Initial state and gluon saturation

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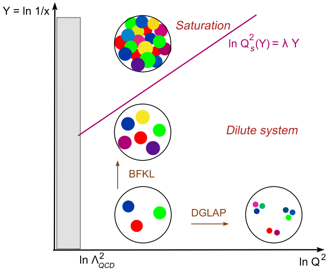
Outline

- Introduction: High-energy QCD formalism at LO+LL accuracy
- Gluon saturation beyond LO+LL
  Ex: DIS at NLO
- IP-Glasma model and proton shape fluctuations
- Flow without hydro in pA??

Kinematical regimes of DIS

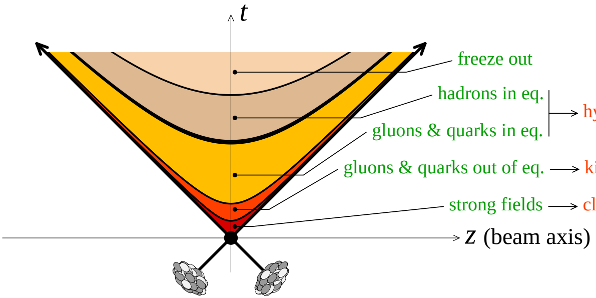


- For $Q^2 \to +\infty$: target more and more dilute due to DGLAP evolution.
  $\Rightarrow$ QCD-improved parton model more and more valid.

- For $x_{Bj} \to 0$: target more and more dense due to BFKL
  $\Rightarrow$ Linear BFKL evolution eventually breaks down, as well as the parton picture.

  Onset of nonlinear collective effects: Gluon saturation!
  $\to$ Regime of strong gauge fields but weak coupling α_s

Space-time picture of heavy ion collisions



Earliest stage of heavy ion collisions:

- Collision the saturated low-x_{Bj} gluons of the incoming nuclei
- Non-linear out-of-equilibrium evolution of the resulting gluon field (Glasma)

$\rightarrow$ Determines initial conditions for later stages: hydro or kinetic theory

$\rightarrow$ Drives the bulk of particle production (soft and semi-hard)

Universality of high-energy/CGC factorization

Many high-energy observables can be written in a factorized way in terms of the same non-perturbative objects (dipole-target amplitude, . . .)

$\Rightarrow$ General program:

ep, eA : Fits of the non-perturbative distributions, using high-energy (N)LL evolution equations

pp, pA : Check of the universality of the high-energy factorization, and further constraints

AA : Calculate *Glasma* initial conditions from first principles and from previous experimental constraints
$\rightarrow$ Use JIMWLK factorization formulae for AA from
Gelis, Lappi, Venugopalan (2008-2009)

Preliminary realization of the complete program (but no LL resummation): IP-Glasma model
Schenke, Tribedy, Venugopalan (2012-2013)

Eikonal dilute-dense scattering

Recipe for *dilute-dense* processes at high-energy, following Bjorken, Kogut and Soper (1971):

- Decompose the projectile on a Fock basis at the time $x^+ = 0$, with appropriate Light-Front wave-functions.
- Each parton n scatters independently on the target via a light-like Wilson line $U_{\mathcal{R}_n}(\mathbf{x}_n)$ through the target:

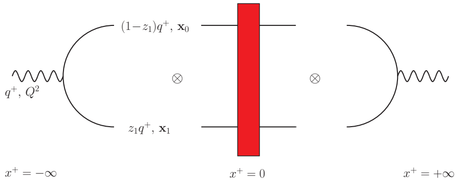
$$U_{\mathcal{R}_n}(\mathbf{x}_n) = \mathcal{P}_+ \exp\left[-ig \int dx^+ \, T^a_{\mathcal{R}_n} \, A_a^-(x^+, \mathbf{x}_n) \right]$$

with $\mathcal{R}_n = A$, F or $\bar{F}$ for g, q or $\bar{q}$ partons.

- Include final-state evolution of the projectile remnants.

→ Light-cone gauge $A_a^+ = 0$ strongly recommended!

