Precision Polarimetric techniques to measure Gas and Vacuum magnetic birefringence
Summary

- Introduction & motivation
- Basic ellipsometric techniques
- Gas measurements
- Towards quantum Vacuum measurements with PVLAS Phase II
- Conclusions
Introduction

- Polarimetric measurements provide an extremely precise and versatile tool to investigate the properties of a medium.

- The basic idea is to pass a light beam having an initially known polarization state through a medium and to measure how this state changes.

- Normally the final polarization state contains information on the intrinsic properties of the medium.

- If the medium is perturbed by some external agent (say a field) the parameters of the final polarization state contain information on the interactions between the field and the bulk of the medium, possibly revealing the medium intimate structure.
Motivation

We discuss here two interesting media having magneto-optical properties which can be investigated with IR-visible wavelengths

GASES -> Magnetic birefringence (Cotton-Mouton effect)

VACUUM (a zero pressure gas...) ->

magnetic birefringence (photon-photon scattering in QED)
Cotton-Mouton effect

Gases subject to a (static) magnetic field become anisotropic optical media, with the field direction defining the optical axis.

The effect of the field is to induce an anisotropy in the hypermagnetizability tensor $\eta$ and in the electric ($\alpha$) and magnetic ($\chi$) moments of the gas molecules, resulting in different refractive indices for light polarized parallel or normal to the external field.

$$\Delta n = n_\parallel - n_\perp = \frac{B^2 P \Delta \eta}{4 \varepsilon_0 k T}$$

spherical molecules

$$\Delta n = n_\parallel - n_\perp = \frac{B^2 P}{4 \varepsilon_0} \left( \frac{\Delta \eta}{k T} + \frac{2 \Delta \alpha \Delta \chi}{15 (k T)^2} \right)$$

axial molecules

($B$ is the magnetic field, $P$ the gas pressure and $T$ the temperature)
Cotton-Mouton effect for several gases


\[
\Delta n_u = \Delta n \left( \frac{T}{B[T]} \right)^2 \left( \frac{P_{atm}}{P} \right)
\]

The unit birefringence is defined as

<table>
<thead>
<tr>
<th>Species</th>
<th>Formula</th>
<th>Reference</th>
<th>( \lambda ) (Å)</th>
<th>( T ) (K)</th>
<th>( \Delta n_u )</th>
<th>( T ) range (K)</th>
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</thead>
<tbody>
<tr>
<td>Helium(^a)</td>
<td>He</td>
<td>30(^g)</td>
<td>5145</td>
<td>273.15</td>
<td>(1.80 ± 0.36) \times 10^{-16}</td>
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</tr>
<tr>
<td>Neon(^d)</td>
<td>Ne</td>
<td>29(^e)</td>
<td>5145</td>
<td>298.15</td>
<td>(2.83 ± 0.15) \times 10^{-16}</td>
<td></td>
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<tr>
<td>Argon(^f)</td>
<td>Ar</td>
<td>18(^g)</td>
<td>5145</td>
<td>273.15</td>
<td>(6.8 ± 1.0) \times 10^{-15}</td>
<td></td>
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<tr>
<td>Krypton(^f)</td>
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<td>5145</td>
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<td>Xenon(^f)</td>
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<td></td>
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<tr>
<td>Hydrogen</td>
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<td>23(^g)</td>
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<td>17</td>
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<td>(−1.90 ± 0.12) \times 10^{-13}</td>
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<td>11(^f)</td>
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<td>(−1.80 ± 0.06) \times 10^{-13}</td>
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<tr>
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<td></td>
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<td>11(^f)</td>
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<td>(−2.37 ± 0.12) \times 10^{-13}</td>
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<td>13</td>
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<td>6328</td>
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<td>(−2.62 ± 0.08) \times 10^{-13}</td>
<td>203–393</td>
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<td>17</td>
<td>6328</td>
<td>294.15</td>
<td>(−2.43 ± 0.12) \times 10^{-13}</td>
<td>203–393</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>5145</td>
<td>298.15</td>
<td>(−2.26 ± 0.10) \times 10^{-13}</td>
<td></td>
</tr>
</tbody>
</table>
Non linearities in the Maxwell equations are predicted by the Heisenberg-Euler effective Lagrangian (1936).

