Black Hole Microstates in Four Dimensions

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"Stringy Origin of 4d Black Hole Microstates", [10.1007/JHEP06(2016)003] M. Bianchi, J.F. Morales, L. Pieri

"Fuzzballs in general relativity: a missed opportunity", [1611.05276] L. Pieri

"More on microstate geometries of 4d black holes", [1701.05520] M. Bianchi, J.F. Morales,L. Pieri, N. Zinnato

Work in progress, S. Giusto, J.F. Morales, L. Pieri, R. Russo

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Plan of the Talk

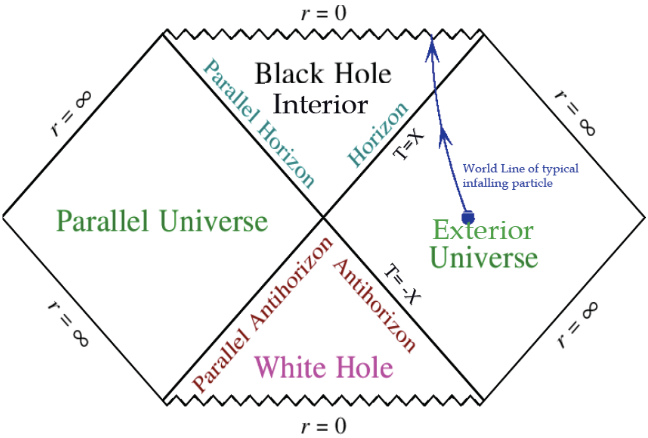
- Classical Black Holes and the Information Paradox
- String Theory and the Fuzzball Proposal
- Stringy Origin of 4d Black Holes Microstates: From Open Strings on D-Branes to Supergravity Fields with scattering amplitudes
- Smooth Horizonless geometries in Supergravity
- From Supersymmetric quantum mechanics to Supergravity

Part I

Classical Black Holes and the Information Paradox

Definition of Black Hole in General Relativity

$$\textit{Black Hole} \equiv [M - J^-(\mathscr{I}^+)]$$



BLACK HOLE IDENTIKIT

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \mathrm{sen}^2\theta d\phi^2$$

- $r = 2M$ (coord. singularity) is the horizon $H = M \cap \dot{J}^-(\mathscr{I}^+)$. Light cannot escape to infinity.
- Singularity Theorems: Trapped Surface $\Rightarrow$ **Singularity**
- Cosmic Censorship: Singularity $\Rightarrow$ **Horizon**
- **Area Theorem**: $\delta A \geq 0$
- **No hair theorem**: Stationary, asymptotically flat black hole solutions are fully characterized by mass M, electric-magnetic charge Q and angular momentum $J \Longrightarrow$ Kerr-Newman

Black Hole Thermodynamics

Law	**Thermodynamics**	**Black Holes**
Zeroth	T constant trough body in thermal equilibrium	κ constant over (Killing) horizon of stationary BHs ($\chi^\mu \nabla_\mu \chi = -\kappa\chi$)
First	$dE = TdS - pdV$	$dM = \frac{1}{8\pi}\kappa dA + \Omega_H dJ$
Second	$\delta S \geq 0$ in any process	$\delta A \geq 0$ in any process
Third	Impossible to achieve $T = 0$ by a physical process	Impossible to achieve $\kappa = 0$ by a physical process

A puzzle arises, indeed a BH in GR doesn't emit at all, therefore:

$$T_{GR}(BH) = 0 \quad (!!??)$$

QFT in Curved Space: Hawking Radiation

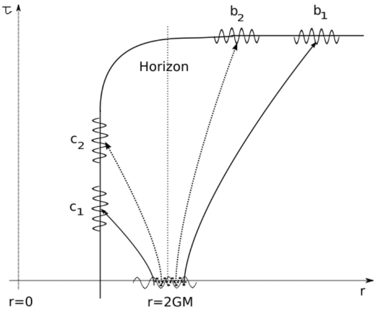
Black hole as a black body:

$$T_{BH} = \frac{\kappa}{2\pi} = \frac{1}{8\pi M} \qquad S_{Bek-Hawk} = \frac{1}{4}A$$

Where are the micro states?

$$S_{GR+QFT}(BH) = log(N_{Microstates}) = log(1) = 0 \quad (!!??)$$

INFORMATION PARADOX



A pure state enters into a BH. The emitted radiation is thermal (no information), but is entangled with the BH. The emitted particle pairs do not depend on the state of earlier produced pairs (why? See later!). When the BH completely evaporates there is nothing to be entangled with. At the end, we have only radiation in a mixed state $\Rightarrow$ lost unitarity.

Information Paradox

In the Black Hole Information Paradox the particle pairs are created from the vacuum!

Information Paradox: Possible Resolutions

The paradox cannot be solved with small corrections to the semi-classical computation, and it's too late to extract the information at the last stages of evaporation (Page Time).

- Loss of unitarity Hawking, Unruh, Wald
- Remnants, Baby Universe Susskind
- Non Local interactions BH-radiation Maldacena-Susskind, Raju-Papadodimas
- Hairs in the asymptotic structure of spacetime Hawking, Perry, Strominger
- The horizon is no more in an "information free vacuum" Fuzzball, Firewall

We will explore the last possibility. Rather than only solving an ad hoc problem, this resolution seems to emerge naturally from string theory, fitting into a bigger picture.

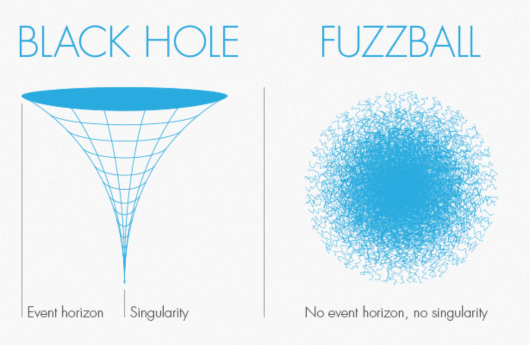
Part II

String Theory and the Fuzzball Proposal

Fuzzball Proposal

Every black hole micro state is dual to a smooth, horizon-less supergravity solution. There is no singularity and quantum gravity effect are horizon-sized thanks to the huge phase space.