Dipole factorization for DIS at LO



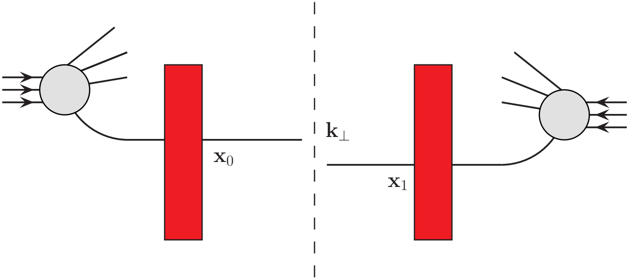
$$\sigma_{T,L}^{\gamma p \to X}(x_{Bj}, Q^2) = \frac{4N_c\, \alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\, \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \times \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1, Q^2) \left[1 - \langle \mathcal{S}_{01} \rangle_Y \right]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

Dipole operator: $\mathcal{S}_{01} = \frac{1}{N_c} \mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1) \right)$

with "rapidity" $Y \sim \log(1/x_{Bj})$ for $x_{Bj} \to 0$.
$\to$ Dependence of $\langle \mathcal{S}_{01} \rangle_Y$ on Y comes from high-energy (low-x_{Bj}) LL resummation.

Forward single-inclusive particle production in pA at LO



$$\frac{\mathrm{d}\sigma^{pA\to q+X}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{k}_\perp} = \frac{1}{(2\pi^2)} \sum_f x\, q_f(x,\mu_F^2) \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1\, e^{-i\mathbf{k}_\perp\cdot(\mathbf{x}_0-\mathbf{x}_1)}\ \langle \mathcal{S}_{01}\rangle_Y$$

with $x = e^y\, |\mathbf{k}_\perp|/\sqrt{s}$ and $Y = y + \log(|\mathbf{k}_\perp|/\sqrt{s})$

Fragmentation functions and gluon channel can be included as well easily.
$\rightarrow$ *Hybrid factorization*

Dumitru, Hayashigaki, Jalilian-Marian (2002-2006)

B-JIMWLK and BK evolutions

RG evolution for the dipole amplitude at LL accuracy:

$$\partial_Y \langle \mathcal{S}_{01} \rangle_Y = \frac{2\alpha_s C_F}{\pi} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \langle \mathcal{S}_{012} - \mathcal{S}_{01} \rangle_Y$$
$$= \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \langle \mathcal{S}_{02}\mathcal{S}_{21} - \mathcal{S}_{01} \rangle_Y$$

with $\bar{\alpha} = N_c \alpha_s / \pi$, and the $q\bar{q}g$ "tripole" operator

$$\mathcal{S}_{012} \equiv \frac{1}{N_c C_F} \mathrm{Tr}\Big(U_F(\mathbf{x}_0) t^a U_F^\dagger(\mathbf{x}_1) t^b \Big)\, U_A^{ba}(\mathbf{x}_2) = \frac{N_c}{2C_F} \left[\mathcal{S}_{02}\, \mathcal{S}_{21} - \frac{1}{N_c^2} \mathcal{S}_{01} \right]$$

New object $\langle \mathcal{S}_{012} \rangle_Y$ or $\langle \mathcal{S}_{02}\mathcal{S}_{21} \rangle_Y$ appears $\Rightarrow$ only the first equation in B-JIMWLK infinite hierarchy.

In practice: truncate the hierarchy with the approx $\langle \mathcal{S}_{02}\mathcal{S}_{21} \rangle_Y \simeq \langle \mathcal{S}_{02} \rangle_Y \langle \mathcal{S}_{21} \rangle_Y$ to get the BK equation.
Balitsky (1996); Kovchegov (1999)

MV model for a large nucleus

Effective content of an ultra-relativistic nucleus:

- Low x_{Bj}: shockwave field $A_a^-(x^+, \mathbf{x})$
- Larger x_{Bj}: eikonal color current $J_a^\mu(x) = \delta^\mu_-\, \rho_a(x^+, \mathbf{x})$

with $-\Delta_\mathbf{x} A_a^-(x^+, \mathbf{x}) = \rho_a(x^+, \mathbf{x})$

In the absence of LL evolution, and in the large nucleus (A) limit:

Correlations of $\rho_a(x^+, \mathbf{x})$ become Gaussian, with

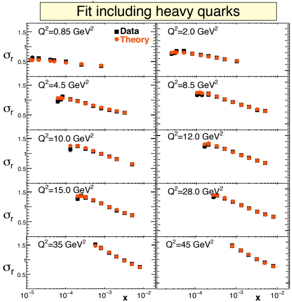
$$\left\langle \rho_a(x^+, \mathbf{x})\, \rho_b(y^+, \mathbf{y}) \right\rangle_{MV} = \delta_{ab}\, \delta(x^+ - y^+)\, \delta^{(2)}(\mathbf{x} - \mathbf{y})\, g^2\, \mu^2(x^+, \mathbf{x})$$

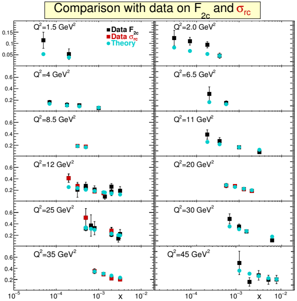
$$g^2 \left\langle A_a^-(x^+, \mathbf{x})\, A_b^-(y^+, \mathbf{y}) \right\rangle_{MV} = \delta_{ab}\, \delta(x^+ - y^+)\, L_{\mathbf{xy}}(x^+)$$

$$\left\langle \mathcal{S}_{\mathbf{xy}} \right\rangle_{MV} = \exp\left[-\frac{C_F}{2} \int dx^+ \Big(L_{\mathbf{xx}}(x^+) + L_{\mathbf{yy}}(x^+) - 2L_{\mathbf{xy}}(x^+) \Big) \right]$$

McLerran, Venugopalan (1994)
→ Suitable initial condition for high-energy LL evolution
Remark: $L_{\mathbf{xy}}(x^+)$ depends on a IR cut-off, or gluon mass

DIS phenomenology





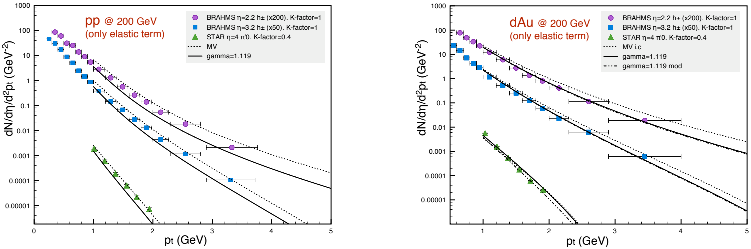
Fits of the reduced DIS cross-section σ_r and its charm contribution σ_{rc} at HERA data with numerical solutions of the running coupling BK equation.

Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
see also: Kuokkanen, Rummukainen, Weigert (2012);
Lappi, Mäntysaari (2013); ...

Good fit, but require a big rescaling of Λ_{QCD} as extra parameter, to slow down the BK evolution.

$\rightarrow$ Mimics missing higher order contributions, like a K-factor.

Phenomenology for single-inclusive particle production



Fits of the single-inclusive hadron or pion production cross-section at forward rapidity in p-p and d-Au collisions at RHIC, using the hybrid factorization at LO, and running coupling BK evolution.

Similar results at LHC (p-p and p-Pb) and Tevatron (p-p) at central rapidity, using $k_\perp$-factorization.

Albacete, Dumitru, Fujii, Nara (2013)

see also: Albacete, Marquet (2010); Lappi, Mäntysaari (2013); . . .

High-energy QCD beyond LO+LL

Higher order corrections needed for higher precision for high-energy QCD with gluon saturation

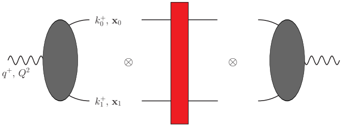
→ Fixed order NLO corrections to observables:

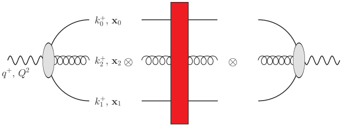
- Forward single inclusive hadron production in pA:
  Chirilli, Xiao, Yuan (2012)
  Original results unstable! Now under control using more consistent scheme for LL factorization.
  Iancu, Mueller, Triantafyllopoulos (2016); Ducloué, Lappi, Zhu (2017)
- DIS structure functions: see next slides