\[ L_{EH} = \frac{1}{2} (E^2 - B^2) + \frac{2\alpha^2}{45m_e^4} \left[ (E^2 - B^2)^2 + 7(E \cdot B)^2 \right] . \]

(in Heaviside-Lorentz natural units)

Photon-photon scattering in QED (also Schwinger, 1951, Adler, 1971)

\[ \Delta n = \frac{6\alpha^2}{45m_e^4} B^2 \]

Polarization selective phase delay. “Detectable” as an induced birefringence on a linearly polarized laser beam propagating in vacuum in an external magnetic field.
Photon-photon scattering in QED

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From birefringence to ellipticity

The quantity that is actually measured in ellipsometry is not $\Delta n$ (normally called the birefringence), but rather the ellipticity $\Psi$, that is the ratio of the semi-minor to the semi-major axes of the polarization ellipse.

\[
\Psi = \frac{a}{b} = \left( \frac{\pi L}{\lambda} \sin 2\theta \right) \Delta n
\]
Some numbers

Assume:

- $B = 2 \, T = 390 \, \text{eV}^2$ (in H.-L. units) $\rightarrow$ a good permanent magnet
- $L = 10^5 \, \text{m} = 5 \cdot 10^{11} \, \text{eV}^{-1}$ $\rightarrow$ a 0.5 m long magnetic zone amplified by a 200000 finesse Fabry-Perot resonator
- $\omega = 1.17 \, \text{eV}$ $\rightarrow$ 1064 nm NdYAG laser

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\Delta n$</th>
<th>$\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne (1 atm)</td>
<td>$2.4 \cdot 10^{-15}$</td>
<td>$7.1 \cdot 10^{-4}$</td>
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<tr>
<td>He (1 atm)</td>
<td>$8.32 \cdot 10^{-16}$</td>
<td>$2.4 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Vacuum</td>
<td>$1.6 \cdot 10^{-23}$</td>
<td>$4.7 \cdot 10^{-12}$</td>
</tr>
</tbody>
</table>
Basic principle of ellipticity measurements
Basic principle of ellipticity measurements

Static detection
Basic principle of ellipticity measurements

Homodyne detection
Basic principle of ellipticity measurements

Heterodyne detection

Homodyne detection
Practical heterodyne ellipticity detection

- Static measurement is excluded:  
  \[ I_{Tr} = I_0 \left[ \sigma^2 + \Psi(t)^2 \right] \]
- Solution: Modulate the effect and add a carrier \( \eta(t) \) to signal at \( \omega_{\text{Mod}} \)
- Keeping the initial polarization fixed and rotating the field at \( \Omega_{\text{Mag}} \) produces an ellipticity at \( 2\Omega_{\text{Mag}} \)

Ideally the transmitted intensity is given by,

\[ I_{Tr} = I_0 \left[ \sigma^2 + (\Psi(t) + \eta(t))^2 \right] = I_0 \left[ \sigma^2 + (\Psi(t)^2 + \eta(t)^2 + 2\Psi(t)\eta(t)) \right] \]

The main frequency components appear at \( \omega_{\text{Mod}} \pm 2\Omega_{\text{Mag}} \) and \( 2\omega_{\text{Mod}} \)
In actual practice, nearly static spurious birefringences generate a 1/f noise at $\omega_{\text{Mod}}$

\[
I_{\text{Tr}} = I_0 \left[ \sigma^2 + (\Psi(t) + \eta(t) + \alpha_s(t))^2 \right] = I_0 \left[ \sigma^2 + (\eta(t)^2 + 2\Psi(t)\eta(t) + 2\alpha_s(t)\eta(t) + \ldots) \right]
\]

- A small, time-varying signal can be extracted from a large noise background with the heterodyne technique

\[
\eta^2/2
\]

\[
\eta \Psi
\]

\[
\omega_{\text{Mod}} - 2\Omega_{\text{Mag}} \quad \omega_{\text{Mod}} + 2\Omega_{\text{Mag}}
\]

**Actual sample spectrum**

- $\omega_{\text{Mod}}$
- $2\omega_{\text{Mod}}$
Basic features of sensitive ellipsometry

- Heterodyne detection (need modulator)
- Good extinction factor polarizers
- Control of spurious birefringences
- Amplification of optical path (need a resonant high-finesse Fabry-Perot)
Basic applications (so far...)