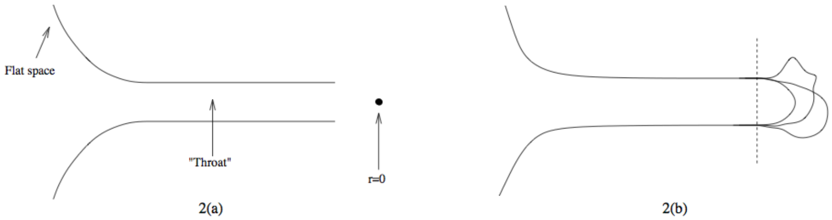
O.Lunin, S.Mathur (2001) *hep* − *th*/0109154



Bena, Giusto, Lunin, Mathur, Ruef, Russo, Shigemori, Skenderis, Taylor, Turton, Warner

Fuzzball Proposal

The "horizon" is no more information free, the BH is similar to a piece of burning coal. The information paradox is solved.



Every microstate has the same asymptotic charges (M, J, Q, P) of the would-be black hole. Classical BH arises as “coarse-grained” description when only the geometry outside the “horizon” is taken into account.

BHs in String Theory: The Naive D1-D5

Black Holes in string theory can be constructed as bound states of intersecting (Dp-M)branes

Brane	t	x_1	x_2	x_3	x_4	y_5	y_6	y_7	y_8	y_9
$D1$	$-$	.	.	.	.	$-$	.	.	.	.
$D5$	$-$	.	.	.	.	$-$	$-$	$-$	$-$	$-$

$$ds^2 = (H_1H_5)^{-1/2}(-dt^2 + dy_5^2) + (H_1H_5)^{1/2}(dx_1^2 + \dots dx_4^2) + H_1^{1/2}H_5^{-1/2}(dy_6^2 + \dots dy_9^2)$$

$$F_{01m} = \partial_m H_1^{-1} \quad F_{0\dots5m} = \partial_m H_5^{-1} \quad e^{\phi} = H_1^{1/2}H_5^{-1/2}$$

$$H_n = 1 + \frac{Q_n}{r^2}$$

D1-D5 Fuzzball

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt-A_i dx^i)^2+(dy_5+B_i dx^i)^2)]+\sqrt{\frac{1+L}{H}}dx_i^2+\sqrt{H(1+K)}dy_a^2$$

$$H^{-1} = 1 + \frac{Q_1}{L_T}\int_0^{L_T}\frac{dv}{|\vec{x}-\vec{F}(v)|^2} \quad K = \frac{Q_1}{L_T}\int_0^{L_T}\frac{dv(\dot{F}(v))^2}{|\vec{x}-\vec{F}(v)|^2}$$

$$A_i = -\frac{Q_1}{L_T}\int_0^{L_T}\frac{dv\dot{F}_i(v)}{|\vec{x}-\vec{F}(v)|^2} \quad dB = -\star_4 dA \quad v = t - y_5$$

For instance $F_1 = cos(\omega v)$, $F_2 = sin(\omega v)$, $F_3 = F_4 = 0$. The metric has only a coordinate singularity on the curve $F(v)$, indeed near curve we recover the metric of a Kaluza-Klein monopole times regular spaces.

The D1D5 fuzzball is horizonless and regular. Instead of going past the "would-be horizon", the throat ends in smooth "cap" depending on $F(v)$, the hair which discriminate between different microstates. There is no "interior space".

Part III

Stringy Origin of 4d Black Holes Microstates: From Open Strings on D-Branes to Supergravity Fields with scattering amplitudes

Stringy Origin of 4d Black Holes Microstates

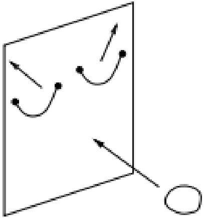
Our goal is to recover the microstate geometries in supergravity from the underlying fundamental string theory description. In particular we consider a bound state of 4 perpendicularly intersecting D3 branes on T^6 in *IIB*. (dual to $D2^3$-D6 in *IIA* or M2-M5-P-KK6 in *M*-theory)

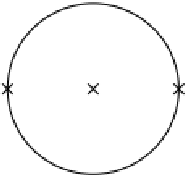
Brane	t	x_1	x_2	x_3	y_1	$\tilde{y}_1$	y_2	$\tilde{y}_2$	y_3	$\tilde{y}_3$
$D3_0$	—	.	.	.	—	.	—	.	—	.
$D3_1$	—	.	.	.	—	.	.	—	.	—
$D3_2$	—	.	.	.	.	—	—	.	.	—
$D3_3$	—	.	.	.	.	—	.	—	—	.

We will indeed derive a 1:1 relation between open string condensates and fields in the bulk for a large class of 4d BPS black holes.

Mixed Open-Closed Scattering Amplitudes

The microstate geometries will be derived from mixed open-closed disk amplitudes, computing the emission rate of massless closed strings from open string condensates binding different stacks of branes.





Closed String Fields

$g_{MN}, b_{MN}, C^{(4)}_{MNPQ}$

Open String Fields

μ^A, ϕ^i

From Amplitudes to Supergravity Fields

$$\mathcal{A}(k) \propto \int \frac{d^{2+n}z}{V_{CKV}} \langle W_{closed}(z,\bar{z}) V_{open}(z_1) \dots V_{open}(z_n) \rangle$$

We will work at the leading order in g_s, take all open string momenta equal to zero and the closed string momentum k only in non compact space directions. This limit is well defined only if the disk diagram cannot factorize via the exchange of open string states, so one must choose the polarization of the open strings in such a way that no factorization diagram is allowed.