→ NLO corrections to the BK and B-JIMWLK equations, to perform NLL resummation: now available
Balitsky, Chirilli (2008-2013); Kovner, Lublinsky, Mulian (2013-2016)

High-energy NLL equations require collinear resummations (like BFKL, see Salam (1998))
Main piece of the collinear resummation already done for NLL BK
G.B. (2014); Iancu et al. (2015)

DIS at NLO: general structure in dipole factorization





$$\sigma_{T,L}(Q^2, x_{Bj}) = \sum_{q\bar{q}\ \text{states}} \left|\widetilde{\Psi}_{q\bar{q}}^{\gamma_{T,L}^*}\right|^2 \Big[1 - \langle \mathcal{S}_{01}\rangle_0\Big] + \sum_{q\bar{q}g\ \text{states}} \left|\widetilde{\Psi}_{q\bar{q}g}^{\gamma_{T,L}^*}\right|^2 \Big[1 - \langle \mathcal{S}_{012}\rangle_0\Big] + O(\alpha_{em}\,\alpha_s^2)$$

- Perturbative building blocks for NLO DIS:
  $\widetilde{\Psi}_{q\bar{q}}^{\gamma_{T,L}^*}$ LFWF at one loop and $\widetilde{\Psi}_{q\bar{q}g}^{\gamma_{T,L}^*}$ LFWF at tree-level
- UV divergences should cancel between $q\bar{q}$ and $q\bar{q}g$ ($\rightarrow$ Dim. Reg.)
- High-energy LL resummation to be performed at the end

G.B. (2016-2017)

DIS at NLO: results

Final results, after cancellation of UV divergences and LL resummation:

$$\sigma^{\gamma}_{T,L} = \sigma^{\gamma}_{T,L}\big|_{\rm dipole} + \sigma^{\gamma}_{T,L}\big|_{q\to g} + \sigma^{\gamma}_{T,L}\big|_{\bar{q}\to g}$$
$$= \sigma^{\gamma}_{T,L}\big|_{\rm dipole} + 2\,\sigma^{\gamma}_{T,L}\big|_{q\to g}$$

Where:

$$\sigma^{\gamma}_{L}\big|_{\rm dipole} = 4N_c\,\alpha_{em}\,\theta(1-2z_{\rm min})\sum_f e_f^2 \int_{z_{\rm min}}^{1-z_{\rm min}} dz\, 4Q^2 z^2(1-z)^2 \int \frac{d^2\mathbf{x}_0}{2\pi}\,\frac{d^2\mathbf{x}_1}{2\pi}$$
$$\times\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]\Big[\mathrm{K}_0\Big(Q\sqrt{z(1-z)}|\mathbf{x}_{01}|\Big)\Big]^2\left\{1+\Big(\frac{\alpha_s\,C_F}{\pi}\Big)\left[\frac{1}{2}\left[\log\Big(\frac{z}{1-z}\Big)\right]^2-\frac{\pi^2}{6}+\frac{5}{2}\right]\right\}$$

with $z_{\rm min} = \frac{Q_0^2}{x_0}\,\frac{x_{Bj}}{Q^2}$

And similar expression for $\sigma^{\gamma}_{T}\big|_{\rm dipole}$

G.B. (2017)

DIS at NLO: results

$$\sigma_L^{\gamma}|_{q\to g} = 4N_c\,\alpha_{em}\,\theta(1-2z_{\min})\frac{\alpha_s C_F}{\pi}\sum_f e_f^2\int_{z_{\min}}^{1-z_{\min}} dz\; 4Q^2z^2(1-z)^2\int_{\frac{z_{\min}}{z}}^{1} d\xi$$

$$\times\int\frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\left\{\xi\,\frac{(\mathbf{x}_{20}\cdot\mathbf{x}_{21})}{x_{20}^2x_{21}^2}\Big(\mathrm{K}_0(QX_{012})\Big)^2\left(1-\left\langle\mathcal{S}_{012}^{(3)}\right\rangle_{Y_2}\right)\right.$$

$$\left.+\frac{[1+(1-\xi)^2]}{\xi}\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}-\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]\left[\Big(\mathrm{K}_0(QX_{012})\Big)^2\left(1-\left\langle\mathcal{S}_{012}^{(3)}\right\rangle_{Y_2}\right)-\Big(\mathbf{x}_2\to\mathbf{x}_0\Big)\right]\right\}$$

with:

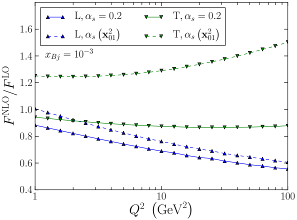
$$X_{012}^2 = (1-\xi)z(1-z)x_{01}^2+\xi(1-\xi)z^2x_{20}^2+\xi z(1-z)x_{21}^2$$

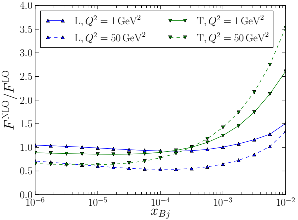
$$z_{\min} = \frac{Q_0^2}{x_0}\frac{x_{Bj}}{Q^2}\quad\text{and}\quad Y_2=\log\left(\frac{\xi\,z}{z_{\min}}\right)$$

And similar (but longer) expression for $\sigma_T^{\gamma}|_{q\to g}$

G.B. (2017)

DIS at NLO: preliminary numerical study





Ducloué, Hänninen, Lappi, Zhu (2017)

Numerical results with a simplified LL factorization scheme:

- NLO results overall well behaved
- At fixed coupling, larger NLO corrections for F_L than F_T
- But: sign of NLO correction to F_T changes sign when switching to running coupling (parent dipole), due to large transient effects in x_{Bj}

$\Rightarrow$ Need to check in case of more realistic RC prescriptions and/or high-energy LL factorization scheme

$\rightarrow$ Next step: new fits to DIS, at NLO

Homogeneous target approximation and beyond

Approx. used in all the practical applications presented so far:

$$\int \mathrm{d}^2\mathbf{x}_0\, \mathrm{d}^2\mathbf{x}_1\; f(\mathbf{x}_{01})\, \Big[1 - \langle \mathcal{S}_{01}\rangle_Y\Big] \;=\; \int \mathrm{d}^2\mathbf{b}\, \mathrm{d}^2\mathbf{r}\; f(\mathbf{r})\, \Big[1 - \Big\langle \mathcal{S}_{\mathbf{b}+\frac{\mathbf{r}}{2},\mathbf{b}-\frac{\mathbf{r}}{2}}\Big\rangle_Y\Big]$$

$$\simeq\; \sigma_0 \int \mathrm{d}^2\mathbf{r}\; f(\mathbf{r})\; \langle N(\mathbf{r})\rangle_Y$$

- Might be justified for a large nucleus
- Simplifies the numerics
- Avoid facing issues of non-pert. QCD arising at large **b**

Problems:

- Cannot address more **b**-sensitive observables
- Insufficient in order to get initial conditions for AA collisions
- Avoid facing issues of non-pert. QCD arising at large **b** (and **r**)

IP-Glasma model for initial conditions

State of the art model for initial conditions in AA collisions: IP-Glasma model Schenke, Tribedy, Venugopalan (2012)

Idea:

- initial conditions for hydro or kinetic theory at $\tau = \tau_0$ from numerical simulations of classical YM
- starting from the collision of two fluctuating shockwaves
- color fluctuations in each incoming nucleus/shockwave driven by the $Q_s(x_{Bj}, \mathbf{b})$ extracted from **b**-dependent model (IP-Sat) fitted on DIS

