-1\right)\left(\frac{1}{1-\xi+\imath 0} + \imath\pi\delta(1-\xi)\right) - 1 + v\,\xi\right] \\
& + 2\,v^{-1}\left(\frac{\xi(1-v)(\xi(1-2v)-1)}{1+\xi} + 1\right) - \frac{(1-2v)\,\xi^3\,(12-7m_{UV}^2-4\xi^2)}{(\xi^2+m_{UV}^2)^{5/2}} \\
& \left. - \frac{2\,\xi^2(m_{UV}^2 + 4\xi^2(1-6v(1-v)))}{(\xi^2+m_{UV}^2)^{5/2}} \right\}
\end{aligned}$$

Higgs amplitudes coefficients

The coefficients $c_i^{(f)}$ are written as $c_i^{(f)} = c_{i,0}^{(f)} + r_f\, c_{i,1}^{(f)}$ with $r_f = s_{12}/M_f^2$, and

$$g_f = \frac{2\, M_f^2}{\langle v \rangle\, s_{12}} \,, \quad c_{23,0}^{(f)} = (d-4)\frac{c_{1,0}^{(f)}}{2} \,, \quad c_{1,0}^{(\phi)} = \frac{2}{d-2} \,, \; c_{1,0}^{(W)} = \frac{4\,(d-1)}{d-2} \,, \quad c_{1,1}^{(W)} = -\frac{2(2\,d-5)}{d-2} \,,$$

$$c_{3,0}^{(\phi)} = 2 \,, \quad c_{1,0}^{(t)} = \frac{8}{d-2} \,, \quad c_{1,1}^{(t)} = -1 \,, \quad c_{3,0}^{(t)} = 8 \,, \quad c_{23,1}^{(W)} = \frac{d-4}{d-2} \,, \quad c_{3,0}^{(W)} = 4(d-1) \,,$$

with $c_2^{(f)} = c_{23}^{(f)} - c_3^{(f)}$ and $c_{1,1}^{(\phi)} = c_{23,1}^{(\phi)} = c_{3,1}^{(\phi)} = c_{23,1}^{(t)} = c_{3,1}^{(t)} = c_{3,1}^{(W)} = 0.$

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]