The deviation from flat space of a closed field is extracted from the string amplitude: $\delta\tilde{\Phi}(k) = \left(-\frac{i}{k^2}\right) \frac{\delta\mathcal{A}(k)}{\delta\Phi}$

$$\delta\tilde{\phi}(k) = -\frac{i}{k^2}\frac{\delta\mathcal{A}(h,k)}{\delta h} \quad \rightarrow \quad \delta\phi(x) = \int \frac{d^3k}{(2\pi)^2}\tilde{\phi}(k)e^{ikx}$$

10D Supergravity Solution

$$ds^2 = -e^{2U}(dt+w)^2 + e^{-2U}\sum_{i=1}^{3} dx_i^2 + \sum_{I=1}^{3}\left[\frac{dy_I^2}{Ve^{2U}Z_I} + Ve^{2U}Z_I\,\tilde{e}_I^2\right]$$

$$C_4 = \alpha_0 \wedge \tilde{e}_1 \wedge \tilde{e}_2 \wedge \tilde{e}_3 + \beta_0 \wedge dy_1 \wedge dy_2 \wedge dy_3 + \frac{1}{2}\epsilon_{IJK}\left(\alpha_I \wedge dy_I \wedge \tilde{e}_J \wedge \tilde{e}_K + \beta_I \wedge \tilde{e}_I \wedge dy_J \wedge dy_K\right)$$

$$P_I = \frac{K_I}{V}, \qquad Z_I = L_I + \frac{|\epsilon_{IJK}|}{2}\frac{K_J K_K}{V}, \qquad \mu = \frac{M}{2} + \frac{L_I K_I}{2\,V} + \frac{|\epsilon_{IJK}|}{6}\frac{K_I K_J K_K}{V^2}$$

$$e^{-4U} = Z_1 Z_2 Z_3 V - \mu^2 V^2, \qquad b_I = P_I - \frac{\mu}{Z_I}, \qquad \tilde{e}_I = d\tilde{y}_I - b_I\, dy_I$$

$$*_3 dA = dV \qquad *_3 dw_I = -d(K_I) \qquad *_3 dv_0 = dM$$

$$*_3 dv_I = dL_I \qquad *_3 dw = V d\mu - \mu dV - V Z_I dP_I$$

$$H_a = \{V, L_I, K_I, M\} \quad I = 1,2,3$$

$$R_{MN} = \frac{1}{4\cdot 4!}F_{MP_1P_2P_3P_4}\,F_N^{\;P_1P_2P_3P_4} \qquad F_5 = *_{10}F_5 \qquad F_5 = dC_4$$

Harmonic Multipole Expansion

$$L_I \approx 1 + \alpha_{D3}\, \frac{N_I}{|x|} \qquad V \approx 1 + \alpha_{D3}\, \frac{N_0}{|x|}$$

$$K_I \approx \alpha_{D3}\, c_i^{K_I}\, \frac{x^i}{|x|^3} \qquad M \approx \alpha_{D3}\, c_i^{M}\, \frac{x^i}{|x|^3} \qquad \alpha_{D3} = 4\,\pi\, g_s\, (\alpha')^2 / V_{T_3}$$

$$H_a(x) = h_a + \textstyle\sum_{n=0}^{\infty} c^a_{i_1 \dots i_n} P_{i_1 \dots i_n}(x)$$

$$\frac{1}{|x+a|} = \textstyle\sum_{n=0}^{\infty} a_{i_1} \dots a_{i_n} P_{i_1 \dots i_n}(x)$$

$$P(x) = \frac{1}{|x|} \qquad P_i(x) = -\frac{x_i}{|x|^3} \qquad P_{ij}(x) = \frac{3 x_i x_j - \delta_{ij} |x|^2}{|x|^5}$$

$$P_{i_1 \dots i_n}(x) = \int \frac{d^3 k}{(2\pi)^3} e^{ikx} \tilde{P}_{i_1 \dots i_n}(k) \qquad \tilde{P}_{i_1 \dots i_n}(k) = \frac{4\pi\, (i)^n}{n!\, k^2} k_{i_1} \dots k_{i_n}$$

The $P_{i_1 \dots i_n}(x)$ are singular, but for an appropriate choice of the coefficients $c_{i_1 \dots i_n}$ the infinite sum of terms produce a fuzzy and smooth geometry.

L Solution

$$V = L(x) \qquad M = K_I = 0 \qquad L_I = 1$$

The L solutions are geometries that fall-off at infinity as Q_i/r, corresponding to a single stack of branes. The "superposition" of this solution together with K and M solutions will give the complete microstate of the SUGRA solution.

At linear order in α_{D3} one finds:

$$\delta g_{MN} dx^M dx^N = \frac{\delta L}{2}\left[dt^2 - \sum_{i=1}^3 (dy_i^2 - dx_i^2 - d\tilde{y}_i^2)\right] + \dots$$
$$\delta C_4 = -\delta L \wedge dt \wedge dy_1 \wedge dy_2 \wedge dy_3 + A \wedge d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3 + \dots$$

with $\delta L = L - 1$ and A both of order α_{D3}. One can take:

$$L = 1 + \frac{\alpha_{D3} N_0}{|x|} + \dots \qquad *_3 dL = dA$$

One boundary Amplitude

$$V_{\xi(\phi)}(x_a) = \sum_{n=0}^{\infty} \xi_{i_1\ldots i_n}\, \partial X^{i_1}(x_1)\, \prod_{a=2}^{n} \int_{-\infty}^{\infty} \frac{dx_a}{2\pi}\, \partial X^{i_a}(x_a)$$
$$W_{NSNS}(z,\bar{z}) = c_{\text{NS}}\,(ER)_{MN}\, e^{-\varphi}\psi^M e^{ikX}(z)\, e^{-\varphi}\psi^N e^{ikRX}(\bar{z})$$
$$\xi(\phi) = \sum_{n=0}^{\infty} \xi_{i_1\ldots i_n}\phi^{i_1}\ldots\phi^{i_n} \qquad E = h + b$$

$$\mathcal{A}_{NS-NS,\xi(\phi)} = \langle c(z)c(\bar{z})c(z_1)\rangle \left\langle W_{NS-NS}(z,\bar{z})V_{\xi(\phi)}\right\rangle = i\, c_{\text{NS}}\ \text{tr}(ER)\xi(k)$$