$\rightarrow$ IP-Glasma + Hydro: very successful in AA collisions

See talks by G. Roland and by P. Bozek

IP-Glasma model for initial conditions

IP-Sat model (proton), fitted on inclusive and exclusive DIS data:

$$\left\langle \mathcal{S}_{\mathbf{b}+\frac{\mathbf{r}}{2},\mathbf{b}-\frac{\mathbf{r}}{2}} \right\rangle_{\log(1/x_{Bj})} = \exp\left[-\mathbf{r}^2\, F(x_{Bj},\mathbf{r})\; T_p(\mathbf{b})\right]$$

$$F(x_{Bj},\mathbf{r}) = \frac{\pi^2}{2N_c}\,\alpha_s(\mu_0^2+4/\mathbf{r}^2)\, x_{Bj}\, g(x_{Bj},\mu_0^2+4/\mathbf{r}^2)$$

$$T_p(\mathbf{b}) = \frac{1}{2\pi B_p}\, e^{\frac{-\mathbf{b}^2}{2B_p}}$$

Nucleus case:

$$T_p(\mathbf{b}) \mapsto T_A(\mathbf{b}) = \sum_{i=1}^{A} T_p(\mathbf{b}-\mathbf{b}_i)$$

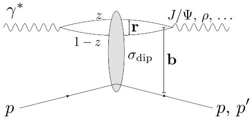
$\Rightarrow$ $Q_s(x_{Bj},\mathbf{b})$ for proton and nuclei

Finally, for each incoming proton or nucleus: MV model

$$\left\langle \rho_a(x^+,\mathbf{x})\,\rho_b(y^+,\mathbf{y})\right\rangle = \delta_{ab}\,\delta(x^+-y^+)\,\delta^{(2)}(\mathbf{x}-\mathbf{y})\,g^2\,\mu^2(\mathbf{x})$$

with $g^4\,\mu^2(\mathbf{x}) \propto Q_s^2(x_{Bj},\mathbf{x})$

Exclusive diffractive vector meson production in DIS



$$\mathcal{A}^{\gamma^* p \to Vp}(x_{\mathbb{P}}, Q^2, \boldsymbol{\Delta}) = \frac{i}{2\pi} \int \mathrm{d}^2\mathbf{r}\, \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\; e^{-i\boldsymbol{\Delta}\cdot[\mathbf{b}-(1-z)\mathbf{r}]} \times \left(\psi_V^*\psi\right)(\mathbf{r}, z, Q^2)\; \left[1 - \mathcal{S}_{\mathbf{b}+\frac{\mathbf{r}}{2},\mathbf{b}-\frac{\mathbf{r}}{2}}\right]$$

Coherent contribution (intact target):

$$\frac{\mathrm{d}\sigma^{\gamma^* p\to Vp}_{\text{coh.}}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\left\langle \mathcal{A}^{\gamma^* p\to Vp}(x_{\mathbb{P}}, Q^2, \boldsymbol{\Delta})\right\rangle_{\log(1/x_{\mathbb{P}})}\right|^2$$

$\rightarrow$ Sensitive to average dipole amplitude

Incoherent contribution (target breaks up):

$$\frac{\mathrm{d}\sigma^{\gamma^* p\to Vp^*}_{\text{incoh.}}}{\mathrm{d}t} = \frac{1}{16\pi}\left[\left\langle\left|\mathcal{A}^{\gamma^* p\to Vp}\right|^2\right\rangle_{\log(1/x_{\mathbb{P}})} - \left|\left\langle\mathcal{A}^{\gamma^* p\to Vp}\right\rangle_{\log(1/x_{\mathbb{P}})}\right|^2\right]$$

$\rightarrow$ Sensitive to fluctuations of the dipole amplitude

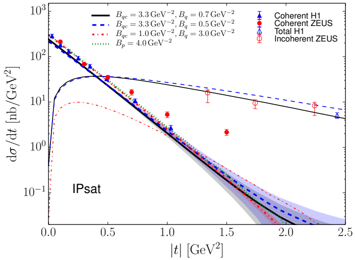
Proton geometric fluctuations from HERA: IP-Sat

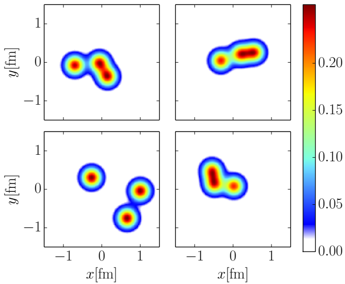
Standard IP-Sat model for proton: No fluctuations $\Rightarrow$ no incoherent contribution

Extension of IP-Sat to proton shape fluctuations:

$$T_p(\mathbf{b}) = \frac{1}{N_q} \sum_{i=1}^{N_q} \frac{1}{2\pi B_p} e^{\frac{-(\mathbf{b}-\mathbf{b}_i)^2}{2B_p}}$$

proton $\simeq$ 3 "constituent quarks"