- Cotton-Mouton effect measurements
- Checks of theoretical models
- Instrument control and calibration
- QED processes in the quantum Vacuum
- ... the Holy Grail ...
Basic applications (so far...)

- Cotton-Mouton effect measurements
- Checks of theoretical models
- Instrument control and calibration
- QED processes in the quantum Vacuum
- ... the Holy Grail ...
C-M of Neon vs. pressure

C-M of Helium vs. pressure

PVLAS Phase II - Table-top heterodyne ellipsometer
Present optics layout schematic

PVLAS Phase II

532 nm
1064 nm

λ/2

OI

λ/2

750 mm

PEM

20 kHz

LIA

FPGA

20 kHz

I_{0}
x

y

I_{sig}

25 mm

PC

Settembre 2008 – Incontro con referee

(F ~ 170000, vicino al valore di progetto)
Detail of the rotating permanent magnet
Test Cotton-Mouton measurements in Nitrogen

\[ \lambda = 532 \text{ nm} \]

\[
\begin{array}{|c|c|c|}
\hline
\text{Equation} & y = a + bx & \\
\text{Adj. R-Square} & 0.99785 & \\
\hline
\text{psi} & \text{Intercept} & 1.09241E-8 & 3.5426E-8 \\
\text{psi} & \text{Slope} & 3.21201E-9 & 6.08806E-11 \\
\hline
\end{array}
\]

\[ |\Delta n_u| = 2.43 \times 10^{-13} \]

Consistent with measurements reported in the literature.
PVLAS Phase II ellipsometer development stages

Prototype (already existing)
- 900 mW at 1064 nm, 20 mW at 532 nm
- Mirror Integrated Modulator
- 1 m long Fabry-Perot with $F \approx 220000$
- 2.3 T, 50 cm long, permanent dipole magnet
- analog frequency locking, environmental screens

Advanced
- intensity stabilization to reduce laser Residual Intensity Noise
- birefringence modulation directly on cavity mirrors
- low noise electronics
- digital frequency locking, improved acoustic isolation

Advanced Power Upgrade
- 600 mW at 532 nm
- light injection and extraction via optical fiber
<table>
<thead>
<tr>
<th>Config.</th>
<th>IR</th>
<th>GREEN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prototype</td>
<td>Advanced</td>
</tr>
<tr>
<td>Sens. [1/√Hz]</td>
<td>10⁻⁸</td>
<td>6 · 10⁻¹⁰</td>
</tr>
<tr>
<td>Min. det. angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in 400 std. days</td>
<td>3 · 10⁻¹²</td>
<td>1.8 · 10⁻¹³</td>
</tr>
<tr>
<td>One magnet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.3 T, L = 0.5 m</td>
<td>$\psi^0_{\text{QED}}$</td>
<td>3.1 · 10⁻¹⁷</td>
</tr>
<tr>
<td></td>
<td>$\psi_{\text{QED}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(F=220000)</td>
<td>4.3 · 10⁻¹²</td>
</tr>
<tr>
<td>Min. meas. time</td>
<td>(std. 8-hr. days)</td>
<td>188</td>
</tr>
<tr>
<td>Two magnets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.3 T, L = 0.5 m</td>
<td>$\psi^0_{\text{QED}}$</td>
<td>6.1 · 10⁻¹⁷</td>
</tr>
<tr>
<td></td>
<td>$\psi_{\text{QED}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(F=220000)</td>
<td>8.6 · 10⁻¹²</td>
</tr>
<tr>
<td>Min. meas. time</td>
<td>(std. 8-hr. days)</td>
<td>47.1</td>
</tr>
</tbody>
</table>

Table IV: Minimum measurement times necessary to detect QED photon-photon scattering for several apparatus configurations.
Conclusions

精准光学偏振度量是一种复杂的技术，需要整合不同领域的专业知识

一旦掌握，它提供了一种强大的方法来研究光学活性介质，并提供了一种敏感的诊断工具

研究磁双折射气体是一个传统应用领域

目前（和未来！）的挑战是完善此技术以达到所需的敏感度来攻击QED微观过程，将粒子物理学带回桌面环境