The asymptotic deviation from the flat metric can be extracted:

$$\delta\tilde{g}_{MN}(k) = \left(-\frac{i}{k^2}\right)\sum_{n=0}^{\infty} \frac{\delta\mathcal{A}_{NS-NS,\phi^n}}{\delta h_{MN}} = c_{\text{NS}}\frac{\xi(k)}{k^2}\,(\eta R)_{MN}$$

After Fourier transform one finds agreement with SUGRA:

$$\delta g_{MN} = \int \frac{d^3k}{(2\pi)^3}\delta\tilde{g}_{MN} = -\frac{1}{2}(\eta R)_{MN}\,\delta L(x)$$

In particular, for a single D3 brane at position $x = a$:

$$\xi(\phi) \sim e^{i\,a\,\phi}$$

K Solution

$K_3 = -M = K(x) \qquad \mu = 0 \qquad L_I = V = 1 \qquad K_1 = K_2 = 0$

The K solutions are geometries that fall-off at infinity as $Q_i Q_j / r^2$. They are associated to fermionic bilinears localized at the intersection of two branes and in general they carry angular momentum. The open string condensates discriminate between different microstates.

At linear order in α_{D3} one finds ($*_3 dw = -dK$):

$$\delta g_{MN} dx^M dx^N = -2\, dt\, w - 2\, K\, dy_3 d\tilde{y}_3 + \ldots$$
$$\delta C_4 = (K\, dt \wedge dy_3 - w \wedge d\tilde{y}_3) \wedge (dy_1 \wedge d\tilde{y}_2 + d\tilde{y}_1 \wedge dy_2)$$

For example one can take K to be

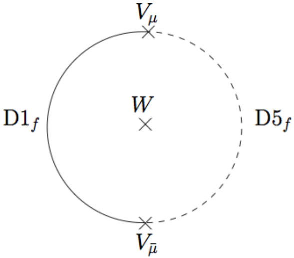
$$K \approx \frac{v_i x_i}{|x|^3} \qquad w \approx \epsilon_{ijk}\, v_i \frac{x_j\, dx_k}{|x|^3}$$

Two boundary Amplitude

$$\mathcal{A}^{NS-NS}_{\mu^2,\xi(\phi)} = \int dz_4 \langle c(z_1)\, c(z_2)\, c(z_3)\rangle \left\langle V_{\bar\mu}(z_1)\, V_\mu(z_2)\, W(z_3,z_4)\, V_{\xi(\phi)}\right\rangle$$

$$V_{\bar\mu}(z_1) = \bar\mu^A\, e^{-\varphi/2}\, S_A\sigma_2\sigma_3 \qquad V_\mu(z_2) = \mu^B\, e^{-\varphi/2}\, S_B\sigma_2\sigma_3$$

$$\left\langle \mathrm{tr}\, \bar\mu^{(A}\mu^{B)}\right\rangle = \frac{c_{\mathcal{O}}}{3!}\, v_{MNP}\, (\Gamma^{MNP})^{AB} \qquad v_{MNP} \in \mathbf{10}\ of\ SO(6)$$



$$\mathcal{A}^{NS-NS}_{\mu^2,\xi(\phi)} = \frac{1}{3!}\, (ER)_{MN} k_P\, v^{MNP}\, \xi(k)$$

One can turn on $v_{y_3\tilde{y}_3 3} = -v_{12t} = 4\pi\, v$, finding SUGRA fields

$$\delta g_{2t} = -v\, \frac{x_1}{|x|^3} \qquad \delta g_{1t} = v\, \frac{x_2}{|x|^3} \qquad \delta g_{y_3\tilde{y}_3} = -v\, \frac{x_3}{|x|^3}$$

M Solution

$$K_2 = M = M(x) \qquad \mu = M \qquad L_I = V = 1 \qquad K_1 = K_3 = 0$$

The M solutions are geometries that fall-off at infinity as $Q_1 Q_2 Q_3 Q_4 / r^3$. This factor appears in the entropy, i.e. the square root of the $E_{7(7)}$ quartic invariant of $\mathcal{N} = 8$ SUGRA in $d = 4$.

$$\delta g_{MN} dx^M dx^N = 2M \left(dy_1 \, \tilde{d}y_1 + dy_3 \, \tilde{d}y_3 \right) + \ldots$$

$$\delta C_4 = -M \, dt \wedge \left(dy_1 \wedge d\tilde{y}_2 \wedge dy_3 + d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3 \right) + w_2 \wedge (dy_1 \wedge dy_2 \wedge dy_3 + d\tilde{y}_1 \wedge dy_2 \wedge d\tilde{y}_3) + \ldots$$

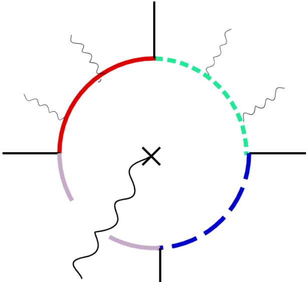
The M solution are actually associated to the vanishing combination $c_i^M + \sum_{i=1}^3 c_i^K = 0$, so they start as $\approx \frac{1}{r^3}$ (no dipole modes). In particular we take the harmonic M to be of the form:

$$M \approx v_{ij} \frac{3 \, x_i \, x_j - \delta_{ij} |x|^2}{|x|^5}$$

Four Boundary Amplitude

Consider the insertions of four fermions μ_a starting on a D3-brane of type (a) and ending on a D3-branes of type $(a+1)$ with $a = 0, 1, 2, 3$ (mod 4), in a cyclic order. The condensate is complex. Indeed, even if each intersection preserves $\mathcal{N} = 2$ SUSY (1/4 BPS), so that each fermion μ_a comes together with its charge conjugate $\bar{\mu}_a$, the overall configuration preserves only $\mathcal{N} = 1$ SUSY (1/8 BPS), so that we have two pairs of opposite chirality.