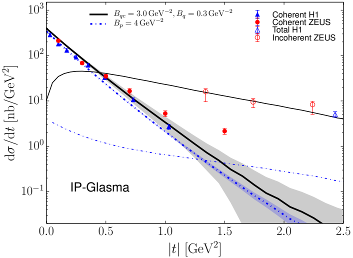


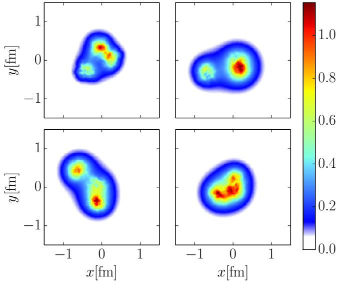
Mäntysaari, Schenke (2016)

Proton geometric fluctuations from HERA: IP-Glasma

Original IP-Glasma model for proton: non-zero but small incoherent contribution from color fluctuations

Update of IP-Glasma from IP-Sat with proton shape fluctuations : ok!

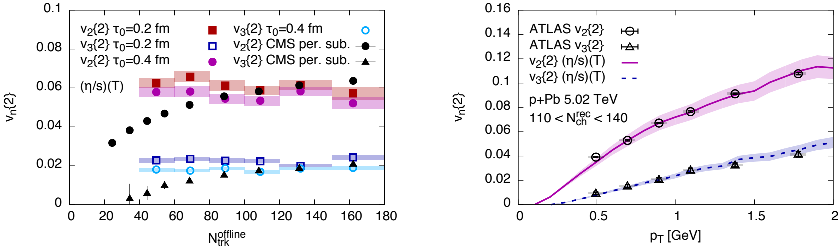




Mäntysaari, Schenke (2016)

Impact of proton shape fluctuations on flow in pA

IP-Glasma init. cond. + Hydro (MUSIC) + hadronic cascade (UrQMD)

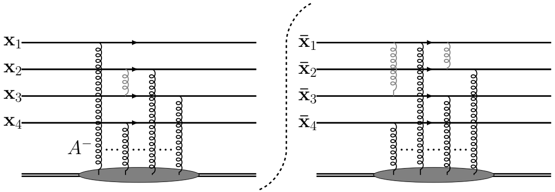


Including proton shape fluctuations fitted on HERA:

$\Rightarrow \sim 5$ times larger v_2 and v_3: now agree with the high multiplicity pA data

Mäntysaari, Schenke, Shen, Tribedy (2017)

Minimalistic MV-based model for pA



- p projectile $\mapsto$ bunch of uncorrelated quarks (color, momentum, ...)
- eikonal scattering of each quark on the shockwave target, event-by-event
- **b**-independent MV model for target averages

$$\frac{\mathrm{d}^m N}{\mathrm{d}^2\mathbf{p}_{\perp 1}\cdots\mathrm{d}^2\mathbf{p}_{\perp m}} = \frac{1}{(4\pi^3 B)^m}\int\prod_{i=1}^{m}\mathrm{d}^2\mathbf{x}_i\,\mathrm{d}^2\bar{\mathbf{x}}_i\; e^{i\mathbf{p}_{\perp i}\cdot(\mathbf{x}_i-\bar{\mathbf{x}}_i)}\; e^{-\frac{\mathbf{x}_i^2+\bar{\mathbf{x}}_i^2}{2B}}\left\langle\prod_{j=1}^{m}\mathcal{S}_{\mathbf{x}_j,\bar{\mathbf{x}}_j}\right\rangle_{MV}$$

Flow coefficients from 2 and 4 particles correlations

$$c_n\{2\} = \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle \quad \Rightarrow \quad v_n\{2\} = \sqrt{c_n\{2\}}$$

$$c_n\{4\} = \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle - 2\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle^2 \;\Rightarrow\; v_n\{4\} = \left[-c_n\{4\}\right]^{\frac{1}{4}}$$

In this model: Flow coefficients from n-particle correlations require the knowledge of the n-dipole correlator

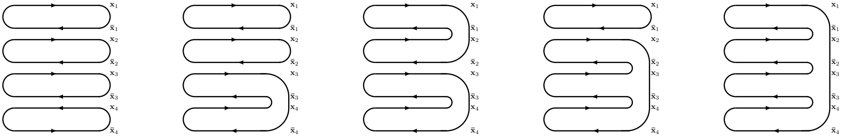
Correlations of $A_a^-(x^+, \mathbf{x})$ local in x^+
$\Rightarrow$ Perform the Gaussian averaging slice by slice in x^+:

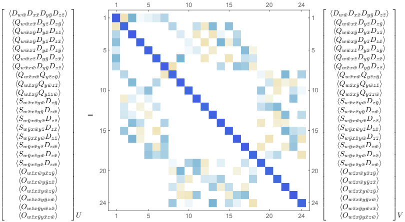
$$\langle \mathcal{S}_{\mathbf{x}_1,\bar{\mathbf{x}}_1}\mathcal{S}_{\mathbf{x}_2,\bar{\mathbf{x}}_2}\rangle_{L^+ + dx^+} = \alpha_{\mathbf{x}_1,\bar{\mathbf{x}}_1,\mathbf{x}_2,\bar{\mathbf{x}}_2}\ \langle \mathcal{S}_{\mathbf{x}_1,\bar{\mathbf{x}}_1}\mathcal{S}_{\mathbf{x}_2,\bar{\mathbf{x}}_2}\rangle_{L^+} + \beta_{\mathbf{x}_1,\bar{\mathbf{x}}_1,\mathbf{x}_2,\bar{\mathbf{x}}_2}\ \langle \mathcal{Q}_{\mathbf{x}_1,\bar{\mathbf{x}}_1,\mathbf{x}_2,\bar{\mathbf{x}}_2}\rangle_{L^+}$$

$\Rightarrow$ Non-trivial mixing between the double dipole and the single-trace quadrupole !

Calculating the 4-dipole correlator

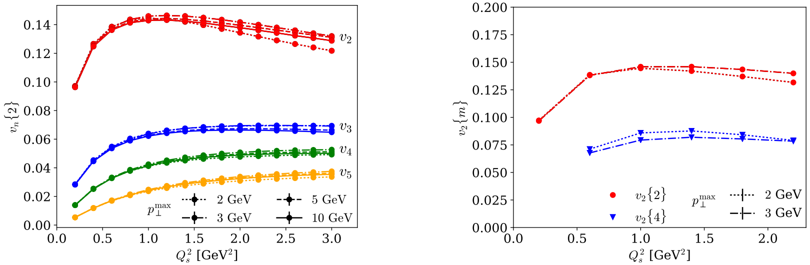
Averaging inside each longitudinal layer in the target mixes the 4-dipole correlator with other objects:





But: exponentiation of the 24*24 transition matrix tractable numerically
Dusling, Mace, Venugopalan (2017)

Results for flow in pA from the MV model



- Hierarchy of $v_n\{2\}$, non-zero for odd n. And $v_2\{2\} > v_2\{4\}$
- Results also available for $p_\perp$-dependent $v_2\{2\}$ and $v_2\{4\}$, and for $SC(n, n')$.
- In abelien/QED approx: MV averages straightforward even with more particles $\Rightarrow$ Result: $v_2\{2\} > v_2\{4\} \simeq v_2\{6\} \simeq v_2\{8\}$

Dusling, Mace, Venugopalan (2017)

Related analytic results at large N_c also available

Fukushima, Hidaka (2017)

Conclusion

1. Ongoing NLO/NLL revolution for high-energy QCD with gluon saturation
   $\rightarrow$ All the ingredients available to perform fits to DIS at NLO+LL accuracy, and possibly even at NLO+NLL

2. State of the art initial conditions for hydrodynamics with maximal QCD content: IP-Glasma
   - Works well for flow observables in AA
   - When including proton shape fluctuations required by incoherent exclusive DIS data, works also for flow observables in pA

3. However, qualitative behavior of flow observables in pA reproduced by a simplistic purely initial-state model

   $\rightarrow$ Is hydrodynamics (and/or QGP) really relevant to pA?