$$\mathcal{A}^{NS-NS}_{\mu^4, \xi(\phi)} = \langle c(z_1)\, c(z_2)\, c(z_4)\rangle \int dz_3 dz_5 dz_6 \left\langle V_{\mu_1}(z_1) V_{\mu_2}(z_2) V_{\mu_3}(z_3) V_{\mu_4}(z_4) W_{NSNS}(z_5, z_6) V_{\xi(\phi)} \right\rangle$$



FOUR BOUNDARY AMPLITUDE

$$\left\langle \mathrm{tr}\, \mu_1^{(\alpha}\, \mu_2^{\beta)}\, \bar{\mu}_3^{(\dot\alpha}\, \bar{\mu}_4^{\dot\beta)} \right\rangle = \frac{2\pi v^{ij}}{c_{\mathrm{NS}}\, \mathcal{I}_1}\, \sigma_i^{\alpha\dot\alpha} \bar{\sigma}_j^{\beta\dot\beta} \quad v^{ij} \in (\mathbf{3},\mathbf{3}) \text{ of } SU_L(2) \times SU_R(2)$$

$$\langle \sigma_2(z_1)\sigma_2(z_2)\sigma_2(z_3)\sigma_2(z_4)\rangle = f\left(\frac{z_{14}z_{23}}{z_{13}z_{24}}\right)\left(\frac{z_{13}z_{24}}{z_{12}z_{23}z_{34}z_{41}}\right)^{1/4}$$

$$f(x) = \frac{\Lambda(x)}{(F(x)F(1-x))^{1/2}} \qquad F(x) = {}_2F_1(1/2, 1/2; 1; x)$$

$$\Lambda(x) = \sum\nolimits_{n_1,n_2} e^{-\frac{2\pi}{\alpha'}\left[\frac{F(1-x)}{F(x)}\, n_1^2 R_1^2 + \frac{F(x)}{F(1-x)}\, n_2^2 R_2^2\right]}$$

$$\mathcal{A}^{NS-NS}_{\mu^4,\xi(\phi)} = \left[(ER)_{[1\bar{1}]} + (ER)_{[3\bar{3}]}\right] k_i k_j\, v^{ij}\, \xi(k)$$

$$\delta\tilde{g}_{1\bar{1}} = \delta\tilde{g}_{3\bar{3}} = -2\pi i\, v^{ij}\, \frac{k_i k_j}{k^2}\, \xi(k)$$

We found again agreement with the SUGRA solution. One can even turn on different condensate to get new SUGRA solutions:

$$\widetilde{\mathcal{O}}^{\alpha\dot\alpha\beta\dot\beta} = \mathrm{tr}\, \mu_1^{(\alpha}\, \bar{\mu}_2^{(\dot\alpha}\, \mu_3^{\beta)}\, \bar{\mu}_4^{\dot\beta)}$$

$$\hat{\mathcal{O}}^{(\alpha\beta\gamma)\dot\beta} = \mathrm{tr}\, \mu_1^{(\alpha}\, \mu_2^{\beta}\, \mu_3^{\gamma)}\, \bar{\mu}_4^{\dot\beta}$$

Part IV

Smooth Horizonless geometries in Supergravity

The 4d solution: STU model

$$\mathcal{L} = \sqrt{g_4}\left(R_4 - \sum_{I=1}^{3} \frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{2 Im U_I{}^2} - \frac{1}{4} F_a \mathcal{I}^{ab} F_b - \frac{1}{4} F_a \mathcal{R}^{ab} \widetilde{F}_b\right)$$

The four-dimensional geometries can be viewed as solutions of an $\mathcal{N} = 2$ truncation of $\mathcal{N} = 8$ supergravity involving the gravity multiplet and three vector multiplets. The scalars U_I parametrise the complex structures of the 2-tori.

$$ds_4^2 = -e^{2U}(dt + w)^2 + e^{-2U}\,|d\vec{x}|^2$$
$$A_a = (A_0, A_I) = w_a + a_a(dt + w)$$
$$U_I = -b_I + i(Ve^{2U} Z_I)^{-1}$$

$$b_I = \frac{K_I}{V} - \frac{\mu}{Z_I} \qquad a_0 = -\mu V^2 e^{4U} \qquad a_I = Ve^{4U}\left(-\frac{Z_1 Z_2 Z_3}{Z_I} + K_I\,\mu\right)$$
$$*_3 dw_0 = dV \qquad *_3\, dw_I = -dK_I$$
$$*_3 dw = \frac{1}{2}(VdM - MdV + K_I dL_I - L_I dK_I)$$

Regularity of the Solutions in 4d

A class of asimptotically $AdS_2 \times S^2 \times T^6$ geometries (IWP) has been shown to be regular in 4d GR ($U_I = 0$). $O.Lunin(2015), arxiv.org/abs/1507.06670$ Remarkably, this result was achieved for harmonic functions written in terms of an arbitrary profile:

$$V = L_I = Im(H) \qquad M = -K_I = Re(H)$$
$$H(\vec{x}) = h_{reg}(\vec{x}) + \int_0^{2\pi} \frac{dv}{2\pi} \frac{1}{|\vec{x}-\vec{F}(v)|} \sqrt{1 + \frac{(\vec{x}-\vec{F})\vec{A}(v)}{|\vec{x}-\vec{F}|^2}}$$

The analog for asymptotically flat solutions turns out to be not possible $L.Pieri(2016), arxiv.org/abs/1507.06670$. Anyway, to evade strong No go theorems for regular solutions in four dimensional gravity ($Gibbons - Warner(2013)$) and to make contact with the physics of AdS_2, these black holes should resemble a wormhole like geometry, that is not what we really want... Let's go to higher dimensions!

The 11d lift (5+6)

The 4d solution lifts to an 11d solution representing intersecting M5-branes with four electric and four magnetic charges.

$$ds^2 = ds_5^2 + ds_{T^6}^2 \qquad \mathbb{R}^{1,3} \times S^1 \times T^6 \to \{t, \vec{x}, \Psi, y_I, \tilde{y}_I\}$$

$$ds_5^2 = -\frac{[dt+\mu(d\Psi+w_0)+w]^2}{(Z_1\, Z_2\, Z_3)^{\frac{2}{3}}} + (Z_1\, Z_2\, Z_3)^{\frac{1}{3}} \left[V^{-1}(d\Psi + w_0)^2 + V|d\vec{x}|^2\right]$$

$$ds_{T^6} = \sum_{I=1}^{3} \left(\frac{Z_1\, Z_2\, Z_3}{Z_I^3}\right)^{\frac{1}{3}} (dy_I^2 + d\tilde{y}_I^2)$$

Micro-states of the four dimensional black holes can be generically defined as smooth geometries with no horizons or curvature singularities in eleven dimensions carrying the same mass and charges as the corresponding black hole.

REGULARITY CONDITIONS IN 5D

Regular solutions can be constructed in terms of multi-center harmonic functions (V, L_I, K_I, M) with the positions of the centers and the charges chosen such that Z_I are finite and $\mu = 0$ near the centers ($r_i = |\vec{x} - \vec{x}_i|$, $i = 1, ...N$).

Multi-center Taub-NUT ansatz

$$V = v_0 + \sum_{i=1}^{N} \frac{q_i}{r_i}, L_I = \ell_{0I} + \sum_{i=1}^{N} \frac{\ell_{I,i}}{r_i}, K^I = k_0^I + \sum_{i=1}^{N} \frac{k_i^I}{r_i}, M = m_0 + \sum_{i=1}^{N} \frac{m_i}{r_i}$$

Near each center, $\mathbb{R} \times \mathbb{R}^4/Z_{|q_i|}$, asymptotically $\mathbb{R}^{1,3} \times S^1_\Psi$.
Geometry factorises, i.e. regular in 5-d, if near the centers

$$Z_I\big|_{r_i \approx 0} \approx \zeta_I^i \; (finite) \quad and \quad \mu\big|_{r_i \approx 0} \approx 0$$

Absence of horizons and closed time-like curves requires

$$Z_I V > 0 \qquad and \qquad e^{2U} > 0$$

Bubble Equations

Z_I finite near the centers if:

$$\ell_{I,i} = -\frac{|\epsilon_{IJK}|}{2}\frac{k_i^J\,k_i^K}{q_i} \quad , \quad m_i = \frac{k_i^1\,k_i^2\,k_i^3}{q_i^2}$$

μ vanishes near the centers if Bubble Equations are satisfied

$$\sum_{j=1}^{N}\frac{\Pi_{ij}}{r_{ij}} + v_0\,\frac{k_i^1\,k_i^2\,k_i^3}{q_i^2} - \sum_{I=1}^{3}\ell_{0I}\,k_i^I - |\epsilon_{IJK}|\,\frac{k_0^I\,k_i^J\,k_i^K}{2\,q_i} - m_0 q_i = 0$$

with $\Pi_{ij} = (q_i q_j)^{-2}\prod_{I=1}^{3}\left(k_i^I q_j - k_j^I q_i\right)$ and $r_{ij} = |\vec{x}_i - \vec{x}_j|$
Bubble equations imply absence of Dirac-Misner strings

$$*_3 dw = \frac{1}{2}\sum_{i,j=1}^{N}\Pi_{ij}\,\left(\frac{1}{r_j} - \frac{1}{r_{ij}}\right) d\frac{1}{r_i} = \frac{1}{4}\sum_{i,j=1}^{N}\Pi_{ij}\,\omega_{ij}$$

with $\omega_{ij} = (\vec{n}_i + \vec{n}_{ij})\cdot(\vec{n}_j - \vec{n}_{ij})d\phi_{ij}/r_{ij}$ free of DM strings along lines between two centers, since numerator vanishes there

Asymptotic Charges

$$\mathfrak{M} = \frac{1}{8\pi G}\int_{S^2_\infty} \star_4 d\xi^{(t)} \quad , \quad J = -\frac{1}{16\pi G}\int_{S^2_\infty} \star_4 d\xi^{(\phi)} \quad ,$$

$$Q^a = \frac{1}{4\pi}\int_{S^2_\infty} (\mathcal{I}^{ab} \star_4 F_b - \mathcal{R}^{ab} F_b) \quad , \quad P_a = \frac{1}{4\pi}\int_{S^2_\infty} F_a$$

Boundary conditions and charges for orthogonal branes ($\ell_I = v = Q/2,\ M = K_I = 0$ is Reissner-Nordstrom BH):

$$V \approx 1 + \frac{v}{r} \qquad L_I \approx 1 + \frac{\ell_I}{r} \qquad K^I \approx M \approx o(r^{-2})$$

$$\mathfrak{M} = v + \ell_1 + \ell_2 + \ell_3\,,\ P = (v,0,0,0)\,,\ Q = (0,\ell_1,\ell_2,\ell_3)$$

Angular Momentum:

$$H = h_0 + \frac{h_1}{r} + \frac{\vec{h}_2 \cdot \vec{x}}{r^3}$$

$$\vec{J} = m_0\,\vec{v}_2 - v_0\,\vec{m}_2 + \ell_{0I}\,\vec{k}^I_2 - k^I_0\,\vec{\ell}_{2I}$$

Scaling solutions

If the coefficients k_i^I satisfy

$$v_0\, m_i - \sum_{I=1}^{3} \ell_{0I}\, k_i^I + k_0^I\, \ell_{Ii} - m_0 q_i = 0$$

invariance under rigid rescaling of the positions of the centers

$$\vec{x}_i \to \lambda \vec{x}_i$$

Multiplying by the positions of the centers $\vec{x}_i$, the solution can be shown to carry zero angular momentum in agreement with (Sen's) expectations for micro-states of single center black holes. In fact this is not obvious (see canonical ensemble interpretation) and usually $\vec{J} \neq 0$ from electric and magnetic charges. As a bonus, stringy interpretation (four boundary diagrams):

$$m_2 + \sum k_{I2} = 0$$

Fuzzball of orthogonally intersecting branes BH

Boundary conditions for single center black hole:

$$\ell_{0I} = v_0 = 1 \qquad m_0 = m = k_0^I = k^I = 0 \qquad \sum_{i=1}^{N} k_i^I = \sum_{i=1}^{N} k_i^1\, k_i^2\, k_i^3 = 0$$

For $q_i = 1$ (to avoid orbifold singularities) and N centers:

$$P_0 = N \quad , \quad Q_I = -\sum_{i=1}^{N} \frac{|\epsilon_{IJK}| k_i^J\, k_i^K}{2}$$

Bubble Equations:

$$\sum_{j \neq i}^{N} \frac{\prod_{I=1}^{3}(k_i^I - k_j^I)}{r_{ij}} + k^1{}_i k^2{}_i k^3{}_i - \sum_{I=1}^{3} k^I{}_i = 0$$

Configurations with one or two centers fail to meet the BPS requirement $Q_I > 0$. Let us start (and end) with three centers fuzzballs.

Three Center Solutions

The bubble equations for three centers can be solved in general by taking (we need $r_{ij} > 0$ and triangle inequalities):

$$r_{12} = \frac{\Pi_{12}\, r_{23}}{\Pi_{23} - r_{23}\,(\Gamma_2 - \Lambda_2)} \qquad r_{13} = \frac{\Pi_{13}\, r_{23}}{-\Pi_{23} + r_{23}\,(\Gamma_1 + \Gamma_2 - \Lambda_1 - \Lambda_2)}$$

$$\Pi_{ij} = \prod_{I=1}^{3}(k_i^I - k_j^I) \qquad \Gamma_i = \sum_{I=1}^{3} k^I_{\ i} \qquad \Lambda_i = k^1_{\ i} k^2_{\ i} k^3_{\ i}$$

$$k^I_{\ i} = \begin{pmatrix} -\kappa_1\,\kappa_2 & -\kappa_1\,\kappa_3 & \kappa_1\,(\kappa_2 + \kappa_3) \\ \kappa_3 & \kappa_2 & -\kappa_2 - \kappa_3 \\ -\kappa_4 & \kappa_4 & 0 \end{pmatrix}$$

$$V = 1 + \sum_{i=1}^{3} \frac{1}{r_i} \qquad M = \kappa_1\kappa_2\kappa_3\kappa_4 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad L_1 = 1 + \kappa_4 \left(\frac{\kappa_3}{r_1} - \frac{\kappa_2}{r_2}\right)$$

$$L_2 = 1 + \kappa_1\kappa_4 \left(-\frac{\kappa_2}{r_1} + \frac{\kappa_3}{r_2}\right) \qquad L_3 = 1 + \kappa_1 \left(\frac{\kappa_2\kappa_3}{r_1} + \frac{\kappa_2\kappa_3}{r_2} + \frac{(\kappa_2 + \kappa_3)^2}{r_3}\right)$$

$$K_1 = \kappa_1 \left(-\frac{\kappa_2}{r_1} - \frac{\kappa_3}{r_2} + \frac{\kappa_2 + \kappa_3}{r_3}\right) K_2 = \frac{\kappa_3}{r_1} + \frac{\kappa_2}{r_2} - \frac{\kappa_2 + \kappa_3}{r_3} \qquad K_3 = \kappa_4 \left(-\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Scaling Solutions

The scaling solution corresponds to the choice:

$$\kappa_2 = 0 \qquad \kappa_1 = 1 \qquad \kappa_3 = \kappa_4 = \kappa$$

One finds

$$k^I{}_i = \begin{pmatrix} 0 & -\kappa & \kappa \\ \kappa & 0 & -\kappa \\ -\kappa & \kappa & 0 \end{pmatrix} \qquad r_{12} = r_{23} = r_{13} = \ell$$

$$P_0 = 3 \qquad Q_1 = Q_2 = Q_3 = \kappa^2$$

for any given ℓ. One can show that $Z_I V > 0$ and $e^{-4U} > 0$. Very peculiar solution: centers in an equilateral triangle with arbitrary size, $\vec{J} = 0$, regular and smooth everywhere and stringy interpetations. There are actually 12 solution, from the allowed permutations of the matrix entries. Bounded in quantum regime? Boer, El-Showk, Messamah, V.d.Bleeken(2008), Bena, Berkooz, De Boer, El-Showk, V.d.Bleeken(2012)

Another (non scaling) solution

$$\kappa_2 = 0, \kappa_1 = 3\,\kappa,\ \kappa_3 = 2\,\kappa, \kappa_4 = \kappa$$

$$k^I{}_i = \begin{pmatrix} 0 & -3\,\kappa & 3\,\kappa \\ \kappa & 0 & -\kappa \\ -2\kappa & 2\kappa & 0 \end{pmatrix}$$

$$r_{12} = \frac{12\,\kappa^2\,r_{23}}{12\,\kappa^2 - r_{23}} \qquad r_{13} = \frac{6\,\kappa^2\,r_{23}}{6\,\kappa^2 - r_{23}}$$

$$P_0 = 3 \qquad Q_1 = 2\,\kappa^2 \qquad Q_2 = 6\,\kappa^2 \qquad Q_3 = 3\,\kappa^2 r_{23} < 6\,(2-\sqrt{2})\,\kappa^2$$

We notice that triangle inequality in this case impose an upper bound on r_{23} leading to a moduli space of finite volume.

Part V

From Supersymmetric quantum mechanics to Supergravity

THE QUANTUM MECHANICS ON THE BRANE WORLD-VOLUME

The 4 intersecting stacks of D-branes enjoy an effective description as a $\mathcal{N} = 4$ SQM in $0+1$

Untwisted Fields:

Adjoint Chiral SuperMultiplet:

$$\Phi_I^{(a)} = \{\phi_I^{(a)},\ \chi_I^{(a)},\ F_I^{(a)}\} \qquad a = 1,2,3,4 \qquad I = 1,2,3$$

Adjoint Vector SuperMultiplet:

$$V^{(a)} = \{x_i^{(a)},\ \lambda^{(a)},\ D^{(a)}\} \qquad i = 1,2,3$$

Twisted Fields:

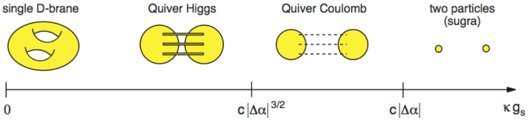
Bifundamental Chiral SuperMultiplet:

$$Z^{(ab)} = \{z^{(ab)},\ \psi^{(ab)},\ F_Z^{(ab)}\}$$

Chowdhury, Garavuso, Mondal, Sen (2015,2016)

From $g_s Q = 0$ to $g_s Q >> 1$

In the limit $g_s \to 0$ we first go from a SUGRA multiple centers configuration in the large scale regime to a Quiver QM description. Indeed after quantum corrections, the Coulomb branch ($x_i^{(a)} \neq 0$) of the QQM is actually identical to the SUGRA "solution space," as both are subject to the Bubble equations.



If we keep on lowering $g_s = 0$, the strings stretched between the branes become tachyonic and the system decay into a configuration with nonzero $z^{(ab)}$: Coulomb $\to$ Higgs. Finally, at $g_s = 0$, the $z^{(ab)}$ can be interpreted as the moduli of a suitable geometric object, a system of intersecting susy D-Branes Denef (2002).

What has been done

$$S = \left[\mathcal{W}^A \mathcal{W}_A\right]_F + \left[\sum_{a,b=1}^{4} (\bar{Z}^{(ab)} e^{V^{(b)} - V^{(a)}} Z^{(ba)})\right]_D + \left[\Phi^* \Phi\right]_D + S_Z + \mathcal{W} + \bar{\mathcal{W}}$$

Explicit counting of the number of ground states for low number of branes in each stack. For generic closed moduli one can read indirectly $\vec{J}$, using the BPS index of D1-D5-KK-P, and find $\vec{J} = 0$.

$$V_F = \sum_{K=1}^{4} \sum_{I=1}^{3} \left| \frac{\partial W}{\partial \phi_I^{(K)}} \right|^2 + \sum_{K=1}^{4} \sum_{I=1}^{4} \left| \frac{\partial W}{\partial z^{(KI)}} \right|^2 \qquad V_D = \frac{1}{2} \sum_{K=1}^{4} \left(\sum_{I=1}^{4} (\bar{z}^{(KI)} z^{(KI)} - \bar{z}^{(IK)} z^{(IK)}) - c^{(K)} \right)^2$$

$$V_{Gauge} = \sum_{i=1}^{3} \sum_{K=1}^{4} \sum_{L=1}^{4} (x_i^{(K)} - x_i^{(L)})(x_i^{(K)} - x_i^{(L)})(\bar{z}^{(KL)} z^{(KL)} + \bar{z}^{(LK)} z^{(LK)})$$

BPS ground states = solutions to $V_F + V_D + V_G = 0$. This analysis is completely classical! Are they different microstates in gravity?

What we want to do (Toy SQM example)

$$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}(\partial_x W(x))^2 + \frac{i}{2}(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi) - \partial_x^2 W(x)\bar{\psi}\psi$$

$$\delta x = \epsilon\bar{\psi} - \bar{\epsilon}\psi \qquad \delta\psi = \epsilon(i\dot{x} + \partial_x W) \qquad \delta\bar{\psi} = \bar{\epsilon}(-i\dot{x} + \partial_x W)$$

$$\delta\int L dt = \int dt\left(-i\dot{\epsilon}Q - i\dot{\bar{\epsilon}}\bar{Q}\right)$$

$$Q = \bar{\psi}(i\dot{x}+\partial_x W) = \bar{\psi}(ip+\partial_x W) \qquad \bar{Q} = \psi(-i\dot{x}+\partial_x W) = \psi(-ip+\partial_x W)$$

$$[x,p] = i \qquad \{\psi\bar{\psi}\} = 1 \qquad \Psi = f_1(x)\left|0\right\rangle + f_2(x)\bar{\psi}\left|0\right\rangle$$

$$Q\Psi = \bar{Q}\Psi = 0 \rightarrow \Psi = e^{-W}\left|0\right\rangle \; or \; \Psi = e^{W}\bar{\psi}\left|0\right\rangle$$

The goal

For every BPS ground states, one can find the associated wavefunction. Then one can compute the condensates appearing in the stringy computation, relating QM with SUGRA.

$$W(s) = W(s^0) + 0 + \frac{1}{2}\left[\frac{\partial^2 W}{\partial s_A s_B}\right]_{s=s^0}(s_A - s_A^0)(s_B - s_B^0) + \ldots$$

$$= W(s^0) + \sum_A c_A^{(0)} (\tau_{(0)}^A)^2$$

$$\Psi^{(0)} = e^{-\sum_A \lambda |c_A^{(0)}| (\tau_{(0)}^A)^2} \prod_{c_B^{(0)}<0} \bar{\xi}^B |0\rangle$$

$$\psi_{\alpha+}^{(14)} \to M_{14}\, \mu_{\alpha+}\, e^{-\frac{\varphi}{2}}\, S^\alpha\, e^{+\frac{i}{2}\varphi_{45}}\, \Delta_{6789} e^{ikX}$$

$$\langle \Psi^{(0)} |\, \psi^{(14)} \psi^{(41)}\, | \Psi^{(0)} \rangle \to \textit{open string condensate}$$

Summary and Conclusions

- Black hole microstates can be identified in gravity by computing the backreaction of the branes with disk scattering amplitudes computations.
- The condensate of open strings binding the branes are actually not arbitrary numbers, but they are fixed by the explicit quantum configuration of the microstate. In a SQM approach one can hope to uncover the difference between microstates. Will the microstates differ for higher order multipoles in the gravity description?
- Large class of regular and horizonless solutions in supergravity have been found. In this work we have focused on four charge geometries. Many puzzles remains, in particular one can ask how much general are these results: is SUGRA enough for fuzzballs?


Thanks for the Attention!